Ab initio calculation of charge-symmetry breaking in $A = 7$ and 8 Λ hypernuclei

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The Λ separation energies of the isospin triplet ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$Be, and the $T = 1/2$ doublet ${}^{8}_{\Lambda}$Li, ${}^{8}_{\Lambda}$Be are investigated within the no-core shell model. Calculations are performed based on a hyperon-nucleon potential derived from chiral effective field theory at next-to-leading order. The potential includes the leading charge-symmetry breaking (CSB) interaction in the ΛN channel, whose strength has been fixed to the experimentally known difference of the Λ separation energies of the mirror hypernuclei ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H. It turns out that the CSB predicted for the $A = 7$ systems is small and agrees with the splittings deduced from the empirical binding energies within the experimental uncertainty. In the case of the $A = 8$ doublet, the computed CSB is somewhat larger than the available experimental value.

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I. INTRODUCTION

Charge symmetry breaking (CSB) in Λ hypernuclei has been experimentally established for many decades. The first and probably most pronounced evidence came from the difference of the Λ-separation energies of the mirror nuclei ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H [1,2], eventually followed by data on other Λ-hypernuclei isospin multiplets up to $A = 16$ [3–6], see also [7,8]. However, a solid theoretical understanding of the CSB effects has been lacking for a long time. Certainly, one of the possible CSB mechanisms, namely Λ-Σ^0 mixing, had already been identified and investigated at an early stage [1]. That mechanism facilitates pion exchange between the Λ and the nucleons, otherwise forbidden by isospin conservation, and thus yields a long-ranged CSB force. However, with Λ-Σ^0 mixing alone, commonly included in elaborate hyperon-nucleon (YN) potentials like those of the Nijmegen group [9], no quantitative description of the observed CSB in the ground (0^+) and excited (1^+) states of ${}^{4}_{\Lambda}$He - ${}^{4}_{\Lambda}$H could be achieved [10]. One could attribute that to the fact that the separation-energy difference $\Delta B_\Lambda(0^+) = B_\Lambda^{0^+}({}^{4}_{\Lambda}\text{He}) - B_\Lambda^{0^+}({}^{4}_{\Lambda}\text{H})$ of 340 keV [3] and $\Delta B_\Lambda(1^+) = 240$ keV [11] accepted at that time are exceptionally large when compared to those found for, say, the mirror nuclei ^{3}H and ^{3}He of about 80 keV after the Coulomb-energy correction [12]. Indeed, they were also large when compared to the CSB effects found for heavier Λ hypernuclei with $A = 7$ and $A = 8$. In fact, cluster model calculations for $A = 7$–10 mirror hypernuclei [13–15], which implemented phenomenological ΛN CSB forces that were tuned to the splittings found for ${}^{4}_{\Lambda}$He - ${}^{4}_{\Lambda}$H, overestimated the CSB splittings for the heavier systems and/or predicted shifts in the wrong direction.

In this work, we present a calculation of the binding energies for the isotriplet ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$Be (${}^{7}_{\Lambda}$Li* denotes the excited state of ${}^{7}_{\Lambda}$Li with isospin $T = 1$), as well as of the $A = 8$ doublet ${}^{8}_{\Lambda}$Li, ${}^{8}_{\Lambda}$Be. The study is motivated by the significant experimental and theoretical progress that has been made since the last extended calculation by Hiyama *et al.* [13]. On the experimental side, there has been a reliable determination of the binding energy of the ${}^{7}_{\Lambda}$He hypernucleus [16]. Moreover, and more importantly, there has been a re-evaluation of CSB in the ${}^{4}_{\Lambda}$He - ${}^{4}_{\Lambda}$H systems. Refined data from experiments at J-PARC [17] and Mainz [18,19] that became available in the years 2015 and 2016 established the splittings to be $\Delta B_\Lambda(0^+) = 233 \pm 92$ keV and $\Delta B_\Lambda(1^+) = -83 \pm 94$ keV [5,20]. Thus, there is a sizable reduction of the CSB effect in the 0^+ state as compared to the former value. In the 1^+ state there is even a change in the sign, and the new value is practically compatible with zero.

With regard to theory, a consistent description of the charge-symmetry preserving and CSB components of the ΛN interaction has been achieved within chiral effective field theory (EFT) applying an appropriate power counting. The resulting potentials yield an excellent description of the available low-energy Λp and ΣN data [21,22]. Earlier studies usually omitted the CSB contact interactions leading to significant dependence of the predictions of the CSB for $A = 4$ hypernuclei on details of the interactions [20]. This problem could be resolved by taking the CSB contact interactions into account and fixing them using the $A = 4\Lambda$-hypernuclei

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data [23]. In addition, microscopic "*ab initio*" calculations of hypernuclei up to $A = 8$ and beyond are feasible now, say, within the no-core shell model (NCSM) [24–27]. As input elementary YN interactions can be used, together with sophisticated nucleon-nucleon (NN) and three-nucleon ($3N$) forces. Specifically, the important coupling between the ΛN and ΣN systems can be fully taken into account and, of course, CSB which induces differences in the Λp and Λn interactions.

The paper is structured in the following way. In Sec. II, we give a brief account of the employed YN interactions. Specifically, we explain how the CSB part is determined from the separation-energy differences in the 0^+ and 1^+ states of the $A = 4$ hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H. In Sec. III, we summarize the treatment of the $A = 4$–8 hypernuclei within the Jacobi no-core shell model. Our results and a discussion of the CSB effects are presented in Sec. IV. Some further details are relegated to the Appendix. The paper ends with a brief summary.

II. HYPERON-NUCLEON INTERACTION INCLUDING CSB

For the present study, we utilize the YN interactions from Refs. [21,22], derived within SU(3) chiral EFT at next-to-leading order (NLO). At that order of the chiral expansion, the YN potential consists of contributions from one- and two-pseudoscalar-meson exchange diagrams (involving the Goldstone boson octet π, η, K) and from four-baryon contact terms without and with two derivatives. The two YN interactions are the result of pursuing different strategies for fixing the low-energy constants (LECs) that determine the strength of the contact interactions. In the YN interaction from 2013 [21], denoted by NLO13 in the following, all LECs have been fixed exclusively by a fit to the available ΛN and ΣN data. The other potential [22] (NLO19) has been guided by the objective to reduce the number of LECs that need to be fixed from the YN data by inferring some of them from the NN sector via the underlying (though broken) SU(3) flavor symmetry. A thorough comparison of the two versions for a range of cutoffs can be found in Ref. [22], where one can see that the two YN interactions yield essentially equivalent results in the two-body sector.

The YN potentials NLO13 and NLO19 do not include any explicit CSB contributions. However, in Ref. [23], we derived the leading CSB interaction within chiral EFT and added it to those YN interactions. At the order considered, CSB contributions arise from a nonzero $\Lambda\Lambda\pi$ coupling constant which is estimated from Λ-Σ^0 and π^0-η mixing, the mass difference between $K^\pm$ and K^0, and from two contact terms that represent short-ranged CSB forces. In the actual calculation, the two arising CSB low-energy constants (LECs) were fixed by considering the known differences in the energy levels of the 0^+ and 1^+ states of the aforementioned $A = 4$ hypernuclei. Then, by construction, the resulting interaction describes all low-energy Λp and ΣN scattering data, the hypertriton and the CSB in ${}^4_\Lambda$He and ${}^4_\Lambda$H accurately.

For a detailed discussion of the CSB effects we refer the reader to [23]. As a main outcome, it turned out that the reproduction of the splittings of $\Delta B_\Lambda(0^+) = 233 \pm 92$ keV and $\Delta B_\Lambda(1^+) = -83 \pm 94$ keV [5] (scenario CSB1 in [23]) requires a sizable difference between the strengths of the Λp and Λn interactions in the ${}^1S_0$ state, whereas the modifications in the ${}^3S_1$ partial wave are much smaller. The effects go also in opposite directions, i.e., while for ${}^1S_0$ the Λp interaction is found to be noticeably less attractive than that for Λn, in the case of ${}^3S_1$ it is slightly more attractive. In terms of the difference in the scattering lengths, $\Delta a^{\rm CSB} = a_{\Lambda p} - a_{\Lambda n}$, a value of 0.62 ± 0.08 fm has been predicted for the ${}^1S_0$ partial wave and -0.10 ± 0.02 fm for the ${}^3S_1$ [23].

Recently, the STAR collaboration has reported a new measurement for the $A = 4$ systems, which suggests somewhat different CSB splittings of the 0^+ and 1^+ states [28]. Their results are $\Delta B_\Lambda(0^+) = 160 \pm 140({\rm stat}) \pm 100({\rm syst})$ keV and $\Delta B_\Lambda(1^+) = -160 \pm 140({\rm stat}) \pm 100({\rm syst})$ keV. Of course, considering the sizable statistical and systematic uncertainties, those values are compatible with the ones cited above, so that quantitative conclusions cannot be drawn at present. Nevertheless, it is interesting to explore the implication of such a possible modification of the CSB in the $A = 4$ Λ hypernuclei for that in the $A = 7$ and 8 systems, though, in view of the uncertainties, we refrain from doing more elaborate calculations at present. Thus, we only readjust the two CSB LECs for the NLO19 potential in order to reproduce the central values of the STAR results. We find that the difference in the Λp and Λn scattering lengths is somewhat reduced in the ${}^3S_1$ partial wave, $\Delta a^{\rm CSB}({}^3S_1) = -0.05$ fm, whereas it slightly increases in the ${}^1S_0$ state, $\Delta a^{\rm CSB}({}^1S_0) = 0.71$ fm. This new set of CSB LECs will be referred to as CSB* and will be employed to explore the impact on the splittings in the $A = 7$ and 8 isospin multiplets. For a recent and detailed overview on the experimental situation regarding the CSB splittings in the ${}^4_\Lambda$He and ${}^4_\Lambda$H systems see Ref. [6].

As shown in previous bound-state calculations, the Λ separation energies of light hypernuclei are not very sensitive to the employed NN interaction [10,22]. Therefore, we use in all of the calculations presented here the same state-of-the-art chiral NN interaction, namely the semilocal momentum-space-regularized (SMS) NN potential of Ref. [29] at order N^{4}LO$^+$ with cutoff $\Lambda_N = 450$ MeV. Indeed, the variation of the separation energy for N^{4}LO$^+$ potentials with other cutoffs is of the order of 100 keV for ${}^4_\Lambda$He / ${}^4_\Lambda$H [22] and within the range expected from calculations based on phenomenological interactions [10]. A recent dedicated study performed within the NCSM, using however only N^{2}LO NN potentials and a LO YN interaction, reported uncertainties of around 100 keV for $A = 4$ and of around 400 keV for $A = 5$ hypernuclei [30]. Earlier, Wirth and Roth [31] found uncertainties of ≈200 keV and ≈400 keV for ${}^7_\Lambda$Li and ${}^9_\Lambda$Be, respectively, utilizing N^{3}LO and N^{4}LO NN potentials but also only LO for the YN interaction. In preliminary calculations, we observed that the sensitivity of the Λ separation energies to the employed NN interactions depends also on the YN interaction itself. For example, for ${}^3_\Lambda$H, we found variations of the separation energy of 18 keV when using the recent SMS N^{2}LO interactions and 13 keV when using the SMS N^{4}LO$^+$ interactions in conjunction with NLO19(650). With the same NN interactions the variation is 60 keV and 23 keV for the LO(650) YN interaction. The surprisingly large dependence of the variation of separation energies on the order of the chiral NN interaction and on the order of the YN interaction might in part explain why our estimate of the dependence of the separation energies on the NN interaction is smaller than other values available in the literature. A more detailed study on this issue is in progress but beyond the scope of this work. We also note that Ref. [30] found that the NN force dependence of the CSB in $A = 4$ hypernuclei is anyhow smaller due to correlations.

In order to accurately describe the parent nuclei, the chiral $3N$ interaction at order N^{2}LO with the regulator of $\Lambda_N = 450$ [32] is also included. Note that such a combination of the NN and $3N$ forces gives a fairly good description for the binding energies of light and medium mass nuclei [32]. It should, however, be stressed that although $3N$ forces contribute moderately to the nuclear and hypernuclear binding energies, their overall effect on the Λ separation energies and, in particular, on the CSB splittings is expected to be rather small for light hypernuclei and ground states of p-shell hypernuclei [22,23,33]. The inclusion of $3N$ forces can improve the description of excited states that are linked to an excited core nucleus.

III. JACOBI NO-CORE SHELL MODEL

We apply the Jacobi NCSM for calculating the Λ-separation (binding) energies of the $A = 4$–8 hypernuclei. A detailed description of the formalism and of the procedure to extract the binding (separation) energies can be found in Ref. [27]. In that reference, and in [34], one can also find results for $^{7}_{\Lambda}$Li based on the YN interactions NLO13 and NLO19 without CSB contribution. As already mentioned, for the current study, we shall employ chiral NN, $3N$, and YN potentials to describe the interactions among the nucleons and between a nucleon and a hyperon, respectively. In all calculations, contributions of the $NN(YN)$ potentials in partial waves up to $J = 6(5)$ are included, while for the $3N$ interaction all partial waves with total angular momentum $J_{3N} \leqslant 9/2$ are taken into account. It has been checked that higher partial waves only contribute negligibly compared to the harmonic oscillator (HO) model space uncertainties. In order to speed up the convergence of the NCSM with respect to the model space, all the employed NN, $3N$, and YN potentials are similarity renormalization group (SRG) evolved to a flow parameter of $\lambda = 1.88\ \text{fm}^{-1}$, see [27] and references therein. The latter is commonly used in nuclear calculations, which, on the one hand yields rather well-converged nuclear binding energies, and on the other hand, minimizes the possible contribution of SRG-induced $4N$ and higher-body forces [32]. Furthermore, in most of the calculations, the SRG-induced YNN interaction with the total angular momentum $J_{YNN} \leqslant 5/2$ is also explicitly included. Based on the contributions of $J_{YNN} \leqslant 1/2$, $3/2$, and $5/2$, the contribution from higher partial waves $J_{YNN} \geqslant 7/2$ is estimated to be negligibly small and therefore is omitted from the calculations. With the proper inclusion of these SRG-induced three-body forces, the otherwise strong dependence of the Λ separation energies B_Λ on the SRG-flow parameter [24,27] is largely removed [31,35].

It should be further noted that, for large systems like $A = 7$ and 8, the extrapolated NCSM separation energies are afflicted with appreciable uncertainties, see [27] and also Table III, which even exceed the experimentally found CSB splittings in these systems. Therefore, it is not advisable to estimate CSB based on the extrapolated separation energies. Instead, one can compute the CSB effects directly from the corresponding nuclear and hypernuclear energy expectation values for each model space $\mathcal{N}_{\max}$ and HO frequency ω. It has been observed that because of the correlations between those binding energies the directly extracted $\Delta B_\Lambda(\omega, \mathcal{N}_{\max})$ converges significantly faster with respect to $\mathcal{N}_{\max}$ and ω than the individual binding energies and to some extent the separation energies, so that a direct comparison with experiment is possible. Accordingly, the separation energies difference, say, for $A = 4$ systems, can be computed as

$$\begin{aligned}\Delta B_\Lambda &= B_\Lambda(^{4}_{\Lambda}\text{He}) - B_\Lambda(^{4}_{\Lambda}\text{H}) \\ &= E(^3\text{He}) - E(^3\text{H}) - (E(^{4}_{\Lambda}\text{He}) - E(^{4}_{\Lambda}\text{H})). \qquad (1)\end{aligned}$$

Let us further separate contributions from the kinetic energy, and from the NN and YN interactions to the total binding energies. This decomposition is justified by the observation that the contributions due to three-body forces are negligibly small. Hence, the CSB splitting in Eq. (1) can finally be expressed as

$$\begin{aligned}\Delta B_\Lambda &= T(^3\text{He}) - T(^3\text{H}) - (T(^{4}_{\Lambda}\text{He}) - T(^{4}_{\Lambda}\text{H})) \\ &\quad + V_{NN}(^3\text{He}) - V_{NN}(^3\text{H}) \\ &\quad - (V_{NN}(^{4}_{\Lambda}\text{He}) - V_{NN}(^{4}_{\Lambda}\text{H})) \\ &\quad - (V_{YN}(^{4}_{\Lambda}\text{He}) - V_{YN}(^{4}_{\Lambda}\text{H})) \\ &= \Delta T + \Delta V_{NN} + \Delta V_{YN}. \qquad (2)\end{aligned}$$

Note that the operators V_{NN} and V_{YN} employed in Eq. (2) are also SRG evolved, like the full Hamiltonian. Furthermore, we will follow the approach in [33] to estimate the individual contributions ΔT, ΔV_{NN}, and ΔV_{YN} perturbatively based on the two (hyper)nuclear wave functions of $^{4}_{\Lambda}$He and ^{3}He (or $^{7}_{\Lambda}$Li* and ^{6}Li, and $^{8}_{\Lambda}$Li and ^{7}Li in cases of $A = 7$ and 8 systems, respectively). The former is computed for the YN interactions that also include the CSB components. Using the wave functions that include CSB effects is strictly speaking a deviation from first order perturbation theory. However, the deviation is of second order and therefore not relevant here. As it has been shown in [33] and will be discussed in the following section, such a perturbative estimate of ΔB_Λ is a good approximation to the exact calculations. The wave functions for $A = 4$, 7, and 8 hypernuclear ($A = 3$, 6, and 7 for nuclear) systems are generated using the largest computationally accessible model spaces, namely $\mathcal{N}_{\max} = 24$, 10, and 9, respectively, and at the optimal $\omega_{\text{opt}} = 16$ MeV that is (or very close to) the variational minimum.

For estimating numerical uncertainties due to the model space truncation, we have performed the same calculations for two-body interactions at the same $\mathcal{N}_{\max}$ and for $\mathcal{N}_{\max} + 2$ and with the same ω_{opt} and $\omega_{\text{opt}} \pm 2$ MeV. The variation of these calculations gave our uncertainty estimate of 10, 30, and 50 keV for the $A = 4$, 7, and 8 isospin multiplets, respectively. Note the larger uncertainty for the $A = 8$ doublet because of the smaller accessible model space.

TABLE I. Λ-separation energies for the ${}^{4}_{\Lambda}$H(0^+, 1^+) states and for ${}^{5}_{\Lambda}$He, computed for the YN potentials NLO13(500) and NLO19(500) including the CSB interaction. Listed are our full results, with inclusion of the corresponding SRG-induced YNN forces, and those with the YN potentials SRG-evolved to the magic flow parameter, $\lambda_{\rm magic} = 0.765$ and 0.823 fm^{-1}, respectively. The B_Λ values are obtained by performing the two-step ω- and $\mathcal{N}$-space extrapolation, see [27] for more details. The FY calculations are performed with the bare NN, $3N$, and YN potentials. Energies are given in MeV.

		${}^{4}_{\Lambda}$H(0^+)	${}^{4}_{\Lambda}$H(1^+)	${}^{5}_{\Lambda}$He
	full	1.551 ± 0.007	0.823 ± 0.003	2.22 ± 0.06
NLO13-CSB	$\lambda = 0.765$	1.29 ± 0.005	0.779 ± 0.02	2.22 ± 0.04
	FY	1.513	0.813	
	full	1.514 ± 0.007	1.27 ± 0.009	3.32 ± 0.03
NLO19-CSB	$\lambda = 0.823$	1.41 ± 0.003	1.131 ± 0.01	3.35 ± 0.02
	FY	1.511	1.268	
Experiment		2.16 ± 0.08 [19]	1.07 ± 0.08 [19]	3.12 ± 0.02 [3]

As already said, we will employ the high-order SMS NN interaction with $\Lambda_N = 450$ MeV [SMS N^{4}LO$^+$(450)] [29] and the N$_2$LO $3N$ force with the same chiral cutoff [32]. Two chiral potentials at next-to-leading order, namely NLO13 and NLO19 [21,22] with a regulator of $\Lambda_Y = 500$ MeV, are chosen for the YN interaction. We know by experience that the SRG evolution for such low cutoff values converges very quickly thanks to the overall small YN potential matrix elements. For larger cutoffs and especially for the NLO13 interaction, which contains sizable off-diagonal potential matrix elements, the ordinary differential equations (ODE) solver used for the SRG evolution demands an extremely small time step for achieving an accurate solution which requires prohibitively large computing resources. The predicted difference in the Λp and Λn scattering lengths, i.e., $\Delta a^{\rm CSB}$, has been found to be basically the same for all cutoffs and for the two realizations of the YN interaction [23]. Obviously, the regulator dependence is efficiently absorbed by the contact terms of the CSB component of the YN potentials, when fixing the pertinent LECs from the $A = 4$ CSB level splittings. Therefore, we expect that the CSB splittings for $A = 7$ and 8 hypernuclei based on those interactions exhibit likewise a fairly weak or even a negligible cutoff dependence. Finally, chiral YNN forces are not considered in the current study, only those from the SRG evolution. In the following "$3N$ forces" stands for (the inclusion of) chiral as well as SRG-induced $3N$ forces, unless explicitly stated otherwise.

IV. RESULTS

A. Charge symmetry breaking in the $A = 4$ systems

The hypernuclei ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He constitute an important test case for our calculations, because here we can directly compare the NCSM predictions with results obtained from solutions of the Faddeev-Yakubovsky (FY) equations [23]. Table I shows the comparison of the separation energies obtained within the two methods. The NLO13 and NLO19 potentials with chiral cutoff of 500 MeV have been employed to describe the YN interaction, while the standard combination of the SMS N^{4}LO$^+$(450) NN and N^{2}LO(450) $3N$ interactions is used [32] for the nucleons. For the NCSM calculations, the employed NN, $3N$, and YN potentials are SRG-evolved to a flow parameter of $\lambda = 1.88\,{\rm fm}^{-1}$. Furthermore, the SRG-induced YNN interaction is taken into account so that the separation energies are practically independent of the SRG-flow parameter [35]. For the FY calculations, the bare NN, $3N$, and YN interactions have been employed. The small discrepancy between the FY results listed in Table I and those provided in [23] is essentially due to the contribution of the $3N$ force, neglected in the latter work, which clearly amounts to less than 50 keV. It is reassuring to observe that for NLO19 the actual separation energies computed within the NCSM approach agree perfectly with the results of the FY equations, for the ground state as well as for the excited state. The extremely small difference could be an indication that the contribution of SRG-induced $YNNN$ forces to the separation energies in the $A = 4$ systems are negligibly small. However, for a more quantitative estimate, well-converged calculations using a wide range of values for the SRG flow parameter are still necessary. For NLO13, the difference of the FY result and the full calculations is more visible and of the order of 40 keV indicating larger contributions of the missing SRG-induced $YNNN$ forces in this case which are probably related to the larger Λ-Σ transition matrix elements [22]. We stress that the agreement of the FY and full calculations are still excellent.

Additionally, the table contains our NCSM results for the Λ separation energies of ${}^{5}_{\Lambda}$He, which are $B_\Lambda({}^{5}_{\Lambda}{\rm He}) = 2.22 \pm 0.06$ MeV and 3.32 ± 0.03 MeV [35], respectively. Evidently, NLO13 significantly underestimates the ${}^{5}_{\Lambda}$He separation energy, while the result for the NLO19 potential is rather close to and only slightly above the experimental value of $B_\Lambda({}^{5}_{\Lambda}{\rm He}) = 3.12$ MeV. The discrepancy between the two NLO13 and NLO19 predictions signals the need for including proper chiral ΛNN and ΣNN three-body forces [36], given that the Λp and ΣN results of those potentials are practically identical. Indeed, three-body forces, with a distinct spin-isospin dependence might also be needed to bring the $A = 4$ results in a better agreement with the experiment.

Finally, we include in Table I results of NCSM calculations where only the NN potential, SRG evolved to $\lambda_{NN} = 1.6$ fm^{-1}, and the two YN potentials, SRG evolved to the "magic" flow parameters, $\lambda_{\rm magic}({\rm NLO19}) = 0.823$ fm^{-1} and $\lambda_{\rm magic}({\rm NLO13}) = 0.765$ fm^{-1}, are employed. The values of $\lambda_{\rm magic}$ for NLO19 and NLO13 are chosen in such a way that the pertinent full NCSM results for the ${}^{5}_{\Lambda}$He separation energy are reproduced. Let us remark that our way of fixing $\lambda_{\rm magic}$

TABLE II. Contributions to CSB for ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H in the 0^+ and 1^+ states, based on the YN potentials NLO13(500) and NLO19(500) (including $3N$ forces and SRG-induced YNN forces). The results are for the original potentials (without CSB force) and for the scenario CSB1 of Ref. [23]. FY indicates the exact CSB results extracted from Faddeev-Yakubovsky calculations which employ the bare NN, $3N$, and YN interactions. All results are in keV. The estimated uncertainty from the NCSM and FY calculations are 10 and 20 keV, respectively. The experimental reference values are $\Delta B_\Lambda(0^+) = 233 \pm 92$ and $\Delta B_\Lambda(1^+) = -83 \pm 94$ keV.

				ΔV_{YN}				
	YN	ΔT	ΔV_{NN}	${}^1S_0$	${}^3S_1$	Total	ΔB_Λ	ΔB_Λ(FY)
	NLO13	17	−12	−3	0	−3	3	43
(0^+)	NLO13-CSB	18	−13	152	76	224	229	252
	NLO19	9	−15	−1	0	−1	−7	10
	NLO19-CSB	9	−16	126	118	245	238	238
	NLO13	6	−5	0	0	−1	0	−9
(1^+)	NLO13-CSB	6	−5	−114	19	−95	−94	−75
	NLO19	5	−15	0	0	−1	−11	5
	NLO19-CSB	5	−15	−114	36	−76	−85	−85

here slightly differs from the strategy in [27] where the experimental value of ${}^{5}_{\Lambda}$He has been used as benchmark. Note that at the SRG parameter of $\lambda_{NN} = 1.6$ fm^{-1}, the parent nuclear cores can be fairly well described even when $3N$ forces are omitted [27]. With λ_{magic} fixed to the actual $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ for NLO13 and NLO19, we observe a fair to good agreement between the $A = 4$ separation energies from the full NCSM calculations and those computed at λ_{magic}, as can be seen in Table I. The small discrepancy between the two results, up to around 200 keV for the 0^+ state and in the order of 100 keV for 1^+, can be attributed again to possible contributions from YNN forces [27,36].

In Table II, we analyze the CSB in the $A = 4$ isodoublet ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H in detail. The results are based on NLO13(500) and NLO19(500) as published originally [21,22] and including a CSB interaction [23] that was adjusted to the experimental splittings $\Delta B_\Lambda(0^+) = 233 \pm 92$ keV and $\Delta B_\Lambda(1^+) = -83 \pm 94$ keV (CSB1 of Ref. [23]). Similarly to [23], we break down the different contributions to the total CSB splitting ΔB_Λ, due to the kinetic energy ΔT, the NN interaction (ΔV_{NN}) and the YN interaction (ΔV_{YN}), see Eq. (2). The perturbatively estimated contributions of the $3N$ and YNN forces are negligibly small and, therefore, omitted in the table. The CSB contribution ΔT is also small when using chiral interactions, but contributes with positive sign to the total CSB. The contribution of the nuclear core ΔV_{NN}, mostly due to the point Coulomb interaction between the protons, is of similar magnitude as ΔT but comes with a negative sign. As expected, ΔV_{YN} for the original YN potentials is insignificant. However, when the CSB interaction [23] is included, ΔV_{YN} becomes sizable and, by construction, the total CSB results for the 0^+ state as well as for 1^+ are in line with the aforementioned empirical information.

Also for the CSB splittings, we can compare our NCSM results with those obtained by solving the FY equations, cf. the last column in Table II. Again, there is good agreement between the two calculations within the estimated uncertainties. Note that the FY values are from an exact solution of the equations. The comparison of perturbative and exact CSB results in Tables 6 and 7 of Ref. [23] reveals that there is very little difference. In addition, for $A = 7$ systems, we have also explicitly studied and observed a discrepancy of only less than 10 keV between the perturbatively estimated CSB and the CSB results that are computed based on the expectation values of the T, V_{NN}, and V_{YN} operators estimated with respect to the corresponding hyper(nuclear) wave functions ${}^{7}_{\Lambda}$Be(^{6}Be) and ${}^{7}_{\Lambda}$Li*(^{6}Li). Let us again stress that due to the large uncertainties of the extrapolated separation energies for the $A \geqslant 7$ systems, see Table III, a direct extraction of CSB splittings based on those separation energies is not useful. One could also calculate the CSB differences for each model space separately and study the model space and ω dependence more carefully. For $A = 4$ and $A = 7$ hypernuclei, our results for this approach were also consistent with the perturbative estimate, but there was still a visible dependence on the model space size and HO frequencies which made the extraction of an uncertainty rather difficult. We therefore favor the perturbative approach which is robust and computationally less demanding and use it for obtaining the CSB effects in the $A = 7$ and 8 Λ hypernuclei below.

Let us now have a closer look at the different contributions of the ${}^1S_0$ and ${}^3S_1$ partial waves, $\Delta V_{YN({}^1S_0)}$ and $\Delta V_{YN({}^3S_1)}$, to the total ΔV_{YN}. From the fourth column in Table II, it follows that $\Delta V_{YN({}^1S_0)}$ and $\Delta V_{YN({}^3S_1)}$ are sizable and of the same sign in the 0^+ state, resulting in a large $\Delta V_{YN(0^+)}$. In the excited state, the two contributions are, however, smaller and of opposite sign so that there is some cancellation. The signs of the two contributions $\Delta V_{YN({}^1S_0)}$ and $\Delta V_{YN({}^3S_1)}$ are directly related to the different strengths of the Λn and Λp interactions in the singlet and triplet states, as manifested by the respective scattering lengths, and to the relative weights of the Λn and Λp components in those spin states. More details are given in the Appendix.

B. Charge symmetry breaking in the $A = 7$ and 8 systems

We now employ the NLO13(500) and NLO19(500) potentials to study CSB in the $A = 7$ isotriplet and the $A = 8$ isodoublet. Predictions for the separation energies of the $(1/2^+, 1)$ mirror hypernuclei ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$Be without CSB terms are provided in Table III. The values listed in the

TABLE III. Λ separation energies for the $A = 7$ and 8 systems, computed for NLO13(500) and NLO19(500) including the SRG-induced YNN forces (full), and at the magic flow parameters (third and fifth columns). Note that the separation energies of $A = 7(8)$ for NLO19 at $\lambda = 0.823$ fm^{-1} have been computed with model spaces up to $\mathcal{N}_{\max} = 12(11)$, whereas the other calculations are performed with $\mathcal{N}_{\max} = 10(9)$. The listed B_Λ values are obtained by performing the two-step ω- and $\mathcal{N}$-space extrapolation, see [27] for more details. Values from emulsion (left) and counter (right) experiments are taken from the compilation in Ref. [4]. Energies are given in MeV.

	NLO19		NLO13		Experiment	
	Full	$\lambda = 0.823$	Full	$\lambda = 0.765$		
${}^{7}_{\Lambda}$Be	5.54 ± 0.22	5.44 ± 0.03	4.30 ± 0.47	4.53 ± 0.34	5.16 ± 0.08	
${}^{7}_{\Lambda}$Li*	5.64 ± 0.28	5.49 ± 0.04	4.42 ± 0.58	4.59 ± 0.34	5.26 ± 0.03	5.53 ± 0.13
${}^{7}_{\Lambda}$He	5.64 ± 0.27	5.43 ± 0.06	4.39 ± 0.54	4.45 ± 0.35		5.55 ± 0.1
${}^{8}_{\Lambda}$Be		7.15 ± 0.10		5.56 ± 0.25	6.84 ± 0.05	
${}^{8}_{\Lambda}$Li	7.33 ± 1.15	7.17 ± 0.10	5.75 ± 1.08	5.57 ± 0.30	6.80 ± 0.03	

second and fourth columns have been obtained with inclusion of both the chiral and SRG-induced $3N$ forces as well as of the SRG-induced YNN interactions, whereas B_Λ displayed in the third and fifth columns is computed at the corresponding λ_{magic}. Obviously, there is a fairly good agreement between the separation energies extracted from the full calculations and the one at λ_{magic}. This confirms our observation in [27,34] that the magic SRG-flow parameters can be utilized to simplify the calculation of light hypernuclear systems. The NLO13 interaction predicts separation energies of $B_\Lambda = 4.30 \pm 0.47$, 4.42 ± 0.58, and 4.39 ± 0.54 MeV for ${}^{7}_{\Lambda}$Be, ${}^{7}_{\Lambda}$Li*, and ${}^{7}_{\Lambda}$He, respectively, and, thus, underestimates the empirical values by about 1 MeV. On the other hand, the results based on NLO19 are rather close to experiment. In particular, the obtained separation energies for the $T_3 = 0$ and $T_3 = -1$ members $B_\Lambda({}^{7}_{\Lambda}\text{Li}^*) = 5.64 \pm 0.28$ MeV and $B_\Lambda({}^{7}_{\Lambda}\text{He}) = 5.64 \pm 0.27$ MeV are perfectly in line with the values of $B_\Lambda({}^{7}_{\Lambda}\text{Li}^*) = 5.53 \pm 0.13$ and $B_\Lambda({}^{7}_{\Lambda}\text{He}) = 5.55 \pm 0.13$ MeV, extracted from counter experiments with an absolute energy calibration [4]. For the ${}^{7}_{\Lambda}$Be hypernucleus, we obtain a separation energy of $B_\Lambda({}^{7}_{\Lambda}\text{Be}) = 5.54 \pm 0.22$ MeV, which exceeds the emulsion result of $B_\Lambda({}^{7}_{\Lambda}\text{Be}) = 5.16 \pm 0.08$ MeV [4]. However, considering the unresolved difference of 270 ± 170 keV between the $B_\Lambda({}^{7}_{\Lambda}\text{Li}^*)$ determinations in counter and emulsion experiments, cf. Table III, the actual discrepancy for $B_\Lambda({}^{7}_{\Lambda}\text{Be})$ could be much smaller. Hopefully, future counter experiments will settle this issue.

The separation energies for the $A = 8$ systems are likewise summarized in Table III. The results for ${}^{8}_{\Lambda}$Li with both $3N$ forces and SRG-induced YNN interactions included are obtained from the full calculations with model space up to $\mathcal{N}_{\max} = 9$. Extending the calculation for model spaces up to $\mathcal{N}_{\max} = 11$ will definitely help to reduce the estimated errors. Unfortunately, such a calculation is very CPU-time consuming and we need to postpone it to a future study. Nevertheless, in spite of the large uncertainty, it clearly follows from Table III that the separation energy for ${}^{8}_{\Lambda}$Li for the NLO13 potential is substantially too low whereas the prediction for NLO19, $B_\Lambda({}^{8}_{\Lambda}\text{Li}) = 7.33 \pm 1.15$ MeV, exceeds the empirical value of $B_\Lambda = 6.80 \pm 0.03$ MeV [4] only moderately. Again, the difference in the predictions of NLO13 and NLO19 can be attributed to possible contributions of chiral YNN forces [22,27,36]. Although full calculations for ${}^{8}_{\Lambda}$Be have not been performed yet, a result for $B_\Lambda({}^{8}_{\Lambda}\text{Be})$ very similar to that for the ${}^{8}_{\Lambda}$Li hypernucleus can be expected. B_Λ values for ${}^{8}_{\Lambda}$Be and ${}^{8}_{\Lambda}$Li computed at the magic SRG-flow parameters are given in the third and fifth columns of Table III. Evidently, the obtained separation energies, e.g., $B_\Lambda({}^{8}_{\Lambda}\text{Li}, \lambda_{\text{magic}}) = 7.17 \pm 0.10$ MeV, is close to the result of $B_\Lambda({}^{8}_{\Lambda}\text{Li}) = 7.33 \pm 1.15$ MeV of the full calculations. This is not too surprising in view of what we had already observed in the pertinent comparison for the $A = 4$ and 7 systems. Note that $B_\Lambda({}^{8}_{\Lambda}\text{Li})$ based on λ_{magic} exceeds the value from the emulsion experiment only by 0.37 ± 0.13 MeV. Anyway, in view of the rather good agreement of our predictions for the $A = 7$ systems with the separation energies from counter experiments, corresponding measurements for $A = 8$ hypernuclei, that could either confirm or revise the emulsion results, are desirable.

Table IV provides a detailed view on the CSB splittings for the three members of the $A = 7$ isotriplet, by comparing ${}^{7}_{\Lambda}$Be - ${}^{7}_{\Lambda}$Li* and ${}^{7}_{\Lambda}$Li* - ${}^{7}_{\Lambda}$He, computed for NLO13 and NLO19 without and with CSB interaction. The $3N$ forces and the SRG-induced YNN interactions are explicitly taken into account. One sees that, despite the substantial discrepancy in the predicted Λ separation energies, the two potentials yield comparable CSB results in the $A = 7$ systems. The overall CSB effect is rather small, with as well as without the CSB part of the potentials, and consistent with the experiment, both in magnitude and sign. It is also interesting to note that, like in the 1^+ state of the $A = 4$ systems, the ${}^{1}S_0$ and ${}^{3}S_1$ states contribute with opposite signs to the total ΔV_{YN}, which, in turn, leads to a small total CSB for the $A = 7$ isotriplet.

We include also results of former studies for the ease of comparison. Those of Gal [37,38] were computed by employing a shell-model approach in combination with an effective $\Lambda\Sigma$ coupling model. The $A = 7$ calculation by Hiyama *et al.* [13] is done within a $(\Lambda + N + N + \alpha)$ four-body cluster model. Surprisingly, our prediction for ${}^{7}_{\Lambda}$Be - ${}^{7}_{\Lambda}$Li* for the original NLO13 potential (without CSB interaction), $\Delta B_\Lambda(\text{NLO13}) = -17$ keV, is identical to the CSB estimated by Gal [37]. However, the individual contributions ΔT, ΔV_{NN}, and ΔV_{YN} differ substantially. For example, the NLO13 potential yields a vanishing ΔV_{YN} (because, as said, there is no CSB part), whereas in Gal's calculation this

TABLE IV. Contributions to CSB in the $A = 7$ and 8 isospin multiplets, based on the YN potentials NLO13(500) and NLO19(500) (including 3N forces and SRG-induced YNN interactions). The results are for the original potentials (without CSB force) and for the scenario CSB1, see text. Results by Gal [37] and by Hiyama *et al.* [13] are included for the ease of comparison. All energies are in keV. The estimated uncertainties for $A = 7$ and 8 systems are 30 and 50 keV, respectively.

		ΔT	ΔV_{NN}	ΔV_{YN}			ΔB_Λ
				$^1S_0$	$^3S_1$	Total	
${}^{7}_{\Lambda}$Be - ${}^{7}_{\Lambda}$Li*	NLO13	7	−24	−1	0	0	−17
	NLO13-CSB	8	−24	−49	26	−24	−40
	NLO19	6	−40	−1	0	0	−34
	NLO19-CSB	6	−41	−43	42	9	−35
	Hiyama [13]		−70			200	150
	Gal [37]	3	−70			50	−17
	Experiment [6]						−100 ± 90
${}^{7}_{\Lambda}$Li* - ${}^{7}_{\Lambda}$He	NLO13	8	−13	0	0	0	−5
	NLO13-CSB	7	−14	−49	26	−24	−31
	NLO19	5	−22	−43	42	0	−17
	NLO19-CSB	5	−21	−38	37	−1	−16
	Hiyama [13]		−80			200	130
	Gal [38]	2	−80			50	−28
	Experiment [6]						−20 ± 230[a] −50 ± 190
${}^{8}_{\Lambda}$Be - ${}^{8}_{\Lambda}$Li	NLO13	12	8	−2	0	−4	16
	NLO13-CSB	12	7	100	56	159	178
	NLO19	7	−11	−1	0	−2	−6
	NLO19-CSB	6	−11	62	79	147	143
	Hiyama [13]		40				160
	Gal [37]	11	−81			119	49
	Experiment [4]						40 ± 60

[a]The difference between $B_\Lambda({}^{7}_{\Lambda}\mathrm{Li}^*)$ and $B_\Lambda({}^{7}_{\Lambda}\mathrm{He})$ is −20 ± 230 keV for the FINUDA and JLab results, but −50 ± 190 keV when the revised SKS and JLab results are used [6].

contribution amounts to 50 keV. ΔV_{YN} evaluated for the actual chiral CSB interaction is of opposite sign and smaller. There is also a large difference in ΔV_{NN} (that quantity includes also the Coulomb effect). Note that ΔV_{NN} used by Gal is taken from the cluster-model study of Hiyama *et al.* [13] whereas our value is calculated consistently within the NCSM.

CSB results for the two $A = 8$ mirror nuclei are listed at the lower end of Table IV. When using the potentials NLO13 and NLO19 without the CSB interaction, a negligibly small CSB is predicted for ${}^{8}_{\Lambda}$Be - ${}^{8}_{\Lambda}$Li, namely $\Delta B_\Lambda = 16 \pm 50$ keV and -6 ± 50 keV, respectively. This is, however, well in line with the empirical CSB of 40 ± 60 keV [4] based on the separation energies determined in emulsion experiments. A similarly small ΔB_Λ was also predicted by Gal in [37], but, in contrast to the rather small ΔV_{NN} contribution in our calculation, e.g., $\Delta V_{NN} = -11$ keV for NLO19, Gal assigned a significantly larger value to ΔV_{NN}, namely $\Delta V_{NN} = -81$ keV. The latter was not computed directly but taken from the shell model calculation by Millener [37].

With the CSB interaction included, both the NLO13 and NLO19 potentials yield rather sizable CSB results, $\Delta B_\Lambda(\mathrm{NLO13}) = 177 \pm 50$ keV and $\Delta B_\Lambda(\mathrm{NLO19}) = 143 \pm 50$ keV. In this case, the $^1S_0$ and $^3S_1$ partial-wave contributions are large, and more importantly, are of the same sign, and, therefore, add up to a pronounced total CSB. This exactly resembles the situation for the 0^+ states of the $A = 4$ mirror hypernuclei discussed in Sec. IV A. Indeed, it is conceivable that a fairly large splitting in the 0^+ state, as presently established, implies automatically a likewise significant CSB splitting in ${}^{8}_{\Lambda}$Be–${}^{8}_{\Lambda}$Li. Interestingly, the predictions of NLO13 and NLO19 with CSB interaction are comparable to the value of $\Delta B_\Lambda = 160$ keV obtained in a $(\Lambda + \alpha + {}^3\mathrm{He}/t)$ three-body cluster calculation by Hiyama *et al.* [13,39]. However, it should be noted that the phenomenological CSB YN interaction used in Ref. [13] was fitted to an outdated CSB splitting in the $A = 4$ systems, namely $\Delta B_\Lambda(0^+) = 350 \pm 60$ keV and $\Delta B_\Lambda(1^+) = 240 \pm 60$ keV. Also, it should be said that, when using only the charge symmetric phenomenological interactions adjusted so that the experimental value of $B_\Lambda({}^{8}_{\Lambda}\mathrm{Li}) = 6.80$ MeV is reproduced, Hiyama *et al.* obtained a separation energy of $B_\Lambda = 6.72$ MeV for ${}^{8}_{\Lambda}$Be. The difference of −80 keV between $B_\Lambda({}^{8}_{\Lambda}\mathrm{Be})$ and $B_\Lambda({}^{8}_{\Lambda}\mathrm{Li})$ was then attributed to the difference of the Coulomb interaction [13], which only amounts to about 10 keV in our calculations.

Finally, for illustration, we compare in Table V CSB results based on calculations with the magic flow parameter λ_{magic},

TABLE V. CSB splittings ΔB_Λ (in keV) for $A = 4$–8 systems. Results for the full calculation and those based on the YN potentials NLO13 and NLO19, SRG-evolved to $\lambda_{\rm magic} = 0.765$ and $\lambda_{\rm magic} = 0.823$ fm^{-1}, respectively, are compared. CSB* corresponds to a CSB interaction adjusted to the new STAR data [28], see text. The estimated uncertainties for $A = 4$, 7, and 8 systems are 10, 30, and 50 keV, respectively.

		${}^4_\Lambda$He $-$ ${}^4_\Lambda$H				
		0^+	1^+	${}^7_\Lambda$Be $-$ ${}^7_\Lambda$Li*	${}^7_\Lambda$Li* $-$ ${}^7_\Lambda$He	${}^8_\Lambda$Be $-$ ${}^8_\Lambda$Li
NLO13-CSB	full	229	−94	−40	−5	178
NLO13-CSB	$\lambda = 0.765$	213	−80	−10	0	204
NLO19-CSB	full	238	−85	−35	−16	143
NLO19-CSB	$\lambda = 0.823$	210	−71	−26	−3	135
NLO19-CSB*	$\lambda = 0.823$	130	−135	−83	−62	74

i.e., of calculations without $3N$ forces and without the SRG-induced YNN interaction, with the full results. Furthermore, we discuss the implications of a somewhat different CSB splitting in the $A = 4$ system, as suggested by a recent STAR measurement [28]. For the latter aspect a new scenario is introduced, called CSB*, where the LECs of the CSB interaction have been readjusted to match the CSB splittings reported by STAR, namely 160 ± 160 keV for the 0^+ state and -160 ± 140 keV for 1^+.

It is reassuring though not surprising that the full $A = 4$ CSB results differ from the values computed at $\lambda_{\rm magic}$ by at most 30 keV. The CSB splittings for the $A = 7$ systems, computed at the magic SRG-flow parameters, are likewise in rather good agreement with the results extracted from the full calculations. The same is also true for the $A = 8$ isodoublet. Apparently, the magic SRG-flow parameter is a fairly reliable starting point for studying the separation energies as well as CSB effects in light hypernuclei. This important observation could help to significantly save computational resources.

Regarding the new STAR data, it clearly sticks out from Table V that the corresponding scenario CSB* yields somewhat larger CSB for ${}^7_\Lambda$Be-${}^7_\Lambda$Li* and ${}^7_\Lambda$Li*-${}^7_\Lambda$He, $\Delta B_\Lambda = -83 \pm 30$ keV and $\Delta B_\Lambda = -62 \pm 30$ keV, respectively, as compared to the values of $\Delta B_\Lambda = -26 \pm 30$ keV and $\Delta B_\Lambda = -3 \pm 30$ keV, predicted by the standard CSB interaction. However, overall, both the CSB* and CSB results are still consistent with the experimental values of $\Delta B_\Lambda({}^7_\Lambda\text{Be}-{}^7_\Lambda\text{Li}^*) = -100 \pm 90$ keV and $\Delta B_\Lambda({}^7_\Lambda\text{Li}^*-{}^7_\Lambda\text{He}) = -20 \pm 230$ keV [6]. Also in case of the $A = 8$ isodoublet the splitting of 74 ± 50 keV predicted for the scenario CSB* is well in line with the experimental value of 40 ± 60 keV [4].

V. SUMMARY

In this work, we have presented results for the separation energies of the isospin triplet ${}^7_\Lambda$He, ${}^7_\Lambda$Li*, ${}^7_\Lambda$Be, and the $T = 1/2$ doublet ${}^8_\Lambda$Li, ${}^8_\Lambda$Be, calculated within the NCSM. The underlying YN interactions, taken from Refs. [21–23], are derived from chiral effective field theory at NLO. The potentials include the leading CSB interaction in the ΛN channel, whose strength has been fixed to the experimental difference of the Λ separation energies of the mirror hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H as established by the J-PARC and Mainz data [17–19]. In order to speed up the convergence of the NCSM with respect to the model space, all included interactions are SRG-evolved and the arising SRG-induced three-body forces are taken into account.

We have found that the YN potential NLO13 [21] produces too low separation energies for the $A = 7$ and 8 systems considered in the present work. However, the predictions for the YN potential from 2019 (NLO19) [22] agree quite well with the experimental values for ${}^7_\Lambda$He and ${}^7_\Lambda$Li*, deduced from counter experiments. On the other hand, separation energies obtained from emulsion experiments for ${}^7_\Lambda$Be and for the $A = 8$ hypernuclei ${}^8_\Lambda$Be and ${}^8_\Lambda$Li are overestimated. For either potentials the discrepancies between theory and experiments and the differences of NLO13 and NLO19 might be a signal for the necessity of chiral ΛNN and ΣNN three-body forces [36], which have been not included so far in our calculations. At the same time, one has to keep in mind that the experimental situation for the hypernuclei studied in the present work is not yet settled, specifically concerning the emulsion data, see the discussion in Refs. [4–6].

With regard to CSB, the predicted values for the $A = 7$ systems are small and agree with the splittings deduced from the empirical binding energies within the experimental uncertainty. In case of the $A = 8$ doublet, the computed CSB is somewhat larger than the available experimental value. We stress that possible YNN three-body forces should have only a minor influence on the calculated CSB splittings. In view of the still uncertain experimental situation, we also considered a scenario motivated by recent data from the STAR collaboration for $A = 4$. We found slightly increased values for the CSB in $A = 7$ and a significant reduction in $A = 8$. The different effects in $A = 7$ and 8 hypernuclei are related to contributions of different sign in the ${}^1S_0$ and ${}^3S_1$ partial waves. Accurate experimental data in these systems will therefore allow one to independently check the CSB deduced from $A = 4$ hypernuclei.

We have also explored in detail the possibility to use the so-called magic flow parameter of the SRG evolution in the actual NCSM computations. In this case, in contrast to the full calculation which includes $3N$ forces and the SRG-induced YNN interaction, only two-body interactions are taken into account. In such a scenario, one can save a significant amount of computational resources. But then, as a consequence, the results depend on the actual value of the SRG-flow parameter. We consider the option to fix its value by requiring that the same ${}^5_\Lambda$He separation energies are obtained as in the full

NCSM calculation. It turned out that the separation energies obtained with that choice of the flow parameter are fairly close to the full results. This suggests that the "magic" SRG-flow parameter is a fairly reliable starting point for studying the separation energies as well as CSB effects in light hypernuclei in an "inexpensive" way.

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APPENDIX: CONTRIBUTION OF $^1S_0$ AND $^3S_1$ PARTIAL WAVES TO ΔV_{YN}

In this Appendix, we provide a brief summary of the contributions from the $^1S_0$ and $^3S_1$ ΛN partial waves to the expectation value of the corresponding YN potentials for the considered $A = 4$–8Λ hypernuclei, see Table VI. The weights of the respective Λp and Λn components are listed, too, which differ, of course, for the mirror hypernuclei in question. Those weights, in combination with the different strengths of the Λn and Λp interactions in the singlet and triplet states as manifested by the respective scattering lengths, see Table 2 in Ref. [23], determine the value for $\langle V_{YN}\rangle$ and, in turn, also the values for $\Delta V_{YN(^1S_0)}$ and $\Delta V_{YN(^3S_1)}$ that are listed in Tables II and IV. Clearly, the signs of the two contributions $\Delta V_{YN(^1S_0)}$ and $\Delta V_{YN(^3S_1)}$ can be the same or the opposite, depending on the concrete interplay realized in a specific mirror hypernucleus.

TABLE VI. Probability (in %) of finding Λp and Λn pairs in the $A = 4$–8 wave functions, and the contributions of the $^1S_0$ and $^3S_1$ ΛN partial waves to the expectation value $\langle V_{YN}\rangle$ (in MeV). The calculations are based on the NLO19(500) YN potential. The SRG-induced YNN interaction is also included in the calculations for $^4_\Lambda$He - $^4_\Lambda$H whereas the $A = 7$ and 8 wave functions were computed at the magic SRG-flow parameter of $\lambda_{\rm magic} = 0.823$ fm^{-1}. The singlet and triplet $\Lambda p\,(\Lambda n)$ scattering lengths predicted by the NLO19(500) are $a_{^1S_0} = -2.649(-3.202)$ fm and $a_{^3S_1} = -1.580(-1.467)$ fm.

	$^1S_0$		$^3S_1$		$\langle V_{YN}\rangle$	
	Λp	Λn	Λp	Λn	$^1S_0$	$^3S_1$
$^4_\Lambda$He(0^+)	13.92	27.60	44.54	0.42	−4.383	−3.916
$^4_\Lambda$H(0^+)	27.1	13.66	0.41	43.79	−4.257	−3.797
$^4_\Lambda$He(1^+)	14.48	0.13	42.47	27.07	−1.383	−5.743
$^4_\Lambda$H(1^+)	0.128	14.48	27.16	42.48	−1.423	−5.8685
$^7_\Lambda$Be	11.13	7.22	33.25	21.67	−3.733	−9.364
$^7_\Lambda$Li*	9.17	9.17	27.44	27.44	−3.768	−9.321
$^7_\Lambda$He	7.22	11.10	21.65	33.13	−3.802	−9.278
$^8_\Lambda$Be	9.49	12.24	28.67	19.33	−5.315	−9.959
$^8_\Lambda$Li	11.71	9.5	19.84	28.58	−5.254	−9.876

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