Dynamical decay of a trinuclear system in the presence of friction forces in the spontaneous fission of ^{252}Cf

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The dynamics of the rupture of the ternary system ^{70}Ni + ^{50}Ca + ^{132}Sn formed at the spontaneous fission of ^{252}Cf has been studied. The heaviest fragment ^{132}Sn is separated first very easily and the behavior of the breakup of the ^{70}Ni + ^{50}Ca system determines final directions of the angular distribution of the final products. The equations of motion with including microscopical friction forces between two contacting nuclei have been calculated. The friction force in the dynamical equations results in increase of decay time of the system only. The results of calculations show that the trajectory of the middle Ca nucleus is nearly perpendicular to the line connecting centers of two massive ^{70}Ni and ^{132}Sn nuclei if initial configuration is not collinear.

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I. INTRODUCTION

Fission of nuclei is a collective rearrangement of nucleons towards an elongated shape influenced by its structure and available excitation energy. The liquid drop aspects and quantal properties of nuclei are responsible for the asymmetric shape of the mass distribution of the binary fission products. Observation of the ternary fission is a very rare phenomenon due to smallness of the sequential fission of a strongly deformed fragment formed at the scission point. At the spontaneous ternary fission of ^{252}Cf, the frequency of occurrence is observed to be greater than 2.2×10^{-6} ternary fission events per binary fission event. Tripartition of ^{252}Cf results preferentially in division into two medium mass particles and one with the small mass number (around 50) [1]. In the thermal-neutron-induced fission of ^{235}U, the frequency of occurrence is observed to be greater than 1.2×10^{-6} ternary fission events per binary fission event. In this case, the light fragment of ternary fission has mass number around 34 [1]. These results indicate that axially asymmetric distortion modes are possible in the prescission configurations of the fissioning nucleus. Another investigation of symmetric ternary spontaneous fission of ^{252}Cf was done by Schall *et al.* [2]. The deduced upper limit for the symmetric ternary to binary fission ratio was found to be 10^{-8}. Some experimental results [3] and [4] showed that ternary fission with emission of light nuclei (such as He, Li, etc.) was more possible, against ternary fission with comparable masses. The extensive search for the ternary decay channel was performed in Flerov Laboratory of Nuclear Reactions (FLNR), JINR (see Ref. [1] and references therein) for the spontaneous fission of ^{252}Cf as well as for thermal neutron-induced fission of ^{235}U. This phenomenon was named "collinear cluster tripartition" since two of the decay products fly apart almost collinearly in opposite directions. It was found that the true ternary fission is a rather probable channel with the yield ratio to the binary one of about 10^{-4}. In the spontaneous fission of ^{252}Cf the following combinations of ternary yields were found: Sn + Ge(or Ni)+S(or Ca).

Early theoretical considerations of ternary fission was based on the liquid drop model [5]. Eventually, three center shell model [6] were developed to take into consideration the shell effects. The remarkable theoretical effort based on three center shell model on ternary fission was made in Ref. [7]. It was shown that ternary decay being strongly suppressed by the macroscopic properties of the potential energy, may, however, be present with a significant probability due to the shell effects. The study of the true ternary fission of ^{252}Cf was performed in Refs. [8–10] based on the consideration of the potential energy surface at contact configuration of three preformed clusters. These studies have confirmed the preference of the collinear configuration of three fragments over the triangular one. So, recent studies show an increasing interest of nuclear physicists in the true ternary fission phenomena [11–15]. The recent review of physics of true ternary fission can be found in Ref. [16].

In recent experiment [17] on spontaneous fission of ^{252}Cf, it was found two similar combinations of ternary fission products, namely, Ni + Ca + Sn and Ca + Ni + Sn (with smaller probability). The authors suggested one scenario of Cf → Ca + Ni + Sn reaction as Ca nucleus rotated left around Ni nucleus after first rupture of Ni + Ca + Sn system (see Fig. 9

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of Ref. [17]). Such a scenario could be explained by the dynamical calculation of tripartition in the presence of friction forces between nuclei of trinuclear system (TNS).

There is not yet theoretical description of the collinearity of the momentum of the two observed tripartition products. In Ref. [18] the possibility of the collinearity was demonstrated for the collinear replacement of the connected three clusters. The current study is inspired from the experimental results presented in Ref. [17], so motivation of the current work is to investigate dynamical trajectories and velocities of fragments in tripartition of TNS (Ni + Ca + Sn) including microscopical friction forces.

In Sec. II, we present the model of the trajectory calculations of true ternary decay process in the presence of microscopically calculated friction forces. In Sec. III, we discuss results of calculations of trajectories and velocities of fragments of ternary spontaneous fission of ^{252}Cf. In Sec. IV, main conclusions of current investigation are described.

II. MODEL

The collinear configuration of TNS, when the third nucleus is placed between the first and the second nuclei, is considered. The motion of these nuclei is considered in the reference frame (x and y coordinates in the Fig. 1) connected with the center of mass (the origin point O), which is at rest, since the spontaneous fission of ^{252}Cf is studied. So the

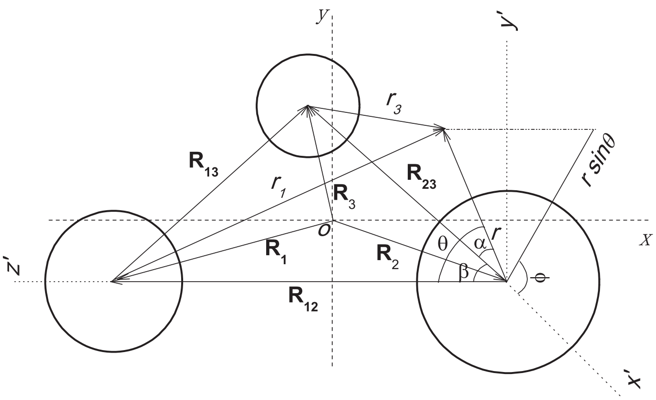

FIG. 1. Point ($\mathbf{R}_k$) and relative ($\mathbf{R}_{ij}$) vectors of trinuclear system. The motion of three nuclei is studied relative to the origin (Point O) of the reference frame (x, y) that corresponds to center of mass.

reference frame is the same as laboratory frame of reference. The used friction forces depend only on relative distances R_{ij} (see Fig. 1) between the centers of nuclei. Therefore, the right-hand side of equation of motion should be converted from the position coordinates R_k to the relative distances R_{ij}. Moreover, as mother nucleus ^{252}Cf decay spontaneously, so fission process occurs in the (x, y) plane. It means that equations of motions will have only x and y components. To investigate the decay of trinuclear system, we solve classical dynamical set of equations:

$$
\begin{aligned}
m_1\dot{\upsilon}_{1x} &= -\frac{R_{12x}}{R_{12}}\left(\frac{\partial V}{\partial R_{12}} - \gamma_{12}(R_{12})\dot{R}_{12}\right) + \frac{R_{13x}}{R_{13}}\left(\frac{\partial V}{\partial R_{13}} - \gamma_{13}(R_{13})\dot{R}_{13}\right)\\
m_2\dot{\upsilon}_{2x} &= \frac{R_{12x}}{R_{12}}\left(\frac{\partial V}{\partial R_{12}} - \gamma_{12}(R_{12})\dot{R}_{12}\right) + \frac{R_{23x}}{R_{23}}\left(\frac{\partial V}{\partial R_{23}} - \gamma_{23}(R_{23})\dot{R}_{23}\right)\\
m_3\dot{\upsilon}_{3x} &= -\frac{R_{23x}}{R_{23}}\left(\frac{\partial V}{\partial R_{23}} - \gamma_{23}(R_{23})\dot{R}_{23}\right) - \frac{R_{13x}}{R_{13}}\left(\frac{\partial V}{\partial R_{13}} - \gamma_{13}(R_{13})\dot{R}_{13}\right)
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
m_1\dot{\upsilon}_{1y} &= -\frac{R_{12y}}{R_{12}}\left(\frac{\partial V}{\partial R_{12}} - \gamma_{12}(R_{12})\dot{R}_{12}\right) + \frac{R_{13y}}{R_{13}}\left(\frac{\partial V}{\partial R_{13}} - \gamma_{13}(R_{13})\dot{R}_{13}\right)\\
m_2\dot{\upsilon}_{2y} &= \frac{R_{12y}}{R_{12}}\left(\frac{\partial V}{\partial R_{12}} - \gamma_{12}(R_{12})\dot{R}_{12}\right) + \frac{R_{23y}}{R_{23}}\left(\frac{\partial V}{\partial R_{23}} - \gamma_{23}(R_{23})\dot{R}_{23}\right)\\
m_3\dot{\upsilon}_{3y} &= -\frac{R_{23y}}{R_{23}}\left(\frac{\partial V}{\partial R_{23}} - \gamma_{23}(R_{23})\dot{R}_{23}\right) - \frac{R_{13y}}{R_{13}}\left(\frac{\partial V}{\partial R_{13}} - \gamma_{13}(R_{13})\dot{R}_{13}\right),
\end{aligned}
\tag{2}
$$

with the friction coefficients γ_{ij}, which have been calculated microscopically as in Refs. [19,20]. The friction coefficients $\gamma_{ij}(R_{ij})$ are between ith and jth nuclei. The similar equations were analyzed in Ref. [18] without friction forces. The friction coefficients were extracted from the dynamics of the coupling between intrinsic motion of nucleons in nuclei and relative motion their center of mass:

$$\gamma(R(t)) = \sum_{i,k}\left|\frac{\partial U_{ik}(R(t))}{\partial R}\right|^2 B_{ik}^{(1)}(t), \tag{3}$$

where $B_{ik}^{(1)}(t)$ is given by:

$$
\begin{aligned}
B_{ik}^{(1)}(t) = \frac{2}{\hbar}\int_0^t dt'(t-t')\exp\left(\frac{t'-t}{\tau_{ik}}\right)\\
\times \sin[\omega_{ik}(R(t'))(t-t')][n_k(t') - n_i(t')],
\end{aligned}
\tag{4}
$$

$$\hbar\omega_{ik} = \varepsilon_i + U_{ii} - \varepsilon_k - U_{kk}. \tag{5}$$

Here ε_i and n_i are energies and occupation numbers of the single-particle states of nucleons in the interacting nuclei, respectively; $U_{ik}(R) = \langle i|U(\mathbf{x}-\mathbf{R})|k\rangle$ are the matrix elements related of the single-particle transitions (particle-hole excitation of nucleons and nucleon transition between interacting nuclei), where $U(\mathbf{x}-\mathbf{R})$ is a nuclear mean field of one of nuclei causing excitation of the nucleons in the quantum states $|i\rangle$ of the other nucleus of the system; in the case of the nucleon exchange between interacting nuclei $U(\mathbf{x}-\mathbf{R})$ is the sum of their mean fields: $U_{ik}(R) = \langle i|U_1(\mathbf{x}-\mathbf{R_1}) + U_2(\mathbf{x}-\mathbf{R_2})|k\rangle$, where wave functions $\langle i|$ and $|k\rangle$ of the single-particle states belong to different interacting nuclei. The friction forces appear when nuclear densities in TNS overlap and their values increase by the increase of the overlap volume. The details of calculations are presented elsewhere [19,21]; $\tau_{ik} = \tau_i\tau_k/(\tau_i+\tau_k)$; τ_i is the lifetime of the quasiparticle excitations in the single-particle state i of the nucleus. It determines the damping of single-particle motion. τ_i is calculated using the results of the quantum liquid theory [22] and the effective nucleon-nucleon forces from [23]:

$$\frac{1}{\tau_i^{(\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha)}}\left[(f_K-g)^2 + \frac{1}{2}(f_K+g)^2\right] \times \left[(\pi T_K)^2 + \left(\tilde{\varepsilon}_i - \lambda_K^{(\alpha)}\right)^2\right] \times \left[1+\exp\left(\frac{\lambda_K^{(\alpha)}-\tilde{\varepsilon}_i}{T_K}\right)\right]^{-1}, \quad (6)$$

where

$$T_K(t) = 3.46\sqrt{\frac{E_K^*(t)}{\langle A_K(t)\rangle}}$$

is the effective temperature determined by the amount of intrinsic excitation energy $E_K^* = E_K^{*(Z)} + E_K^{*(N)}$ and by the mass number $\langle A_K(t)\rangle$ [with $\langle A_K(t)\rangle = \langle Z_K(t)\rangle + \langle N_K(t)\rangle$]. In addition, $\lambda_K^{(\alpha)}(t)$ and $E_K^{*(\alpha)}(t)$ are the chemical potential and intrinsic excitation energies for the proton ($\alpha = Z$) and neutron ($\alpha = N$) subsystems of the nucleus $K[K = 1(\text{projectile}), 2(\text{target})]$, respectively. Furthermore, the finite size of the nuclei and the difference between the numbers of neutrons and protons makes it necessary to use the following expressions for the Fermi energies [23]:

$$\varepsilon_{F_K}^{(Z)} = \varepsilon_F\left[1 - \frac{2}{3}(1+2f'_K)\frac{\langle N_K\rangle - \langle Z_K\rangle}{\langle A_K\rangle}\right], \quad (7)$$

$$\varepsilon_{F_K}^{(N)} = \varepsilon_F\left[1 + \frac{2}{3}(1+2f'_K)\frac{\langle N_K\rangle - \langle Z_K\rangle}{\langle A_K\rangle}\right], \quad (8)$$

where $\varepsilon_F = 37$ MeV,

$$f_K = f_{in} - \frac{2}{\langle A_K\rangle^{1/3}}(f_{in} - f_{ex}), \quad (9)$$

$$f'_K = f'_{in} - \frac{2}{\langle A_K\rangle^{1/3}}(f'_{in} - f'_{ex}) \quad (10)$$

and $f_{in} = 0.09$, $f'_{in} = 0.42$, $f_{ex} = -2.59$, $f'_{ex} = 0.54$, $g = 0.7$. The constants f_{in}, f'_{in}, f_{ex}, f'_{ex}, and g were introduced by Migdal [23] to obtain the dependence of the effective nucleon-nucleon interaction on the nucleon density of the interacting nuclei. The values of these constants used in our work were found in Ref. [23] by the description of the experimental data.

According to the conservation law of the linear momentum, the velocity of the third nucleus υ_3 can be found by the velocity values of the first υ_1 and second υ_2 nuclei. So, the last equation in each system of equations (1) and (2) can be skipped. Finally, we will get the following system of equations of motion

$$\begin{aligned}
m_1\dot{\upsilon}_{1x} &= -\frac{R_{12x}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{13x}}{R_{13}}\left(\frac{\partial V}{\partial R_{13}} - \gamma_{13}(R_{13})\dot{R}_{13}\right)\\
m_2\dot{\upsilon}_{2x} &= \frac{R_{12x}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{23x}}{R_{23}}\left(\frac{\partial V}{\partial R_{23}} - \gamma_{23}(R_{23})\dot{R}_{23}\right)\\
m_1\dot{\upsilon}_{1y} &= -\frac{R_{12y}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{13y}}{R_{13}}\left(\frac{\partial V}{\partial R_{13}} - \gamma_{13}(R_{13})\dot{R}_{13}\right)\\
m_2\dot{\upsilon}_{2y} &= \frac{R_{12y}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{23y}}{R_{23}}\left(\frac{\partial V}{\partial R_{23}} - \gamma_{23}(R_{23})\dot{R}_{23}\right). \quad (11)
\end{aligned}$$

Here, m_i is the mass of the ith nucleus, υ_{ix} and υ_{iy} are x and y components of velocities of the ith nucleus. Also, $\frac{R_{ijx}}{R_{ij}}$ and $\frac{R_{ijy}}{R_{ij}}$ represent components of a certain force ($\frac{\partial V}{\partial R_{ij}}$) on x and y axes, respectively. In Fig. 1 we introduce (x', y', z') coordinates in order to calculate the total interaction potential by the spherical coordinates of variables r, θ, and ϕ. For the collinear configuration of the system, x and z' axes lie on the same line.

The total interaction potential of the TNS V for the certain charge and mass configurations is a sum of interaction potential between two interacting nuclei:

$$V(R_{13}, R_{23}, R_{12}) = V_{13}(R_{13}) + V_{23}(R_{23}) + V_{12}(R_{12}).$$

Each V_{ij} potential includes the nuclear V_N and Coulomb V_C potentials, so

$$V_{ij}(R_{ij}) = V_{N,ij}(R_{ij}) + V_{C,ij}(R_{ij}). \quad (12)$$

The nuclear potential $V_{N,ij}$ between two nuclei is calculated by the double folding procedure with effective Migdal nucleon-nucleon forces [23]. This method is widely used to calculate nucleus-nucleus interaction and it can be found in Refs. [18,20]. But Coulomb V_C part between two nuclei is calculated as two pointlike charges. The details of calculation of $\frac{\partial V}{\partial R_{ij}}$ in Eq. (11) are presented in Ref. [18].

III. RESULTS AND DISCUSSION

The trajectories and velocities of decay fragments have been calculated using the system of equations (11). The initial conditions are chosen so that the three nuclei (^{70}Ni, ^{50}Ca, and ^{132}Sn) located close and in the single line, which corresponds to the minimum value of total interaction potential due to presence of the strong nuclear interaction. As three nuclei are located in the minimum of the total interaction potential (see Ref. [18] for more details), so initially they are supposed to be at rest ($\upsilon_{ix} = \upsilon_{iy} = 0$) and have the coordinates $R_{1x} = -12.42$ fm, $R_{2x} = 7.71$ fm, $R_{3x} = -2.97$ fm of nuclei 1, 2, and 3 [in the (x, y) reference frame on Fig. 1], respectively. For these values of the R_{ix} coordinates the relative distances

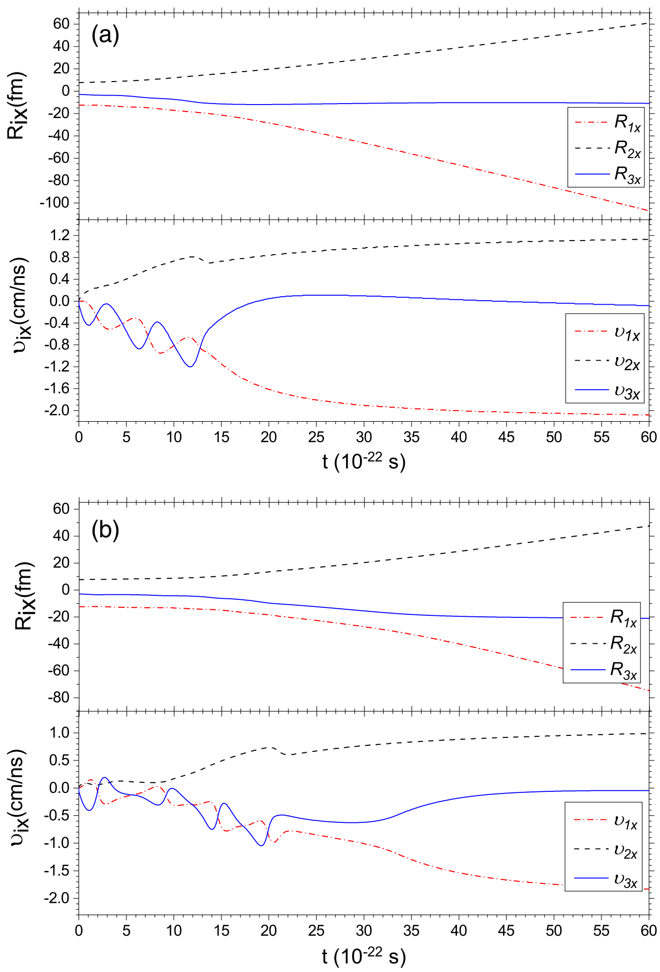

FIG. 2. Trajectories and velocities of the TNS fragments with the friction coefficients (a) $\gamma_{ij} = 0$ and (b) γ_{ij} which were calculated as in Ref. [20].

between the mass centers of these nuclei can be $R_{13x} = 9.45$ fm, $R_{23x} = -10.68$ fm, $R_{12x} = -20.13$, correspondingly. For simplicity, the nuclei of TNS are assumed to be spherical.

Figure 2(a) shows the positions R_{ix} and velocities υ_{ix} of three nuclei calculated without friction forces as a function of time. The results of calculations with the same initial conditions for the trajectory and velocities of three fragments with friction forces are presented in Fig. 2(b). The friction coefficients γ_{ij} ($i \neq j$) have been by the method presented in Ref. [20].

Comparison of these results shows that the TNS decay time calculated by the friction forces is around three times larger than the one found without friction ($\gamma_{ij} = 0$). Oscillations in the relative velocity between fragments 1 and 2 before the decay of TNS show the momentum transfer between TNS nuclei.

The increase of the friction coefficients γ_{ij} by 50% leads the TNS to stable oscillations without decay due to the friction forces between the TNS nuclei (see Fig. 3). So, proper calculation of friction forces is very important in dynamics of the TNS decay.

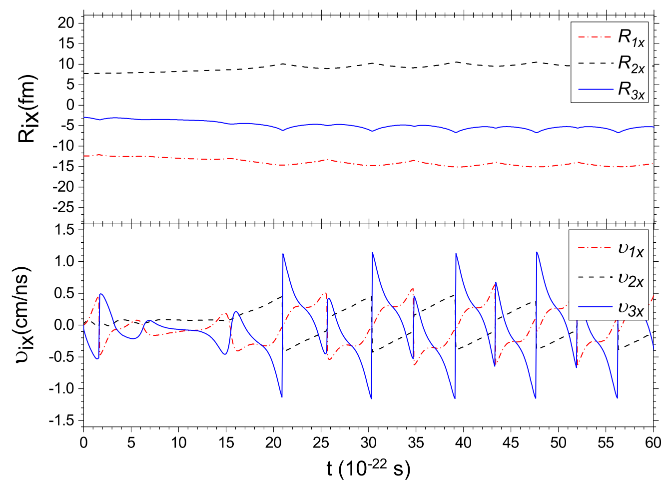

FIG. 3. Trajectories and velocities of the TNS fragments with the increased friction coefficients γ_{ij} by 50% then the one calculated as in Ref. [20].

A possibility of the rotation of the middle nucleus (in our case Ca nucleus) around Ni in the clockwise (or counterclockwise) direction to be observed as a border fragment during decay of the Ni + Ca system, we take as initial conditions $R_{3yi} = 0.5$ fm and $\upsilon_{3yi} = 0$ for ^{50}Ca at solving Eq. (11). The results are presented in Fig. 4 which shows that Sn nucleus moves away from Ni + Ca system and after 23×10^{-22} s Ni and Ca nuclei are separated without rotation together.

IV. CONCLUSION

The possibility of the observation the light cluster Ca as one of reaction channels of the collinear cluster tripartion at the spontaneous fission of ^{252}Cf has been explored. This kind of events were extracted from their analysis of the measured data in Ref. [17]. As a mechanism leading to the rotation of the middle nucleus (in our case Ca nucleus) around Ni in

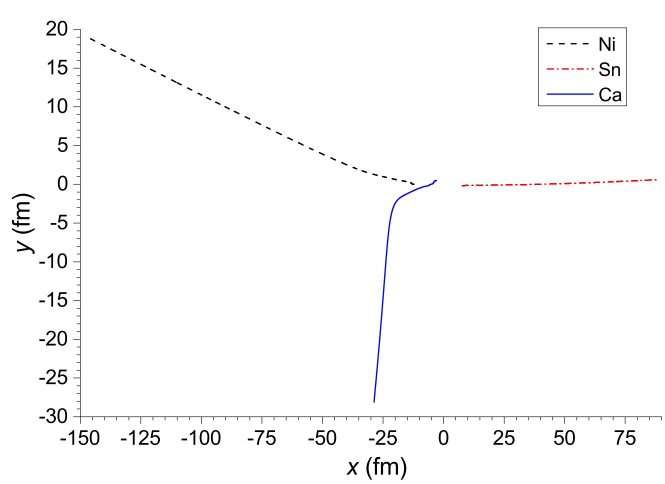

FIG. 4. Trajectories of the noncollinear fission of the TNS fragments with the friction forces having coefficients γ_{ij} calculated as in Ref. [20].

the clockwise (or counterclockwise) direction for the large angles ≈180° due to the Coulomb repulsion of ^{132}Sn has been considered. As a result the Ca nucleus could be placed beyond Ni as the border product in the collinear line, which is registered in a detector of an experiment. This suggestion has motivated us to make investigation of the possibility appearance of the Ca nucleus as a border fragment flying in the opposite direction to the Sn nucleus. Indeed, we have got the noncollinear decay of TNS as it was obtained in Ref. [18].

The results of the dynamical calculations with microscopic friction forces show that after formation of a certain collinear TNS, a middle nucleus cannot rotate around another border nucleus so that mutual replacement of the middle and border fragments of the collinear TNS does not occur during its decay. The initial deviation of the TNS from the collinear configuration leads to the noncollinear decay, even the trajectories are calculated including the friction forces in the equation of motions. Including the friction forces results the increase of decay time of the system only.

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