Pairing reentrance in odd nuclei at finite temperature

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Pairing correlations typically decrease as temperature increases, but in some nuclei, pairing is enhanced within a specific temperature range. This phenomenon, known as pairing reentrance, often occurs in nuclei with an odd number of protons and/or neutrons and is possibly due to the configurations of single-particle levels, especially those close to the Fermi energy. In this study, we examine various nuclear single-particle level configurations that can produce this effect. Our findings show that, although configurations with an odd number of nucleons cause the pairing reentrance, this effect may decrease or even disappear by decreasing the distance between levels located close to the one occupied by the odd nucleon. This reduction leads to an increase in the Pauli blocking effect, which is likely attributed to the participation of nucleons in the neighboring levels into the blocking effect, particularly when the distance between these levels and that occupied by the odd nucleon is very small. Our calculations for realistic calcium isotopes confirm the presence of this phenomenon.

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I. INTRODUCTION

Pairing correlation plays a fundamental role in the superconducting properties of many-body systems. These systems range from large ones such as neutron stars to tiny ones, such as atomic nuclei. The nature of pairing correlations was first explained by the Bardeen-Cooper-Schrieffer theory (BCS) [1], which is used to describe the superconductivity in superconductors. Since then, this microscopic theory has been successfully applied to describe the pairing effect in many-body systems, including atomic nuclei [2,3].

The BCS theory suggests that nucleons, like electrons in superconductors, tend to form Cooper pairs and move in time-reversal orbits. These pairing correlations represent an essential aspect of nuclear structure in addition to nucleon-nucleon interactions. The pairing gap, which is built based on the pairing energy, pairing strength, and single-particle occupation numbers, can be used to theoretically explore the nuclear pairing property.

Previous works have indicated that nuclear pairing decreases with increasing temperature. In the BCS theory, pairing correlations (characterized by the pairing gap) vanish at a critical temperature, which denotes the phase transition from the superfluid phase to the normal one [4–9]. However, in other frameworks, such as the modified BCS (MBCS), the modified Hartree-Fock-Bogoliubov (MHFB) [10], Lipkin-Nogami method [11], or the finite-temperature exact pairing method (FTEP) [12,13], which include the thermal fluctuations due to the finiteness of the nuclear system, pairing correlations decrease monotonically and still persist even at high temperatures ($T > 4$ MeV). This nonvanishing pairing gap was first pointed out and developed by Moretto in the 1970s [14].

As the pairing gap decreases with increasing temperature, a special phenomenon of enhanced pairing correlations, which is called the pairing reentrance, in even-odd nuclei was predicted at low temperature ($T < 1$ MeV) [10,15–19].

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In even nucleon systems, the pairing reentrance is related to the energy of resonant states [9,15,16]. On the other hand, the enhancement of the pairing gap in odd nucleon systems arises due to the weakening of the Pauli blocking effect in the level occupied by the odd nucleon, which is referred to as the odd level hereafter [9,19]. The pairing reentrance appears to be more prominent in nuclei near the drip line, where the boundary of stability is unclear [15,16]. This phenomenon also appears in hot rotating nuclei [17,20], where the effect of thermal fluctuations and angular momentum acts together [9].

In the present work, we focus on the thermal pairing reentrance phenomenon in nonrotating hot nuclei with odd nucleon numbers. This phenomenon was first predicted in Ref. [18] in odd systems by using the number-parity projected gaps within the extended BCS in the N-odd system. That result was confirmed in the latter work [19] via a proposed treatment at finite temperature for the Pauli blocking effect within both BCS and FTEP methods. The authors have provided a plausible explanation for the phenomenon of thermal pairing reentrance, which involves the weakening of the blocking effect by the odd level k_0. This results in a reduction of the occupancy of the k_0 level, which is set to be 1 in traditional approaches [21–23], thereby increasing the ability of coupling between two nucleons at low temperature. However, the relation between the single-particle levels around the odd k_0 level and the pairing reentrance effect was not clarified. The present work investigates this relation in schematic models for odd systems as well as in some realistic nuclei, such as ^{37,39,41,43,45}Ca, by using the FTEP method. We aim to point out the microscopic origin of the pairing reentrance and its behavior depending on the distance between the levels closed to the odd level.

II. FORMALISM

A. Exact pairing solution

The solution of the pairing problem, which can be obtained by directly diagonalizing the pairing Hamiltonian, is referred to as the exact pairing solution. This approach was first proposed by Richardson *et al.* [24], and later simplified by Volya *et al.* [12]. By using the special unitary group SU(2), the authors of Ref. [12] introduced the quasispin angular momentum operators that allow to rewrite the pairing Hamiltonian [Eq. (1)] in a form that is simple and easy to be diagonalized. To employ this method, we consider the original pairing Hamiltonian in the nuclear system, which can be expressed as

$$\hat{H} = \sum_j \epsilon_j a^\dagger_{jm} a_{jm} - G \sum_{mm'} a^\dagger_{jm} a^\dagger_{j\widetilde{m}} a_{j'\widetilde{m}'} a_{j'm'}, \tag{1}$$

where $a^\dagger_{jm}$ and a_{jm} are the creation and annihilation operators of a nucleon on the jth orbital, whose projections, degeneracies, and single-particle energies are $\pm m$, $\Omega_j = j + 1/2$, and ϵ_j, respectively. The symbol $\sim$ denotes the time-reversal operator $a_{j\widetilde{m}} = (-1)^{j-m} a_{j-m}$.

In order to simplify the pairing Hamiltonian, one utilizes the quasispin operators $\hat{L}_j$, which are operators that represent the nuclear pairing correlations and are derived by using SU(2) operators [12]. These operators can be expressed as

$$\hat{L}^-_j = \sum_m \tilde{a}_{jm} a_{jm}, \tag{2}$$

$$\hat{L}^+_j = (\hat{L}^-_j)^\dagger = \sum_m a^\dagger_{jm} \tilde{a}^\dagger_{jm}, \tag{3}$$

$$\hat{L}^z_j = \frac{1}{2} \sum_m \left(a^\dagger_{jm} a_{jm} - \frac{1}{2} \right) = \frac{1}{2}(\hat{N}_j - \Omega_j), \tag{4}$$

where $\hat{N}_j$ is the particle number operator. These operators satisfy the commutation relations

$$[\hat{L}^+_j, \hat{L}^-_{j'}] = 2\delta_{jj'}\hat{L}^z_j, [\hat{L}^z_j, \hat{L}^+_{j'}] = \delta_{jj'}\hat{L}^+_j, [\hat{L}^z_j, \hat{L}^-_{j'}] = -\delta_{jj'}\hat{L}^-_j. \tag{5}$$

By utilizing the quasispin operators $\hat{L}_j$ as defined in Eqs. (2)–(4), the original pairing Hamiltonian given by Eq. (1) can be expressed in a simplified form [12]

$$H = \sum_j \epsilon_j \Omega_j + 2 \sum_j \epsilon_j \hat{L}^z_j + G \sum_{jj'} \hat{L}^+_j \hat{L}^-_{j'}. \tag{6}$$

With each $\hat{L}_j$, the operator $\hat{L}^2_j = \hat{L}^+_j \hat{L}^-_j - \hat{L}^z_j + (\hat{L}^z_j)^2$ commute with the Hamiltonian. It has been shown that the value of quasispin L_j within the $\hat{L}^2_j$ eigenvalue $[L_j(L_j+1)]$ is a good quantum number. Thus, the L_j and its projection L^z_j are expressed in terms of particle number N_j (the number of particles on the jth level) and seniority s_j (the number of unpaired particles on the jth level) as [12]

$$L_j = \tfrac{1}{2}(\Omega_j - s_j),\ Ł^z_j = \tfrac{1}{2}(N_j - \Omega_j), \tag{7}$$

and

$$\hat{L}_j^{\pm} |L_j, L^z_j\rangle = \sqrt{\left(L_j \mp L^z_j\right)\left(L_j \pm L^z_j + 1\right)} |L_j, L_j \pm 1\rangle. \tag{8}$$

The pairing states of the nuclear system are denoted as $|L_j, L^z_j\rangle$, which can be described in terms of the basis states $|k\rangle \equiv |s_j, N_j\rangle$ by using Eq. (7). The matrix element of the pairing Hamiltonian (6) within the basis $\{s_j, N_j\}$ can be then exactly calculated as the diagonal and off-diagonal matrix elements corresponding to Eqs. (9) and (10) in the form

$$\langle \{s_j\}, \{N_j\} | H | \{s_j\}, \{N_j\} \rangle = \sum_j \left(\epsilon_j N_j - \frac{G}{4}(N_j - s_j)(2\Omega_j - s_j - N_j + 2) \right), \tag{9}$$

$$\langle \{s_j\}, \ldots N_j + 2, \ldots N_{j'} - 2, \ldots | H | \{s_j\}, \ldots N_j, \ldots N_{j'}, \ldots \rangle = -\frac{G}{4}[(N_{j'} - s_{j'})(2\Omega_{j'} - s_{j'} - N_{j'} + 2)(2\Omega_j - s_j - N_j) \times (N_j - s_j + 2)]^{1/2}. \tag{10}$$

By diagonalizing the above pairing matrix (9)–(10), one obtains the eigenvalues $\mathcal{E}_s$ and eigenstates $|s\rangle$. These eigenvalues correspond to the energy at the total seniority s (total number of unpaired particles) with $s = \sum_j s_j$, while the eigenstates $|s\rangle = \sum_k C^s_k |k\rangle$ are the products of the basic states $|k\rangle$ and their coefficients $(C^s_k)^2$. Here $(C^s_k)^2$, with the normalization condition $\sum_k (C^s_k)^2 = 1$, are the weights of the eigenvector components obtained from the diagonalization of the pairing matrix (9)–(10). The degeneracy of the states $|s\rangle$ is given as [13]

$$d_s = \prod_j \left[\frac{(2\Omega_j)!}{s_j!(2\Omega - s_j)!} - \frac{(2\Omega_j)!}{(s_j - 2)!(2\Omega - s_j + 2)!} \right]. \quad (11)$$

The exact single-particle occupation number of the jth level corresponding to each $|s\rangle$ state is computed via the average of the partial occupation numbers N_j^k overall the basic states $|k\rangle$ [13], namely

$$f_j^s = \frac{\sum_k N_j^k \left(C_k^s\right)^2}{\sum_k \left(C_k^s\right)^2} = \sum_k N_j^k \left(C_k^s\right)^2. \quad (12)$$

B. Exact pairing solution at finite temperature

At zero temperature, the exact pairing solution provides a set of quantities, including the eigenvalues $\mathcal{E}_s$ and single-particle occupation numbers f_j^s [12]. For finite temperatures, the partition function in the canonical ensemble (CE) can be constructed based on the eigenvalues $\mathcal{E}_s$ at zero temperature as [13]

$$Z(T) = \sum_s d_s e^{-\mathcal{E}_s/T}. \quad (13)$$

From the partition function (13), one can easily derive various thermodynamic quantities, including the free energy $\mathcal{F}$, total energy $\mathcal{E}$, heat capacity $\mathcal{C}$, and pairing gap Δ, namely

$$\mathcal{F} = -T \ln Z(T), \qquad \mathcal{S} = -\frac{\partial \mathcal{F}}{\partial T}, \quad (14)$$

$$\mathcal{E} = \mathcal{F} + T\mathcal{S}, \qquad \mathcal{C} = \frac{\partial \mathcal{E}}{\partial T}, \quad (15)$$

$$\Delta = \sqrt{-G\mathcal{E}_{\text{pair}}}, \quad \mathcal{E}_{\text{pair}} = \mathcal{E} - 2\sum_j \Omega_j \left[\epsilon_j - \frac{G}{2} f_j\right] f_j, \quad (16)$$

where f_j are the temperature-dependent single-particle occupation numbers, which are calculated from the state-dependent occupation numbers f_j^s as

$$f_j = \frac{1}{Z} \sum_s d_s f_j^s e^{-\mathcal{E}_s/T}. \quad (17)$$

Knowing the single-particle occupation numbers f_j, one can evaluate the effect of pairing correlations on the single-particle spectra at finite temperatures. Specifically, the value of f_j for the odd level can be used to study the reentrance effect of pairing and/or the unblocking phenomenon that arises due to the weakening of the Pauli blocking by the odd particle at $T \neq 0$.

III. NUMERICAL CALCULATIONS AND RESULTS

A. Ingredients of the numerical calculations

In order to study the pairing correlations in an odd-nucleon system, we first perform the calculations within a schematic equidistant multilevel pairing model. The model consists of a single-particle spectrum with $N = 9$ particles and $\Omega = 10$ equidistant levels. The single-particle energy

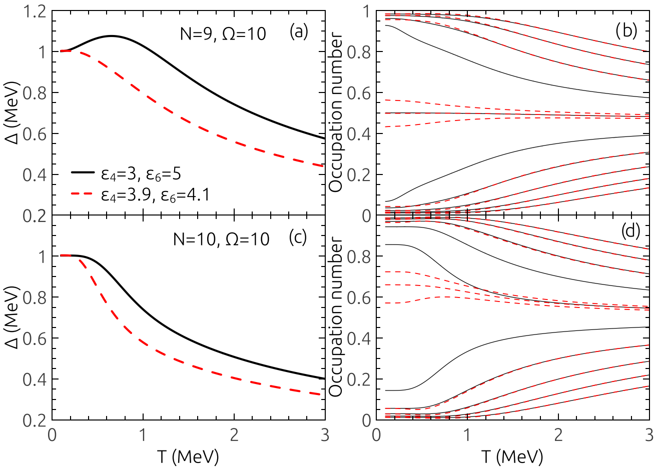

FIG. 1. Temperature-dependent pairing gaps and occupation numbers of odd [(a) and (b)] and even [(c) and (d)] nucleon configurations obtained within the FTEP. The unit of ϵ_4 and ϵ_6 is MeV.

for each level k is set to $\epsilon_k = k - 1$ MeV, where $k = 1, \ldots, 10$.[1] Each of the first four levels is occupied by two particles, while the fifth level ($k = 5$) is occupied by the odd one. For this configuration, we choose a constant pairing parameter G such that the value of the zero-temperature pairing gap $\Delta(T = 0)$ is always equal to 1 MeV. In order to study the effect of the relative position of the single-particle levels on the pairing correlation, we modify the single-particle energy of two levels, which are located just below and above the odd level ($k = 5$ with $\epsilon_k = 4$ MeV), i.e., the levels $k = 4$ and $k = 6$, as in the following cases $\{\epsilon_4 = 3\,\text{MeV}, \epsilon_6 = 5\,\text{MeV}\}$, $\{\epsilon_4 = 2.1\,\text{MeV}, \epsilon_6 = 5.9\,\text{MeV}\}$, $\{\epsilon_4 = 3\,\text{MeV}, \epsilon_6 = 4.1\,\text{MeV}\}$, $\{\epsilon_4 = 3.9\,\text{MeV}, \epsilon_6 = 5\,\text{MeV}\}$, and $\{\epsilon_4 = 3.9\,\text{MeV}, \epsilon_6 = 4.1\,\text{MeV}\}$. The remaining levels are left unchanged. We also perform some calculations with realistic nuclei, namely the calcium ^{37,39,41,43,45}Ca isotopes. These isotopes have the proton magic number, therefore only neutron pairing needs to be considered. Moreover, the levels around the odd-neutron level in ^{43,45}Ca are very close to it, enabling a comparison of the results with those obtained for ^{37,39,41}Ca. The single-particle spectra for these nuclei are calculated using the axially deformed Woods-Saxon potential [25].

B. Pairing reentrance in odd nuclei at finite temperature

1. Appearance and disappearance of pairing reentrance in odd nucleon configuration at finite temperature

In the first part of this work, we investigate an equidistant level configuration with $N = 9$, $\Omega = 10$, and $\epsilon_k = k - 1$ MeV, as mentioned earlier. We employ the FTEP method to obtain the pairing gap and occupation numbers at $T \neq 0$, as shown in Figs. 1(a) and 1(b). For comparison, we also examine the corresponding even nucleon configuration with $N = 10$, and

[1]We use the notation k to simulate the single-particle scheme in deformed nuclei.

the results are presented in Figs. 1(c) and 1(d). This allows us to analyze the effect of the odd particle on the pairing correlation in the system.

The results obtained without modified levels (solid line in Fig. 1) show a clear difference between the pairing gaps for odd and even configurations at finite temperatures. The phenomenon of pairing reentrance (enhanced pairing gap) is only observed in the odd configuration [Fig. 1(a)]. This is caused by the weakening of the Pauli blocking in the odd level ($k = 5$), whose occupation number, which is 0.5 at $T = 0$, slightly decreases with increasing T, in the same way as that previously discussed in Ref. [19].

To know in detail about the variation of pairing with the level positions (distances), we adjust two levels with $k = 4$ and 6 that are adjacent to the odd level with $k = 5$, by moving them closer to the latter, namely $\{\epsilon_4 = 3.9\,\text{MeV}, \epsilon_5 = 4\,\text{MeV}, \epsilon_6 = 4.1\,\text{MeV}\}$. The odd configuration in this case no longer exhibits a pairing reentrance [dashed line in Fig. 1(a)] as the enhancement of the pairing gap at low temperatures is completely washed out. Meanwhile, no change is observed in the behavior of the pairing gap for the configuration with an even particle number, except its decrease starts at a lower temperature. By examining the single-particle occupation numbers of the levels on the right panel of Fig. 1, we see a strongly associated scattering in the levels $k = 4$, 5, and 6. These levels share their particles with each other due to a very small single-particle energy gap (about 0.1 MeV). For $k = 4$, 5, and six levels in the odd system, the average occupation number is approximately 0.5 in the entire temperature region from $T = 0$ MeV up to $T = 4$ MeV [Fig. 1(b)]. This finding strongly suggests that these levels contribute to the blocking effect so that the pairing reentrance is suppressed. The corresponding levels in the even configuration exhibit an average occupation number larger than 0.5 [Fig. 1(d)], indicating a strong propensity for pairs to scatter to these levels. As a result, pairs that scatter to these higher-energy levels are more susceptible to breaking at lower temperatures, causing a more rapid decrease in the pairing gap. Furthermore, in the even configuration, there is no odd-level intermediary to facilitate the transmission of the blocking effect to its neighboring levels, as is the case in the odd configuration, as we shall see in a more detailed discussions in the next section.

2. *Impact of odd-neighboring levels on pairing reentrance effect*

The results presented above for the odd nucleon configuration highlight the significant influence of level positions on the pairing reentrance effect, particularly in relation to the odd level. The proximity of neighboring levels to the odd level can significantly diminish or even eliminate the reentrance phenomenon in an odd system. To elucidate this issue, we investigate changes in the level positions following different scenarios. For instance, we alternately move two adjacent levels ($k = 4$ and 6) closer to the odd level ($k = 5$). The results obtained within the FTEP are shown in Fig. 2, where, one can see that, when the $k = 6$ level is moved closer to the $k = 5$ one (ϵ_6 is reduced from 5 to 4.1 MeV), the pairing reentrance clearly decreases but is still not washed out [Fig. 2(a)]. By

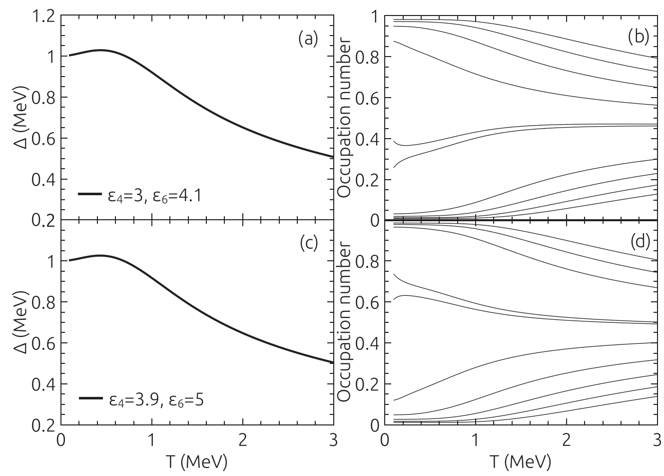

FIG. 2. The pairing gaps and occupation numbers of the odd nucleon configuration obtained within the FTEP for two cases $\{\epsilon_4 = 3\,\text{MeV}, \epsilon_5 = 4\,\text{MeV}, \epsilon_6 = 4.1\,\text{MeV}\}$ [(a) and (b)] and $\{\epsilon_4 = 3.9\,\text{MeV}, \epsilon_5 = 4\,\text{MeV}, \epsilon_6 = 5\,\text{MeV}\}$ [(c) and (d)].

examining the occupation numbers in Fig. 2(b), one observes a strong fluctuation, which is caused by the strong asymmetry in the position of these levels. In this case, it appears that the odd particle is shared between the $k = 5$ and $k = 6$ levels, causing both of them to be blocked. This increase in the blocking leads to a decrease in the pairing reentrance. A similar phenomenon can be observed in Figs. 2(c) and 2(d) when the $k = 4$ level is brought closer to the odd one ($k = 5$). It is reasonable to expect that bringing more levels closer to the level with the odd particle will result in a stronger suppression of the pairing reentrance, eventually up to the point of entirely eliminating it.

Shown in Fig. 3 are results obtained with various shifts of $k = 4$ and $k = 6$ levels. As they move closer to the odd level, the enhancement of the pairing gap at low temperature ($T < 1$ MeV) gradually decreases and finally vanishes when $\epsilon_4 = 3.9$ MeV and $\epsilon_6 = 4.1$ MeV, that is when they are located at the positions closest to the odd level. The position of the maximum of the pairing gaps is shifted towards the $T = 0$ region

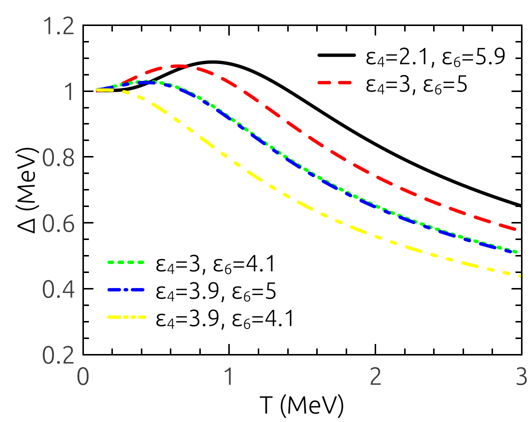

FIG. 3. The correlation between the level positions and the changes in the pairing gap obtained within the FTEP. The unit of ϵ_4 and ϵ_6 is MeV.

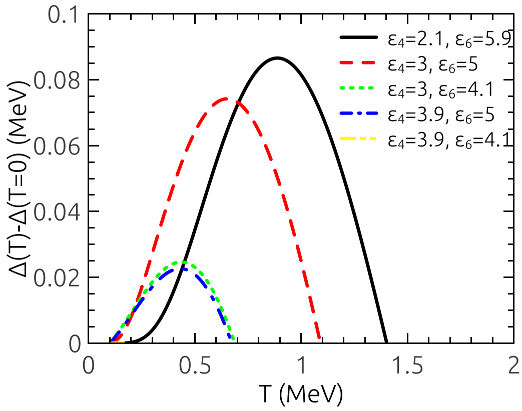

FIG. 4. Magnification of the pairing reentrance extracted from Fig. 3. The unit of ϵ_4 and ϵ_6 is MeV.

when the level distance $\Delta\epsilon = \epsilon_k - \epsilon_5$ $(k = 4, 6)$ decreases. When the energy levels get close together, it suggests that the pairing reentrance occurs and concludes more rapidly. With narrower level distances, the pair breaking process becomes more sensitive to temperature. On the other hand, the presence of a larger $\Delta\epsilon$ makes it easier for nucleons to couple to the odd level (creating the enhancement in the pairing gap) rather than to scatter to higher orbits.

The pairing reentrance process can be magnified by using the quantity $\Delta(T) - \Delta(T = 0)$ as shown in Fig. 4. This figure clearly displays the starting point, ending point, maximum point, and weight of the pairing reentrance process. The case with a smaller $\Delta\epsilon$ experiences an earlier starting and ending point compared to the case with a larger $\Delta\epsilon$.

To examine in more details the effect of changing the odd-neighboring levels on the pairing reentrance as reported in Fig. 3, we perform a series of changes on the $k = 3, 4, 6$, and 7 levels. Thus, the resulting level distance $\Delta\epsilon = \epsilon_k - \epsilon_5$, falls within the range of 0–3 MeV. For example, for the $k = 3$ level, its energy is varied from $\epsilon_3 = 2.1$ MeV to $\epsilon_3 = 3.9$ MeV with a constant step of $\delta\epsilon = 0.1$ MeV. Consequently, the corresponding energy distance from $k = 3$ level to odd $k = 5$ level $(\Delta\epsilon = |\epsilon_3 - \epsilon_5|)$ also varies from 1.1–2.9 MeV. Similar variations of the energy distance from $k = 4$, 6, and 7 levels to the odd one can be seen in Table I. This Table I clearly shows that the maximum temperature $T_{\max}$, at which the pairing gap reaches its maximum value $(\Delta_{\max})$, changes between 0.58 MeV and 0.67 MeV, when varying the energy of $k = 3$ and $k = 7$ levels. The value of $\delta\Delta_{\max} = \Delta_{\max} - \Delta(T = 0)$ in this case oscillates around 0.074 MeV. This result implies that changing the energy of $k = 3$ and $k = 7$ levels insignificantly affects the pairing reentrance. Considering the two closest levels to the odd one ($k = 4$ and 6 levels), one can see from Table I that both $T_{\max}$ and $\delta_{\max}$ significantly increases with the energy distance $\Delta\epsilon$ from 0.1–1.9 MeV, namely $T_{\max}$ increases from 0.43 MeV–0.78 MeV and $\delta\Delta_{\max}$ raises from around 0.02 MeV–0.08 MeV.

In short, the above analysis of the pairing reentrance within the framework of a schematic model indicates that the pairing reentrance in an odd-nucleon system can be attributed to the weakening of the Pauli blocking effect at finite temperature caused by the presence of the odd particle. As the energy

TABLE I. The maximum temperature $T_{\max}$ at which the pairing gap reaches its maximum value and relative pairing gap $\delta\Delta_{\max} = \Delta_{\max} - \Delta(T = 0)$ obtained within the FTEP by varying the energy distance $\Delta\epsilon$ between the $k = 3, 4, 6$, and 7 levels and the odd $k = 5$ one. $\Delta\epsilon$, $T_{\max}$, and $\delta\Delta_{\max}$ have the same unit of MeV.

	$k = 4$		$k = 6$		$k = 3$		$k = 7$		
$\Delta\epsilon$	$T_{\max}$	$\delta\Delta_{\max}$	$T_{\max}$	$\delta\Delta_{\max}$	$\Delta\epsilon$	$T_{\max}$	$\delta\Delta_{\max}$	$T_{\max}$	$\delta\Delta_{\max}$
0.1	0.43	0.0231	0.44	0.0286	1.1	0.67	0.0708	0.58	0.0760
0.2	0.46	0.0316	0.46	0.0356	1.2	0.67	0.0714	0.58	0.0765
0.3	0.48	0.0413	0.49	0.0463	1.3	0.67	0.0719	0.60	0.0764
0.4	0.50	0.0504	0.51	0.0527	1.4	0.67	0.0721	0.61	0.0762
0.5	0.54	0.0573	0.54	0.0590	1.5	0.67	0.0721	0.62	0.0762
0.6	0.56	0.0630	0.56	0.0653	1.6	0.67	0.0728	0.63	0.0758
0.7	0.59	0.0671	0.59	0.0676	1.7	0.66	0.0728	0.64	0.0755
0.8	0.61	0.0702	0.61	0.0743	1.8	0.66	0.0736	0.64	0.0752
0.9	0.63	0.0722	0.63	0.0728	1.9	0.66	0.0737	0.65	0.0747
1.0	0.65	0.0745	0.65	0.0758	2.0	0.66	0.0745	0.65	0.0741
1.1	0.67	0.0763	0.67	0.0791	2.1	0.65	0.0748	0.66	0.0737
1.2	0.69	0.0770	0.69	0.0784	2.2	0.64	0.0749	0.66	0.0736
1.3	0.71	0.0781	0.70	0.0780	2.3	0.64	0.0758	0.67	0.0729
1.4	0.72	0.0786	0.72	0.0822	2.4	0.63	0.0760	0.67	0.0724
1.5	0.74	0.0794	0.73	0.0823	2.5	0.62	0.0762	0.67	0.0722
1.6	0.75	0.0797	0.75	0.0828	2.6	0.61	0.0762	0.67	0.0716
1.7	0.76	0.0802	0.76	0.0794	2.7	0.60	0.0766	0.67	0.0715
1.8	0.77	0.0801	0.77	0.0804	2.8	0.59	0.0763	0.67	0.0711
1.9	0.78	0.0804	0.78	0.0817	2.9	0.58	0.0760	0.67	0.0708

TABLE II. The neutron single-particle energies (in MeV) of $^{37-45}$Ca obtained within the axially deformed Woods-Saxon potential [25] with the deformation parameter β_2 collected from Ref. [27]. The bold values indicate the energies of the odd level.

Level	^{37}Ca ($\beta_2 = -0.021$)	^{39}Ca ($\beta_2 = 0.011$)	^{41}Ca ($\beta_2 = -0.021$)	^{43}Ca ($\beta_2 = 0.011$)	^{45}Ca ($\beta_2 = -0.011$)
$k = 1$	−20.898	−20.046	−19.287	−15.001	−13.646
$k = 2$	−20.722	−19.954	−15.436	−13.851	−13.511
$k = 3$	−20.638	−15.922	−14.147	−13.721	−8.805
$k = 4$	−16.474	−14.347	−13.872	−9.052	−8.719
$k = 5$	**−14.707**	**−14.208**	**−9.490**	**−9.022**	**−8.663**
$k = 6$	−14.418	−9.737	−9.321	−8.962	−8.635
$k = 7$	−10.292	−9.705	−9.211	−8.873	−5.183
$k = 8$	−10.115	−9.642	−9.157	−5.366	−5.081
$k = 9$	−10.000	−9.500	−5.621	−5.259	−2.941
$k = 10$	−9.944	−5.826	−5.422	−3.001	−1.545

levels neighboring to the odd level are shifted closer to the latter, the level distance $\Delta\epsilon$ decreases, resulting in a decrease in the pairing reentrance effect. The explanation for this phenomenon is that, when the neighboring levels get too close to the odd one, they participate in the Pauli blocking effect in a similar way to that for the odd level. As a result, pairing reentrance is prevented due to the increased blocking effect. In the next section, we shall further examine this effect in realistic nuclei.

3. *Pairing reentrance in Ca isotopes*

We consider five odd Ca isotopes ranging from ^{37}Ca to ^{45}Ca. Since these isotopes have a magic number of protons, only neutron pairing is considered. Some of them have the single-particle spectra, which are obtained within the axially deformed Woods-Saxon potential [25], with some energy levels very close to the odd one (see Table II and Fig. 5). We select the same $\Omega = 10$ levels as in the analysis with the schematic model. The pairing strength G_N for neutrons is adjusted to reproduce the empirical pairing gap $\Delta_N(T = 0) \approx 11.56.N^{-0.552}$, which is extracted from the odd-even mass difference [26].

Figure 5 illustrates the density of the neutron single-particle spectra for the calcium isotopes $^{37-45}$Ca. Upon examining the figure, we can observe that in ^{43,45}Ca, there are very dense levels above and below the odd level, whereas this distribution only occurs on one side in the remaining isotopes. As anticipated, the pairing reentrance effect is almost imperceptible in the ^{43,45}Ca isotopes, whereas it is evident in the remaining ones (see Fig. 6).

The above investigation with realistic Ca isotopes suggests that the occurrence of pairing reentrance is not universal in odd nuclei. In nuclei such as ^{43,45}Ca, where there are very dense levels surrounding the odd level, this effect may not be present. The key factor to the explanation is the increase in Pauli blocking effect resulting from sharing the blocking property of the odd level with the neighboring ones. In the case of ^{43,45}Ca, up to four levels participate in the blocking effect ($k = 4$–7 for ^{43}Ca and $k = 3$–6 for ^{45}Ca, as seen in Table II.

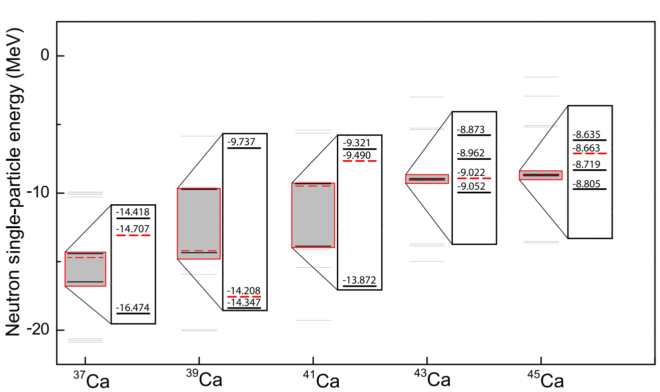

FIG. 5. The neutron single-particle energies of $^{37-45}$Ca obtained within the axially deformed Woods-Saxon potential [25]. The dash line presents the odd level.

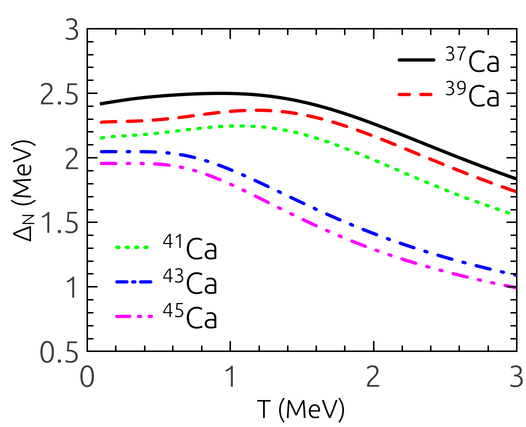

FIG. 6. The temperature-dependent pairing gaps obtained within the FTEP for Ca isotopes.

IV. CONCLUSION

The present paper proposes a method for testing the pairing properties of odd nucleon configurations at finite temperatures, utilizing the finite-temperature exact pairing method. The configurations under consideration are the schematic single-particle spectra with the levels adjacent to the odd level being shifted in different scenarios. Our findings indicate that the pairing reentrance effect, which occurs due to the weakening of blocking by the odd particle at finite temperature, can become depleted or even disappear, depending on the position of single-particle levels surrounding the odd level. When these levels are very dense, not only the odd level but also the neighboring levels participate in the blocking effect. The resulting increase in Pauli blocking effect outweighs the weakening effect and ultimately prevents the occurrence of pairing reentrance. This phenomenon is also observed in realistic nuclei such as calcium isotopes $^{37–45}$Ca, where the pairing reentrance effect is suppressed in ^{43,45}Ca due to the participation of up to four levels in the blocking effect.

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