The $\pi^- p \to a_0^- \eta p$ reaction in an effective Lagrangian model

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In this work, we perform a study of the $\pi^- p \to a_0^- \eta p$ reaction within an effective Lagrangian approach and isobar model. In our model, we consider the excitation of the N^* and Δ^* resonances in the intermediate states and background terms. Based on our model, it is found that this reaction is dominated by the production of the $N(1535)$ in the near threshold region. And the contributions from other well-established N^* or Δ^* resonances only play minor roles. Besides, we discuss the possibility of verifying the existence of a narrow nucleon resonance $N(1685)/N(1700)$ proposed by some recent works in the present reaction. With adopting different quantum numbers, we find that it may contribute significantly in this reaction. Thus this reaction may provide a good platform to study the properties of nucleon resonances, e.g., $N(1535)$ and $N(1685)/N(1700)$, which couple strongly with the ηN channel. To compare with future experiments, the predictions of the total cross sections, the angular distributions, and invariant mass spectrums of final particles are also presented. Furthermore, the off-shell effects due to high-spin states are also discussed.

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I. INTRODUCTION

The studies of the properties of nucleon resonances have always been an interesting topic in hadronic physics, which offers important information about the properties of quantum chromodynamics (QCD) in the nonperturbative regime and test our understanding of strong interactions. Most of our current knowledge of nucleon resonances has come from πN scattering data [1–3]. While, due to the rapid experimental progress on the electron and photon induced reactions, much more accurate and high statistic data on nucleon resonances in these processes were accumulated in recent years [4–11], which have significantly advanced our understanding of the properties of nucleon resonances [12–19]. At the same time, current πN scattering data were obtained almost more than 30 years ago [20,21], which usually have large uncertainties and low statistics. As discussed in Ref. [22], such data may cause systematical uncertainties on the results of modern energy dependent partial wave analysis on the meson production processes in πN and γN scatterings. Here, it is worth noting that in Ref. [22] the authors mainly concentrated on the single meson production processes. While, as we know multimeson production processes in πN scatterings may also provide important information about nucleon resonances. In fact, both the analysis of the old πN scattering data and recent studies in photon induced reactions have shown that the studies of multimeson production processes can also offer interesting and important opportunities to study the properties of resonances [17–19,22]. Therefore, high quality πN scattering data in both the single meson and multimeson production processes are helpful for improving our knowledge of baryon resonances.

Till now, due to the poor quality of the available data theoretical studies on multimeson production processes in πN scatterings are still rather limited. Therefore, it is desirable to further explore the potential of such processes in studying baryon resonances, and such studies may also offer motivations for experimental studies at next generation of pion beam facility. In fact, due to special reaction mechanisms some multimeson production processes may have special advantages in studying the properties of nucleon resonances, which are usually difficult to be investigated in single meson production process. In this work, we shall investigate the $\pi^- p \to a_0^- \eta p$ reaction within an effective Lagrangian approach and isobar model. In this reaction, nucleon and delta resonances may be excited as intermediate states and decay to the ηp or $a_0 p$ channels. Due to the strong $a_0 \pi \eta$ coupling it is expected that this reaction may offer a good place to study the nucleon and delta resonances which have strong coupling to $N\eta$ or Na_0 channels [23]. For well established nucleon resonances, we consider the $N(1535)$, $N(1650)$, and $N(1710)$ in the intermediate states and find the $N(1535)$ gives dominant contribution in the near threshold region. For the Δ resonance, we only consider the possible contribution from the $\Delta(1920)$ in the intermediate state due to the rather poor knowledge of the coupling of Δ resonances with the $a_0 p$ channel [24]. Based on our calculations, it is found that the $\Delta(1920)$ contribution is negligible compared to the contribution of the nucleon resonances. Therefore, this reaction offers a good place to study the properties of nucleon resonances having large coupling with the $N\eta$ channel.

Another motivation of this work is to discuss the possibility of looking for the signal of a new nucleon resonance proposed

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in some recent works. In Ref. [25], Kuznetsov *et al.* reported a narrow structure ($\Gamma \approx 10$ MeV) in the distribution of the invariant mass $M_{\eta N}$ at 1678 MeV [denoted as $N(1685)$] in the $\gamma N \to N\pi\eta$ reaction. However, such a structure was not confirmed by a series of following works [24,26,27]. Very recently, the CBELSA/TAPS collaboration [28] reported their analysis on the same reaction as in Ref. [25]. They concluded that the $N(1685)$ was not observed either. Instead, they reported a new structure in the $M_{\eta N}$ distribution near 1700 MeV [denoted as $N(1700)$] with a width of $\Gamma \approx 50$ MeV. In their analysis, they suggested that this structure was most likely caused by the triangle singularity mechanism. However, the resonance production scenario is still not excluded. Due to its narrow width, this structure is also unlikely induced by the production of the well-established nucleon resonance $N(1710)$. Regarding that the disputes about the new structure are still unsettled and all the relevant experiments are done in the photoproduction processes, further investigations on the nature of the structure in some other reactions should be helpful. Here, we suggest that the $\pi^- p \to a_0^- \eta p$ reaction may offer a good place to test various scenarios of the newly observed structure. On the one hand, because the kinematic conditions for the triangle singularity mechanism are not satisfied in this reaction, if this scenario is correct the structure in the $M_{p\eta}$ spectrum should disappear, which can be verified by future experiment. On the other hand, if the new state indeed exists, since it was firstly found in its decay to $N\eta$ channel, it is natural to expect that it may have a relatively large coupling with the $N\eta$ channel. Then the new state N_X^*, which represents $N(1685)$ or $N(1700)$ hereafter for convenience, should also be observed in the $\pi^- p \to a_0^- \eta p$ reaction since this reaction is suitable for studying the N^*s having strong coupling with the $N\eta$ channel. Therefore, in this work we calculate the possible contribution from the new state by considering various quantum numbers and estimate its effects on the invariant mass spectrum and angular distributions of final particles, which should be helpful for future experimental studies on this reaction.

This paper is organized as follows. In Sec. II, the theoretical framework and amplitudes are presented for the reaction $\pi^- p \to a_0^- \eta p$. In Sec. III, the numerical results are presented with some discussions. Finally, this paper ends with a short summary in Sec. IV.

II. MODEL AND INGREDIENTS

In the present work, we study the $\pi^- p \to a_0^- \eta p$ reaction within an effective Lagrangian approach and isobar model. The Feynman diagrams considered in this work are depicted in Fig. 1. Here we consider the excitation of the $N(1535)$, $N(1650)$, $N(1710)$, and a possible new state N_X^* in the intermediate states, which subsequently decay into the final $N\eta$. Due to their relatively large coupling to the $N\eta$ channel, we expect these nucleon resonances may give significant contributions in the reaction. For the t-channel excitation of the intermediate nucleon resonances (Fig. 1(a)), we only consider the η meson exchange because of the strong $a_0\pi\eta$ coupling [23]. In addition to the contributions from the t-channel excitation of the nucleon resonances, there are also

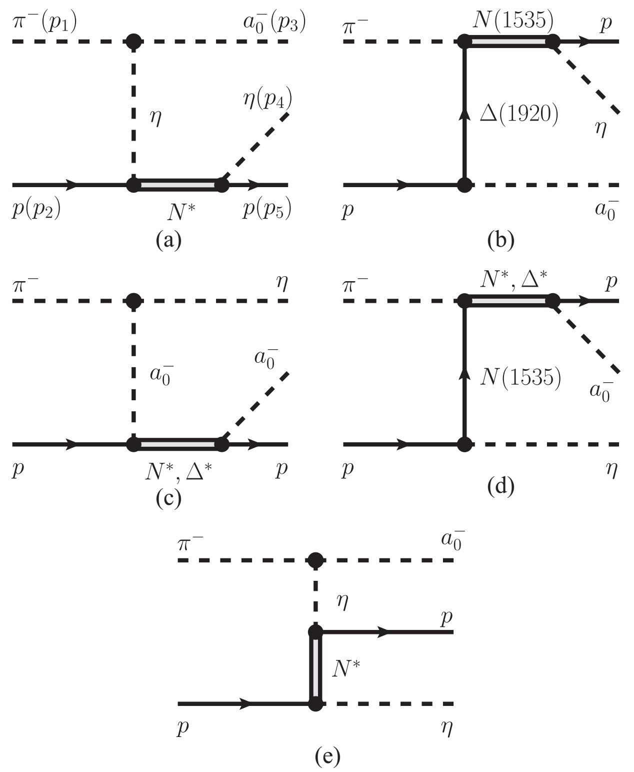

FIG. 1. Feynman diagrams for the $\pi^- p \to a_0^- \eta p$ reaction. The intermediate state N^* for (a) and (e) denotes the $N(1535)$, $N(1650)$, and $N(1710)$. The N^* and Δ^* for (c) and (d) denote the $N(1880)$ and $\Delta(1920)$, respectively.

some other processes may contribute. For the u-channel process (Fig. 1(b)), we find that $\Delta(1920)$ may couple to $a_0 p$ and $N(1535)\pi$ as suggested in the Particle Data Group (PDG) book [29]. On the other hand, since the center of mass energy at threshold is around 2.5 GeV, we ignore the s-channel resonance excitation processes.

In the $a_0\eta$ channel, we only find the experimental evidence for the coupling of $\pi(1800)$ with the $a_0\eta$ channel [29]. However, the present knowledge about this resonance still have large uncertainties and the information of its coupling to various channels is still absent. Therefore, we ignore the resonance contribution in the $a_0\eta$ channel. In the $a_0 p$ channel, we find the excitation of the $N(1880)$ and $\Delta(1920)$ in intermediate states may contribute. While, since the $\Delta(1920)N(1535)\pi$ and $\Delta(1920)Na_0$ couplings are rather weak, one can expect that the contribution from the $\Delta(1920)$ is small. Finally, the background contribution is modeled by the Feynman diagram shown in Fig. 1(e) in this work.

With the discussions presented above, the effective Lagrangian densities needed in this work can be given as [17,18,23]

$$\mathcal{L}_{a_0\eta\pi} = g_{a_0\eta\pi}\vec{a}_0\vec{\pi}\eta, \tag{1}$$

$$\mathcal{L}_{N(1535)N\eta} = ig_{N^*N\eta}\bar{N}^*N\eta + \text{h.c.}, \tag{2}$$

$$\mathcal{L}_{N(1650)N\eta} = ig_{N^*N\eta}\bar{N}^*N\eta + \text{h.c.}, \tag{3}$$

$$\mathcal{L}_{N(1710)N\eta} = -\frac{g_{N^*N\eta}}{m_{N^*}+m_N}\bar{N}^*\gamma_5\gamma_\mu N\partial^\mu\eta + \text{h.c.}, \tag{4}$$

TABLE I. Coupling constants used in this work. The experimental decay widths are taken from PDG book [29].

State	Width (MeV)	Decay channel	Adopted branching ratio	$g^2/4\pi$
$a_0(980)$	100	$\eta\pi$	0.84	0.51
$N(1535)$	150	$N\eta$	0.42	0.28
$N(1650)$	125	$N\eta$	0.25	7.63×10^{-2}
$N(1710)$	140	$N\eta$	0.30	2.03
$N(1880)$	300	Na_0	0.03	$2.58\times10^{-2*}$[a]
		$N(1535)\pi$	0.08	1.67×10^{-2}
$\Delta(1920)$	300	Na_0	$0.01^{\dagger}$[b]	1.03^*
		$N(1535)\pi$	$0.02^{\dagger}$	2.42×10^{-3}

[a]Values of $g^2/4\pi$ with an asterisk mean that they are obtained considering the finite width effect of the mother particle.
[b]Values of branching ratio with a dagger mean the values are obtained with taking a upper limit value suggested by the PDG due to the absence of more accurate data.

$$\mathcal{L}_{N(1880)Na_0} = g_{N^*Na_0}\bar{N}^*(\vec{\tau}\vec{a}_0)N + \text{h.c.}, \tag{5}$$

$$\mathcal{L}_{N(1880)N(1535)\pi} = g_{N^*N(1535)\pi}\bar{N}^*(\vec{\tau}\vec{\pi})N(1535) + \text{h.c.}, \tag{6}$$

$$\mathcal{L}_{\Delta(1920)Na_0} = \frac{g_{\Delta^*Na_0}}{m_{a_0}}\bar{\Delta}^{*\mu}\Theta_{\mu\nu}(Z)\gamma_5(\vec{\tau}\partial^\nu\vec{a}_0)N + \text{h.c.}, \tag{7}$$

$$\mathcal{L}_{\Delta(1920)N(1535)\pi} = \frac{g_{\Delta^*N^*\pi}}{m_\pi}\bar{\Delta}^{*\mu}\Theta_{\mu\nu}(Z)\gamma_5(\vec{\tau}\partial^\nu\vec{\pi})N^* + \text{h.c.} \tag{8}$$

Here, for the spin 3/2 particle $\Delta(1920)$ we have introduced the off-shell parameter Z in the Lagrangian to include the off-shell effect of high-spin particles [30,31]. And the $\Theta_{\mu\nu}(Z)$ is defined as

$$\Theta_{\mu\nu}(Z) = g_{\mu\nu} - \left(Z + \tfrac{1}{2}\right)\gamma_\mu\gamma_\nu, \tag{9}$$

where the off-shell parameter Z is in principle arbitrary and should be determined by fitting experiment data. On the other hand, the coupling constants in Lagrangian densities can be determined from the partial decay widths (see Table I) through the following formulas:

$$\Gamma[a_0 \to \eta\pi] = \frac{g^2_{a_0\eta\pi}}{8\pi}\frac{|\vec{p}|}{m^2_{a_0}}, \tag{10}$$

$$\Gamma[N(1535) \to N\eta] = \frac{g^2_{N^*N\eta}}{4\pi}\frac{(E_N + m_N)}{m_{N^*}}|\vec{p}|, \tag{11}$$

$$\Gamma[N(1650) \to N\eta] = \frac{g^2_{N^*N\eta}}{4\pi}\frac{(E_N + m_N)}{m_{N^*}}|\vec{p}|, \tag{12}$$

$$\Gamma[N(1710) \to N\eta] = \frac{g^2_{N^*N\eta}}{4\pi}\frac{(E_N - m_N)}{m_{N^*}}|\vec{p}|, \tag{13}$$

$$\Gamma[N(1880) \to Na_0] = \frac{3g^2_{N^*Na_0}}{4\pi}\frac{(E_N + m_N)}{m_{N^*}}|\vec{p}|, \tag{14}$$

$$\Gamma[N(1880) \to N(1535)\pi] = \frac{3g^2_{N^*N(1535)\pi}}{4\pi}|\vec{p}|\frac{(E_{N(1535)}+m_{N(1535)})}{m_{N^*}}, \tag{15}$$

$$\Gamma[\Delta(1920) \to Na_0] = \frac{g^2_{\Delta^*Na_0}}{4\pi}\frac{(E_N - m_N)}{m_{\Delta^*}m^2_{a_0}}|\vec{p}|^3, \tag{16}$$

$$\Gamma[\Delta(1920) \to N(1535)\pi] = \frac{g^2_{\Delta^*N(1535)\pi}}{4\pi}|\vec{p}|^3\frac{(E_{N(1535)}-m_{N(1535)})}{m_{\Delta^*}m^2_\pi}, \tag{17}$$

where p denotes the magnitude of the momentum of final particles in the center-of-mass frame. For the $N(1880)$ and $\Delta(1920)$ decaying to the Na_0 channel, due to their mass lying very close to the Na_0 threshold it is necessary to take into account their finite width in the calculations. For example, for the $N(1880)Na_0$ vertex we shall use the following formula to include the finite width effect [19,32,33]:

$$\Gamma_{N^*\to Na_0} = -\frac{1}{\pi}\int_{(M_{N^*}-2\Gamma_{N^*})^2}^{(M_{N^*}+2\Gamma_{N^*})^2}\Gamma_{N^*\to Na_0}(\sqrt{s}) \times \Theta\left(\sqrt{s} - M_N - M_{a_0}\right) \times \text{Im}\left\{\frac{1}{s - M^2_{N^*} + iM_{N^*}\Gamma_{N^*}}\right\}ds. \tag{18}$$

The obtained coupling constants are listed in Table I.

To take into account the internal structure of hadrons, we have introduced form factors in the calculations. In this work, the form factors for intermediate meson and baryon are taken as [17,18,34,35]

$$F_M(q_{ex}, m_{ex}) = \left(\frac{\Lambda^2_M - m^2_{ex}}{\Lambda^2_M - q^2_{ex}}\right)^2, \tag{19}$$

$$F_B(q_{ex}, m_{ex}) = \left(\frac{\Lambda^4_M}{\Lambda^4_M + \left(q^2_{ex} - m^2_{ex}\right)^2}\right)^2, \tag{20}$$

where q_{ex} and m_{ex} are the momentum and mass of the exchanged particles. Here, we take $\Lambda_\eta = 1.3$ GeV [17] and $\Lambda_{N^*} = \Lambda_{\Delta^*} = 0.9$ GeV [34,35], respectively. The propagators for the exchanged particles are adopted as below:

$$G_0(q) = \frac{i}{q^2 - m^2} \tag{21}$$

for η,

$$G_{\frac{1}{2}}(q) = \frac{i(\not{q} + m)}{q^2 - m^2 + im\Gamma} \tag{22}$$

for N^* with spin being 1/2, and

$$G^{\mu\nu}_{\frac{3}{2}}(q) = \frac{i(\not{q} + m)}{q^2 - m^2 + im\Gamma}\left[-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3m}(\gamma^\mu q^\nu - \gamma^\nu q^\mu) + \frac{2}{3m^2}q^\mu q^\nu\right] \tag{23}$$

for $\Delta(1920)$ or N^* with spin being 3/2 [36,37], where q, m, and Γ are the four-momentum, mass, and width of the exchanged particle.

With the ingredients presented above, the amplitudes for the $\pi^- p \to a_0^- \eta p$ reaction shown in Fig. 1 can be obtained in a standard way, and we get

$$\mathcal{M}^a_{N^*_{\frac{1}{2}^-}} = -g_{a_0\eta\pi} g^2_{N^*N\eta} \bar{u}(p_5, s_5) G_{\frac{1}{2}}(q_{N^*}) u(p_2, s_2) \times G_0(q_\eta) F_B(q_{N^*}, m_{N^*}) F_M(q_\eta, m_\eta), \quad (24)$$

$$\mathcal{M}^a_{N^*_{\frac{1}{2}^+}} = \frac{g_{a_0\eta\pi} g^2_{N^*N\eta}}{(m_{N^*}+m_N)^2} \bar{u}(p_5, s_5)\gamma_5 \not{p}_4 G_{\frac{1}{2}}(q_{N^*})\gamma_5 \not{q}_\eta \times u(p_2, s_2) G_0(q_\eta) F_B(q_{N^*}, m_{N^*}) \times F_M(q_\eta, m_\eta), \quad (25)$$

$$\mathcal{M}^b_{N(1535)} = i\frac{2g_{\Delta^* N a_0} g_{\Delta^* N\pi} g_{N^* N\eta}}{m_\pi m_{a_0}} \bar{u}(p_5, s_5) G_{\frac{1}{2}}(q_{N^*}) \times \gamma_5 p_{1\mu} G^{\mu\nu}_{\frac{3}{2}}(q_{\Delta^*}) p_{3\nu}\gamma_5 u(p_2, s_2) \times F_B(q_{N^*}, m_{N^*}) F_B(q_{\Delta^*}, m_{\Delta^*}), \quad (26)$$

$$\mathcal{M}^c_{N(1880)} = 2g_{a_0\eta\pi} g^2_{N^*Na_0} \bar{u}(p_5, s_5) G_{\frac{1}{2}}(q_{N^*}) u(p_2, s_2) \times G_0(q_{a_0}) F_B(q_{N^*}, m_{N^*}) F_M(q_{a_0}, m_{a_0}), \quad (27)$$

$$\mathcal{M}^c_{\Delta(1920)} = \frac{2g_{a_0\eta\pi} g^2_{\Delta^* N a_0}}{m^2_{a_0}} \bar{u}(p_5, s_5)\gamma_5 p_3^\nu \Theta_{\nu\mu}(Z) \times G^{\mu\alpha}_{\frac{3}{2}}(q_{\Delta^*})\Theta_{\alpha\beta}(Z) q^\beta_{a_0}\gamma_5 u(p_2, s_2) \times G_0(q_{a_0}) F_M(q_{a_0}, m_{a_0}) F_B(q_{\Delta^*}, m_{\Delta^*}), \quad (28)$$

$$\mathcal{M}^d_{N(1880)} = i2g_{N(1535)N\eta} g_{N^* N(1535)\pi} g_{N^* N a_0} \bar{u}(p_5, s_5) \times G_{\frac{1}{2}}(q_{N^*}) G_{\frac{1}{2}}(q_{N(1535)}) u(p_2, s_2) \times F_B(q_{N^*}, m_{N^*}) F_B(q_{N(1535)}, m_{N(1535)}), \quad (29)$$

$$\mathcal{M}^d_{\Delta(1920)} = \frac{i2g_{N^*N\eta} g_{\Delta^* N^*\pi} g_{\Delta^* N a_0}}{m_{a_0} m_\pi} \bar{u}(p_5, s_5)\gamma_5 p_3^\nu \Theta_{\nu\mu}(Z) \times G^{\mu\alpha}_{\frac{3}{2}}(q_{\Delta^*})\Theta_{\alpha\beta}(Z) p_1^\beta \gamma_5 G_{\frac{1}{2}}(q_{N^*}) u(p_2, s_2) \times F_B(q_{\Delta^*}, m_{\Delta^*}) F_B(q_{N^*}, m_{N^*}), \quad (30)$$

$$\mathcal{M}^e = -g_{a_0\eta\pi} g^2_{N^*N\eta} \bar{u}(p_5, s_5) G_{\frac{1}{2}}(q_{N^*}) u(p_2, s_2) \times G_0(q_\eta) F_B(q_{N^*}, m_{N^*}) F_M(q_\eta, m_\eta), \quad (31)$$

where p_i and s_i represent the four-momentum and helicity of individual particles as denoted in Fig. 1, respectively.

As discussed in the Introduction, one motivation of the present work is to discuss the possibility to verify the existence of a new state in the present reaction. In Ref. [25], the mass and width of this state were obtained as $M = 1678 \pm 0.8 \pm 10$ MeV and $\Gamma \sim 10$ MeV. However, in Ref. [28] a structure around 1.7 GeV with a width $\Gamma \approx 50$ MeV was found instead. In the following calculations, we shall consider both these two scenarios from Refs. [25,28]. In addition, since this state was first found in the $N\eta$ decay channel, one may expect that it should have a relatively large coupling to $N\eta$ channel. As the properties of this state, e.g., its decay branch ratios and quantum numbers, are still not well known, we just assume the $Br(N^*_X \to N\eta)$ is 0.3 in this work. At the same time, various assignments of the quantum numbers of N^*_X, i.e., $J^P = 1/2^\pm$ or $3/2^\pm$, will be considered. Then the corresponding Lagrangian densities for the $N^*_X N\eta$ vertex can be adopted as

TABLE II. Coupling constants for the $N^*_X N\eta$ vertex with adopting various quantum numbers. Here, the branching ratio of $N^*_X \to N\eta$ is taken as $Br(N^*_X \to N\eta) = 0.3$.

J^P	$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{3}{2}^+$
$g^2_{N(1685)N\eta}/4\pi$	6.81×10^{-3}	0.18	1.13	4.27×10^{-2}
$g^2_{N(1700)N\eta}/4\pi$	3.24×10^{-2}	0.77	4.30	0.18

$$\mathcal{L}^{\frac{1}{2}^-}_{N^*_X N\eta} = ig_{N^*_X N\eta} \bar{N}^*_X \eta N + \text{h.c.}, \quad (32)$$

$$\mathcal{L}^{\frac{1}{2}^+}_{N^*_X N\eta} = -\frac{g_{N^*_X N\eta}}{m_N + m_{N^*_X}} \bar{N}^*_X \gamma_5\gamma_\mu \partial^\mu \eta N + \text{h.c.}, \quad (33)$$

$$\mathcal{L}^{\frac{3}{2}^-}_{N^*_X N\eta} = \frac{g_{N^*_X N\eta}}{m_\eta} \bar{N}^{*\mu}_X \Theta_{\mu\nu}(Z)\gamma_5 \partial^\nu \eta N + \text{h.c.}, \quad (34)$$

$$\mathcal{L}^{\frac{3}{2}^+}_{N^*_X N\eta} = \frac{g_{N^*_X N\eta}}{m_\eta} \bar{N}^{*\mu}_X \Theta_{\mu\nu}(Z) \partial^\nu \eta N + \text{h.c.} \quad (35)$$

Here, for the cases $J^P = 3/2^\pm$ we have also introduced the off-shell parameter Z in the Lagrangian densities. The coupling constants can be obtained in the same way as above through the following formulas:

$$\Gamma[N^*_{X,\frac{1}{2}^\pm} \to N\eta] = \frac{g^2_{N^*_X N\eta}}{4\pi} \frac{(E_N \mp m_N)}{m_{N^*_X}} |\vec{p}|, \quad (36)$$

$$\Gamma[N^*_{X,\frac{3}{2}^\pm} \to N\eta] = \frac{g^2_{N^*_X N\eta}}{12\pi} \frac{(E_N \pm m_N)}{m_{N^*_X} m^2_\eta} |\vec{p}|^3. \quad (37)$$

The obtained coupling constants for the $N^*_X N\eta$ vertex with different quantum numbers are summarized in Table II.

The excitation of the N^*_X state is taken into account by considering the Fig. 1(a) process. Here, we present the corresponding amplitudes for the $J^P = 3/2^\pm$ cases as an example,

$$\mathcal{M}^t_{N^*_{X,\frac{3}{2}^-}} = \frac{g_{a_0\eta\pi} g^2_{N^*_X N\eta}}{m^2_\eta} \bar{u}(p_5, s_5)\gamma_5 p_4^\nu \Theta_{\nu\mu}(Z) \times G^{\mu\alpha}_{\frac{3}{2}}(q_{N^*_X})\Theta_{\alpha\beta}(Z) q^\beta_\eta \gamma_5 u(p_2, s_2) G_0(q_\eta) \times F_B(q_{N^*_X}, m_{N^*_X}) F_M(q_\eta, m_\eta), \quad (38)$$

$$\mathcal{M}^t_{N^*_{X,\frac{3}{2}^+}} = \frac{g_{a_0\eta\pi} g^2_{N^*_X N\eta}}{m^2_\eta} \bar{u}(p_5, s_5) p_4^\nu \Theta_{\nu\mu}(Z) \times G^{\mu\alpha}_{\frac{3}{2}}(q_{N^*_X})\Theta_{\alpha\beta}(Z) q^\beta_\eta u(p_2, s_2) G_0(q_\eta) \times F_B(q_{N^*_X}, m_{N^*_X}) F_M(q_\eta, m_\eta). \quad (39)$$

The total amplitude $\mathcal{M}$ is obtained by the summation of the individual amplitudes. With the total amplitude and the appropriate formalism for describing the off-shell effects, various observables and the off-shell effects of high-spin states can be investigated and compared with data once the experimental data are available in the future. The differential and total cross

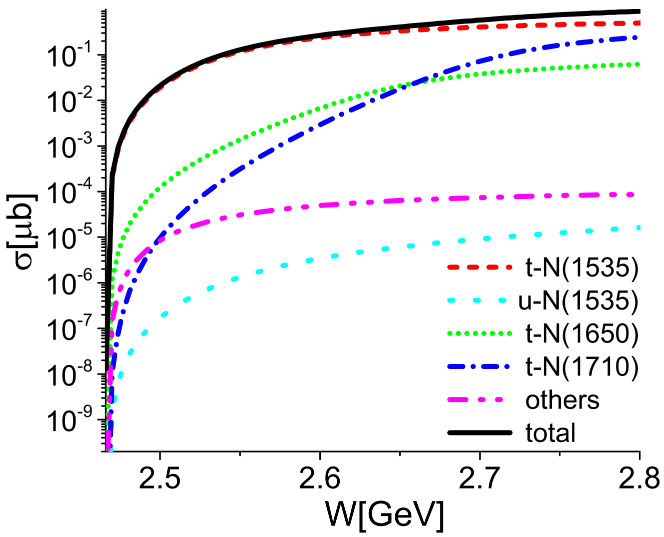

FIG. 2. Total cross section vs center of mass energy W for the $\pi^- p \to a_0^- \eta p$ reaction. The red dashed, cyan long dashed, green dotted, and blue dash-dotted lines denote the contributions from t-channel $N(1535)$, u-channel $N(1535)$, t-channel $N(1650)$, t-channel $N(1710)$ excitations, respectively. The contributions from other processes and the total contributions are represented by the magenta dash-dotted-dotted and black solid lines, respectively.

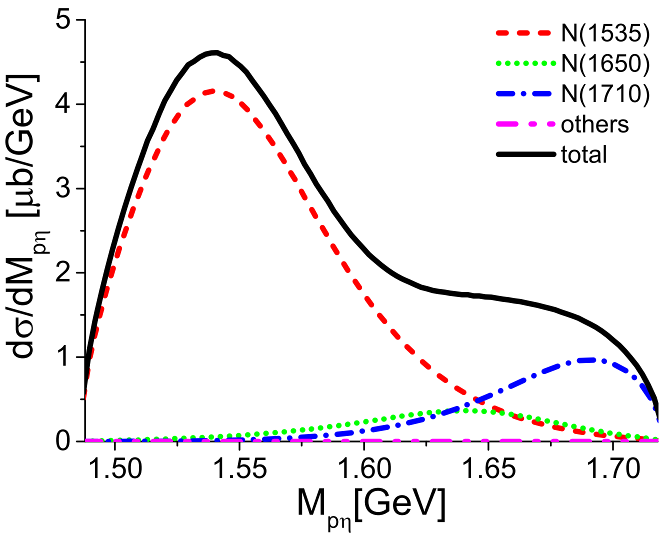

FIG. 3. Invariant mass spectrum of the final $p\eta$ pair at $W = 2.7$ GeV. The notations are same as Fig. 2.

sections for this reaction can be calculated through

$$d\sigma = \frac{1}{4}\frac{m_N^2}{\sqrt{(p_1 p_2)^2 - m_\pi^2 m_N^2}}\frac{1}{(2\pi)^5}\sum_{s_2,s_5}|\mathcal{M}_{fi}|^2 \times \frac{d^3 p_3}{2E_{a_0}}\frac{d^3 p_4}{2E_\eta}\frac{d^3 p_5}{E_N}\delta^4(P_i - P_f), \tag{40}$$

where $\mathcal{M}_{fi}$ represents the total amplitude, P_i and P_f represent the total momenta in the initial and final states, respectively. The s_2 and s_5 are the helicities of the initial and final protons.

III. RESULTS AND DISCUSSION

In this section, we present the calculated results and discussions for the $\pi^- p \to a_0^- \eta p$ reaction based on the model described above. The discussions are roughly separated into three parts. At first, we shall consider the case without the contribution from the possible new state N_X^*. Then we will further include the contribution of the N_X^* and explore its influences on the observables. Finally, the possible off-shell effects will be discussed for the cases when taking the spin of N_X^* as 3/2.

At the first step, we shall concentrate on the roles of the $N(1535)$, $N(1650)$, and $N(1710)$ in this reaction. In Fig. 2, we show the total cross sections ranging from threshold to the center of mass energy $W = 2.8$ GeV, where both the total and individual contributions are shown. The results show clearly that the $N(1535)$ gives the dominant contribution in the whole energy region under study. The dominant role of the $N(1535)$ is mainly attributed to its large coupling to the $N\eta$ channel. At higher energies, the contributions from the excitation of the $N(1650)$ and $N(1710)$ become more and more important with the increasing W. While, in the entire energy region under consideration their contributions only play minor roles. Furthermore, because all of the contributions from other processes are small, we only show the sum of them[1] in Fig. 2. In Figs. 3 and 4, we study the $p\eta$ invariant mass spectrum and the a_0 angular distribution in the center of mass frame at $W = 2.7$ GeV, where the $N(1535)$ gives the dominant contribution. As can be seen in Fig. 3, there is a clear peak caused by the $N(1535)$ contribution. Furthermore, there is also a shoulder in the distribution appearing at around $M_{p\eta} = 1.65$ GeV, which is produced by the contributions from the $N(1650)$ and $N(1710)$. This shows that even their contributions in total cross sections are small compared to that of the $N(1535)$ their roles can still be investigated through studying the invariant mass spectrum of the $p\eta$ system. In Fig. 4, the dominant role of the nucleon resonance excitation in the t channel is clearly shown by the forward enhancement in the angular distribution of the a_0^- in the center of mass frame. The results presented here show that without the N_X^*'s contribution the t-channel $N(1535)$ production process plays the dominant role in the present reaction in the near threshold region. At the same time, other resonances' contributions also show significant effects in the $p\eta$ invariant mass spectrum. So this reaction may serve as a good place for studying these nucleon resonances. Next, let us turn to the role of a possible new state N_X^*, which represents $N(1685)$ or $N(1700)$, in this reaction. As mentioned above, till now the quantum numbers of the N_X^* are still not known. So here we will consider four sets of J^P quantum numbers for the N_X^*, i.e., $1/2^\pm$ and $3/2^\pm$. For the $J^P = 3/2^\pm$ cases, in this step we ignore the off-shell effects and set the off-shell parameter $Z = -0.5$ in the cal-

[1]Here, it should be noted that in our calculations we find the contributions from the intermediate $\Delta(1920)$ exchanges are negligible. Therefore, for simplicity we set the off-shell parameter $Z = -0.5$ for the $\Delta(1920)$ exchanges throughout this work.

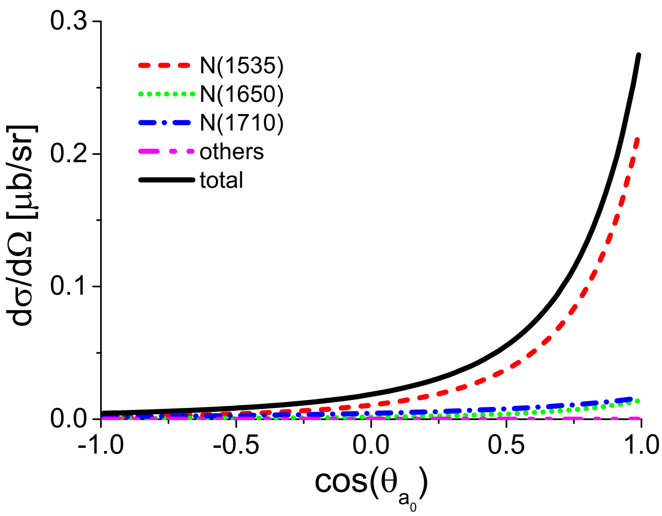

FIG. 4. Angular distribution of a_0^- in the c.m. frame at $W = 2.7$ GeV. The θ_{a_0} is defined as the angle of the a_0 momentum relative to the beam direction. The notations are same as Fig. 2.

culations. The possible off-shell effects will be discussed in the next part. In Fig. 5, we present the contribution of the N_X^* for the total cross sections with adopting various J^P quantum numbers mentioned above and compare it with the $N(1535)$'s contribution. It is shown that the contribution from the N_X^* becomes significant at about $W = 2.658$ GeV/2.68 GeV for Figs. 5(a)/5(b), which corresponds the $N_X^* a_0$ threshold, and increases with the total center of mass energy W increasing.

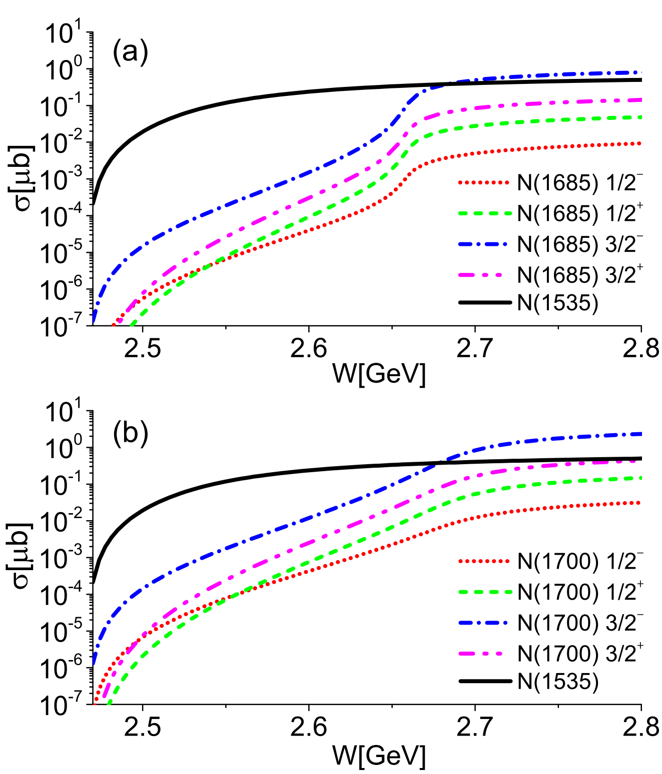

FIG. 5. Total cross section vs center of mass energy W for the $\pi^- p \to a_0^- \eta p$ reaction. The lines represent the corresponding results of the $N(1535)$ and the N_X^* with adopting various quantum numbers.

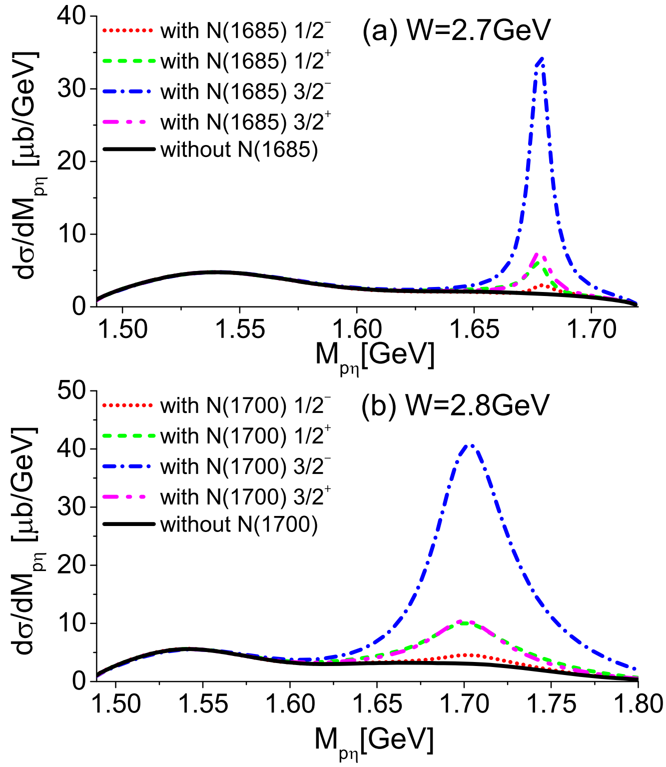

FIG. 6. Invariant mass spectrum of the final $p\eta$ with or without N_X^*'s contribution.

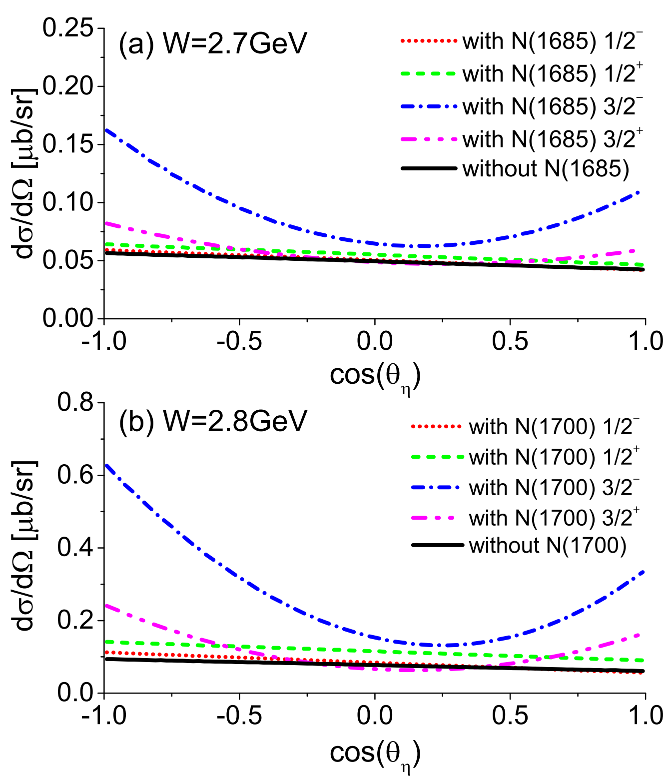

FIG. 7. Angular distribution of η in the ηp rest frame with or without N_X^*'s contribution. The θ_η is defined as the angle of the η momentum relative to the beam direction.

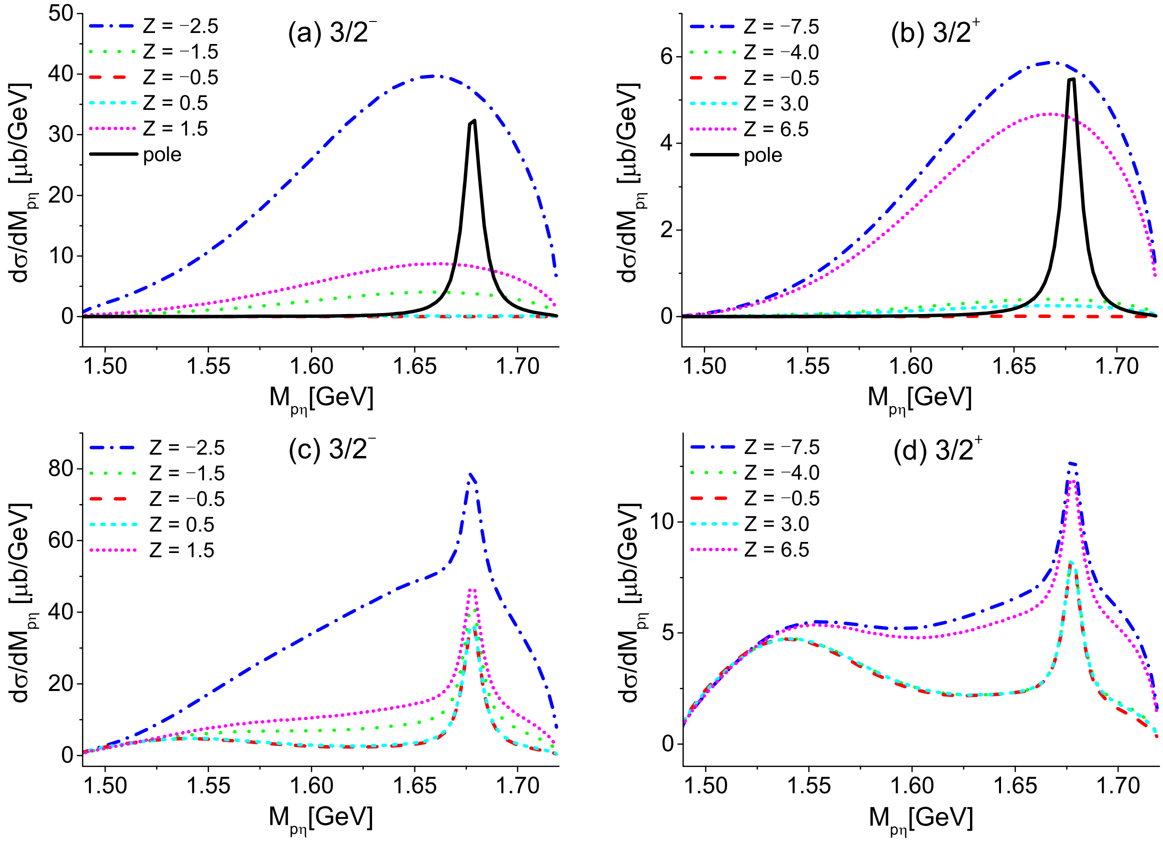

FIG. 8. Invariant mass spectrum of the final $p\eta$ with taking into account of the off-shell effects at $W = 2.7$ GeV. The results of pole and nonpole contributions of the $N(1685)$ with adopting various values of the off-shell parameter Z are depicted in (a) and (b) for $3/2^-$ and $3/2^+$ cases, respectively. The corresponding full results are presented in (c) and (d), respectively.

Especially, if the J^P quantum numbers of the N_X^* is $3/2^-$ its contribution may exceed the $N(1535)$'s contribution at about $W = 2.67$ GeV. In all the cases, we find that the N_X^* contribution has a rapid growing in the total cross sections at the $N_X^* a_0$ threshold. The appearance of this structure is mainly attributed to the opening of the $N_X^* a_0$ threshold and the narrow width of the N_X^*. Such a structure is more evident for the $N(1685)$ case due to its smaller width. A similar phenomena was also found in the study of the reaction $\pi^- p \to D^- D^0 p$ [38], where the $\Lambda_c^+(2940)$ having $\Gamma = 17.5$ MeV was produced. In the $M_{\eta p}$ distribution shown in Fig. 6, the peak structure caused by the N_X^* is prominent for taking J^P quantum number as $1/2^+$ and $3/2^\pm$, which correspond to P and D wave couplings with the $N\eta$ channel. While if the N_X^* has $J^P = 1/2^-$, i.e., a S wave coupling with the ηN channel, its contribution becomes relatively weak and only causes a small bump in the invariant mass spectrum. The enhancement for the higher partial wave coupling is mainly because of the relatively large threshold momentum compared to the $\gamma N \to N\pi\eta$ reaction. At the N_X^* production vertex, the vertex function of the $N_X^* N\eta$ vertex is roughly proportional to p_{th}^L at the near threshold region, where p_{th} is the magnitude of the threshold momentum in the total center of mass frame and L is the relative orbital angular momentum of $N\eta$ system in the $N_X^* \to N\eta$ process. Therefore for higher partial wave couplings, their contribution may be enhanced due to a large value of p_{th} [19,39]. The angular distribution of the η in the $p\eta$ rest frame is presented in Fig. 7, where the θ_η is defined as the angle of the η momentum relative to the beam direction. As expected, the η angular distribution is roughly flat if N_X^* has $J = 1/2$. While, a bowl structure appears in the case that the spin of N_X^* is 3/2 [40].

Finally, we shall discuss the possible off-shell effects on the results. Here, we will concentrate on the influence of the off-shell effects on the $M_{p\eta}$ spectrum. Our analysis mainly follows the ideas of Ref. [31], i.e., the amplitude of N_X^* ($J^P = 3/2^\pm$) can be divided into pole (P) and nonpole (NP) parts. The contribution of the pole part is independent of the off-shell parameter Z. While, the nonpole part includes the contribution of the off-shell effects, which is dependent on the value of the off-shell parameter. As mentioned above, the value of the off-shell parameter should be determined by fitting the experimental data. Due to the absence of data at present, to estimate the value of the off-shell parameter we follow the arguments in Ref. [31] that the nonpole contribution might not dominate the pole contribution, which offers a guideline of the values of the off-shell parameter adopted in this work. To illustrate the off-shell effects on the $M_{p\eta}$ spectrum, we use the $N(1685)$ parameters for the N_X^* as an example, and we have checked that the main results are similar for the $N(1700)$ case.

The results of the $M_{p\eta}$ spectrum at $W = 2.7$ GeV with adopting a set of values of the off-shell parameter Z are shown in Fig. 8. The pole and non-pole contributions of the $N(1685)$ are presented in Fig. 8(a) and 8(b) for $3/2^-$ and $3/2^+$ cases, respectively. The corresponding $M_{p\eta}$ spectrum for the full results are depicted in Fig. 8(c) and 8(d). As can be seen from the figures, with including the off-shell effects the shape of the invariant mass spectrum can indeed be changed. While such changes are dependent on the value of the off-shell parameter, for which it is still not well determined. However, if we accept the opinion that the nonpole term contribution might not dominate the pole term contribution, our results show that the appearance of a peak structure in the $M_{p\eta}$ spectrum due to the possible new resonance is not affected by considering the off-shell effects. Of course, more careful studies on the off-shell effects are still needed when the experimental data are available in the future.

Based on the discussions presented above, we conclude that if the N_X^*, i.e., $N(1685)$ or $N(1700)$, exists we expect it should show signal in the $M_{p\eta}$ spectrum in the reaction $\pi^- p \to a_0^- \eta p$. If the N_X^* has J^P quantum numbers as $1/2^+$ or $3/2^{\pm}$, it may show a clear peak in the $M_{p\eta}$ spectrum. While, if the N_X^* has $J^P = 1/2^-$, it may only show a small bump in the $M_{p\eta}$ spectrum. The signal of the new resonance can also be found in the angular distribution of the η in the $p\eta$ rest frame if its spin is 3/2. In other cases, it does not have significant effects on the η angular distribution.

IV. SUMMARY

In this work, we investigate the $\pi^- p \to a_0^- \eta p$ reaction within an effective Lagrangian approach and isobar model. We study the roles of various intermediate nucleon and Δ resonances and background contribution in this reaction. Specifically, we discuss the possibility to verify the existence of a new nucleon resonance, i.e., $N(1685)/N(1700)$ observed in the $\gamma N \to \pi\eta N$ reaction, in the present reaction. We find that without considering the new resonance's contribution the $N(1535)$ plays a dominant role in the near threshold region. While, if the new state exists, it may also give significant contributions in this reaction and can show clear signals in the $p\eta$ invariant mass spectrum. In particular, we find that, depending on the value of the off-shell parameter Z, the off-shell effects may change the $M_{p\eta}$ spectrum significantly for high-spin cases. At present, such effects still can not be well determined due to the absence of the data and deserve further studies when the data are available in the future. We also show that the analysis of the angular distribution of the final η in the $N\eta$ rest frame is helpful to determine its quantum numbers. Therefore, this reaction can offer a good place to study the $N(1535)$ and verify the existence of the $N(1685)/N(1700)$ proposed in recent experiments.

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