Enhancement of incoherent bremsstrahlung in proton-nucleus scattering in the Δ-resonance energy region

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Emission of the bremsstrahlung photons in the scattering of protons off nuclei in the Δ-resonance energy region is studied. Including properties of Δ resonance in the target nucleus in the bremsstrahlung model, the following issues are found. (1) The ratio between incoherent emission and coherent emission is about 10^6–10^7 for $p + {}^{197}$Au (without Δ resonance) at the proton beam energy E_p of 190 MeV, where the calculated full bremsstrahlung spectrum is in good agreement with experimental data. This confirms the importance of incoherent processes in the study of Δ resonances in this reaction, which have not been studied yet. The coherent and incoherent contributions, electric and magnetic contributions, and full bremsstrahlung spectra for the scattering of protons on the ^{12}C, ^{40}Ca, ^{208}Pb nuclei at $E_p = 800$ MeV are found, and conditions for the most intensive bremsstrahlung emission are found. (2) The transition $pN \to \Delta^+ N$ in the target nucleus reinforces emission of bremsstrahlung photons in that reaction at $E_p = 800$ MeV. The difference between the spectra for normal nuclei and nuclei with included Δ resonance is larger for lighter nuclei, but the spectra are larger for heavier nuclei. (3) Taking short-lived Δ resonance into account, It is found that the spectrum for the target nucleus with Δ resonance is essentially larger in the high energy photon region than the spectrum without this Δ resonance (calculations for ${}^{12}_{\Delta}$C, ${}^{40}_{\Delta}$Ca, ${}^{208}_{\Delta}$Pb in comparison with ^{12}C, ^{40}Ca, ^{208}Pb are provided). Such an aspect is recommended for registration of Δ resonances in nuclei in future experiments.

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I. INTRODUCTION

Non-nucleon degrees of freedom in nuclei are often studied in nuclear physics [1,2]. In recent years some attention has been focused on hadron excitation of Δ(1232) resonances in nuclear matter (for brevity, the term "Δ resonance in nuclei" will be used in this paper) [3,4]. Protons, electrons, photons, and pions can be used as probes to study such Δ resonances in nuclei. But, the easiest way to study Δ resonances in nuclei is based on analysis of the scattering of beams of pions on nuclei and photoinduced reactions [5]. Recently proton nucleus scattering was used as a main reaction for this study, where Δ resonances are formed as a result of virtual pion interaction with one nucleon of the target nucleus. As studied in Ref. [6], coherent photon production in proton-nucleus scattering is another similar reaction that can be used also for such a study. There is interest in the emission of photons because such photons can be measured and new information about the physics of the formation of Δ resonances in nuclear matter can be extracted.

Another possibility to study Δ resonances in nuclei is to use analysis of bremsstrahlung photons which are emitted during proton-nucleus scattering with the formation of Δ resonances. Many effects, such as dynamics of the nuclear process, interactions of nucleons, nuclear forces, structure of nuclei, quantum effects, anisotropy (deformations), etc., can be included in the model describing the bremsstrahlung emission (for example, see Refs. [7–10] for general properties of bremsstrahlung in α decay, Ref. [11] for extraction of information about deformation of nuclei in α decay from experimental bremsstrahlung data, Ref. [12] for bremsstrahlung in nuclear radioactivity with emission of protons, Ref. [13] for bremsstrahlung in the spontaneous fission of ^{252}Cf, Ref. [14] for bremsstrahlung in the ternary fission of ^{252}Cf, and Ref. [15] for bremsstrahlung in pion-nucleus scattering from our research; there are many investigations from other researchers). Note the perspectives on studying electromagnetic observables of light nuclei based on chiral effective field theory [16] (see also research [17] for pp bremsstrahlung). Experimental measurements of such photons and analysis provide useful information about these aspects. Model suitability can be therefore verified.

So, the bremsstrahlung photon is a useful tool to investigate all the above questions. This paper is devoted to studying Δ resonances in nuclei using bremsstrahlung analysis.

In Ref. [6] the authors estimated cross sections, where coherent processes were included in the model. However, an experimental study of bremsstrahlung in proton-nucleus scattering by the TAPS Collaboration [18] indicates that incoherent processes play an important role in the bremsstrahlung

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emission ($p + {}^{197}$Au at the proton beam energy of $E_p = 190$ MeV was studied). The conclusion about more intensive incoherent bremsstrahlung emission than coherent bremsstrahlung emission for proton-nucleus scattering comes from (a) agreement between experimental data [18] and calculations obtained with the highest accuracy, (b) an explanation of the existence of a plateau in experimental data [18] (which cannot be explained only by coherent emission), (c) theoretical unified estimation of both coherent and incoherent contributions (and estimated ratios of contributions), (d) the difference between intensity of incoherent emission and intensity of coherent emission being huge (that makes it impossible to remove the incoherent emission via modeling the coherent emission in different ways).

One of the reasons to conclude that incoherent emission is more intensive than coherent one is the existence of a so-called plateau in the experimental data in Ref. [18] (in the middle part of spectrum). That plateau is explained if one adds the incoherent contribution to the formalism. Without this incoherent contribution, the model has only coherent terms and gives a spectrum with a logarithmic type shape (i.e., without a plateau). In Ref. [19] the ratio between incoherent and coherent contributions was extracted on the basis of analysis of experimental data [18]. It was concluded that incoherent bremsstrahlung is intensive (for heavy nuclei used in experiments).

Then, the unified formalism was constructed, which includes both incoherent and coherent contributions. That model estimated ratios for different nuclei and energies of the proton beam. From calculations of such ratios it was concluded that the incoherent contribution is essentially larger for heavy nuclei than the coherent one. Now it is estimated that incoherent and coherent contributions have comparable values for light nuclei (like ^{4}He, etc.; those processes can occur in stars). Subsequent research includes Refs. [20,21]. Measurements of bremsstrahlung photons in proton-nucleus scattering are also presented in those works. Here one can find the spectra with a plateau. However, accuracy of data in [20,21] is smaller than in the data of [18]. So, before the analysis described above, there was little clarity about incoherent emission. So, Refs. [20,21] provided the first indications about the important role of the incoherent bremsstrahlung in the proton-nucleus processes.

As shown in [19], including in the model relations between spin of the scattered proton and moments of nucleons of the target nucleus allows one to essentially improve agreement between theory and experimental data. Without such an inclusion it is impossible to explain the presence of a plateau in experimental data [18]. For the studied reaction the incoherent bremsstrahlung contribution is 10^6–10^7 times larger than the coherent contribution in the full bremsstrahlung.

As a detailed formalism of incoherent emission of bremsstrahlung photons during proton-nucleus scattering has been constructed [19,22,23], in this paper that model is applied for a new situation where Δ resonances in the target nucleus can be produced. The Role of the incoherent processes will be analyzed.

Photons emitted from incoherent processes have been studied in many topics of nuclear and particle physics. A physical picture of the studied reaction becomes more complete after the inclusion of such photons. The authors of Ref. [24] found a non-negligible role of incoherent photons in photoproduction of heavy quarks (charm, bottom) and heavy quarkonia [J/ψ, $\Upsilon(1S)$] in ultraperipheral collisions of heavy nuclei Pb-Pb and in proton-proton collisions at high energies (see also Refs. [25,26] and review [27]). Incoherent photons also have an important place in the study of interactions between dark matter and nuclear matter [28]. Generally, it is difficult to perform calculations of spectra when including the incoherent contribution. Researchers apply different approximations to calculate coherent processes or incoherent ones [29]. So, focus will be given to construction of a unified formalism (with corresponding calculations and analysis) with a joint description of the coherent and incoherent bremsstrahlung emission. With detailed analysis, it is found that the role of incoherent emission is important and the calculated spectra are essentially changed after taking this type of emission into account.

Important parameters in incoherent emission are magnetic moments of nucleons of the nucleus and their spins. The spin of a Δ resonance is $3/2$, while spins of protons and neutrons of nucleus are $1/2$. So, one can suppose that bremsstrahlung emission after the inclusion of transition $NN \to \Delta N$ can be changed visibly. This is another motivation of this research. It is intended to estimate how large is the difference in emissions of photons. Note that the investigation in this paper of Δ resonances in nuclei on the basis of bremsstrahlung analysis (tested on existing experimental data for similar processes) is performed for the first time.

The paper is organized in the following way. In Sec. II the new model of the bremsstrahlung photons emitted during proton nucleus scattering is presented. In Sec. III the results of studying the scattering of protons off the ^{12}C, ^{40}Ca, ^{208}Pb nuclei at a proton beam energy of 800 MeV with possible formation of Δ resonance are presented. Conclusions are summarized in Sec. IV. Previously unpublished details of the model and some useful new calculations are presented in Supplemental Material [30].

II. MODEL

A. Generalized Pauli equation for nucleons in the proton-nucleus system with Δ resonance and operator of emission of photons

Let us consider scattering of a proton on a nucleus in the laboratory frame, where the nucleus consists $A - 1$ nucleons and one short-lived Δ resonance. One writes the Hamiltonian of such a system with emission of photons as a many-nucleon generalization of the Pauli equation, as (obtained from Eq. (1.3.6) in Ref. [31], p. 33; this formalism follows Refs. [12,15,19,22,32] and references therein)

$$\hat{H} = \left\{ \frac{1}{2m_p} \left(\hat{\mathbf{p}}_p - \frac{z_p e}{c} \mathbf{A}_p \right)^2 + z_p e A_{p,0} - \frac{z_p e \hbar}{2m_p c} \boldsymbol{\sigma} \cdot \mathbf{rot}\, \mathbf{A}_p \right\} + \sum_{j=1}^{A-1} \left\{ \frac{1}{2m_j} \left(\hat{\mathbf{p}}_j - \frac{z_j e}{c} \mathbf{A}_j \right)^2 + z_j e A_{j,0} - \frac{z_j e \hbar}{2m_j c} \boldsymbol{\sigma} \cdot \mathbf{rot}\, \mathbf{A}_j \right\}$$
$$+ \left\{ \frac{1}{2m_\Delta} \left(\hat{\mathbf{p}}_\Delta - \frac{z_\Delta e}{c} \mathbf{A}_\Delta \right)^2 + z_\Delta e A_{\Delta,0} - f_\Delta \cdot \frac{z_\Delta e \hbar}{2m_\Delta c} \boldsymbol{\sigma} \cdot \mathbf{rot}\, \mathbf{A}_\Delta \right\} + V(\mathbf{r}_1, \ldots, \mathbf{r}_{A-1}, \mathbf{r}_\Delta, \mathbf{r}_p). \tag{1}$$

Here, m_i and z_i are mass and electric charge of a nucleon with number i, $\hat{\mathbf{p}}_i = -i\hbar\, \mathbf{d}/\mathbf{dr}_i$ is the momentum operator for a nucleon with number i, $V(\mathbf{r}_1, \ldots, \mathbf{r}_{A-1}, \mathbf{r}_\Delta, \mathbf{r}_p)$ is a general form of the potential of interactions between nucleons of the nucleus, the Δ resonance, and the scattered proton, $\boldsymbol{\sigma}$ are Pauli matrices, $A_i = (\mathbf{A}_i, A_{i,0})$ is the potential of the electromagnetic field formed by a moving nucleon with number i or Δ resonance, and A in the summation is the mass number of the target nucleus. A new parameter f_Δ is introduced which is related to the Dirac equation for fermions with spin 1/2, while the Δ resonance has spin 3/2. For Δ resonance an analog of the Pauli equation with a different term of spin is used (in this I assume $f_\Delta = 3$, as the spin of the Δ resonance is 3 times larger than that of the nucleon).

The Hamiltonian (1) can be rewritten as

$$\hat{H} = \hat{H}_0 + \hat{H}_\gamma, \tag{2}$$

where

$$\hat{H}_0 = \frac{1}{2m_p} \hat{\mathbf{p}}_p^2 + \sum_{j=1}^{A-1} \frac{1}{2m_j} \hat{\mathbf{p}}_j^2 + \frac{1}{2m_\Delta} \hat{\mathbf{p}}_\Delta^2 + V(\mathbf{r}_1, \ldots, \mathbf{r}_{A-1}, \mathbf{r}_\Delta, \mathbf{r}_p),$$
$$\hat{H}_\gamma = \left\{ -\frac{z_p e}{m_p c} \hat{\mathbf{p}}_p \cdot \mathbf{A}_p + \frac{z_p^2 e^2}{2m_p c^2} \mathbf{A}_p^2 - \frac{z_p e \hbar}{2m_p c} \boldsymbol{\sigma} \cdot \mathbf{rot}\mathbf{A}_p + z_p e A_{\mathrm{p},0} \right\}$$
$$+ \sum_{j=1}^{A-1} \left\{ -\frac{z_j e}{m_j c} \hat{\mathbf{p}}_j \cdot \mathbf{A}_j + \frac{z_j^2 e^2}{2m_j c^2} \mathbf{A}_j^2 - \frac{z_j e \hbar}{2m_j c} \boldsymbol{\sigma} \cdot \mathbf{rot}\mathbf{A}_j + z_j e A_{j,0} \right\}$$
$$+ \left\{ -\frac{z_\Delta e}{m_\Delta c} \hat{\mathbf{p}}_\Delta \cdot \mathbf{A}_\Delta + \frac{z_\Delta^2 e^2}{2m_\Delta c^2} \mathbf{A}_\Delta^2 - f_\Delta \cdot \frac{z_\Delta e \hbar}{2m_\Delta c} \boldsymbol{\sigma} \cdot \mathbf{rot}\mathbf{A}_\Delta + z_\Delta e A_{\Delta,0} \right\}. \tag{3}$$

Here, $\hat{H}_0$ is the Hamiltonian describing evolution of nucleons of the nucleus and the Δ resonance in the scattering of a proton (without emission of photons); $\hat{H}_\gamma$ is operator describing emission of bremsstrahlung photons in this scattering.

To include magnetic moments of particles, Dirac's magnetic moment $\mu_i^{(\mathrm{Dir})} = z_i e\hbar/(2m_i c)$ for each particle with number i is changed as $\mu_i^{(\mathrm{Dir})} \to \mu_i^{(\mathrm{an})}\mu_N$, where $\mu_N = e\hbar/(2m_p c)$ is the nuclear magneton, $\mu_p^{(\mathrm{an})} = 2.792\,847\,344\,62$ is the anomalous magnetic moment for the proton, $\mu_{\mathrm{n}}^{(\mathrm{an})} = -1.913\,042\,73$ is the anomalous magnetic moment for the neutron, and μ_j are magnetic moments of protons or neutrons of the nucleus (measured in units of nuclear magneton μ_N; see Ref. [33]). Terms at $\mathbf{A}_j^2$ and $A_{j,0}$ are neglected, and the Coulomb gauge is used.[1] The operator of emission (3) is transformed to

$$\hat{H}_\gamma = -\frac{z_p e}{m_p c} \hat{\mathbf{p}}_p \cdot \mathbf{A}_p - \mu_N\, \mu_p\, \boldsymbol{\sigma} \cdot \hat{\mathbf{H}}_p + \sum_{j=1}^{A} \left\{ -\frac{z_j e}{m_j c} \hat{\mathbf{p}}_j \cdot \mathbf{A}_j - \mu_N\, \mu_j\, \boldsymbol{\sigma} \cdot \hat{\mathbf{H}}_j \right\}, \tag{4}$$

where

$$\hat{\mathbf{H}} = \mathbf{rot}\, \mathbf{A} = [\nabla \times \mathbf{A}] \tag{5}$$

[1] In QED the gauge for the four-potential of a electromagnetic field can be written down as $A'_\nu(x) = A_\nu(x) + \partial_\nu f(x)$ ($\nu = 0, 1, 2, 3$). Function $f(x)$ can be varied (which does not change equations of motion); it can be found in any fixed frame at $A_0 = 0$ (this is not an approximation). Coulomb gauge $\div \mathbf{A} = 0$ is used (see Ref. [34], p. 37). QED is a perturbative theory, where matrix elements based on terms with $\mathbf{A}^2$ [related to fine-structure constant $\alpha = e^2/(\hbar c) = 1/137$] are smaller than matrix elements with $\mathbf{A}$ (related to $\alpha^2 = 1/137^2$). Electromagnetic processes are described in QED using the S matrix, where different orders of matrix elements are characterized by α with different powers. Following this logic, terms with $\mathbf{A}^2$ are neglected in the paper. Ways to neglect terms with A_0 and $\mathbf{A}^2$ were used by many authors in the study of bremsstrahlung in different nuclear processes (for example, see Refs. [35,36]).

and $\mu_{\Delta} = f_{\Delta} \cdot \mu_{p,n}$ ($f_{\Delta} = 3$).[2] This expression is a many-nucleon generalization of the operator of emission $\hat{W}$ in Eq. (4) in Ref. [32] with included magnetic moments for nucleons and Δ resonance.

B. Formalism in space representation

Substituting the definition for the potential of an electromagnetic field,

$$\mathbf{A} = \sum_{\alpha=1,2} \sqrt{\frac{2\pi \hbar c^2}{w_{\rm ph}}}\, \mathbf{e}^{(\alpha),*} e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}}, \tag{6}$$

one obtains

$$\begin{aligned} \hat{\mathbf{H}} &= [\nabla \times \mathbf{A}] \\ &= \sqrt{\frac{2\pi \hbar c^2}{w_{\rm ph}}} \sum_{\alpha=1,2} \{-i\, e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}}\, [\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha),*}] + e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}}\, [\nabla \times \mathbf{e}^{(\alpha),*}]\}. \end{aligned} \tag{7}$$

Here, $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are two independent unit vectors of polarizations for a photon [$\mathbf{e}^{(\alpha),*} = \mathbf{e}^{(\alpha)}$, $\alpha = 1, 2$], $\mathbf{k}_{\rm ph}$ is wave vector of the photon, and $w_{\rm ph} = k_{\rm ph} c = |\mathbf{k}_{\rm ph}| c$. Vectors $\mathbf{e}^{(\alpha)}$ are perpendicular to $\mathbf{k}_{\rm ph}$ in Coulomb gauge, satisfying Eq. (8) in Ref. [23]. One can develop a simpler formalism in the system of units where $\hbar = 1$ and $c = 1$, but constants $\hbar$ and c will be written explicitly. There are the properties:

$$\begin{aligned} &[\mathbf{k}_{\rm ph} \times \mathbf{e}^{(1)}] = k_{\rm ph}\, \mathbf{e}^{(2)}, \\ &[\mathbf{k}_{\rm ph} \times \mathbf{e}^{(2)}] = -k_{\rm ph}\, \mathbf{e}^{(1)}, \\ &[\mathbf{k}_{\rm ph} \times \mathbf{e}^{(3)}] = 0, \\ \sum_{\alpha=1,2,3} &[\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha)}] = k_{\rm ph}\, (\mathbf{e}^{(2)} - \mathbf{e}^{(1)}). \end{aligned} \tag{8}$$

Substituting formulas (6) and (7) in Eq. (4) for the operator of emission, one obtains

$$\begin{aligned} \hat{H}_{\gamma} = &\sqrt{\frac{2\pi \hbar c^2}{w_{\rm ph}}}\, \mu_N \sum_{\alpha=1,2} e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}_K} \{i\, 2z_p\, \mathbf{e}^{(\alpha)} \cdot \nabla_p + \mu_p\, \boldsymbol{\sigma} \cdot (i\, [\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha)}] - [\nabla_p \times \mathbf{e}^{(\alpha)}])\} \\ &+ \sqrt{\frac{2\pi \hbar c^2}{w_{\rm ph}}}\, \mu_N \sum_{j=1}^{A} \sum_{\alpha=1,2} e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}_j} \left\{ i\, \frac{2z_j m_p}{m_{Aj}}\, \mathbf{e}^{(\alpha)} \cdot \nabla_j + \mu_j\, \boldsymbol{\sigma} \cdot (i\, [\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha)}] - [\nabla_j \times \mathbf{e}^{(\alpha)}]) \right\}. \end{aligned} \tag{9}$$

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[2] The formalism with Dirac magnetic moments of nucleons is in agreement with gauge invariance in the first relativistic approximation (like the Pauli equation is the first relativistic approximation of the Dirac equation for one nucleon inside an external field). The region of applicability of the first relativistic approximation is up to 1.6 GeV (for a beam of protons on a heavy target nucleus). So, the first approximation can be used in the current study. One can obtain the second relativistic correction for one nucleon inside an external field (for example, see Ref. [30]; gauge invariance exists inside such an approximation). However, the number of terms of bremsstrahlung is dramatically increased from the second correction, and has different physical interpretations [this is extension of Eq. (15) in the paper; i.e., the sense of using terminology of "coherent" and "incoherent" is lost]. It is not an easy problem to connect many nucleons with gauge invariance. But, following the idea of relativistic corrections above, this problem is resolved and is used in the previous formalism. However, Dirac magnetic moments of protons and neutrons of a nucleus, even with gauge invariance, cannot describe experimental data [18] from proton-nucleus scattering with good agreement. But, anomalous magnetic moments of nucleons describes these experimental data with essentially better agreement (compared to other approaches), and allow one to explain the presence of a plateau in experimental data, in contrast to the formalism with Dirac magnetic moments of nucleons. So, this motivates going from Dirac magnetic moments to anomalous ones of nucleons. This is a really important issue. One can use the composition of nucleons on quarks with Dirac magnetic moments, and obtain anomalous magnetic moments for nucleons that can be in agreement with gauge invariance. This could be better resolution of this question, but such calculations will be more overloaded by additional complicated formulas. Note that in the shell model of a nucleus the full magnetic moments for protons and neutrons in nucleus depend on the shells of the nucleus (for example, see Eqs. (118.12)–(118.14), p. 583–591 in book [37]). This formulation violates gauge invariance. However, the level of accuracy of determination of spectroscopic characteristics of nuclei in frameworks of the nuclear shell model can be good.

This expression coincides with the operator of emission $\hat{W}$ in the form (6) in Ref. [32] in the limit case of the problem of one nucleon with charge $Z_{\rm eff}$ in an external field [taking Eqs. (8), $\hbar = 1$ into account]. Terms for Δ resonance are included in the summation in Eq. (9) and further in this paper, for convenience.

C. Operator of emission with relative coordinates

The following notations and definitions will be used in the paper. Coordinates of center of mass for the nucleus and for the complete system are $\mathbf{R}_A$, $\mathbf{R}$, the relative coordinate between the scattered proton and center of mass of target nucleus is $\mathbf{r}$, and relative coordinates of nucleons (and Δ resonance) of the nucleus relative to its center of mass are $\boldsymbol{\rho}_{Aj}$. Following Ref. [32] (see Eqs. (3) and (4) and explanation in the text of that paper; see Appendix A in Ref. [23]), one can find new relative coordinate $\mathbf{r} = \mathbf{r}_p - \mathbf{R}_A$ and new relative coordinates $\boldsymbol{\rho}_{Aj} = \mathbf{r}_{Aj} - \mathbf{R}_A$ for nucleons of the nucleus. Momenta $\hat{\mathbf{P}}$, $\hat{\mathbf{p}}$, $\tilde{\mathbf{p}}_{Aj}$ are calculated corresponding to independent variables $\mathbf{R}$, $\mathbf{r}$, $\boldsymbol{\rho}_{Aj}$ at $j = 1, \ldots, A-1$ (defined as $\hat{\mathbf{p}}_i = -i\hbar\, \mathbf{d}/\mathbf{d}\mathbf{r}_i$), and the formalism above is rewritten.[3]

Following Ref. [23], the operator of emission of bremsstrahlung photons in the scattering of a proton off a nucleus in the laboratory frame is calculated as

$$\hat{H}_\gamma = \hat{H}_P + \hat{H}_p + \Delta \hat{H}_{\gamma E} + \Delta \hat{H}_{\gamma M} + \hat{H}_k, \tag{11}$$

where operators $\hat{H}_P$, $\hat{H}_p$, $\hat{H}_k$ are calculated in Eqs. (16)–(19) in Ref. [23] (at $m_\alpha \to m_p$ replacement) and (see solutions for $\Delta \hat{H}_{\gamma E}$, $\Delta \hat{H}_{\gamma M}$ in Appendix A in Ref. [38]):

$$\Delta \hat{H}_{\gamma E} = -\sqrt{\frac{2\pi c^2}{\hbar w_{\rm ph}}}\, 2\, \mu_N\, e^{-i\,\mathbf{k}_{\rm ph}\mathbf{R}} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)} \Biggl\{ e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}} \sum_{j=1}^{A-1} \frac{z_j\, m_p}{m_{Aj}}\, e^{-i\,\mathbf{k}_{\rm ph}\boldsymbol{\rho}_{Aj}}\, \tilde{\mathbf{p}}_{Aj} - \frac{m_p}{m_A}\, e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}} \sum_{j=1}^{A} z_j\, e^{-i\,\mathbf{k}_{\rm ph}\boldsymbol{\rho}_{Aj}} \sum_{k=1}^{A-1} \tilde{\mathbf{p}}_{Ak} \Biggr\}, \tag{12}$$

$$\begin{aligned} \Delta \hat{H}_{\gamma M} = & -i\sqrt{\frac{2\pi c^2}{\hbar w_{\rm ph}}}\, \mu_N\, e^{-i\,\mathbf{k}_{\rm ph}\mathbf{R}} \sum_{\alpha=1,2} \Biggl\{ e^{i\,\mathbf{k}_{\rm ph} c_p\,\mathbf{r}} \sum_{j=1}^{A-1} \mu_j\, e^{-i\,\mathbf{k}_{\rm ph}\boldsymbol{\rho}_{Aj}}\, \boldsymbol{\sigma}\cdot[\tilde{\mathbf{p}}_{Aj} \times \mathbf{e}^{(\alpha)}] \\ & - e^{i\,\mathbf{k}_{\rm ph} c_p\,\mathbf{r}} \sum_{j=1}^{A} \mu_j\, \frac{m_{Aj}}{m_A}\, e^{-i\,\mathbf{k}_{\rm ph}\boldsymbol{\rho}_{Aj}} \sum_{k=1}^{A-1} \boldsymbol{\sigma}\cdot[\tilde{\mathbf{p}}_{Ak} \times \mathbf{e}^{(\alpha)}] \Biggr\}. \end{aligned} \tag{13}$$

Here, new coefficients $c_A = \frac{m_A}{m_A+m_p}$ and $c_p = \frac{m_p}{m_A+m_p}$ are introduced, where m_p and m_A are masses of scattered proton and target nucleus, m_{Aj} is the mass of a nucleon of nucleus with number j, and ($\mathbf{e}^{(\alpha)}$ are perpendicular to $\mathbf{k}_{\rm ph}$ in the Coulomb gauge, satisfying Eq. (8) in Ref. [23]. A in the summations is mass number of the target nucleus.

D. Matrix elements of emission of bremsstrahlung photons

The wave function of the full nuclear system is defined as $\Psi = \Phi(\mathbf{R}) \cdot \Phi_{p\text{-nucl}}(\mathbf{r}) \cdot \psi_{\rm nucl}(\beta_A)$ following the formalism in Ref. [19] for the proton-nucleus scattering [see Sec. II B, Eqs. (10)–(13)], and many-nucleon structure of the nucleus is described as in Ref. [22]. Here, β_A is the set of numbers $1, \ldots, A$ of nucleons of the nucleus, $\Phi(\mathbf{R})$ is the function describing motion of the center of mass of the full nuclear system in the laboratory frame, $\Phi_{p\text{-nucl}}(\mathbf{r})$ is the function describing relative motion of the scattered proton with respect to to nucleus (without description of internal relative motions of nucleons in the nucleus), and $\psi_{\rm nucl}(\beta_A)$ is the many-nucleon function of the nucleus, defined in Eq. (12) Ref. [19] on the basis of one-nucleon functions $\psi_{\lambda_s}(s)$. One-nucleon functions $\psi_{\lambda_s}(s)$ represent the multiplication of space and spin-isospin functions as $\psi_{\lambda_s}(s) = \varphi_{n_s}(\mathbf{r}_s)\, |\, \sigma^{(s)}\tau^{(s)}\rangle$, where φ_{n_s} is the space function of the nucleon with number s, n_s is the number of state of the space function of the nucleon with number s, and $|\, \sigma^{(s)}\tau^{(s)}\rangle$ is the spin-isospin function of the nucleon with number s.

The matrix element of emission is defined using the wave functions Ψ_i and Ψ_f of the full nuclear system in states before emission of photons (i state) and after such emission (f state), as

$$F = \langle \Psi_f\, |\, \hat{H}_\gamma\, |\, \Psi_i \rangle. \tag{14}$$

Integration in this matrix element should be obtained over all independent variables, i.e., space variables $\mathbf{R}$, $\mathbf{r}$, $\boldsymbol{\rho}_{Am}$. Space representation of all moments $\hat{\mathbf{P}}$, $\hat{\mathbf{p}}$, $\tilde{\mathbf{p}}_{Am}$ should be taken into account (as $\hat{\mathbf{P}} = -i\hbar\, \mathbf{d}/\mathbf{d}\mathbf{R}$, $\hat{\mathbf{p}} = -i\hbar\, \mathbf{d}/\mathbf{d}\mathbf{r}$, $\tilde{\mathbf{p}}_{Am} = -i\hbar\, \mathbf{d}/\mathbf{d}\boldsymbol{\rho}_{Am}$). Using formulas (11)–(13) for the operator of emission, one can calculate [see Sec. I in Supplemental Material [30] for details, Eqs. (1) and (33)–(37)]

$$\begin{aligned} \langle \Psi_f\, |\, \hat{H}_\gamma\, |\, \Psi_i \rangle &= \sqrt{\frac{2\pi\, c^2}{\hbar w_{\rm ph}}}\, M_{\rm full}, \\ M_{\rm full} &= M_P + M_p^{(E)} + M_p^{(M)} + M_k + M_{\Delta E} + M_{\Delta M}, \end{aligned} \tag{15}$$

[3] One can write useful formulas for a nucleon with number A of the target nucleus as

$$\begin{aligned} \boldsymbol{\rho}_{AA} &= -\frac{1}{m_{AA}} \sum_{k=1}^{A-1} m_{Ak}\, \boldsymbol{\rho}_{Ak}, \\ \hat{\mathbf{p}}_{AA} &= \frac{m_{AA}}{m_A + m_p}\, \hat{\mathbf{P}} - \frac{m_{AA}}{m_A}\, \hat{\mathbf{p}} - \frac{m_{AA}}{m_A} \sum_{k=1}^{A-1} \tilde{\mathbf{p}}_{Ak}. \end{aligned} \tag{10}$$

where

$$M_p^{(E)} = 2\,i\hbar\,(2\pi)^3 \frac{m_p}{\mu}\,\mu_N \sum_{\alpha=1,2} \int \Phi^{*}_{p\text{-nucl},f}(\mathbf{r})\, e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}}\, Z_{\rm eff}(\mathbf{k}_{\rm ph},\mathbf{r})\, \mathbf{e}^{(\alpha)}\, \frac{\mathbf{d}}{\mathbf{dr}} \Phi_{p\text{-nucl},i}(\mathbf{r})\, \mathbf{dr},$$

$$M_p^{(M)} = -\,\hbar\,(2\pi)^3 \frac{m_p}{\mu}\,\mu_N \sum_{\alpha=1,2} \int \Phi^{*}_{p\text{-nucl},f}(\mathbf{r})\, e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{M}_{\rm eff}(\mathbf{k}_{\rm ph},\mathbf{r}) \biggl[\frac{\mathbf{d}}{\mathbf{dr}} \times \mathbf{e}^{(\alpha)}\biggr] \Phi_{p\text{-nucl},i}(\mathbf{r})\, \mathbf{dr}, \tag{16}$$

$$\begin{aligned} M_P = {} & \frac{\hbar\,(2\pi)^3}{m_A+m_p}\,\mu_N \sum_{\alpha=1,2} \int \Phi^{*}_{p\text{-nucl},f}(\mathbf{r})\, \{2\,m_p\,[e^{-i\,c_A\,\mathbf{k}_{\rm ph}\mathbf{r}} F_{p,\,\rm el} + e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}} F_{A,\,\rm el}]\, \mathbf{e}^{(\alpha)}\, \mathbf{K}_i \\ & + i\,[e^{-i\,c_A\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{F}_{p,\,\rm mag} + e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{F}_{A,\,\rm mag}][\mathbf{K}_i \times \mathbf{e}^{(\alpha)}]\}\, \Phi_{p\text{-nucl},i}(\mathbf{r})\, \mathbf{dr}, \end{aligned} \tag{17}$$

$$M_k = i\,\hbar\,(2\pi)^3 \mu_N \sum_{\alpha=1,2} [\mathbf{k}_{\rm ph}\times\mathbf{e}^{(\alpha)}] \int \Phi^{*}_{p\text{-nucl},f}(\mathbf{r})\, \{e^{-i\,c_A\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{D}_{p,\,\rm k} + e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{D}_{A,\,\rm k}\}\, \Phi_{p\text{-nucl},i}(\mathbf{r})\, \mathbf{dr}, \tag{18}$$

$$M_{\Delta E} = -\,(2\pi)^3\, 2\,\mu_N \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)} \int \Phi^{*}_{p\text{-nucl},f}(\mathbf{r}) \biggl\{ e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{D}_{A1,\,\rm el} - \frac{m_p}{m_A}\, e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \mathbf{D}_{A2,\,\rm el} \biggr\} \Phi_{p\text{-nucl},i}(\mathbf{r})\, \mathbf{dr}, \tag{19}$$

$$M_{\Delta M} = -\,i\,(2\pi)^3\,\mu_N \sum_{\alpha=1,2} \int \Phi^{*}_{p\text{-nucl},f}(\mathbf{r})\, \{e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, D_{A1,\,\rm mag}(\mathbf{e}^{(\alpha)}) - e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, D_{A2,\,\rm mag}(\mathbf{e}^{(\alpha)})\}\, \Phi_{p\text{-nucl},i}(\mathbf{r})\, \mathbf{dr} \tag{20}$$

and $\mathbf{K}_i = \mathbf{K}_f + \mathbf{k}_{\rm ph}$. Here, $\mu = m_p m_A/(m_p+m_A)$ is reduced mass and the effective electric charge and magnetic moment are (see Eqs. (28) and (29) in Supplemental Material [30])

$$\begin{aligned} Z_{\rm eff}(\mathbf{k}_{\rm ph},\mathbf{r}) &= e^{i\,\mathbf{k}_{\rm ph}\mathbf{r}} \biggl[e^{-i\,c_A\mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{m_A}{m_p+m_A}\, F_{p,\,\rm el} - e^{i\,c_p\mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{m_p}{m_p+m_A}\, F_{A,\,\rm el} \biggr], \\ \mathbf{M}_{\rm eff}(\mathbf{k}_{\rm ph},\mathbf{r}) &= e^{i\,\mathbf{k}_{\rm ph}\mathbf{r}} \biggl[e^{-i\,c_A\mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{m_A}{m_p+m_A}\, \mathbf{F}_{p,\,\rm mag} - e^{i\,c_p\mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{m_p}{m_p+m_A}\, \mathbf{F}_{A,\,\rm mag} \biggr]. \end{aligned} \tag{21}$$

Here, $F_{p,\,\rm el}$, $F_{A,\,\rm el}$, $\mathbf{F}_{p,\,\rm mag}$, $\mathbf{F}_{A,\,\rm mag}$, $\mathbf{D}_{A1,\,\rm el}$, $\mathbf{D}_{A2,\,\rm el}$, $D_{A1,\,\rm mag}$, $D_{A2,\,\rm mag}$, $\mathbf{D}_{p,\,k}$, $\mathbf{D}_{A,\,\rm k}$, $D_{p,P\,\rm el}$, $D_{A,P\,\rm el}$, $\mathbf{D}_{p,P\,\rm mag}$, $\mathbf{D}_{A,P\,\rm mag}$ are electric and magnetic form factors defined in Sec. I in Supplemental Material [30] [see Eqs. (16), (21), (23), and (25)].

E. Dipole approximation of effective electric charge and magnetic moment of the nuclear system

After calculations, the matrix elements are obtained for the coherent bremsstrahlung [see Sec. II A in Supplemental Material [30], Eqs. (48)],

$$\begin{aligned} M_p^{(E,\,\rm dip)} &= i\hbar\,(2\pi)^3 \frac{2\,\mu_N\,m_p}{\mu}\, Z_{\rm eff}^{(\rm dip)} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)}\, \mathbf{I}_1, \\ M_p^{(M,\,\rm dip)} &= \hbar\,(2\pi)^3\, \frac{\mu_N}{\mu}\, \alpha_M (\mathbf{e}_{\rm x}+\mathbf{e}_{\rm z}) \sum_{\alpha=1,2} [\mathbf{I}_1 \times \mathbf{e}^{(\alpha)}], \end{aligned} \tag{22}$$

and for incoherent bremsstrahlung [see Secs. II B and II C in Supplemental Material [30], Eqs. (57), (55), and (60)],

$$\begin{aligned} M_{\Delta E} &= 0, \\ M_{\Delta M} &= i\,\hbar\,(2\pi)^3\,\mu_N\, f_1 |\mathbf{k}_{\rm ph}|\, Z_{\rm A}(\mathbf{k}_{\rm ph}) I_2, \\ M_k &= -\,i\,\hbar\,(2\pi)^3\,\mu_N\, k_{\rm ph}\, z_p\, \mu_p\, I_3 - \frac{\bar{\mu}_{pn}}{f_1} M_{\Delta M}. \end{aligned} \tag{23}$$

Integrals are

$$\begin{aligned} \mathbf{I}_1 &= \langle\, \Phi_{p\text{-nucl},f}(\mathbf{r})\, |\, e^{-i\,\mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{\mathbf{d}}{\mathbf{dr}} |\, \Phi_{p\text{-nucl},i}(\mathbf{r})\, \rangle_{\mathbf{r}}, \\ I_2 &= \langle \Phi_{p\text{-nucl},f}(\mathbf{r})\, |\, e^{i\,c_p\,\mathbf{k}_{\rm ph}\mathbf{r}}\, |\, \Phi_{p\text{-nucl},i}(\mathbf{r})\, \rangle_{\mathbf{r}}, \\ I_3 &= \langle \Phi_{p\text{-nucl},f}(\mathbf{r})\, |\, e^{-i\,c_A\,\mathbf{k}_{\rm ph}\mathbf{r}}\, |\, \Phi_{p\text{-nucl},i}(\mathbf{r})\, \rangle_{\mathbf{r}}. \end{aligned} \tag{24}$$

The effective electric charge and magnetic moment (21) are [see Eqs. (40) and (49) in Supplemental Material [30]]

$$\begin{aligned} Z_{\rm eff}^{(\rm dip)}(\mathbf{k}_{\rm ph}) &= \frac{m_A\, z_p - m_p\, Z_{\rm A}(\mathbf{k}_{\rm ph})}{m_p+m_A}, \\ \mathbf{M}_{\rm eff}^{(\rm dip)}(\mathbf{k}_{\rm ph}) &= [z_p\, m_A\, \mu_p - Z_{\rm A}(\mathbf{k}_{\rm ph})\, m_p\, \bar{\mu}_{pn}] \\ & \quad \times \frac{m_p}{m_p+m_A}\, (\mathbf{e}_{\rm x}+\mathbf{e}_{\rm z}), \end{aligned} \tag{25}$$

and one can find

$$\begin{aligned} \alpha_M &= [Z_{\rm A}(\mathbf{k}_{\rm ph})\, m_p\, \bar{\mu}_{pn} - z_p\, m_A\, \mu_p] \frac{m_p}{m_p+m_A}, \\ f_1 &= \frac{A-1}{2A}\, \bar{\mu}_{pn}. \end{aligned} \tag{26}$$

Here, $\bar{\mu}_{pn} = \mu_p + \kappa\,\mu_{\rm n}$, $\kappa = (A-N)/N$, A and N are numbers of nucleons and neutrons in the nucleus, and μ_p and $\mu_{\rm n}$ are magnetic moments of proton and neutron.

Calculation of integrals (24) is straightforward:

$$
\begin{aligned}
M_p^{(E,\,\mathrm{dip})} &\simeq -\hbar\,(2\pi)^3\,\frac{2\,\mu_N\,m_p}{\mu}\,Z_{\mathrm{eff}}^{(\mathrm{dip})}\,\frac{\sqrt{3}}{6}J_1(0,1,0),\\
M_p^{(M,\,\mathrm{dip})} &\simeq -i\,\hbar\,(2\pi)^3\,\frac{\mu_N\,\alpha_M}{\mu}\,\frac{\sqrt{3}}{6}J_1(0,1,0),\\
M_{\Delta M} &\simeq \hbar\,(2\pi)^3\,\mu_N\,f_1\,k_{\mathrm{ph}}\,Z_{\mathrm{A}}(\mathbf{k}_{\mathrm{ph}})\frac{\sqrt{3}}{2}\tilde{J}\,(-c_p,0,1,1),\\
M_k &\simeq -\hbar\,(2\pi)^3\,\mu_N k_{\mathrm{ph}}\,z_p\,\mu_p^{(\mathrm{an})}\frac{\sqrt{3}}{2}\tilde{J}\,(c_A,0,1,1)\\
&\quad -\frac{\bar{\mu}_{pn}^{(\mathrm{an})}}{f_1}M_{\Delta M},
\end{aligned}
\tag{27}
$$

where

$$
\begin{aligned}
J_1(l_i,l_f,n) &= \int_0^{+\infty}\frac{dR_i(r,l_i)}{dr}\,R_f^*(l_f,r)\,j_n(k_{\mathrm{ph}}r)\,r^2dr,\\
\tilde{J}\,(c,l_i,l_f,n) &= \int_0^{+\infty} R_i(l_i,r)\,R_f^*(l_f,r)\,j_n(c\,k_{\mathrm{ph}}r)\,r^2dr.
\end{aligned}
\tag{28}
$$

Here, $R_{i,f}$ is the radial part of wave function $\Phi_{p\text{-nucl}}(\mathbf{r})$ in the i state or f state, and $j_n(k_{\mathrm{ph}}r)$ is the spherical Bessel function of order n.

The cross section of the emitted bremsstrahlung photons is defined on the basis of the full matrix element p_{fi} in the framework of the formalism given in Refs. [19,22,32] (see Eq. (22) in Ref. [22] and references therein) and it is not repeated in this paper. Finally, the bremsstrahlung cross section is obtained as

$$
\begin{aligned}
\frac{d\sigma}{dw_{\mathrm{ph}}} &= \frac{e^2}{2\pi\,c^5}\,\frac{w_{\mathrm{ph}}\,E_i}{m_p^2\,k_i}\,|p_{fi}|^2,\\
\frac{d^2\sigma}{dw_{\mathrm{ph}}\,d\cos\theta} &= \frac{e^2}{2\pi\,c^5}\,\frac{w_{\mathrm{ph}}\,E_i}{m_p^2\,k_i}\left\{p_{fi}\frac{d\,p_{fi}^*}{d\,\cos\theta}+\mathrm{c.\,c.}\right\},\\
M_{\mathrm{full}} &= -\frac{e}{m_p}\,p_{fi},
\end{aligned}
\tag{29}
$$

where c. c. is complex conjugation. The different contributions of the emitted photons to the full bremsstrahlung spectrum are calculated. For estimation of the interesting contribution, the corresponding matrix elements of emission are used. In this paper the matrix elements are calculated on the basis of wave functions with quantum numbers $l_i = 0$, $l_f = 1$, and $l_{\mathrm{ph}} = 1$ [here, l_i and l_f are orbital quantum numbers of wave function $\Phi_{p\text{-nucl}}(\mathbf{r})$ for states before and after photon emission, and l_{ph} is the orbital quantum number of photon in the multipole approach].

III. ANALYSIS

A. The spectra for Δ nuclei: Coherent contributions vs incoherent ones

The emission of photons in the scattering of protons on the ^{12}C, ^{40}Ca, ^{208}Pb nuclei in the Δ-resonance energy region was analyzed [6]. To test calculations of emission of photons, the scattering of $p + {}^{197}$Au is chosen at the proton beam energy of 190 MeV. Such a choice is explained by the following. It is estimated that accuracy of measurements of bremsstrahlung photons and presented data are the highest for the nucleus ^{197}Au (see Ref. [18]) in comparison with other measurements of bremsstrahlung in proton-nucleus scattering. To compare those data with other measurements of bremsstrahlung photons for other nuclear reactions (α decay, fission, α-nucleus scattering, nucleus-nucleus scattering), data in Ref. [18] were obtained with the highest accuracy. So, those data are a good basis for tests of models. Fo that reason, calculations in this paper are compared with experimental data [18] for nucleus ^{197}Au (at proton beam energies used in experiments). Those calculations are performed as a test, before their next use in the paper.

Note the experimental bremsstrahlung data for $p + {}^{208}$Pb at proton beam energy 140 MeV obtained by Edington and Rose [39], and for $p + {}^{64}$Cu, $p + {}^{107}$Ag at proton beam energy 72 MeV obtained by Kwato Njock *et al.* [40]. One can find some disagreement between those data and the ones of Ref. [18]. But, once again, accuracy of the data in Ref. [18] is higher, so data from [18] are chosen for tests.

The wave function of relative motion between the proton and the center of mass of nucleus is calculated numerically using to the proton-nucleus potential in the form of $V(r) = v_c(r) + v_N(r) + v_{\mathrm{so}}(r) + v_l(r)$, where $v_c(r)$, $v_N(r)$, $v_{\mathrm{so}}(r)$, and $v_l(r)$ are Coulomb, nuclear, spin-orbital, and centrifugal components, respectively. Parameters of the potential are defined in Eqs. (46)–(47) in Ref. [19]. The bremsstrahlung cross section is calculated by Eq. (29), where the matrix elements of coherent emission, $M_p^{(E,\,\mathrm{dip})}$ and $M_p^{(M,\,\mathrm{dip})}$, are included in Eqs. (22), and the matrix elements of incoherent emission, $M_{\Delta M}$ and M_k, are included in Eqs. (23).

Results of previous studies of bremsstrahlung emission [19,22,23,32] show that incoherent emission is essentially larger than coherent emission. For this reason the calculation is started for ^{197}Au, where experimental data exist. Results of such a calculation with inclusion of coherent and incoherent contributions in comparison with experimental data are presented in Fig. 1(a). The calculated spectra are normalized on one point of these experimental data. This shows that the bremsstrahlung model with the coherent and incoherent contributions provides a spectrum in good agreement with experimental data. Note that the first point in the experimental data is not explained in a satisfactory way by models with included incoherent bremsstrahlung contribution. Without the incoherent contribution, the calculated renormalized spectrum is in disagreement with experimental data. The presence of such a point could indicate the existence of some unknown processes forming more intensive coherent bremsstrahlung emission at low photon energies. That problem can motivate future investigations and developments of the model or more precise measurements of emission of photons at low energies in possible future experiments. But, in the current research, the analysis of this problem is omitted.

One can obtain the following formula:

$$
M_p^{(E,\,\mathrm{dip})} + M_p^{(M,\,\mathrm{dip})} = M_p^{(E,\,\mathrm{dip})}\left\{1 + i\,\frac{\alpha_M}{2\,m_p\,Z_{\mathrm{eff}}^{(\mathrm{dip})}}\right\}. \tag{30}
$$

This formula shows the role of magnetic emission on the basis of electric emission in the full coherent emission of photons.

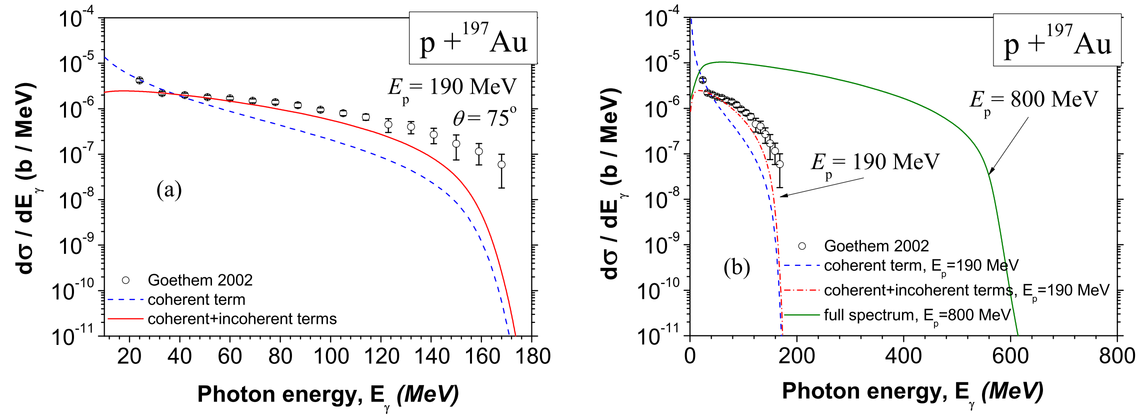

FIG. 1. (a) The calculated bremsstrahlung spectra (with coherent and incoherent terms) in the scattering of protons off the ^{197}Au nuclei at proton beam energy $E_p = 190$ MeV in comparison with experimental data [18] [matrix elements are defined in Eqs. (22)–(23), $Z_A(k_{\rm ph}) \simeq Z_A$ is the electric charge of the nucleus, and each calculated spectrum is normalized on the second point of experimental data]. Here, experimental data given by open circles (Goethem 2002) are extracted from Ref. [18], the blue dashed line is the coherent contribution defined by $M_p^{(E,\,{\rm dip})}$, and red solid line is the full spectrum with coherent and incoherent contributions defined by $M_p^{(E,\,{\rm dip})}$, $M_p^{(M,\,{\rm dip})}$, $M_{\Delta M}$, and M_k. (b) New calculated spectrum of full bremsstrahlung for ^{197}Au at proton beam energy $E_p = 800$ MeV in comparison with the bremsstrahlung spectrum for the same reaction at proton beam energy $E_p = 190$ MeV shown in (a).

From previous research it is known that increasing the energy of the proton beam in nuclear scattering increases intensity of bremsstrahlung emission. Indeed, new calculations for the same scattering $p + {}^{197}$Au but atproton beam energy $E_p =$ 800 MeV are presented in Fig. 1(b), confirming this property of bremsstrahlung. Also from this figure one can see that the difference in intensity of bremsstrahlung for these two cases is significant.

The ratio of the full incoherent contribution to the full coherent contribution in the bremsstrahlung emission during this scattering process at $E_p = 190$ MeV is shown in Fig. 2. From this result one can see that role of incoherent emission is essentially larger than coherent emission.

Let us analyze how much the bremsstrahlung spectra are different for different nuclei. Such calculations for the ^{12}C, ^{40}Ca, ^{197}Au, ^{208}Pb nuclei at proton beam oenergy $E_p =$ 800 MeV are presented in Fig. 3. From this figure one can conclude that the probability of emission is larger for heavier nuclei, and the difference between the spectra for light and heavy nuclei is significant.

Now suppose the possibility of formation of Δ resonance in the target nucleus and emission of photons in the scattering, following Ref. [6].[4] One can estimate how much the bremsstrahlung emission is changed after inclusion of Δ resonance in the formalism. For that, one can suppose that such photons are emitted by protons in the beam and the modified nucleus, where transition of one nucleon to Δ resonance takes place (the transition $pN \to \Delta^+ N$ in the nucleus is taken for analysis in this paper, while another case, $nN \to \Delta^0 N$, can be studied in an analogous way).

The first question is, in which nuclei is it most convenient to find a larger difference between the spectra for normal nuclei and nuclei with included Δ resonance? Analyzing the formalism, one can find that such difference can be from

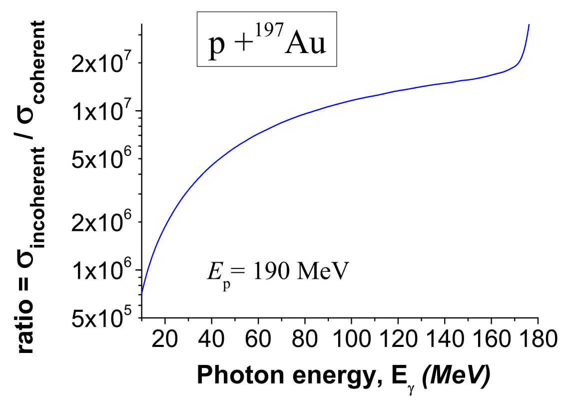

FIG. 2. Ratio between incoherent and coherent bremsstrahlung contributions in the scattering of protons off the ^{197}Au nuclei at proton beam energy $E_p = 190$ MeV [matrix elements are defined in Eqs. (22)–(23); $Z_A(k_{\rm ph}) \simeq Z_A$ is the electric charge of the nucleus]. In the coherent bremsstrahlung, the magnetic emission based on $M_p^{(M,\,{\rm dip})}$ is almost the same as electric emission based on $M_p^{(E,\,{\rm dip})}$: $\frac{\sigma_{\rm mag}^{\rm (coh)}}{\sigma_{\rm el}^{\rm (coh)}} = 3.3213$ is obtained in the full energy region of photons. In the incoherent bremsstrahlung, the role of background emission based on M_k is a little larger than the magnetic contribution based on $M_{\Delta M}$: $\frac{\sigma_{\rm background}^{\rm (incoh)}}{\sigma_{\rm mag}^{\rm (incoh)}} = 4.04$ is obtained in the full energy region of photons.

[4]However, the formalism in Ref. [6] does not include quantum fluxes, which can be useful in analysis of scattering. As an example, one can note the study of fusion in capture of α particles by nuclei (see Refs. [41,42] and references therein on that method and its applications to other reactions). In particular, the cross section of capture can be essentially changed in dependence on different variants of fusion and quantum fluxes inside the target nucleus. Those processes and effects are not taken into account in the nuclear formalism in Ref. [6].

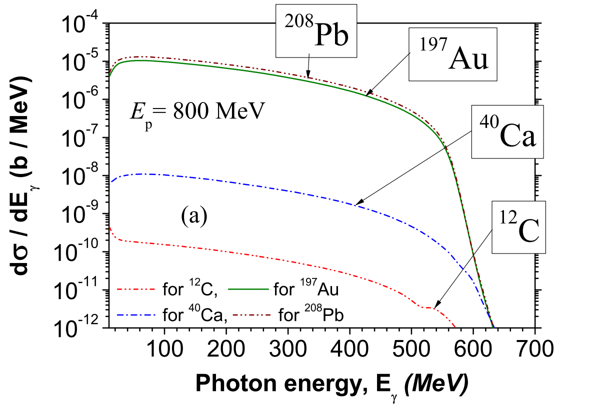

FIG. 3. The calculated full bremsstrahlung spectra in the scattering of protons off the ^{12}C, ^{40}Ca, ^{197}Au, ^{208}Pb nuclei at proton beam energy $E_p = 800$ MeV.

effective electric charge and magnetic moment of the proton-nucleus system. The difference between such characteristics is larger for lighter nuclei. However, the probability of photons for lighter nuclei is smaller (see Fig. 3). So, the situation is unclear for finding the proper direction. In Fig. 4 calculations of the spectra for the ^{12}C, ^{40}Ca nuclei are shown in comparison with the $^{12}_{\Delta}$C, $^{40}_{\Delta}$Ca nuclei. From such results one can conclude that the bremsstrahlung emission after formation of a Δ resonance from one of the nucleons of the nucleus does not change much betweem light and heavy nuclei, in general.

B. Nuclei with highest enhancement of bremsstrahlung due to formation of Δ resonance

One can find nuclei where the transition from a nucleon of the nucleus to a Δ resonance maximally reinforces the bremsstrahlung emission in the proton-nucleus scattering. Parameters important in this task are mass of the Δ resonance, its spin, and its magnetic moment. The change of mass of a baryon in transition from nucleon to Δ resonance is not very sensitive in calculations of the bremsstrahlung probability, so it will be ignored for clearer understanding of the model. From analysis of the model above one can find that the matrix elements $M_{\Delta M}$ and M_k defined in Eqs. (23) give the largest contributions to the bremsstrahlung emission. On such a basis, from Eqs. (23) one can find that the most important parameter in this task is $\bar{\mu}_{pn}$. One can see that the highest change of the probability of bremsstrahlung at the transition from nucleon to Δ resonance is in case when $\bar{\mu}_{pn}$ is minimal for a normal nucleus. In particular, this is a condition when $\bar{\mu}_{pn}$ is equal to zero for a normal nucleus, and

$$\frac{Z}{N} = \frac{\mu_p}{|\mu_n|} = \frac{2.792\,847\,34}{1.913\,042\,73} = 1.459\,898, \tag{31}$$

where Z and N are numbers of protons and neutrons for the normal target nucleus. From Eq. (31) one can find the simpler condition $Z > N$. Note that unstable nuclei mainly satisfy this condition and so cannot be used as targest. However, condition (31) indicates a limit, which can be used in analysis to find the proper nucleus from stable isotopes. One can consider Eq. (31) as condition of the highest enhancement of the bremsstrahlung emission due to production of Δ resonance in the target nucleus.

For example, let us look at isotopes of carbon. Nucleus ^{12}C does not satisfy the simpler condition $Z/N = 1$. But, unstable nuclear system ^{10}C satisfies to that condition, and even condition (31) ($Z/N = 1.5$ for ^{10}C). Results of calculations of the bremsstrahlung spectra for ^{10}C and $^{10}_{\Delta}$C are shown in Fig. 5. In this figure one can see that the spectrum for $^{10}_{\Delta}$C is larger than the spectrum for ^{10}C. Comparing with results in Fig. 4(a), one can see that the difference between the spectra for normal nucleus ^{10}C and Δ nucleus $^{10}_{\Delta}$C is larger than the difference between the spectra for ^{12}C and $^{12}_{\Delta}$C.

C. Bremsstrahlung emission for short-lived Δ resonance in a nucleus

The analyzed maximal enhancement of bremsstrahlung emission presented above is difficult to realize in experiments, as condition $Z > N$ is not satisfied for stable nuclei. Moreover, a Δ resonance is a short-lived state of a baryon (mean lifetime is about 5.58×10^{-24} s). However, one can recall the possibility of emission of bremsstrahlung photons during tunneling in α decay of heavy nuclei analyzed in

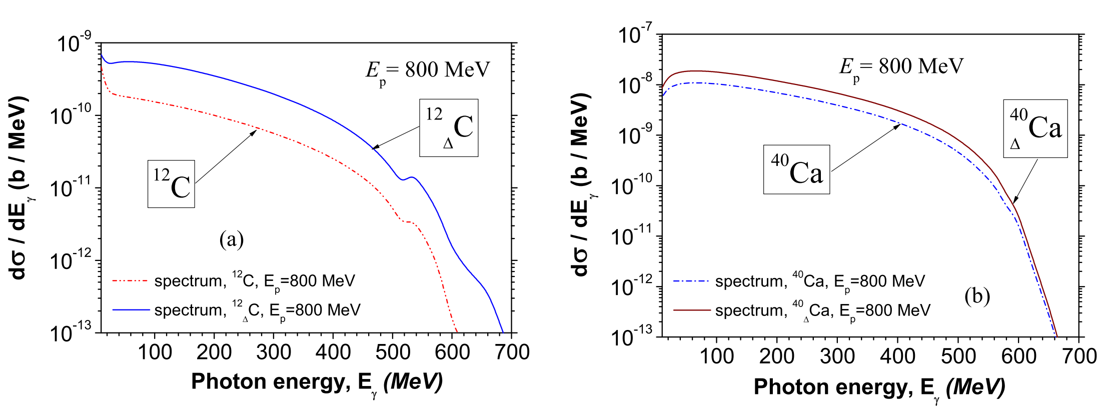

FIG. 4. The calculated full bremsstrahlung spectra in the scattering of protons off the ^{12}C, $^{12}_{\Delta}$C nuclei (a) and ^{40}Ca, $^{40}_{\Delta}$Ca nuclei (b) at proton beam energy $E_p = 800$ MeV.

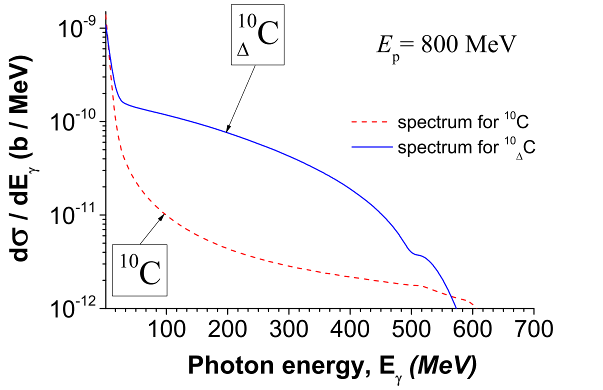

FIG. 5. The calculated bremsstrahlung spectra in the scattering of protons on ^{10}C and $^{10}_{\Delta}$C at proton beam energy $E_p = 800$ MeV. One can see that the difference between the spectra for ^{10}C and $^{10}_{\Delta}$C is larger than the difference between the spectra for ^{12}C and $^{12}_{\Delta}$C shown in Fig. 4(a).

Refs. [8–11]. As shown in Ref. [9] (see Fig. 4 in that paper for ^{214}Po and ^{226}Ra), exclusion of the possibility of emission of photons from the external space region outside the potential barrier of α decay does not decrease the full bremsstrahlung spectrum, but but has the opposite effect. In particular, there is destructive interference between bremsstrahlung emission from the tunneling region and bremsstrahlung emission from the external region outside the barrier. Such an effect for the α decay of the ^{226}Ra nucleus is reproduced in Fig. 6(a). In this figure one can see that emission from the tunneling region (green dashed line in that figure) is larger than the full bremsstrahlung emission (blue solid line in that figure) at higher photon energies. But the full bremsstrahlung emission is in good agreement with experimental data that confirms existence of such an effect of destructive interference. This picture is general for bremsstrahlung in the α decay of different nuclei.

This property can be used in the current research of study of Δ resonances in nuclei. An Δ resonance is a short-lived baryon, so it is formed in the target nucleus only during propagation of the scattered proton (from the beam) through the space region of this nucleus. So, only this space region of the proton-nucleus system will be taken into account in calculation of matrix elements of emission. The energy of a proton in the beam is much larger than barrier of the proton-nucleus potential. However, in calculation of the bremsstrahlung matrix elements two wave functions of proton-nucleus scattering are used: these are wave functions in states before and after emission of a photon. In particular, after emission of a photon, energy of relative motion of the proton with respect to to nucleus is reduced by the energy of the photon emitted. So, at enough high energies of the emitted photons the wave function in the final state reaches below-barrier energies, with tunneling through the barrier. So, in the high energy region of photons there is phenomenon of destructive interference between contributions of emission from the tunneling region and from the external region outside barrier. In particular, on the basis of this logic one can suppose that the spectrum for the short-lived Δ resonance in the target nucleus should be larger in the high energy photon region than in the spectrum of the full emission in the previous figures. Results of such calculations of the spectrum for $^{12}_{\Delta}$C in comparison with previous results are shown in Fig. 6(b). Similar calculations of the spectra for the $^{40}_{\Delta}$Ca and $^{208}_{\Delta}$Pb nuclei are presented in next in Fig. 7. Calculations in these figures confirm the above conclusions.

D. Correction of the spectra on the basis of difference in masses for nucleon and Δ resonance

The Δ resonance is nearly 300 MeV heavier than the nucleon. This mass difference is of the same order of magnitude as the photon and incoming-proton energies under study. One can analyze whether neglecting this difference in masses is a good approximation.

The reduced mass of proton-nucleus system is a main parameter, which is changed after inclusion of a correction such as the difference between masses for proton and Δ resonance. From previous study, one concludes that relative motion of proton (in beam) and nucleus is the most important process

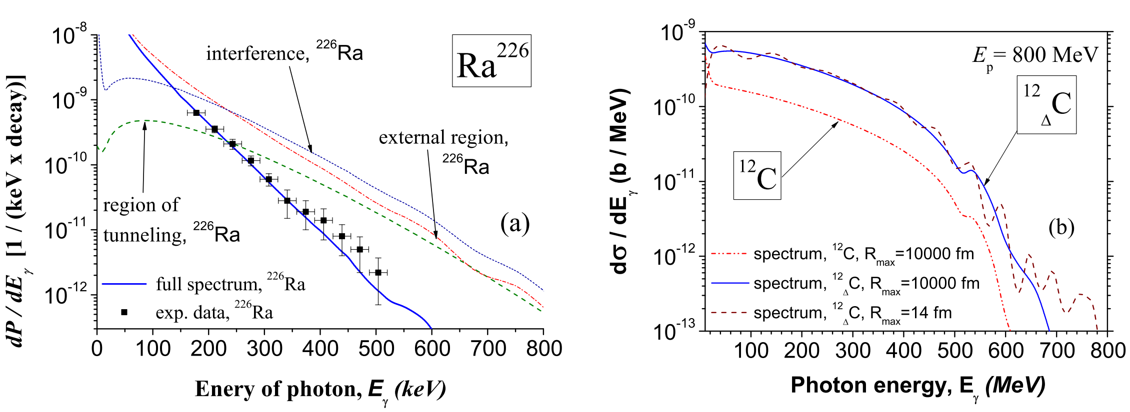

FIG. 6. (a) The calculated bremsstrahlung contributions in α decay of the ^{226}Ra nucleus. (b) The calculated bremsstrahlung spectra in the scattering of protons on the ^{12}C and $^{12}_{\Delta}$C nuclei at proton beam energy $E_p = 800$ MeV in different space regions of integration. One can see that the spectrum for $R_{\max} = 14$ fm in the high energy region (red dashed line) is larger than the spectra in the full space region (brown solid line).

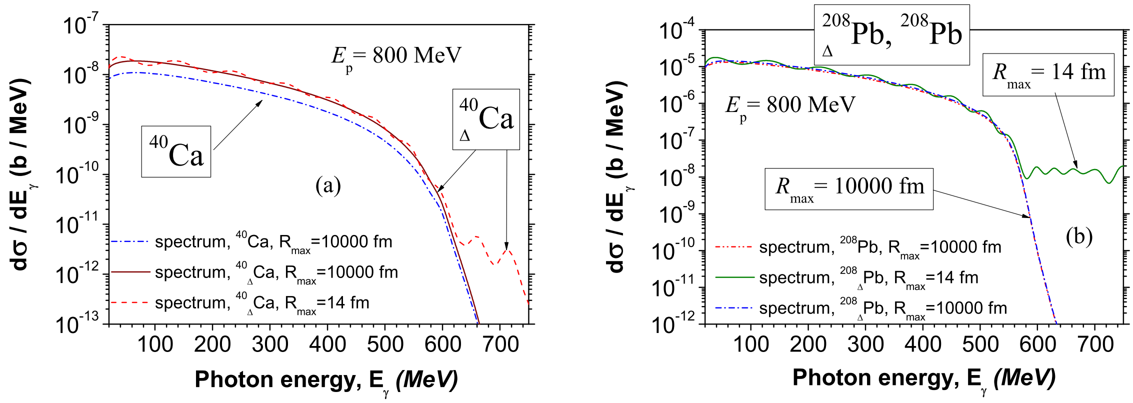

FIG. 7. The calculated bremsstrahlung spectra in the scattering of protons on the ^{40}Ca and $^{40}_{\Delta}$Ca nuclei (a) and the ^{208}Pb and $^{208}_{\Delta}$Pb nuclei (b) at proton beam energy $E_p = 800$ MeV in different space regions of integration. One can see that the spectrum for at $R_{\max} = 14$ fm in the high energy region is larger than the spectra in the full space region.

forming the largest emission of photons. Here, reduced mass is an important parameter (it is used in calculations of wave functions of relative motion for states before and after emission of photons, and also in formulas for operator of emission of photons). One can consider the two following cases.

(i) One of the protons of the nucleus is changed to a Δ resonance. The new reduced mass $\mu^{(\mathrm{cor},\,1)} = m_p\, m_A^{(\mathrm{cor})}/(m_p + m_A^{(\mathrm{cor})})$ is obtained, where $m_A^{(\mathrm{cor})} = m_\Delta + (Z-1)m_p + (A-Z)\,m_{\mathrm{n}}$.

(ii) A proton from the beam is changed to a Δ resonance. The new reduced mass $\mu^{(\mathrm{cor},\,2)} = m_\Delta m_A/(m_\Delta + m_A)$ is obtained.

Here, m_p, m_{n}, m_Δ are masses of proton, neutron, and Δ resonance, m_A is the mass of the nucleus, and Z and A are numbers of protons and nucleons in the nucleus. Calculations of the bremsstrahlung spectra on the basis of these two corrected reduced masses are presented in the Fig. 8(a) for $^{12}_{\Delta}$Ca [for comparison, two old spectra from Fig. 4(a) are used also]. The spectrum for the first case almost coincides with the old spectrum for $^{12}_{\Delta}$Ca [see the purple dash-dotted line in Fig. 8(a)] (the difference between the two spectra is in the second or third digits). But, the spectrum for the second case is visibly different from the previous spectrum for $^{12}_{\Delta}$Ca [see the green dashed line in Fig. 8(a)]. The difference between old spectra [in Fig. 4(a)] is larger.

Another transition, $pN \to \Delta + N$, is also possible in this problem, in which the final nucleon can be different from the initial one (e.g., $pp \to \Delta^{++}n$). To analyze this transition, one can consider the two following processes.

(i) Two protons of the nucleus are changed to a Δ^{++} resonance and a neutron in this nucleus.

(ii) A proton from the beam is changed to a Δ^{++} resonance, and one neutron of the nucleus is changed to a proton.

One could suppose that the second case essentially changes the bremsstrahlung spectrum. Interesting analysis can be obtained from such modifications of the reaction under study. The electric charge of the scattered fragment is increased almost twice. As a result, the effective electric charge Z_{eff} of the nuclear system is increased almost 4 times in dependence on the target nucleus (i.e., $Z_{\mathrm{eff}}^{(\mathrm{new})} = 1.304\,065$ and $Z_{\mathrm{eff}}^{(\mathrm{old})} = 0.303\,992\,8$ for $p + {}^{12}$C). This effective charge is included

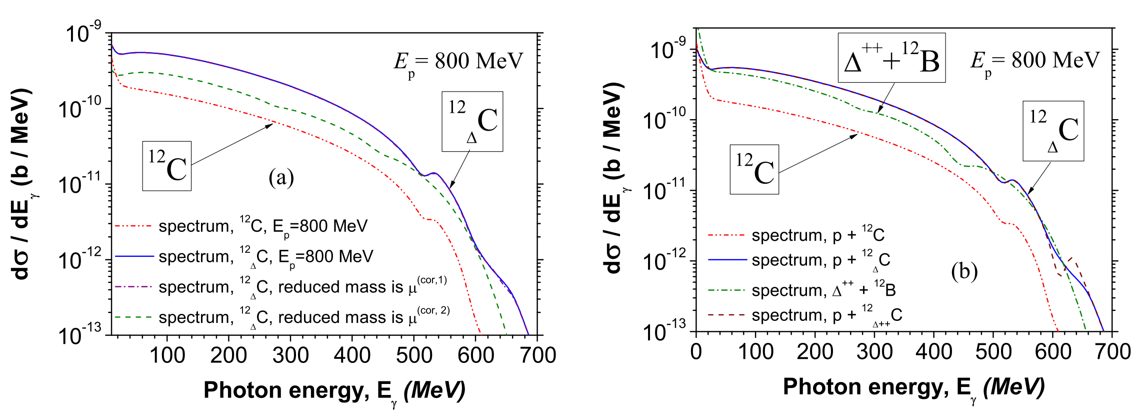

FIG. 8. The calculated full bremsstrahlung spectra in the scattering of protons off ^{12}C, $^{12}_{\Delta^+}$C, $^{12}_{\Delta^{++}}$C nuclei (a) and in the scattering $\Delta^{++} + {}^{12}$B (b) at beam energy $E = 800$ MeV. (a) Corrections of the spectra on the basis of taking into account difference between masses of Δ resonance and nucleon in calculations. (b) The new spectra at transition $pp \to \Delta^{++}n$ in the nuclear system (see text for explanation).

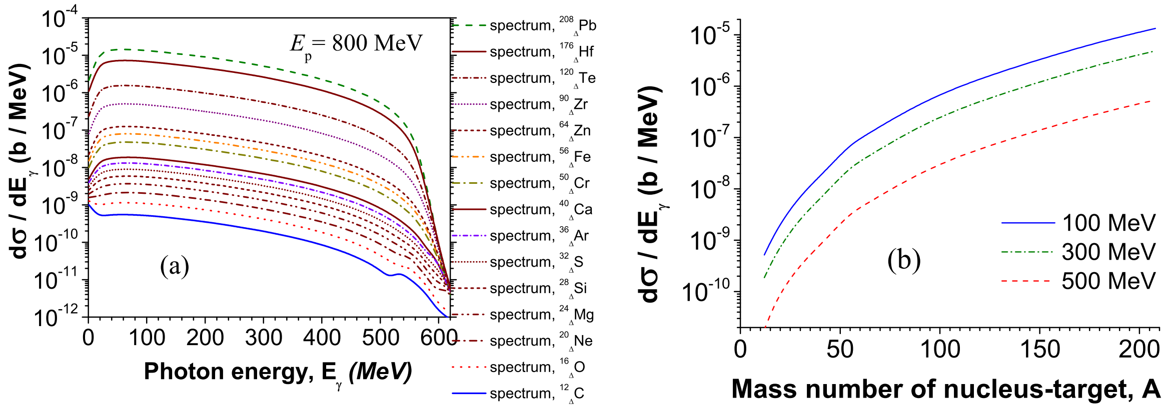

FIG. 9. The calculated full bremsstrahlung spectra in dependence on mass of the target nucleus (with included Δ resonance) in the scattering of protons at proton beam energy $E_p = 800$ MeV.

in the matrix element of coherent bremsstrahlung emission [see Eqs. (24)–(28)]. So, this contribution is increased almost 4 times. However, the incoherent bremsstrahlung contribution is larger than the coherent one. The incoherent term does not include effective charge. So, in summary the full spectrum is not deformed much. This situation is shown by the green dash-dotted line in Fig. 8(b). In short, one can suppose that the magnetic field of the nuclear system is more important than the electric field in emission of photons.

The first process is calculated for $p + [\delta^{++}]C$ at beam energy $E_p = 800$ MeV, and is shown in Fig. 8(b) by the brown dashed line. One can see that this spectrum is very close to blue solid line in this figure.

Calculations of the bremsstrahlung spectra in dependence on mass of the target nucleus (where Δ resonance is included) are presented in Fig. 9. One can see that the cross section of bremsstrahlung increases monotonically with increasing mass of the target nucleus. The wave function of relative motion is calculated on the basis of the proton-nucleus potential, which is determined using numbers of protons and neutrons in the target nucleus, according to Ref. [43] (that potential is also extrapolated for the region of light nuclei). Dependence of the bremsstrahlung spectra on the numbers of protons and neutrons in the target nucleus can be explained by (1) calculations of proton-nucleus wave functions on the basis of such a potential, and (2) dependence of the operator of emission of photons on the numbers of protons and neutrons in the target nucleus.

E. Bremsstrahlung emission with taking into account production of Δ resonances during proton-nucleus scattering

Previously, emission of bremsstrahlung photons was studied at a stage when Δ resonance had already been produced in the proton-nucleus scattering. However, an important question is also how much the process of production of Δ resonances during scattering changes the bremsstrahlung spectra estimated above. In this section these processes are analyzed and estimated.

Note that a detailed theory describing processes of production of Δ resonances in nucleon-nucleon scattering and in nuclear matter for light nuclei has been developed (for example, see Refs. [2,44]; also Ref. [45] for elastic and inelastic collisions, and references therein). So, in this paper the focus is on the connection between a current approach including a detailed study of the coherent and incoherent processes in bremsstrahlung and that theory. Let us estimate changes of the bremsstrahlung spectra on the example reaction $p\,p \to n\,\Delta^{++}$; for that a deep analysis and formalism can be found in Ref. [44] (this Δ resonance in bremsstrahlung emission was studied in Sec. III D above; see the calculated spectra in Fig. 8 for $p + [\delta^{++}]C$).

Following Ref. [44] [see Eqs. (4) and (7) in that paper], the matrix element is calculated for that reaction, squared and summed over polarizations of the final particles, and averaged over the polarizations of the initial particles for direct π exchange as

$$M^2(t) = \frac{1}{4}\left(-\sqrt{2}\,\frac{g_\pi\, f_\pi^*}{m_\pi}\right)^2 \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}\right)^4 \frac{-2\,t}{\left(t - m_\pi^2\right)^2} \times \frac{[(m_\Delta + m_p)^2 - t]^2}{3m_\Delta^2}\,[(m_\Delta - m_p)^2 - t], \qquad (32)$$

where parameters are (also see Ref. [2])

$$g_\pi = 13.61, \quad \frac{f_\pi}{m_\pi} = \frac{g_\pi}{2m}, \quad \frac{f_\pi^*}{f_\pi} = 2.0, \quad \Lambda = 650\ \text{MeV}/c. \qquad (33)$$

The expression of the matrix element for the exchange process in the Δ production in pp collisions can be obtained from Eq. (32) by the replacement $t \to u = (k_2 - k_3)^2$. Direct and exchange graphs for Δ production are shown in Fig. 1 in Ref. [44]. The interference term between direct and exchange matrix elements can be found in Eq. (8) in Ref. [44] (for brevity, these processes are omitted in the analysis).

Matrix element M characterizes the amplitude of transition from the initial state with two nucleons to the final state with a nucleon and the produced Δ resonance (these nucleons and Δ resonance are described by one-nucleon wave functions in the formalism of this paper). Taking into account the many-nucleon representation of the full wave function for proton-nucleus scattering above (see Sec. II D), one can determine the cross section of bremsstrahlung emission at the initial stage

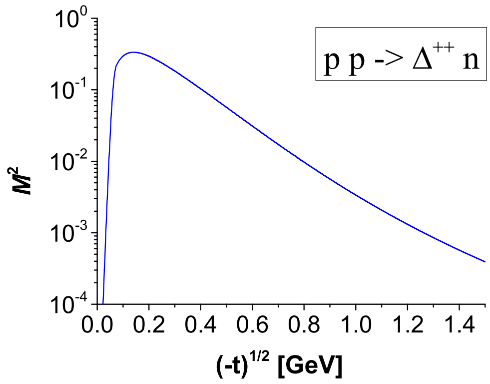

FIG. 10. The squared matrix element M^2 in dependence on four-momentum transfer t in the center-of-mass frame for the reaction $p\,p \to n\,\Delta^{++}$ [the squared matrix element is defined in Eq. (32)].

before production of the Δ resonance in the proton-nucleus scattering as

$$\frac{d\sigma}{dw_{\rm ph}} = |M(t)|^2 \frac{d\sigma^{\rm (old)}}{dw_{\rm ph}}, \quad \frac{d\sigma^{\rm (old)}}{dw_{\rm ph}} = \frac{e^2}{2\pi\, c^5}\, \frac{w_{\rm ph}\, E_i}{m_p^2\, k_i}\, |p_{fi}|^2, \tag{34}$$

where $d\sigma^{\rm (old)}/dw_{\rm ph}$ is the bremsstrahlung cross section defined in Eqs. (29) with included Δ resonance in the proton-nucleus scattering.

Note that the matrix element M in Eq. (32) is defined in the center-of-mass frame, while energy E_p of the proton beam for the proton-nucleus scattering is defined in the laboratory frame. Energy $E_{\rm cmf}$ of the relative motion of a proton in beam and the target nucleus in the center-of-mass frame (squared four-momentum transfer t is related with this energy after taking into account the number of nucleons in the nucleus) is connected with E_p as $E_{\rm cmf} = A/(1 + A)E_p$, where A is the mass number of the target nucleus. So, the difference for these energies for center-of-mass frame and laboratory frame is small for heavy nuclei (for example, for target nucleus ^{208}Pb one can obtain $E_{\rm cmf} = 208/209 \times E_p = 0.995\,215\,3\, E_p$). So, for heavy nuclei one can use $E_{\rm cmf} \approx E_p$ for estimations with good accuracy.

Calculations of the squared matrix element M^2 in dependence on four-momentum transfer in the center-of-mass frame are presented in Fig. 10 (experimental cross sections for this process with production of the Δ resonance can be described on the basis of such a distribution of the squared matrix element; see Fig. 5 in Ref. [2]). The bremsstrahlung spectra taking into account production of Δ resonances during the scattering of protons off the ^{40}Ca and ^{208}Pb nuclei are presented in Fig. 11. From these calculations one can see that, after taking production of Δ resonances into account, the bremsstrahlung cross section is decreased. However, the role of the incoherent bremsstrahlung processes in the full bremsstrahlung is essentially larger (for example, see the ratio in Fig. 2 for $p + {}^{197}$Au for 190 MeV proton beam), and their inclusion enhances the full spectrum even after inclusion of production of Δ resonances in proton-nucleus scattering.

IV. CONCLUSIONS AND PERSPECTIVE

In this paper emission of bremsstrahlung photons in the scattering of protons off nuclei in the Δ-resonance energy region is investigated. A focus is on the question of how much the bremsstrahlung spectrum is changed after transition of one nucleon in a nucleus to a Δ resonance. The previous bremsstrahlung formalism is improved (see Ref. [46] and references therein), including properties of the Δ resonance in the target nucleus. On such a basis the following aspects are found.

(i) The bremsstrahlung spectra in the scattering of protons on the ^{12}C, ^{40}Ca, ^{208}Pb nuclei at proton beam energy E_p of 800 MeV are calculated (see Fig. 3). Coherent and incoherent bremsstrahlung contributions are included in the calculations, and the formalism and calculations are tested for the ^{197}Au nucleus at $E_p = 190$ MeV on the experimental data [18] [see Fig. 1(a)]. Incoherent emission is essentially more intensive than coherent emission (see Fig. 2; the ratio between such contributions is about 10^6–10^7 for

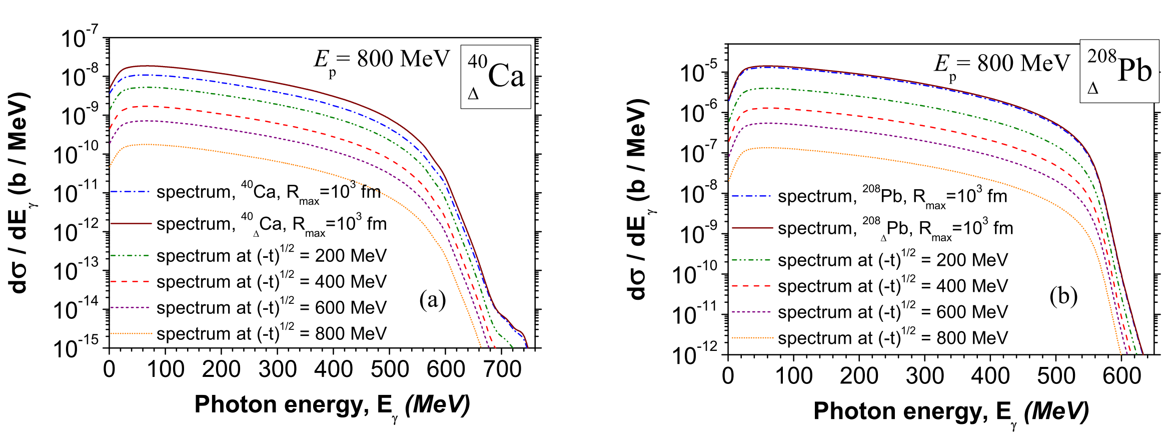

FIG. 11. The calculated bremsstrahlung spectra in dependence on four-momentum transfer t in the scattering of protons off ^{40}Ca (a) and ^{208}Pb (b) at proton beam energy $E_p = 800$ MeV [the bremsstrahlung cross section is defined in Eq. (34) and the squared matrix element is defined in Eq. (32)].

$p + {}^{197}$Au at $E_p = 190$ MeV, which is in agreement with results in Refs. [19,22,23]). This incoherent contribution is dependent on magnetic moments of nucleons of the target nucleus [see Eqs. (22), (23), and (26)]. This confirms that inclusion of incoherent processes in the study of Δ resonances in proton-nucleus scattering is important. This aspect has not been studied yet.

(ii) It is shown that increasing the energy of the proton beam in the nuclear scattering increases the intensity of bremsstrahlung emission [see Fig. 1(b) for a comparison and test with experimental data]. Bremsstrahlung emission is larger for heavier nuclei, and the difference between the spectra for light and heavy nuclei is significant (see Fig. 3 for ^{12}C, ^{40}Ca, ^{197}Au, ^{208}Pb; the difference is about 10^5 times between the spectra for ^{208}Pb and ^{12}C at $E_p = 800$ MeV).

(iii) The coherent and incoherent contributions and the electric and magnetic contributions in the full bremsstrahlung are analyzed for different nuclei and proton beam energies [see Eqs. (22)–(23)]. In the coherent bremsstrahlung, the magnetic emission is almost the same as electric emission [$\sigma_{\rm mag}^{\rm (coh)}/\sigma_{\rm el}^{\rm (coh)} = 3.3213$ for ^{197}Au at $E_p = 190$ MeV for 10–180 MeV photons; see Eq. (30)]. In the incoherent bremsstrahlung, the role of background emission based on M_k is a little larger than that of the magnetic contribution based on $M_{\Delta M}$ ($\sigma_{\rm background}^{\rm (incoh)}/\sigma_{\rm mag}^{\rm (incoh)} = 4.04$ for ^{197}Au at $E_p = 190$ MeV for 10–180 MeV photons). The ratio between incoherent emission and coherent emission is increased with increasing photon energy (see Fig. 2, for ^{197}Au at $E_p = 190$ MeV).

(iv) To include in the model the transition from proton to Δ^+ resonance ($pN \to \Delta^+ N$) in the target nucleus in proton-nucleus scattering, the scheme of coherent processes from Ref. [6] is used. It is found that emission of bremsstrahlung photons in reactions with one Δ resonance in the target nucleus is more intensive than for a normal nucleus (see Fig. 4 for ^{12}C, $^{12}_{\Delta}$C and ^{40}Ca, $^{40}_{\Delta}$Ca at $E_p = 800$ MeV). This result confirms reinforcement of bremsstrahlung in proton-nucleus scattering. It is found that the difference between the spectra for normal nuclei and nuclei with an Δ resonance is larger for light nuclei, but the spectra are larger for heavier nuclei (see Fig. 4 for ^{12}C, ^{40}Ca in comparison with $^{12}_{\Delta}$C, $^{40}_{\Delta}$Ca).

(v) To find nuclei with maximal reinforcement of bremsstrahlung due to transition $pN \to \Delta^+ N$, condition (23) using ratio between protons and neutrons for a normal nucleus is obtained. Using the example of $^{10}_{\Delta^+}$C, it is shown that the difference between the spectra for ^{10}C and $^{10}_{\Delta^+}$C is essentially larger than the difference between the spectra for ^{12}C and $^{12}_{\Delta^+}$C (see Fig. 5). However, stable nuclei (like ^{12}C, ^{40}Ca, and ^{208}Pb) do not satisfy condition (23). As parameter $\bar{\mu}_{pn}$ has the highest influence on reinforcement of bremsstrahlung of such a type, it confirms the importance of incoherent processes (i.e., there is no enhancement of bremsstrahlung due to the transition $pN \to \Delta^+ N$, if incoherent processes are not included in the model).

(vi) One can take into account the property of a Δ resonance as a short-lived baryon, which is formed in the target nucleus during propagation of the scattered proton (from the beam) through the space region of the nucleus. On such a basis it is found that the spectrum for the short-lived Δ resonance in the target nucleus is larger in the high energy photon region than the spectrum without production of Δ resonance in the target nucleus [see Figs. 6(b) and 7 for $^{12}_{\Delta}$C, $^{40}_{\Delta}$Ca, and $^{208}_{\Delta}$Pb). This effect has the same origin as the phenomenon of destructive interference between emission from the tunneling region and the external region investigated for bremsstrahlung in the α decay [see Fig. 6(a) for ^{226}Ra, and also Ref. [9]).

Comparing the spectra for normal nuclei with the spectra for nuclei with the short-lived Δ resonance, one can see an essential difference between these spectra in the high energy region of photons. This property can be used for proposal of future experiments with measurements of photons as tools to distinguish processes of formation of Δ resonances in the target nuclei.

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