Transparency of the $\gamma(n, p)\pi^-$ reaction in nuclei

Swapan Das
*Nuclear Physics Division, Bhabha Atomic Research Centre, Trombay, Mumbai 400085, India
and Homi Bhabha National Institute, Anushakti Nagar, Mumbai 400094, India*

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The transparency of the hadrons produced in the $\gamma(n, p)\pi^-$ reaction in nuclei is calculated using the Glauber model modified by including the Fermi motion of the nucleon in the nucleus. Because the calculated results underestimate the measured transparency for the ^{4}He nucleus, the Glauber model is further modified by incorporating the short-range correlation of the nucleon and the color transparency of the hadron in the nucleus. The nuclear transparency of the $\gamma(n, p)\pi^-$ reaction is calculated for $\theta_{\pi^-}(\mathrm{c.m.}) = 50^\circ$, 70°, and 90°. The calculated results are compared with the data reported for the ^{4}He nucleus.

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I. INTRODUCTION

The nuclear transparency T_A of a hadron h can be defined by the ratio of the hadron-nucleus total cross section σ_t^{hA} to the mass number A (of the nucleus) times the hadron-nucleon total cross section σ_t^{hN} in the free space. The interaction of the hadron with the nucleus reduces σ_t^{hA} compared to $A\sigma_t^{hN}$, i.e., $T_A(=\frac{\sigma_t^{hA}}{A\sigma_t^{hN}}) < 1$. According to the Glauber model [1], the hadron-nucleus cross section arises because of the multiple scattering or the interaction of the hadron with nucleons in the nucleus. Therefore, the hadron-nucleon total cross section in the nucleus (denoted by σ_t^{*hN}) can be probed by studying T_A of the hadron. The measured values of T_A for the ω and ϕ mesons produced in the photonuclear reaction [2] show both $\sigma_t^{*\omega N}$ and $\sigma_t^{*\phi N}$ are larger than the respective free space values, i.e., $\sigma_t^{*\omega N} > \sigma_t^{\omega N}$ and $\sigma_t^{*\phi N} > \sigma_t^{\phi N}$ [3].

One of the publicized issues in nuclear physics is to search the color transparency of the hadron produced in the nucleus. The transverse size $d_\perp$ of a hadron produced in the nucleus due to the spacelike four-momentum transfer Δ^2 is shrunken as $d_\perp \approx 1/\Delta$ [4,5]. The reduced hadron (in size) is referred to as the pointlike configuration (PLC) [5]. The color neutral PLC, according to quantum chromodynamics, has reduced interaction with the nucleon in a nucleus [5,6]. The PLC expands to the physical size of the hadron as it moves up to a length (≈1 fm), called the hadron formation length l_h [5]. The interaction of the PLC with the nucleon in a nucleus increases as the size of the PLC enlarges during its passage up to l_h. The decrease of σ_t^{*hN} compared to σ_t^{hN} leads to the enhancement in σ_t^{hA} [1]. As a result, T_A of the hadron increases. This phenomenon is referred to as the color transparency (CT) of a hadron. Since Δ^2 is involved, the CT is an energy-dependent phenomenon. The physics of CT for hadrons has been discussed elaborately in Refs. [6,7].

It should be mentioned that σ_t^{hA} can also be altered without the modification of σ_t^{hN} in the nucleus. As explained by the Glauber model [1], σ_t^{hA} increases because of the short-range correlation (SRC) of the nucleon in the nucleus. The SRC originates because of the repulsive (short-range) interaction between the nucleons in the nucleus. The repulsive interaction keeps the bound nucleons apart (≈1 fm), which is called nuclear granularity [8]. Therefore, the SRC prevents the shadowing of the hadron-nucleon interaction due to the surrounding nucleons present in the nucleus. This phenomenon causes the enhancement in σ_t^{hA}, i.e., T_A increases due to the SRC of the nucleon. Unlike the color transparency, the SRC is independent of the energy momentum of the hadron propagating through the nucleus.

The color transparency of the proton (pCT) is not found in both the $A(p, pp)$ and $A(e, e'p)$ reactions. The nuclear transparency for protons in the previous reaction was measured at Brookhaven National Laboratory (BNL) [9] in the beam momentum region $k_p = 5.9$–14.5 GeV/c. The measured spectra could not be reproduced by the results calculated using the pCT in the Glauber model [8]. The data can be understood by other mechanisms for the pp scattering in the nucleus. Brodsky and de Teramond [10] have considered two $J = S = L = 1$ dibaryon resonances to elucidate the oscillation in the energy dependence of the pp scattering data. Ralston and Pire [11] have described the measured transparency of the reaction by the oscillating color transparency, which arises due to two subprocesses in the perturbative quantum chromodynamics (pQCD). The proton transparency in the $A(e, e'p)$ reaction was measured at the Stanford Linear Accelerator Center (SLAC) [12] and Jefferson Laboratory (JLab) [13] for the photon virtuality $Q^2 = 0.64$–14.2 GeV^2. The data are well described by the short-range correlation of the bound nucleon included in the Glauber model calculation [14]. The data are also analyzed for $Q^2 < 10$ GeV^2 by the other calculations which do not consider pCT [15,16].

The data for the nuclear transparency of the meson are realized by the inclusion of the meson color transparency (mCT) in the calculations. The ρCT is reported in the experiment of the ρ°-meson electroproduction from nuclei [17]. There exist calculated results for the ρCT in the energy region available at JLab [18]. The nuclear transparency of the K^+ meson in the $A(e, e'K^+)$ reaction evaluated using the KCT in the Glauber model calculation [19] is in good agreement with the data reported from JLab for $Q^2 = 1.1$–3.0 GeV^2 [20]. The color transparency is also found in the nuclear diffractive dissociation of the pion (of 500 GeV/c) to dijets at Fermi National Accelerator Laboratory (FNAL) [21]. The πCT in the (π^-, l^+l^-) reaction on nuclei is estimated for $p_\pi = 5$–20 GeV/c [22], which can be measured at the forthcoming facilities in the Japan Proton Accelerator Research Complex (J-PARC) [23]. This reaction provides information complementary to that obtained from the $A(\gamma^*, \pi)$ reaction, i.e., the $A(e, e'\pi)$ reaction [24].

The nuclear transparency of the π^+ meson produced in the (e, e') reaction on nuclei was measured at JLab for $Q^2 = 1.1$–4.7 GeV^2 [24]. The data have been understood due to the inclusion of the πCT in the Glauber model calculation [14,22]. The πCT in the electronuclear reaction is also studied by Cosyn *et al.* [25] and Kaskulov *et al.* [26]. The calculated results due to them reproduce the data [24] in the energy region available at JLab. The quoted transparency has been measured to explore the πCT in the higher Q^2 region, i.e., up to $\approx$10 GeV^2 [6,27]. The data will be reported in future. The calculated results for it can be seen in Ref. [14]. Considering the πCT in this reaction, the dependence of the pion transparency on the momentum of the pion is also studied [28].

The meson is a quark-antiquark bound state, whereas the baryon is a composite state of three quarks. Therefore, it can be thought that the PLC formation of a two-quark system is more probable than that of a three-quark system. The transparency of the meson and the baryon can be studied simultaneously in the nuclear reaction. Miller and Strikman [29], considering both πCT and pCT in the Glauber transparencies for the hadrons, have shown large enhancement in the transparency of the $A(\pi, \pi p)$ reaction at the energy 200 GeV available at the CERN COMPASS experiment. Jain *et al.* [30], using pQCD-based two- and three-component models, calculate the cross sections which describe well the 90° data of the $\pi^-p \to \pi^-p$ (for the c.m. energy square $s = 4.36$–38.2 GeV^2) and $\gamma p \to \pi^+n$ (for $5.5 < s < 15$ GeV^2) reactions. They have predicted the oscillation in the color transparency in the $\pi^-A \to \pi^-p(A-1)$ and $\gamma A \to \pi^+n(A-1)$ reactions for $s > 6$ GeV^2. As mentioned earlier, the oscillating color transparency is also envisaged in the $A(p, pp)$ reaction [11]. In the recent past, the nuclear transparency of the $\gamma(n, p)\pi^-$ and $\gamma(n, p)\rho$ reactions in ^{4}He and ^{12}C nuclei were measured at JLab for large s [27]. The data of those reactions (not yet published) will be useful to confirm the prediction for the oscillatory phenomenon in the nuclear transparency.

The measured transparency of the $\gamma(n, p)\pi^-$ process in ^{4}He at low energy, i.e., $\sqrt{s} = 1.99$–2.95 GeV, is reported from JLab [31]. The data are plotted versus the four-momentum transfer 0.79–3.5 GeV^2. The experimental results in the quoted energy region do not show the oscillation in the transparency. The nuclear transparency of the $\gamma(n, p)\pi^-$ reaction in ^{4}He and ^{12}C nuclei is investigated using the Glauber model. The nuclear phenomena, i.e., Fermi motion and short-range correlation of the nucleon, are used to modify this model. The color transparency of the hadron is also incorporated in the modified Glauber model.

II. FORMALISM

The hadron produced in the nuclear reaction interacts with the bound nucleons while propagating through the nucleus. This process at high energy can be described by the Glauber model [1]. Using this model, the differential cross section of the elementary $\gamma(n, p)\pi^-$ reaction in a nucleus [32] can be written as

$$\frac{d\sigma(\gamma A)}{d\Delta^2} = \int d\mathbf{r}\varrho_n(\mathbf{r})P_{\pi^-}(\mathbf{r})P_p(\mathbf{r})\left\langle\frac{d\sigma_{\gamma n\to\pi^-p}}{d\Delta^2}(\sqrt{s})\right\rangle, \quad (1)$$

where Δ^2 is the spacelike four-momentum transfer in the elementary reaction. $\varrho_n(\mathbf{r})$ is the density of the neutron in the nucleus, normalized to the number of neutrons in the nucleus. $P_h(\mathbf{r})$ denotes the Glauber transparency of the hadron h (i.e., the survival probability of h) when it traverses through the nucleus.

The quantity $\langle\frac{d\sigma_{\gamma n\to\pi^-p}}{d\Delta^2}(\sqrt{s})\rangle$ represents the differential cross section of the elementary $\gamma n \to \pi^-p$ reaction in the nucleus. Because the bound nucleon possesses Fermi motion, it can be expressed as [32]

$$\left\langle\frac{d\sigma_{\gamma n\to\pi^-p}}{d\Delta^2}(\sqrt{s})\right\rangle = \iint d\mathbf{k}_i d\epsilon_i P(k_i, \epsilon_i)\frac{d\sigma_{\gamma n\to\pi^-p}}{d\Delta^2}(\sqrt{s}), \quad (2)$$

where $P(k_i, \epsilon_i)$ denotes the spectral function of the target nucleus, normalized to unity. It represents the probability of finding a nucleon of momentum $\mathbf{k}_i$ and binding energy ϵ_i in the nucleus [32,33]. $P(k_i, \epsilon_i)$ has been discussed elaborately in Ref. [34]. $\frac{d\sigma_{\gamma n\to\pi^-p}}{d\Delta^2}(\sqrt{s})$ is the cross section of the $\gamma n \to \pi^-p$ reaction in the free space occurring at the c.m. energy $\sqrt{s} = \sqrt{(E_\gamma + E_N)^2 + (\mathbf{k}_\gamma + \mathbf{k}_i)^2}$. The energy of the nucleon in the nucleus is $E_N = m_A - \sqrt{(-\mathbf{k}_i)^2 + (m_A - m_N + \epsilon_i)^2}$, where m_N and m_A are the masses of the nucleon (in the free state) and the nucleus, respectively.

The survival probability of the hadron, i.e., $P_h(\mathbf{r})$ in Eq. (1), propagating through the uncorrelated nucleons in the nucleus [35] is given by

$$P_h(\mathbf{r}) = \exp\left\{-\int_0^\infty dl\varrho(\mathbf{r} + \hat{\mathbf{k}}_h l)\sigma_t^{hN}\right\}, \quad (3)$$

where σ_t^{hN} is the hadron-nucleon total cross section in the free space. l is the distance in the nucleus traveled by the hadron h in the direction of its momentum $\hat{\mathbf{k}}_h$. ϱ is the density distribution, normalized to the mass number of the nucleus.

$P_h(\mathbf{r})$ increases because of the correlated nucleon [i.e., short-range correlation (SRC) of the nucleon] in the nucleus. Therefore, the hadron-nucleus cross section (as discussed earlier) increases due to the SRC of the nucleon. This occurs since the SRC of the nucleon in the nucleus modifies its density distribution ϱ in Eq. (3) [8] as

$$\varrho(\mathbf{r} + \hat{\mathbf{k}}_h l) \to \varrho(\mathbf{r} + \hat{\mathbf{k}}_h l)C(l), \quad (4)$$

where $C(l)$ represents the correlation function. Using the nuclear matter estimate, it is given by

$$C(l) = \left[1 - \frac{h(l)^2}{4}\right]^{1/2} [1 + f(l)], \tag{5}$$

with $h(l) = 3\frac{j_1(k_F l)}{k_F l}$ and $f(l) = -e^{-\alpha l^2}(1 - \beta l^2)$. The Fermi momentum k_F is chosen to be equal to 1.36 fm^{-1}. The value of $C(l)$ with the parameters $\alpha = 1.1\ \mathrm{fm}^{-2}$ and $\beta = 0.68\ \mathrm{fm}^{-2}$ agrees well that derived from the many-body calculations [8].

The hadron-nucleon cross section in a nucleus, as mentioned earlier, can be less than that in the free space due to the color transparency of the hadron (hCT). The reduction in the cross section causes the enhancement in $P_h(\mathbf{r})$ in Eq. (3), and hence the nuclear transparency of the hadron increases. To look for the hCT, σ_t^{hN} in $P_h(\mathbf{r})$ (according to quantum diffusion model [4,5]) has to be replaced by that in the nucleus, i.e., σ_t^{*hN}:

$$\sigma_t^{*hN} = \sigma_{t,\mathrm{CT}}^{hN}(\Delta^2, l) = \sigma_t^{hN}\left[\left\{\frac{l}{l_h} + \frac{n_q^2\langle k_t^2\rangle}{\Delta^2}\left(1 - \frac{l}{l_h}\right)\right\} \times \theta(l_h - l) + \theta(l - l_h)\right], \tag{6}$$

where $\sigma_{t,CT}^{hN}(\Delta^2, l)$ refers to σ_t^{*hN} due to the hCT, and n_q denotes the number of the valence quarks or quark-antiquarks present in the hadron, e.g., $n_q = 2(3)$ for pion (proton) [4]. $\langle k_t^2\rangle^{1/2} (= 0.35\ \mathrm{GeV}/c)$ illustrates the transverse momentum of the (anti)quark. l is that defined in Eq. (3). As illustrated earlier, l_h represents the hadron formation-length which is expressed [22] as

$$l_h = \frac{2k_h}{\Delta M^2}, \tag{7}$$

where k_h is the momentum of the hadron in the laboratory frame. ΔM^2 is related to the mass difference of the hadronic states originating due to the fluctuation of the (anti)quark in the PLC of the hadron [4,5].

III. RESULTS AND DISCUSSION

The transparency of the hadrons produced in the $\gamma(n, p)\pi^-$ process in nuclei has been calculated. The data for the ^{4}He nucleus are reported by Dutta *et al.* [31] in the four-momentum transfer region 0.79–3.5 GeV^2. They have extracted transparency from the measured and Monte Carlo yields from ^{4}He and d (deuteron) nuclei, using the relation [31]

$$T(^4\mathrm{He}) = \frac{\frac{\mathrm{Yield}_{\mathrm{Data}}(^4\mathrm{He})}{\mathrm{Yield}_{\mathrm{MonteCarlo}}(^4\mathrm{He})}}{\frac{\mathrm{Yield}_{\mathrm{Data}}(d)}{\mathrm{Yield}_{\mathrm{MonteCarlo}}(d)}} T(d). \tag{8}$$

The ratio removes the uncertainties in the results. The transparency for d, i.e., $T(d)$, is obtained from the measured proton transparency in the $d(e, e'p)$ reaction and the π^- meson transparency in the deuteron nucleus. The value of $T(d)$ is around 0.8, as tabulated in Ref. [31].

The cross section of the $\gamma(n, p)\pi^-$ reaction in the nucleus is calculated using Eq. (1) to estimate the transparency of the reaction. Following Eq. (8), the quoted transparency is

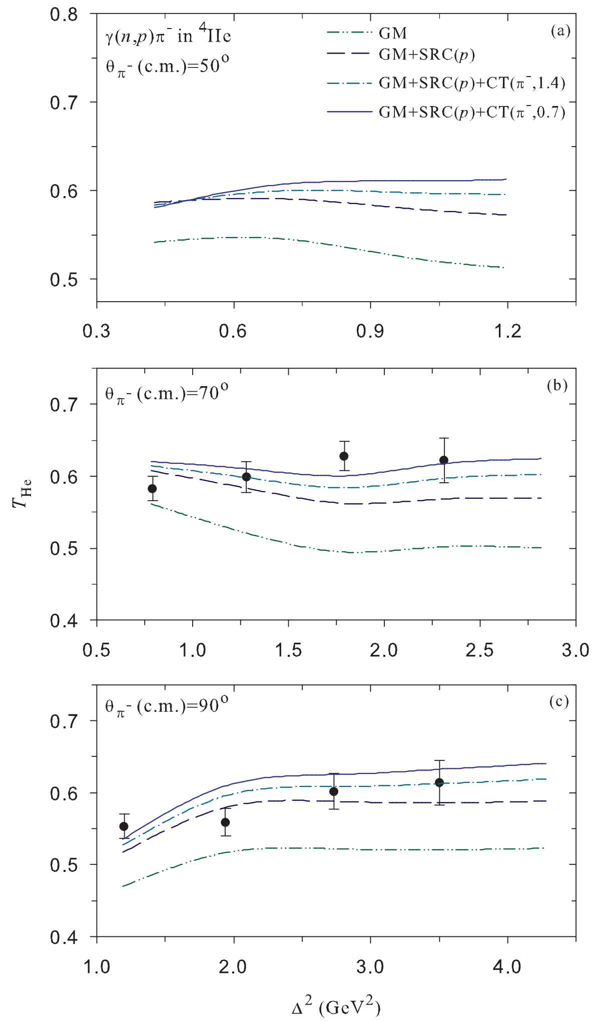

FIG. 1. The transparency T_{He} of the pion and the proton in ^{4}He vs the four-momentum transfer Δ^2. The data are taken from Ref. [31]. The curves appearing in the figure are explained in the text.

calculated as

$$T(A) = \frac{\frac{d\sigma(\gamma A)/d\Delta^2}{d\sigma(\gamma A)/d\Delta^2_{\mathrm{PWIA}}}}{\frac{d\sigma(\gamma d)/d\Delta^2}{d\sigma(\gamma d)/d\Delta^2_{\mathrm{PWIA}}}} T(d). \tag{9}$$

The suffix PWIA stands for the plane wave impulse approximation where the final state interactions of the hadrons are neglected.

The differential cross sections for the four-momentum transfer distribution of the $\gamma(n, p)\pi^-$ reaction are measured by Zhu *et al.* [36] at θ_{π^-} (c.m.) = 50°, 70°, and 90°. Those cross sections are used in Eq. (2) to calculate the transparency of the $\gamma(n, p)\pi^-$ reaction in the nucleus. The measured total cross sections $\sigma_t^{\pi^- N}$ and σ_t^{pN} [37] are used to evaluate the survival probabilities [in Eq. (3)] for the pion and the proton, respectively. The density distribution of the deuteron is generated using the Hulthèn wave function [38], whereas that for other nuclei (as reported from the electron scattering experiment) is taken from Ref. [39].

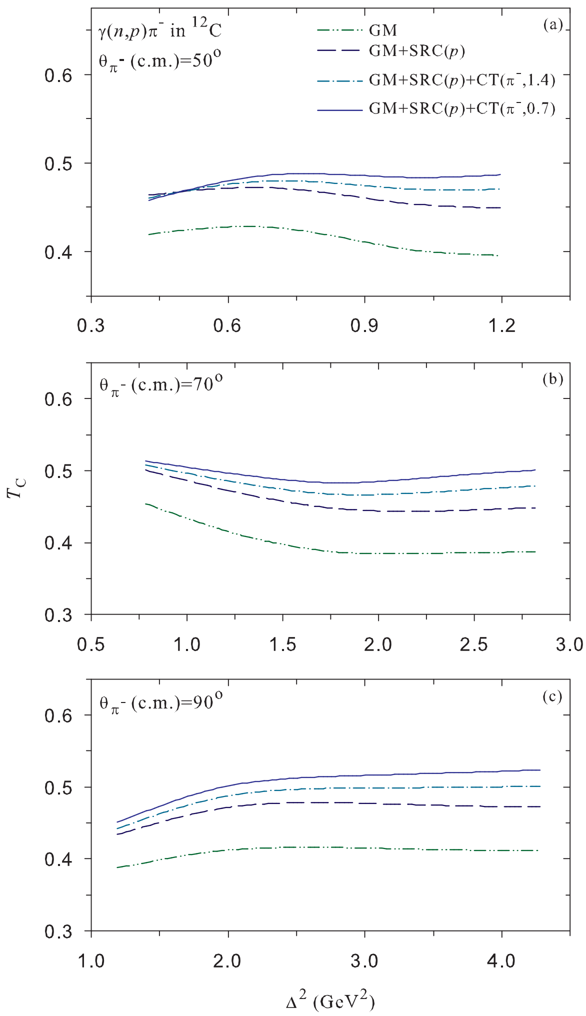

FIG. 2. Same as those described in Fig. 1, but for the transparency T_{C} presented for the ^{12}C nucleus.

As described earlier, the nuclear transparency of the proton measured in the $A(e, e'p)$ reaction for the wide range of the four-momentum transfer [12,13] does not show the color transparency (CT) of the proton. The data are reproduced by considering the short-range correlation (SRC) of the nucleon, as described in Eq. (5), in the Glauber model calculation [14] (also see the references therein). Therefore, the SRC (but not the CT) is included in the Glauber model to estimate the survival probability of the proton, i.e., $P_p(\mathbf{r})$ in Eq. (3). The measured pionic transparency in the $A(e, e'\pi^+)$ reaction [24] is reproduced well due to the incorporation of the pion color transparency πCT, as illustrated in Eq. (6), in the Glauber model calculation [14,22]. Therefore, the πCT in the Glauber model is used to evaluate the survival probability of the pion [see $P_{\pi^-}(\mathbf{r})$ in Eq. (3)].

The calculated nuclear transparency for the $\gamma(n, p)\pi^-$ reaction in the ^{4}He nucleus, denoted by T_{He}, is shown in Fig. 1 along with the data for $\theta_{\pi^-}(\mathrm{c.m.}) = 70^\circ$ and 90° [31]. The dot-dot-dashed curve (labeled as GM) arises due to the Glauber model (GM) calculation. The long-dashed curve [labeled as GM + SRC(p)] describes the transparency evaluated using the SRC of the bound nucleon in the Glauber model calculation for the proton survival probability $P_p(\mathbf{r})$ in Eq. (3). The dot-dashed curve [labeled as GM + SRC(p) + CT(π^-, 1.4)] represents the calculated results, where the SRC is incorporated in the Glauber model to estimate $P_p(\mathbf{r})$, and πCT with $\Delta M^2 = 1.4$ GeV^2 [defined in Eq. (7)] is included in the Glauber model to evaluate $P_{\pi^-}(\mathbf{r})$ in Eq. (3). The solid curve [labeled as GM + SRC(p) + CT(π^-, 0.7)] illustrates that of the dot-dashed curve except ΔM^2 is taken equal to 0.7 GeV^2.

The transparency of the $\gamma(n, p)\pi^-$ reaction in the ^{12}C nucleus, denoted by T_{C}, has been calculated using the (modified) Glauber model, as that is done for the ^{4}He nucleus. As shown in Fig. 2, the calculated T_{C} are qualititavely similar to T_{He} presented in Fig. 1. The magnitude of T_{C} is less than that of T_{He}, since the survival probability reduces for the hadrons propagating through the large nucleus.

IV. CONCLUSIONS

The nuclear transparency of the hadrons produced in the $\gamma(n, p)\pi^-$ reaction has been calculated for ^{4}He and ^{12}C nuclei using the Glauber model where the Fermi motion of the nucleon in the nucleus is incorporated. The calculated results underestimate the data reported for the ^{4}He nucleus. Therefore, the Glauber model is further modified by including the short-range correlation of the nucleon to evaluate the proton survival probability and the color transparency to determine the pion survival probability. Those modifications are done based on the earlier studies. The calculated results are compared with the data reported for the ^{4}He nucleus in the four-momentum transfer region 0.79–3.5 GeV^2, which shows more data are required to conclude the color transparency of the pion. The ongoing analysis of the experimental results for ^{4}He and ^{12}C nuclei for the wide range of the four-momentum transfer may confirm this issue in the future.

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