Letter

Nucleon-nucleon short-ranged correlations, β decay, and the unitarity of the CKM matrix

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The influence of nucleon-nucleon short-ranged correlations (SRC) on nuclear superallowed β decay is examined. The need for this is driven by the observed depletion of spectroscopic strength obtained in studies of (e, e') and $(d, {}^3\mathrm{He})$ reactions on a wide variety of nuclei. We show that the influence of SRC is model dependent, but may be very substantial. The ^{46}V nucleus is used as an example. The resulting impact on studies of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element is discussed.

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The dominant contribution to the unitarity test of the standard model (SM) Cabibbo-Kobayashi-Maskawa (CKM) matrix comes from the up-down quark matrix element V_{ud}. The value of V_{ud} has been extracted by Hardy and Towner (HT) [1–10] with the highest precision from $0^+ \rightarrow 0^+$ decays from nuclei ranging from ^{10}C to ^{74}Rb. The remarkably consistent nature of the values of V_{ud} obtained from many different decays has led to a very small uncertainty. Their latest paper [10] states

$$V_{ud} = 0.97373 \pm 0.00031. \tag{1}$$

Despite the considerable success of the HT approach, the crucial importance of the process in testing the standard model has long mandated that the theory behind the analysis be continually reexamined, an especially urgent process now because a more recent evaluation [11] of an electroweak radiative correction claims a 4 standard deviation violation of unitarity. Our focus is on the isospin-breaking correction δ_C. A variation of this quantity, $\Delta\delta_C$, would cause a change in V_{ud} given by

$$\frac{\Delta\left(V_{ud}^2\right)}{V_{ud}^2} \approx \Delta\delta_C. \tag{2}$$

Consider the result $\delta_C = 0.960(63)\%$ for the $0f_{7/2}$ orbital of ^{42}Ti [10]. A 20% change, for example, in that number is about 0.2% and V_{ud} would be changed by half that, 10^{-3}, a number that is 3.5 times the uncertainty quoted in Eq. (1). The Particle Data Group [12] find a similar central value of V_{ud}, but a smaller uncertainty of ± 0.00014. In that case, the 20% change in δ_C would be almost 8 times the uncertainty.

The purpose of this Letter is to argue that the influence of short-ranged correlations between nucleons, unaccounted for by Towner and Hardy [6] (TH), may cause changes in the values of δ_C, which are large on the scale of the desired accuracy. This means that depending on future theoretical and experimental work, either the uncertainty in the value of V_{ud} is significantly larger than that of Eq. (1) or that the value is itself changed significantly.

Superallowed β decays are generated by the isospin operator $\boldsymbol{\tau}$ obeying the usual commutation relations. The theoretical formalism of TH is based on using a weak interaction operator different than $\boldsymbol{\tau}$, which does not obey these commutation rules [13,14]. The operator of TH was designed to reduce the size of the necessary small shell-model space. Corrections to the TH formalism based on the collective isovector monopole state were presented in [15,16]. Work on the effects of short-ranged correlations appears in [17] that concludes, "we present a new set of isospin-mixing corrections . . . , different from the values of Towner and Hardy. A more advanced study of these corrections should be performed."

The TH restriction is motivated by a shell-model picture in which radial excitations of energy $2\hbar\omega$ and higher above the relevant orbitals can be neglected. This approach specifically eliminates the influence of short-ranged nucleon-nucleon correlations that involve nucleons in orbitals high above the given shell-model space. This strong interaction effect reduces the probability that a decaying nucleon is in a valence single-particle orbital and suggests that the magnitude of δ_C is smaller than that of previous calculations.

An exact formalism for evaluating δ_C was presented in [13,14]. The present effort presents an extension of that formalism, focusing on the influence of short-ranged correlations, now known to be important because of recent significant experimental and theoretical work.

In the time since TH started their epic sequence of calculations, many new experimental and theoretical results have obtained unambiguous evidence that nucleon-nucleon short-ranged correlations do exist in an observable fashion [18–45]. The effects of short-ranged correlations between nucleons, predicted long ago, have finally been measured and are significant. Such correlations involve the excitations of nucleons to intermediate states of high energy. Consequently, radial excitations are now known to be important in nuclear physics.

Spectroscopic factors, essentially the occupation probability of a single-particle, shell-model orbital, play an important role in what follows. As reviewed in Ref. [21], electron scattering experiments typically observe only about 60–70% of the expected number of protons. This depletion of the spectroscopic factor was observed over a wide range of the periodic table at relatively low-momentum transfer for both valence nucleon knockout using the $(e, e'p)$ reaction [46] and stripping using the $(d,^3\mathrm{He})$ reaction [47]. See, also, the (e, e') work of Ref. [48]. The missing strength of 30–40% implies the existence of collective effects (long-range correlations) and short-range correlations in nuclei. Substantial theoretical analyses [49–54] used detailed many-body evaluations to find that including the effects of both long- and short-range correlations must be included to reproduce the results of experiments that measure spectroscopic factors.

Reference [30] made a quantitative effort to analyze the separate long- (LRC) and short-range (SRC) contributions to the quenching of the spectroscopic factors. Their result is that the SRC contribution amounts to $22\% \pm 8\%$ and the LRC contribution to $\delta = 14\% \pm 10\%$. This is in accordance with expectations [18–21,29] and with the results of [40,49–52,55]. In the following, we argue that in analogy with the $(e, e'p)$ and $(d,^3\mathrm{He})$ reactions, the superallowed β decay measurements are impacted by the short-ranged correlations that reduce the spectroscopic strength by about 20%.

It is well known that that short-range physics, at low energy, can be embedded into effective operators. The standard shell model has been doing this for more than 70 years through fitting phenomenological interactions, effective charges, and empirical quenching g_A. It successfully reproduces a large body of experimental data. However, recent work shows that it is possible to understand the quenching of g_A by using modern techniques such as the in-medium similarity renormalization group, as explained, for example, in Ref. [56]. This is necessary to understand other processes such as neutrinoless double-β decay. The problem here is that modern techniques have not yet been applied to the analysis of superallowed β decays. The work of TH is no longer consistent with modern nuclear theory, yet is still used to limit the effects of possible nonstandard model interactions.

Therefore, we reexamine the calculations of superallowed β decay rates with an eye toward including the effects of short-ranged correlations absent in the TH formalism. Doing this precisely requires separating the long-range correlations inherent in the shell model of TH from the missing short-range correlations. This challenging task leads to the goal of first providing a plausibility argument, rather than a detailed evaluation. We rely on simple arguments, starting from the basics.

The shell model is the starting point for nuclear physics. In its simplest form, the nucleons are in single-particle orbitals and the β decay matrix element is simply an overlap between neutron and proton wave functions. If the Hamiltonian commutes with all components of the isospin operator, the spatial overlap will be unity. But the noncommuting interactions, such as Coulomb interaction and the nucleon mass difference, cause the overlap to be less than unity. This leads to a nonzero value of the isospin correction known as δ_C.

There must be a further modification of the value of the matrix element because there is no fundamental single-nucleon, mean-field potential in the nucleus. The mean field that binds the orbitals is only a first approximation to nuclear binding. The mean field arises from the average of two- (or more-) body interactions, but residual two- (or more-) nucleon effects must remain. There are residual interactions that cause long-range correlations, such as particle-vibration coupling and those that cause the short-ranged correlations mentioned above.

The fundamental theory for the Fermi interaction of proton β decay involves the isospin operator τ_+ and the Fermi matrix element is then given by $M_F = \langle f|\tau_+|i\rangle$, $|i\rangle$ and $|f\rangle$ the exact initial and final eigenstates of the full Hamiltonian $H = H_0 + V_C$, with energy E_i and E_f, respectively, and V_C denotes the sum of all interactions that do not commute with the vector isospin operator.

Here we extend the formalism of Refs. [13,14] by first developing an effective β-decay one-body operator that includes the dominant isospin-violating effects and then evaluating its matrix element in a strongly correlated system. Consider single-particle proton p and neutron n orbitals denoted by $|v, p\rangle$ and $|v, n\rangle$, in which the index v denotes the space-spin quantum numbers. These are eigenstate of a Hamiltonian, $h = h_0 + U_C(p)$, with a Coulomb potential, $U_C(p)$ that acts only on protons. The eigenkets of h_0 are denoted with rounded brackets and those of h with the usual Dirac notation. Then, using Wigner-Brillouin perturbation theory in U_C, one has

$$|v, p\rangle = \sqrt{Z_C}|v, p) + \frac{1}{E_v - \Lambda_v h_0 \Lambda_v}\Lambda_v U_C|v, p\rangle, \tag{3}$$

with $Z_C = 1 - \langle v, p|U_C \frac{1}{(E_v-\Lambda_v h_0 \Lambda_v)^2} U_C|v, p\rangle$ and $[v, (n, p)|\Lambda_v = 0$ with $|v, n\rangle = |v, n]$.

The single-particle superallowed β-decay matrix element, $M_{\rm sp} \equiv (v, n|\tau_+|v, p\rangle$, is given by the overlap $(v|v\rangle$,

$$M_{\rm sp} = \sqrt{Z_C}, \tag{4}$$

with the proton to neutron matrix element of τ_+ evaluated as unity. Evaluating $\sqrt{Z_C}$ to second order in U_C leads to

$$M_{\rm sp} \approx 1 - \frac{1}{2}\left(v|U_C \frac{1}{(E_v - \Lambda_v h_0 \Lambda_v)^2}\Lambda_v U_C|v\right), \tag{5}$$

with the second term as the isospin correction. This result repeats the well-known results that the electromagnetic corrections are of second order [13,14,57,58]. The dominant isospin correction of TH is twice the second term.

Next we turn to nuclear superallowed β decay. It is useful to define the one-body Coulomb-correction operator that appears in Eq. (5) as $\widehat{\mathcal{O}}_C(v) \equiv U_C \frac{1}{(E_v-\Lambda_v h_0 \Lambda_v)^2}\Lambda_v U_C$. Consider, as a first step, a simplified situation in which the initial nucleus i consisting of a proton in a valence orbital v outside an isospin-0 core state of A nucleons β decays to a neutron outside the same state, f. Then, the $p \to n$ matrix of τ_+ is still unity and is not mentioned below. The core of the state f is taken to be the same as that of the state i, so that their overlap does not influence the β-decay matrix element. This is an accurate treatment because the only isospin-violating influence on the wave function is caused by the external valence proton, a negligible $\mathcal{O}(1/A)$ effect.

Then the largest Coulomb correction is obtained by taking the matrix element $\delta_C(v)$ of the operator $\widehat{\mathcal{O}}_C(v)$. In coordinate space and suppressing spin indices, this quantity is given by

$$\delta_{C0}(v) = \int d^3r d^3r' \phi_v^*(\mathbf{r}) \mathcal{O}_C(\mathbf{r}, \mathbf{r}') \phi_v(\mathbf{r}'). \tag{6}$$

The simple single-particle state leading to Eq. (6) is only a first, mean-field approximation to the nuclear wave function. This is because the valence proton (neutron) undergoes strong interactions with the core nucleons that involve both long- and short-ranged correlations. Focusing on short-ranged, two-nucleon aspects, we need to compute the two-nucleon wave function given by

$$|v, \alpha\rangle = \sqrt{Z_S(v, \alpha)}|v, \alpha) + Q\frac{G}{e}|v, \alpha), \tag{7}$$

with α being one of the occupied orbitals of the $T = 0$ core state. Here, G[1] is the antisymmetrized [59] reaction matrix operator that sums ladder diagrams involving two-nucleon interactions. The factor Z_S insures the normalization, e represents an energy denominator, and iterations of the potential that correct the state $|i_0)$ are included in the schematic factor $Q\frac{G}{e}$. The Hermitian projection operator Q obeys $Q|v, \alpha) = 0$. and is constructed to exclude the long-ranged correlations so that *only* the short-ranged correlations are included in the correction that we study. Equation (7) includes only the leading-order term in the linked-cluster expansion of Refs. [59–63]. Defining an operator $\Omega \equiv Q\frac{G}{e}$, one has $Z_S(v, \alpha) = 1 - (v, \alpha|\Omega^\dagger\Omega|v, \alpha)$. Then,

$$\delta_C(v) = Z_S(v)[v|\widehat{\mathcal{O}}_C(v)|v] + \sum_\alpha \big(\{v, \alpha|\sqrt{Z_S(v, \alpha)}[\widehat{\mathcal{O}}_C(v)\Omega + \Omega^\dagger\widehat{\mathcal{O}}_C(v)] + \Omega^\dagger\widehat{\mathcal{O}}_C(v)\Omega|v, \alpha\}\big), \tag{8}$$

with $Z_S(v) \equiv 1 - \sum_\alpha (v, \alpha|\Omega^\dagger\Omega|v, \alpha) \equiv 1 - \kappa(v)$. This is the occupation probability, known always to be <1.

In this first analysis of the effect of SRC on superallowed β-decay, we rely on the existing literature that indicates that $Z_S(v) \approx 0.8$ for many states v, although with dependence on the specific state, nucleus, and interactions. This number comes from many experimental measurements and theoretical calculations cited above. To be specific, consider the case of a single proton in a $0f_{7/2}$ state outside an inert core of charge $Z = 22$, schematically representing the calculation for ^{46}V. A search of the literature finds a review [40] for $Z_S(0f_{7/2})$ reveals a value of $\kappa(0f_{7/2}) = 0.14$ for that state in ^{55}Ni. This is for a Hamiltonian with "mild short-range repulsion effects." The original paper [64] shows that $\kappa(0f_{7/2}) = 0.05$, obtained by including 10 harmonic oscillator shells. However, the shell-model space of TH is just 2 $\hbar\omega$. Thus, 0.14 is used here as a reasonable number to motivate an estimate. This value is consistent with the results of the independent phenomenological analysis of [30]. In the following, we use $\kappa = 1 - Z_S$ to simplify the notation.

Next we turn to an evaluation of the matrix element $[v|\widehat{\mathcal{O}}_C(v)|v]$ that provides a proof of the validity of Eq. (6). Our numerical results and Hartree-Fock calculations [65] show that the valence radial wave function is very well approximated by that of a three-dimensional harmonic oscillator. We therefore use harmonic-oscillator single-particle wave functions with the parameters of [65]. The overlap between the realistic wave functions of [65] is about 0.998.

The nuclear Coulomb potential arises from the convolution of $Z\alpha/(|\mathbf{r} - \mathbf{r}'|)$ with the charge density $\rho_C(r')$. If we take the latter to be a constant within $r \leqslant R_C$, the one-body Coulomb potential takes the form used by TH. The value of R_C is chosen to match the Coulomb potential obtained with a Fermi shape using $R_A = 1.1A^{1/3}$ fm and $a = 0.54$ fm. Our estimate takes the state $|v)$ to be in the single-particle orbit with radial quantum number $n = 0$ and angular momentum $l = 3$ appropriate for the state appearing in the first line of Table I for ^{46}V of Ref. [6]. The matrix element of U_C between the valence state and the state with n, l is $(0l|U_c|nl)$ so that, using Eq. (6), we find

$$\delta_{C0}(l) = \sum_{n>0} \frac{|(0l|U_c|nl)|^2}{4n^2\omega^2}. \tag{9}$$

Using this yields $\delta_{C0} = 0.267\%$, in agreement with the result in Table I for ^{46}V in Ref. [6].

At this stage, the result is that the leading Coulomb correction of TH is multiplied by the factor Z_S, potentially a very substantial reduction in terms of present accuracy requirements.

Now we turn towards the remaining terms of Eq. (8). First note that the operator Ω contains the projection operator Q that projects away from the initial state. The mean-field state $|v\alpha)$ is changed by the action of QG to one in which one or mainly both nucleons are above the Fermi sea. Then the one-body operator $\widehat{\mathcal{O}}_C(v)$ cannot connect the intermediate state to the mean-field state. Thus the terms of Eq. (8) that are linear in the operator Ω vanish and one has

$$\delta_C(v) = Z_S(v)[v|\widehat{\mathcal{O}}_C(v)|v] + \sum_\alpha [v, \alpha|\Omega^\dagger\widehat{\mathcal{O}}_C(v)\Omega|v, \alpha]. \tag{10}$$

Next we estimate the matrix elements of $\Omega^\dagger\widehat{\mathcal{O}}_C(v)\Omega$, the second-order terms. This is implemented here by modeling the operator Ω using the Jastrow correlation [62,63,66–73] approximation to the nuclear wave function. The most important and best-measured SRC involve two nucleons, in which the correlated wave function $\psi^{(2)}$ is related to the mean-field approximation $\phi^{(2)}$ with $\psi^{(2)} = (1 + f)\phi^{(2)}$. In the present situation, one of the nucleons is the decaying proton in orbital v and the other is any nucleon in the occupied orbital α, so $\phi^{(2)}$ represents the product state $|v\alpha)$. The correlations (including Pauli) are represented by a function $f(s)$, in which s is the separation distance. A schematic notation, in which various quantum numbers of the two-nucleon wave function are not explicitly specified, simplifies the presentation. Then, the operator Ω and $f(s)$ are related by $f(s) = \langle s|\Omega|\phi^{(2)}\rangle$ [62,63]. The function $f(s)$ is meant to represent *only* the short-ranged correlations, as mandated by the proper construction of the operator Q. The operator Ω acts only in two-nucleon states allowed by the Pauli principle. A first-principles calculation would explicitly state angular momentum and isospin

[1]For two-nucleon interactions, G is the two-nucleon T matrix evaluated at negative energy and modified by Pauli blocking effects.

quantum numbers L,J,S,T dependence of the functions f. Indeed, several partial-wave contributions enter in the computation of $\kappa = \sum_v \kappa(v)/A$ [74].

The nuclear force is repulsive at small separations and attractive at large separations. This means that in all existing models, f is negative for small values of s, rises to 0 or slightly above as values of s increase towards the region of attraction, and then falls to within 1 fm or so [69,73,75–79]. The details of $f(s)$ are model dependent, but the previous sentence holds. The function $f(s)$ is substantial only for small values of s.

Next we use the short-ranged nature of $f(s)$ to compute the second-order terms of Eq. (10). Defining that term as $\Delta\delta_{C0}(v)$, and evaluating in coordinate-space leads to the following expression:

$$\Delta\delta_{C0}(v) = \int d^3r d^3r' \phi_v(\mathbf{r}) I(\mathbf{r},\mathbf{r}') \mathcal{O}_C(\mathbf{r},\mathbf{r}')\phi_v(\mathbf{r}'), \quad (11)$$

with

$$I(\mathbf{r},\mathbf{r}') \equiv \int d^3r_2 \rho(\mathbf{r}_2) f(|\mathbf{r}-\mathbf{r}_2|) f(|\mathbf{r}'-\mathbf{r}_2|). \quad (12)$$

An examination of the integrals shows that $\Delta\delta_{C0}(v) > 0$, and tends to compensate for the reduction caused by using $Z(v) < 1$. A simple general analysis that focuses on the short-distance repulsion is used. Let us first suppose that the short-ranged correlations are captured by using $f(s) = -\lambda g(s)$, with $0 < \lambda \leqslant 1$ and $g(0) = 1$, and $g(s)$ is vanishing for values of s larger than a reasonable range, r_0, of order 1 fm or less. This form is a reasonable and flexible representation of the short-distance properties of all of the models [69,76–82] that allows us to study the model dependence. The values of λ and the function $g(s)$ are chosen to reproduce the value of κ via

$$\kappa = \rho_0 \lambda^2 \int d^3s\, g^2(s), \quad (13)$$

where ρ_0 is the density of nuclear matter ≈ 0.167 fm^3. Our philosophy is to take the value of κ as determined by experimentally measured spectroscopic factors and independent theory.

For the ^{46}V example used here, $\kappa = 0.14$. Then, using a Gaussian form, $g(s) = e^{-s^2/2r_0^2}$, leads to $r_0 = 0.532/\lambda^2$ fm, and using a square shape, $g(s) = \Theta(r_0 - s)$, leads to $r_0 = 0.585/\lambda^2$ fm.

Now turn to the evaluation of $I(\mathbf{r},\mathbf{r}')$. Note that because of the short-ranged nature of the correlations, the integrand of $I(\mathbf{r},\mathbf{r}')$ is substantial only when both $\mathbf{r}$ and $\mathbf{r}'$ are close to $\mathbf{r}_2$, and therefore close to each other. Because r_0 is much less than the nuclear radius, we approximate as a three-dimensional δ function via

$$g(s) = \delta(\mathbf{s}) \int d^3s g(s). \quad (14)$$

Numerical analysis shows that using this simplification provides an excellent approximation to the exact calculation. The ratio

$$\gamma \equiv \frac{\int d^3s\, g(s)}{\int d^3s\, g^2(s)} \quad (15)$$

is an important parameter in the following treatment.

Using Eq. (14) and Eq. (13) to evaluate $I(\mathbf{r},\mathbf{r}')$ leads to the result

$$I(\mathbf{r},\mathbf{r}') \approx \frac{\gamma^2}{\lambda^2}\left(\frac{\kappa}{\rho_0}\right)^2 \rho(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}'). \quad (16)$$

Using this expression to compute $\Delta\delta_{C0}(v)$ of Eq. (11) leads to

$$\delta_C(v) = Z_S(v)\delta_{C0}(v) + \frac{\gamma^2\kappa^2}{\lambda^2\rho_0^2}\int d^3r \phi_v^*(\mathbf{r}) \times \mathcal{O}_C(\mathbf{r},\mathbf{r}')\phi_v(\mathbf{r}')\rho(\mathbf{r}), \quad (17)$$

showing that the second-order term tends to compensate for the depletion caused by the factor $Z_S(v) < 1$. Furthermore, there is strong sensitivity to the value of $\gamma\kappa/\lambda$.

Next, use Eq. (16), to compute $\Delta\delta_{C0}(v)$ of Eq. (11), which is also the second term of Eq. (17). The ^{46}V model and parameters used to compute δ_{C_0} are again used. Then,

$$\Delta\delta_{C0}(v) = \frac{\gamma^2}{\lambda}\left(\frac{\kappa}{\rho_0}\right)^2 \frac{2l+1}{4\pi}\sum_{n=1} \frac{\int r^2 dr R_{0l}^2(r) R_{nl}^2(r) U_C^2(r)\rho(r)}{4n^2\omega^2}$$
$$= \frac{\gamma^2}{\lambda^2}\left(\frac{\kappa}{\rho_0}\right)^2 \frac{2l+1}{4\pi} 0.087\delta_{C0}, \quad (18)$$

with numerical evaluation and the results of using Eq. (9). For the case of interest ($l = 3, \kappa = 0.14$), we find

$$\Delta\delta_{C0}(v) = \frac{\gamma^2}{\lambda^2} 0.034\, \delta_{C0}. \quad (19)$$

The value of γ is determined by the shape of $g(s)$. Using a square shape yields $\gamma^2 = 1$ and using a Gaussian yields $\gamma^2 = 8$. With the former (and $\lambda = 1$), one finds that $\Delta\delta_{C0}(v)$ is negligible, but with the latter, the correction is $0.27\delta_{C0}$ and the effects of short-ranged correlations are to provide an overall increase of about 13%. One may use a Fermi function, $g(s) = 1/[1 + \exp(s - r_0)/a]$. In that case, varying the value of a from small values to about 1.9 fm smoothly interpolates the values of g^2 between 1 and 8. For the specific example, one obtains either an 11% decrease with $\gamma^2 = 1$ or a 13% increase with $\gamma^2 = 8$. If the value of κ is taken to be 0.2 for stronger short-range repulsion, this spread goes from a 20% decrease to a 30% increase. More generally, the resulting electromagnetic corrections to superallowed β decay can be increased or decreased substantially by the influence of short-ranged correlations.

Our considerations are limited to one state. The nuclear dependence of δ_C is important, as established by TH. Roughly speaking, our trend is very similar to theirs because the driving effect is the increase of the Coulomb interaction with increasing nuclear size. The cited theory and measurement references on the A dependence of spectroscopic factors indicate that the influence of SRC is likely to have little A dependence. Thus, trends similar to that of TH are to be expected.

In summary, the key result is that computations of the isospin correction are strongly sensitive to the effects of short-ranged correlations. The detailed features of the short-ranged correlations determine whether the influence is an increase, decrease, or no change. This is true despite the schematic nature of the present calculations. The correct evaluation of this effect can only be assessed precisely by doing detailed calculations with different models that account for the experimentally measured spectroscopic factors. This is important because tests of the unitarity of the CKM matrix demand very high accuracy. Doing more detailed state-of-the-art nuclear calculations of superallowed β decay is a high priority for nuclear theorists.

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