Letter | Editors' Suggestion

Effect of the N3LO three-nucleon contact interaction on *p-d* scattering observables

L. Girlanda [1,2] E. Filandri [1,2] A. Kievsky [3] L. E. Marcucci [3,4] and M. Viviani [3]

[1] *Department of Mathematics and Physics "E. De Giorgi", University of Salento, I-73100 Lecce, Italy*
[2] *INFN-Lecce, I-73100 Lecce, Italy*
[3] *INFN-Pisa, I-56127, Pisa, Italy*
[4] *Department of Physics "E. Fermi", University of Pisa, I-56127 Pisa, Italy*

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A unitary transformation allows to remove redundant terms in the two-nucleon ($2N$) contact interaction at the fourth order (N3LO) in the low-energy expansion of chiral effective field theory. In so doing a three-nucleon ($3N$) interaction is generated. We express its short-range component in terms of five combinations of low-energy constants (LECs) parametrizing the N3LO $2N$ contact Lagrangian. Within a hybrid approach, in which this interaction is considered in conjunction with the phenomenological AV18 $2N$ potential, we show that the involved LECs can be used to fit very accurate data on polarization observables of low-energy *p-d* scattering, in particular the A_y asymmetry. The resulting interaction is of the right order of magnitude for a N3LO contribution.

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Introduction. The effective field theory (EFT) framework is nowadays the standard setting to address the nuclear interaction problem. Starting from the choice of low-energy active degrees of freedom, it allows to express physical observables in terms of low-energy constants (LECs) in a systematic expansion in powers of a small parameter representing the separation of scales [1–4]. All the short-distance effects from the frozen degrees of freedom are effectively encoded in the values of the LECs parametrizing contact interactions. Being unconstrained by the imposed symmetries, the LECs have to be fitted from experimental data, at the cost of the predictive power of the EFT. From another perspective, they may provide the needed flexibility to accurately model the nuclear interaction. For example, chiral EFT (ChEFT) makes use of the approximate chiral symmetry of strong interaction to severely constrain the interaction of pions and nucleons. According to common wisdom, in the isospin limit there are two LECs contributing at the leading order (LO) in the two-nucleon ($2N$) sector, traditionally called C_S and C_T, seven ($C_{i=1,\dots,7}$) at the next-to-leading order (NLO), and 15 more ($D_{i=1,\dots,15}$) at the next-to-next-to-next-to leading order (N3LO). The three-nucleon ($3N$) sector is much more constrained. The first contributions arise at the next-to-next-to leading order (N2LO) [5,6], parametrized by two LECs, C_D and C_E, the former of which is actually a weak $2N$ LEC, contributing, e.g., to the muon capture from deuterium [7–10]. At the fifth order (N4LO) we find the contribution of 13 more LECs, $E_{i=1,\dots,13}$ [11]. However, the distinction between $2N$ and $3N$ LECs is, to some extent, a matter of convention, depending on the arbitrary choice for the nucleon interpolating field: nonlinear field redefinitions may change a seemingly $2N$ interaction into a $3N$ one, without changing the predictions for on-shell quantities. Thus, it was observed in Ref. [12] that three out of the 15 $2N$ independent contact interactions arising at N3LO can be made to vanish by a suitable unitary transformation, inducing specific modifications of the $3N$ interaction. In Ref. [13] we exhibited the precise form of the induced $3N$ interaction. Moreover, we identified two additional $2N$ contact LECs at N3LO parametrizing momentum dependent interactions allowed by Poincaré symmetry, which we named D_{16} and D_{17}. They can also be transformed, through a unitary transformation, into a $3N$ interaction. Thus, contrary to widespread belief [14,15], five adjustable LECs parametrize the $3N$ interaction at N3LO of the chiral expansion. This could be the explanation of all failed attempts to improve the accuracy in $3N$ systems (particularly for scattering observables) when going from N2LO to N3LO [16]. On the other hand, the inclusion of the N4LO $3N$ contact interaction has already proved to be of great importance in reducing existing discrepancies between theory and experimental data [17,18]. In the present paper we provide quantitative evidence that the five extra LECs at N3LO ensure sufficient flexibility to drastically improve the description of low-energy *p-d* scattering polarization observables, most notably the A_y asymmetry, which constitutes a longstanding problem for most nuclear interaction models. We do this in a hybrid approach in which the induced $3N$ force (restricted to its shortest range part) is considered in conjunction with the phenomenological AV18 $2N$ potential [19]. This allows us to speed up the minimization procedures using the pair-correlated hyperspherical harmonic (PHH) basis [20], which is formulated for local potentials in coordinate space. Moreover, a fully consistent ChEFT calculation would also require to consider the unitarily transformed one-pion exchange potential [13]. We defer such a study to future work. Here, using the hyperspherical harmonics method [21,22], we fit the five N3LO LECs to polarized *p-d* scattering data at 2 MeV center-of-mass energy [23].

N3LO 2N contact Hamiltonian. The N3LO $2N$ contact potential was originally considered in Refs. [24,25] as consisting of 15 LECs. After careful scrutiny of the constraints imposed by Poincaré symmetry, two further LECs emerge [13], leading to the following expression in the general reference frame:

$$\begin{aligned} V_{NN}^{(4)} &= D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\boldsymbol{k}\times\boldsymbol{Q})^2 \\ &\quad + [D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\boldsymbol{k}\times\boldsymbol{Q})^2](\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\ &\quad + \frac{i}{2}(D_9 k^2 + D_{10} Q^2)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{Q}\times\boldsymbol{k}) \\ &\quad + (D_{11} k^2 + D_{12} Q^2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}) \\ &\quad + (D_{13} k^2 + D_{14} Q^2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{Q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{Q}) \\ &\quad + D_{15}\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}\times\boldsymbol{Q})\,\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}\times\boldsymbol{Q}) \\ &\quad + iD_{16}\,\boldsymbol{k}\cdot\boldsymbol{Q}\,\boldsymbol{Q}\times\boldsymbol{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) \\ &\quad + D_{17}\,\boldsymbol{k}\cdot\boldsymbol{Q}\,(\boldsymbol{k}\times\boldsymbol{P})\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) \end{aligned} \tag{1}$$

with $\boldsymbol{k}=\boldsymbol{p}'-\boldsymbol{p}$ and $\boldsymbol{Q}=\frac{\boldsymbol{p}'+\boldsymbol{p}}{2}$, $\boldsymbol{p}$ and $\boldsymbol{p}'$ being the initial and final relative momenta, and $\boldsymbol{P}=\boldsymbol{p}_1+\boldsymbol{p}_2$ the total pair momentum. However, as it was pointed out in Ref. [12], only 12 independent LECs survive on shell and can thus be determined from $2N$ scattering data. This redundancy amounts to a unitary ambiguity, i.e., to the possibility of generating shifts of the LECs by unitary transforming the one-body kinetic energy operator $H_0 \to U^\dagger H_0 U$, where U is the most general unitary two-body contact transformation depending on five arbitrary parameters α_i,

$$U = \exp\left[\sum_{i=1}^{5}\alpha_i T_i\right], \tag{2}$$

and the independent generators T_i were given explicitly in Ref. [13] [Eqs. (1)–(5) of that reference]. The transformation (2) entails a shift of the N3LO contact LECs, $D_i \to D_i + \delta D_i$, with $\delta D_3 = -\delta D_4 = -4\alpha_1/m$, $\delta D_7 = -4\alpha_2/m$, $\delta D_8 = (4\alpha_2+2\alpha_3)/m$, $\delta D_{12}=\delta D_{13}=\delta D_{15}=-4\alpha_3/m$, $\delta D_{16} = -2\alpha_4/m$, $\delta D_{17} = -(4\alpha_3+2\alpha_5)/m$, and the remaining ones being zero. Here, m is the nucleon mass. By choosing

$$\alpha_1 = \frac{m}{16}(16D_1 + D_2 + 4D_3), \tag{3}$$

$$\alpha_2 = \frac{m}{16}(16D_5 + D_6 + 4D_7), \tag{4}$$

$$\alpha_3 = \frac{m}{32}(D_{14} + 16D_{11} + 4D_{12} + 4D_{13}), \tag{5}$$

$$\alpha_4 = \frac{m}{2}D_{16}, \tag{6}$$

$$\alpha_5 = \frac{m}{16}(8D_{17} - D_{14} - 16D_{11} - 4D_{12} - 4D_{13}), \tag{7}$$

the N3LO contact potential is brought in the form of Eq. (4) of Ref. [12], with the following replacements:

$$\begin{aligned} &D_3 \to D_3 + D_4, \quad D_4 \to D_5, \quad D_5 \to D_6, \\ &D_6 \to D_7 + D_8 + \frac{1}{16}D_{14} + D_{11} + \frac{1}{4}D_{12} + \frac{1}{4}D_{13}, \\ &D_7 \to D_9, \quad D_8 \to D_{10}, \quad D_9 \to -4D_{11}, \\ &D_{10} \to -\frac{1}{8}D_{14} - 2D_{11} + \frac{1}{2}D_{12} - \frac{1}{2}D_{13}, \\ &D_{11} \to -\frac{1}{8}D_{14} - 2D_{11} - \frac{1}{2}D_{12} + \frac{1}{2}D_{13}, \\ &D_{12} \to D_{15} - 2D_{11} - \frac{1}{2}D_{12} - \frac{1}{2}D_{13} - \frac{1}{8}D_{14}. \end{aligned} \tag{8}$$

Induced 3N contact interactions. When applied to the LO $2N$ contact Hamiltonian,

$$V_{NN}^{(0)} = C_S + C_T\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \tag{9}$$

the unitary transformation (2) induces additional $3N$ interactions [13] which can be viewed as a modification of the subleading $3N$ contact interaction entering at N4LO of the low-energy expansion

$$V_{3N}^{(2)} = \sum_{i=1}^{13} E_i\, O_i, \tag{10}$$

as in Eq. (15) of Ref. [11]. Specifically, we have

$$U^\dagger V_{NN}^{(0)} U = \sum_{i=1}^{13}\delta E_i O_i \tag{11}$$

with[1]

$$\delta E_1 = C_S(\alpha_1+\alpha_2) + C_T(\alpha_1 - 2\alpha_2), \tag{12}$$

$$\delta E_2 = C_T(3\alpha_2 + 2\alpha_3 - 8\alpha_4 + 2\alpha_5), \tag{13}$$

$$\begin{aligned} \delta E_3 &= 2C_S\left(\alpha_2 + \frac{2}{3}\alpha_3\right) \\ &\quad + C_T\left(2\alpha_1 - \alpha_2 - \frac{2}{3}\alpha_3 + 8\alpha_4 - 2\alpha_5\right), \end{aligned} \tag{14}$$

$$\delta E_4 = \frac{2}{3}C_S\alpha_2 + \frac{C_T}{3}(2\alpha_1 - 7\alpha_2 - 2\alpha_3 + 8\alpha_4 - 2\alpha_5), \tag{15}$$

$$\begin{aligned} \delta E_5 &= 2C_S\left(\alpha_2 + \frac{2}{3}\alpha_3\right) \\ &\quad + 2C_T\left(\alpha_1 - 2\alpha_2 - \frac{1}{3}\alpha_3 + 4\alpha_4 - \alpha_5\right), \end{aligned} \tag{16}$$

$$\delta E_6 = \frac{2}{3}C_S\alpha_2 + \frac{2}{3}C_T(\alpha_1 - 2\alpha_2 - \alpha_3 + 4\alpha_4 - \alpha_5), \tag{17}$$

$$\delta E_7 = 24 C_T\alpha_4, \tag{18}$$

$$\delta E_8 = \frac{1}{3}\delta E_7, \tag{19}$$

$$\begin{aligned} \delta E_9 &= C_S(3\alpha_2 + 2\alpha_3 - \alpha_4 + 2\alpha_5) \\ &\quad + C_T(3\alpha_1 - 6\alpha_2 - 4\alpha_3 + 11\alpha_4 - 4\alpha_5), \end{aligned} \tag{20}$$

$$\delta E_{10} = C_S(\alpha_2 - \alpha_4) + C_T(\alpha_1 - 2\alpha_2 + 5\alpha_4), \tag{21}$$

$$\begin{aligned} \delta E_{11} &= C_S(3\alpha_2 + 2\alpha_3 + \alpha_4 - 2\alpha_5) \\ &\quad + C_T(3\alpha_1 - 6\alpha_2 - 4\alpha_3 - 11\alpha_4 + 4\alpha_5), \end{aligned} \tag{22}$$

$$\delta E_{12} = C_S(\alpha_2 + \alpha_4) + C_T(\alpha_1 - 2\alpha_2 - 5\alpha_4), \tag{23}$$

$$\delta E_{13} = -4C_T(4\alpha_4 - \alpha_5). \tag{24}$$

[1]We correct here some wrong factors in Ref. [13].

With the specific choice for the unitary transformation encoded in Eqs. (3)–(7), the $3N$ contact LECs E_i in Eq. (10) are shifted to

$$E_i \to \tilde{E}_i = E_i + \delta E_i, \tag{25}$$

where the induced contributions δE_i are enhanced as compared to the genuine ones E_i, due to the presence of the nucleon mass factor, scaling as $m \sim O(\Lambda_\chi^2/p)$ in the Weinberg counting [26], which effectively promotes them to N3LO. From now on, the LECs E_i will be thought of as constituted only of the induced contributions, $E_i = \delta E_i$. Thus, at N3LO the $3N$ contact interaction depends on five combinations of the $2N$ LECs D_i, appearing in Eqs. (3)–(7), which cannot be determined from $2N$ scattering data, but have to be fitted to experimental observables in $A > 2$ systems. In the following we explore the sensitivity of polarization observables in low-energy N-d scattering to these five combinations of LECs. Since we take the phenomenological AV18 as representative of a realistic $2N$ interaction, we should clarify the meaning of the LECs C_S and C_T in this framework. As a reasonable estimate, based on studies of universal behavior [27], we take them from a fit of the LO $2N$ contact interaction (9)

$$V^{(0)}_{NN,\Lambda} = [C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2] Z_\Lambda(r) \tag{26}$$

to the singlets and triplets n-p scattering lengths as predicted by the AV18 potential. In other words, we treat the contact potential (26) as a very low-energy representation of the AV18 potential. In the above expression a local cutoff has been introduced,

$$Z_\Lambda(r) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} F(\mathbf{p}^2;\Lambda) \tag{27}$$

with

$$F(\mathbf{p}^2,\Lambda) = \exp\left[-\left(\frac{\mathbf{p}^2}{\Lambda^2}\right)^2\right], \tag{28}$$

and $\Lambda = 500$ MeV. From this procedure we get

$$C_S = -66.53\ \mathrm{GeV}^{-2}, \quad C_T = -3.47\ \mathrm{GeV}^{-2}. \tag{29}$$

The same cutoff is also used in the coordinate space expression of the induced $3N$ contact interaction $V^{(2)}_{3N,\Lambda}$ which can be read in Eq. (18) of Ref. [11]. Notice that the above representation for the "LO" AV18 potential is valid only at very low-energy: the resulting effective ranges are about 1 fm in both S-wave channels, too small compared to the actual values. Indeed, the effective ranges would only be described at NLO in a pionless EFT. So they exhibit a strong cutoff dependence at LO. It would be possible to choose a cutoff Λ such that also the effective range is reproduced, in addition to the scattering length. Thus, for $\Lambda = 192$ MeV, the LECs

$$C_S = -233.1\ \mathrm{GeV}^{-2}, \quad C_T = -32.34\ \mathrm{GeV}^{-2} \tag{30}$$

reproduce the ${}^1S_0$ effective range, while for $\Lambda = 275$ MeV, the LECs

$$C_S = -139.5\ \mathrm{GeV}^{-2}, \quad C_T = -13.46\ \mathrm{GeV}^{-2} \tag{31}$$

reproduce the ${}^3S_1$ effective range, in addition to both channel scattering lengths. We will adopt these three "LO" models of the AV18, while maintaining the $3N$ cutoff at $\Lambda = 500$ MeV, to study the cutoff dependence of our results, namely the possibility to accurately describe p-d scattering observables with the induced $3N$ interaction at N3LO. Notice that, in principle this cutoff dependence could be sizable, due to the hybrid nature of the calculation.

Low-energy p-d scattering observables within the HH method. The scattering observables at a given energy E are obtained from the N-d transition matrix M, which is composed of the Coulomb amplitude $f_c(\theta_{\mathrm{c.m.}})$ and a nuclear term

$$\begin{aligned} M^{SS'}_{\nu\nu'}(\theta_{\mathrm{c.m.}}) = {}& f_c(\theta_{\mathrm{c.m.}})\delta_{SS'}\delta_{\nu\nu'} + \frac{\sqrt{4\pi}}{q}\sum_{LL'J}\sqrt{2L+1} \\ &\times \langle L0,\, S\nu|J\nu\rangle\langle L'M',\, S'\nu'|J\nu\rangle \\ &\times e^{i(\sigma_L+\sigma_{L'}-2\sigma_0)} T^J_{LS,L'S'} Y_{L'M'}(\theta_{\mathrm{c.m.}},0), \end{aligned} \tag{32}$$

where $\theta_{\mathrm{c.m.}}$ is the center-of-mass scattering angle and $M^{SS'}_{\nu\nu'}(\theta_{\mathrm{c.m.}})$ a 6×6 matrix corresponding to the couplings of the spin 1 of the deuteron and the spin 1/2 of the proton, to $S, S' = 1/2$ or $3/2$ with projections ν, ν'. The quantum numbers L, L' are the relative proton-deuteron orbital angular momenta and J is the total angular momentum, while σ_L are the Coulomb phase shifts. The matrix elements $T^J_{LS,L'S'}$ form the T matrix of a Hamiltonian containing the nuclear plus Coulomb interactions.[2] The corresponding S matrix, $S = 1 - 2i\pi T$, is computed through the complex Kohn variational principle [29,30], with trial variational functions $\Psi_{LSJJ_z} = \Psi_C + \Psi_A$, composed of an internal part Ψ_C, expanded in the PHH functions [21], and an asymptotic part, Ψ_A, describing the relative motion between the proton and the deuteron at large distance. The latter is a linear combination of the ingoing and outgoing solutions of the Coulomb p-d Schrödinger equation, $\Omega^{\mp}_{LSJJ_z}$, regularized at small distances,

$$\Psi_A = \Omega^-_{LSJJ_z} + \sum_{L'S'} \mathcal{S}^J_{LS,L'S'}(q)\,\Omega^+_{L'S'JJ_z}. \tag{33}$$

The Hamiltonian is decomposed as

$$H = H_{NN} + V^{(0)}_{3N,\Lambda} + V^{(2)}_{3N,\Lambda}, \tag{34}$$

where H_{NN} contains the kinetic energy plus the AV18 $2N$ interaction with Coulomb potential and $V^{(0)}_{3N,\Lambda} + V^{(2)}_{3N,\Lambda}$ contain the $3N$ interaction. Specifically, we consider, in addition to the induced contact interaction $V^{(2)}_{3N,\Lambda}$, as in Eq. (18) of Ref. [11], a leading order contact interaction

$$V^{(0)}_{3N,\Lambda} = E_0 \sum_{ijk} Z_\Lambda(r_{ij}) Z_\Lambda(r_{ik}). \tag{35}$$

The problem is thus reduced to a linear one (see [17] for further details). Using the PHH expansion a convergence of observables to less than 1% accuracy [31–33] is attained with

[2] The effects of other components of the electromagnetic interaction are discussed in Ref. [28].

matrices of approximately 2000×2000. In this way computing the phase shifts and mixing angles requires few seconds on an ordinary desktop for each channel.

Fit results. The observables used in the fitting procedure are the *p-d* differential cross section, the two vector analyzing powers A_y and iT_{11}, the three tensor analyzing powers T_{20}, T_{21}, T_{22} and the doublet and quartet *n-d* scattering lengths. In particular we determine the leading contact LEC E_0 from the experimental triton binding energy. Then, we fit the experimental doublet and quartet *n-d* scattering lengths [34,35] and the six *p-d* scattering observables at center-of-mass energy $E_{\rm c.m.} = 2$ MeV [23], amounting to 282 experimental data. The theoretical observables are calculated from the transition matrix M of Eq. (32) (see Ref. [36]), starting from the S matrix in Eq. (33). At the energy considered, states up to $L = 2$ are calculated using the full Hamiltonian, whereas for $L > 2$ the three-body potential was neglected due to its short-range character (see also Ref. [37]), while the strong two-body potential was included up to a maximum value of $L = 6$ in the partial wave expansion of the observables, which is enough at the energy of interest.

For the differential cross section we include in the χ^2 definition an overall normalization factor Z of the data points, i.e.,

$$\chi^2 = \sum_i \frac{\left(d_i^{\rm exp}/Z - d_i^{\rm th}\right)^2}{\left(\sigma_i^{\rm exp}/Z\right)^2} \tag{36}$$

with Z obtained from the minimization condition as

$$Z = \frac{\sum_i d_i^{\rm exp} d_i^{\rm th} / \left(\sigma_i^{\rm exp}\right)^2}{\sum_i \left(d_i^{\rm th}\right)^2 / \left(\sigma_i^{\rm exp}\right)^2}. \tag{37}$$

In Eqs. (36) and (37) $d_i^{\rm exp/th}$ are the experimental data points and their theoretical predictions, while $\sigma_i^{\rm exp}$ is the experimental error. In our study we have checked that Z never differs from 1 by more than 2% [38]. For the other observables, we treat the normalization $Z = 1.00 \pm 0.01$ as an additional experimental datum since, according to Ref. [23], the systematic uncertainty is estimated as 1%.

For an initial random set of the five α_i parameters of Eqs. (3)–(7), we solve the scattering and bound states problem and calculate the corresponding observables. The corresponding LECs characterizing the induced $3N$ interaction are obtained from Eqs. (12)–(24) with the values of C_S and C_T taken from Eqs. (29)–(31). The corresponding fits will be denoted, respectively, as "fit A", "fit B", and "fit C". At each step the LEC E_0 in Eq. (35) is determined to reproduce the triton binding energy. Using the POUNDerS algorithm [39] we start an iterative procedure to minimize the global χ^2. Using different initial random input of α_i values, we repeat the algorithm trying to localize the deepest minimum. The results are displayed in Table I for the choices A, B, and C of the $2N$ LO contact LECs. The quality of the fits is largely independent of the above choice, corresponding for all models to χ^2/d.o.f. =1.7, of the same quality as the most accurate multiparameter fits to the same data performed so far [17]. Also shown in the same table (within brackets) are the results from three-parameter fits, which ignore the **P**-dependent $2N$ contact interaction, i.e., with $\alpha_4 = \alpha_5 = 0$, in order to assess the relevance of the LECs D_{16} and D_{17}, which were never considered before. No spin-orbit operators, of the kind proposed in Ref. [40], are present in this case, and the minimum χ^2/d.o.f. increases to 2.3.

TABLE I. Results of the five-parameter (three-parameter) fits, the latter one obtained ignoring the **P**-dependent $2N$ contact interaction, i.e., setting $\alpha_4 = \alpha_5 = 0$. Columns A, B, and C refer, respectively, to values (29), (30), and (31) for the LECs of the LO pionless model for the AV18 $2N$ potential. See text for more explanations.

Fit	A	B	C
χ^2/d.o.f.	1.7 [2.3]	1.7 [2.4]	1.7 [2.4]
e_0	0.685 [−1.570]	−0.377 [−2.117]	0.239 [−1.844]
$\tilde\alpha_1 C_S$	1.410 [−3.611]	−0.485 [−4.120]	0.516 [−4.183]
$\tilde\alpha_2 C_S$	0.211 [−0.483]	0.190 [−0.531]	0.218 [−0.375]
$\tilde\alpha_3 C_S$	−0.370 [0.209]	0.267 [0.583]	−0.113 [0.377]
$\tilde\alpha_4 C_S$	1.735 [0]	1.549 [0]	1.513 [0]
$\tilde\alpha_5 C_S$	2.266 [0]	3.412 [0]	2.840 [0]
$^2a_{nd}$ [fm]	0.648 [0.647]	0.650 [0.622]	0.642 [0.633]
$^4a_{nd}$ [fm]	6.31 [6.32]	6.32 [6.32]	6.31 [6.32]

Figure 1 shows the best fit curves (fit A) for the A_y and iT_{11} analyzing power in $\vec{p}$-d and $\vec{d}$-p scattering, compared to the predictions from the purely $2N$ AV18 interaction and from the addition of the Urbana IX $3N$ interaction. We conclude that the effective N3LO induced $3N$ contact interaction allows to solve the longstanding A_y problem. Also the description of the vector analyzing power iT_{11} is drastically improved. We also show in the same figure the corresponding curves obtained from the three-parameter fits which do not include the α parameters of the **P**-dependent N3LO $2N$ contact interaction, i.e., with $\alpha_4 = \alpha_5 = 0$. In Fig. 2 we show the same curves for the tensor analyzing powers of $\vec{d}$-p elastic scattering and for

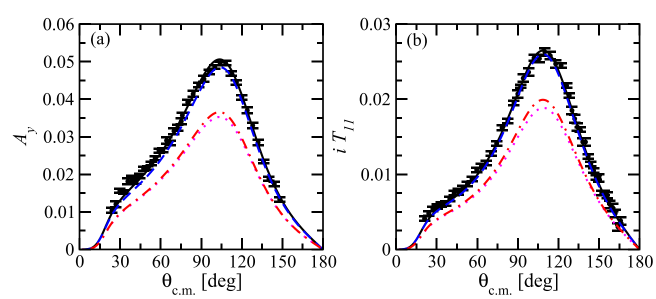

FIG. 1. Proton and deuteron analyzing power in $\vec{p}$-d and $\vec{d}$-p scattering at $E_{\rm c.m.} = 2$ MeV. The full (black) lines result from a global five-parameter fit, the dashed (blue) lines from a three-parameter fit excluding the **P**-dependent $2N$ interaction, the dotted (pink) lines are the predictions from the $2N$ AV18 potential, while the dashed-dotted (red) lines are the predictions including also the $3N$ Urbana IX interaction. Experimental data are from Ref. [23]. Here, we only show the results from fit A, since fit B and C yield very similar results.

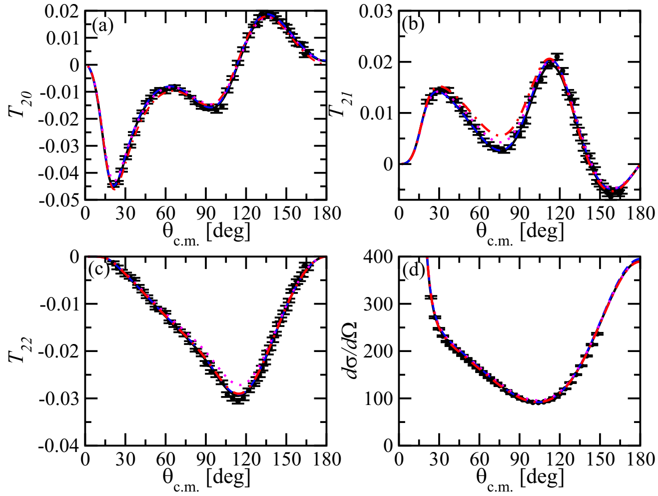

FIG. 2. Same as Fig. 1 but for T_{20}, T_{21}, T_{22} tensor observables in $\vec{d}$-p scattering and for the unpolarized differential cross section at $E_{\mathrm{c.m.}} = 2$ MeV.

the differential cross section. By inspection of the figures, we can conclude that all the observables are nicely reproduced.

The fitted parameters α_i are displayed in Table I, together with the corresponding values of the LO $3N$ contact LEC E_0, in natural units, dictated by naive dimensional analysis [41,42], i.e.,

$$e_0 = E_0 F_\pi^4 \Lambda, \quad \tilde{\alpha}_i = \alpha_i F_\pi^4 \Lambda^3, \tag{38}$$

where $F_\pi = 92.4$ MeV is the pion decay constant and $\Lambda = 500$ MeV is the cutoff in the $3N$ interaction. Also shown in the table are the doublet and quartet n-d scattering lengths, to be compared with the experimental values ${}^2a_{nd} = (0.645 \pm 0.003 \pm 0.007)$ fm [34] and ${}^4a_{nd} = (6.35 \pm 0.02)$ fm [35]. It is interesting to observe that the fitted $3N$ interaction parameters are of a natural size for a N3LO contribution. In order to see this, we can translate the values of the α_i's into combinations of the N3LO $2N$ LECs D_i's using Eqs. (3)–(7). This is done in Table II for the two fitting procedures. As a reference, we report in the same table the corresponding combinations of LECs obtained from $2N$ data in Ref. [4], and used in the Idaho N3LO $2N$ chiral potential with $\Lambda = 500$ MeV. The comparison of the actual values has little meaning, also due to the hybrid character of our calculation. However it is interesting to observe that the orders of magnitude are the same. In particular, for the five-parameter fit, the LECs combinations are not larger than those obtained in the Idaho N3LO chiral potential.We advocate that, were those combinations fitted in the $A = 3$ system, the A_y puzzle would be solved at N3LO. However this remains to be seen explicitly in a consistent chiral calculation.

Conclusions. A suitable choice of unitary transformation allows to reduce the number of LECs parametrizing the N3LO $2N$ contact interaction to twelve. This procedure generates a $3N$ interaction depending on five unconstrained LECs. In the present paper we examined the effect of this induced $3N$ interaction on polarization observables of p-d scattering below the breakup threshold. We showed that the

TABLE II. Estimation of some N3LO LECs combinations from fit A, B, and C. The D_i are in units of 10^4 GeV^{-4}, and we have defined $\tilde{D}_{13} = 16D_1 + D_2 + 4D_3$, $\tilde{D}_{14} = 16D_5 + D_6 + 4D_7$, and $\tilde{D}_{15} = D_{14} + 16D_{11} + 4D_{12} + 4D_{13}$. We show between brackets the values obtained from the three-parameter fits and, in the last column, the values obtained in Ref. [4] and used for the Idaho N3LO $2N$ potential with $\Lambda = 500$ MeV.

LECs	Fit A	Fit B	Fit C	Ref. [4]
$\tilde{D}_{13}$	-3.96 [10.15]	0.39 [3.31]	-0.69 [5.61]	6.41
$\tilde{D}_{14}$	-0.59 [1.36]	-0.15 [0.43]	-0.29 [0.50]	4.05
$\tilde{D}_{15}$	2.08 [-1.17]	-0.43 [-0.94]	0.30 [-1.01]	-3.04
D_{16}	-0.61 [0]	-0.16 [0]	-0.25 [0]	–
D_{17}	-0.54 [-0.15]	-0.40 [-0.12]	-0.44 [-0.13]	–

LECs can be adjusted allowing to solve the longstanding A_y puzzle.

The induced $3N$ interaction can be thought of as a specific off-shell extension of the $2N$ interaction, leaving the $2N$ observables unchanged. Such off-shell extension of the $2N$ potentials were considered in the past (see, e.g., Ref. [43]) and found to have a prominent role in the N-d A_y puzzle [44]. We remark in passing that a satisfactory fit (with χ^2/d.o.f. =1.8) can be obtained even without including any $3N$ interaction except for the induced one, i.e., with $E_0 = 0$. We emphasize that the novelty of our proposal lies in the identification of its precise form in the context of a systematic low-energy expansion, where it starts to contribute at N3LO. This statement has also a quantitative content, despite all the limitations of our hybrid calculation, in light of the comparison of the magnitudes of the involved LECs with those inferred within the ChEFT framework of the $2N$ interaction, as shown in Table II.

It will be interesting to repeat the above analysis in a fully consistent ChEFT framework for $2N$ and $3N$ interactions. In this respect, also the induced $3N$ interaction from the unitary transformation of the one-pion exchange $2N$ potential has to be taken into account. To the best of our knowledge such contribution, first worked out in Ref. [13], has never been considered in the literature so far. In the present work it was implicitly taken into account through the values of the LECs C_S and C_T, by considering a pionless representation of the AV18 potential. It will be also necessary to explore the energy dependence of the predicted p-d scattering observables and confront it with experimental data. Such exploration has been pursued in Ref. [17] to energies lower than $E_{\mathrm{c.m.}} = 2$ MeV using a restricted form for the subleading $3N$ contact interaction, leading to quite satisfactory results. Finally, the same shuffling of contact operators between the $2N$ and $3N$ sectors applies to the pionless formulation of the EFT. The counting of the induced $3N$ operators examined in the present paper should follow from the corresponding counting of the $2N$ operators. A further peculiarity in this case is the promotion of the $3N$ force to LO. Thus the appropriate counting should be re-examined in this perspective (see also Ref. [45]). Work along these lines is deferred to forthcoming investigations.

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