Letter | Editors' Suggestion

In-medium Λ isospin impurity from charge symmetry breaking in the ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He mirror hypernuclei

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The Λ separation energies in the mirror hypernuclei ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He exhibit large charge symmetry breaking (CSB). Analyzing this CSB within pionless effective field theory while using partially conserved baryon-baryon SU(3) flavor symmetry, we deduce a Λ-Σ^0 induced in-medium admixture amplitude $\mathcal{A}_{I=1} \approx 1.5\%$ in the dominantly isospin $I = 0$ Λ hyperon. Our results confirm the free-space value $\mathcal{A}^{(0)}_{I=1}$ inferred directly within the SU(3) baryon octet by Dalitz and von Hippel in 1964 and reaffirmed in a recent QCD+QED lattice calculation. Furthermore, exploring the consequences of SU(3) flavor symmetry on the Λ-nucleon interaction, we find that CSB is expected to impact the $S = 1$ and $S = 0$ spin channels in opposite directions, with the latter dominating by an order of magnitude. These observations explain a recent deduction of Λ-nucleon CSB strengths.

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Introduction. Renormalization of hadron decay constants in nuclear matter is a recurring theme in hadronic physics. Well known examples are the roughly 30% in-medium quenching of the pion decay constant f_π and the weak-decay axial-vector constant g_A. The quenching of f_π was inferred from deeply bound π^--atom levels in heavy nuclei [1], soon shown by Friedman to hold over the whole periodic table [2], in line with a chiral-symmetry partial restoration argument by Weise [3]. The quenching of g_A was noticed by Wilkinson in nuclear Gamow-Teller β decays [4] and soon attributed by Rho [5] to spin-isospin correlations manifest in the Ericson-Ericson-Lorentz-Lorenz renormalization of the p-wave pion-nucleus optical potential [6].

Less explored is the $\mathcal{S} = -1$ strange hadronic sector, where the poorer database of Λ hypernuclei [7] limits the deduction of in-medium trends. A particularly interesting question is whether and how far the Dalitz–von Hippel (DvH) [8] relatively large amplitude $\mathcal{A}^{(0)}_{I=1} \approx 1.5\%$ of an $I = 1$ admixture in the dominantly $I = 0$ Λ hyperon gets renormalized in dense matter. $\mathcal{A}^{(0)}_{I=1}$ was inferred by DvH from the Λ-Σ^0 mass-mixing matrix element $M_{\Sigma^0\Lambda}$ related in SU(3)-flavor [SU(3)$_f$] symmetry to octet baryon electromagnetic mass differences $\delta M_{BB'} = M_B - M_{B'}$:

$$M_{\Sigma^0\Lambda} = \frac{1}{\sqrt{3}}(\delta M_{\Sigma^0\Sigma^+} - \delta M_{np}) = 1.14 \pm 0.05 \text{ MeV}, \quad (1)$$

leading to the free-space value

$$\mathcal{A}^{(0)}_{I=1} = M_{\Sigma^0\Lambda}/\delta M_{\Lambda\Sigma^0} = -0.0148 \pm 0.0006. \quad (2)$$

We note that this result holds also in quark models, where SU(3) octet and decuplet baryons are assigned to the **56** SU(6) multiplet, by assuming only one-quark mass terms and two-quark interaction terms [9] but no three-quark interaction terms. More recently, Eq. (2) was confirmed in a QCD+QED lattice calculation [10], although with considerably larger uncertainty of order 30%.

Appreciable ΛN charge symmetry breaking (CSB) is implied by this isospin impurity of the Λ hyperon. Unfortunately, the poorly known two-body ΛN scattering data are limited to Λp. In Λ hypernuclei [7], CSB affects mirror states ($N \leftrightarrow Z$), e.g., the $0^+_{\text{g.s.}}$ and $1^+_{\text{exc.}}$ ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He levels where differences of Λ separation energies $B_\Lambda(J^\pi)$ are nonzero: $\Delta B_\Lambda(0^+_{\text{g.s.}}) = 233 \pm 92$ keV and $\Delta B_\Lambda(1^+_{\text{exc.}}) = -83 \pm 94$ keV; see Fig. 1. A particularly precise measure of CSB is given by their difference, $\Delta\Delta B_\Lambda = 316 \pm 20$ keV, equal to the difference ΔE_γ between the two γ-ray energies marked red in the figure. This value is about four times larger than the nuclear CSB splitting $\Delta B_{\text{CSB}}({}^3\text{H}\text{-}{}^3\text{He}) = 67 \pm 9$ keV in the mirror core nuclei ^{3}H-^{3}He [14]. The $A = 4$ hypernuclear CSB B_Λ splittings have been studied recently within chiral effective field theory (χEFT) at leading order

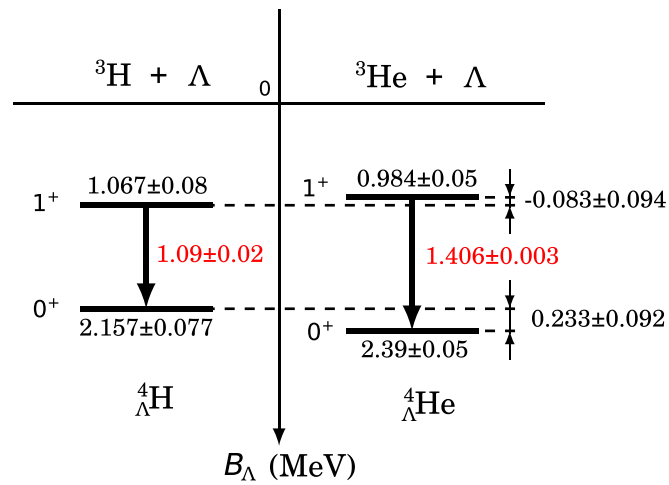

FIG. 1. $A = 4$ hypernuclear level scheme [11–13] with γ-ray energies [11] marked in red. CSB splittings are shown in MeV to the right of the ${}^{4}_{\Lambda}$He levels. Figure adapted from Ref. [13].

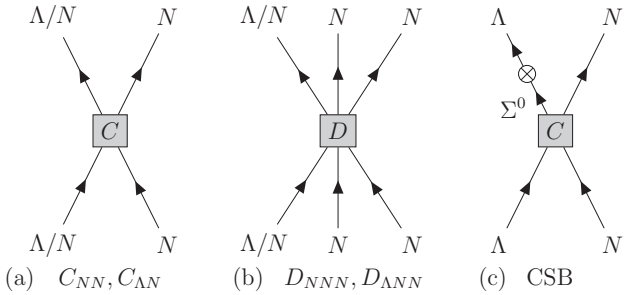

FIG. 2. Λ hypernuclear CS two-body (a) and three-body (b) contact-term diagrams and their associated low-energy constants C and D, respectively, plus in (c) a $\Lambda N \to \Sigma^0 N$ contact-term diagram, followed by a cross for Λ-Σ^0 mixing, to illustrate a $\not\pi$EFT(LO) realization of the CSB ansatz (3).

(LO) [15] and next-to-leading order (NLO) [16]. In the latter work, the ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He splittings were used to estimate the impact of CSB on the ΛN interaction. It was found that CSB acts predominantly on the spin $S = 0$ channel and that it affects the $S = 1$ and $S = 0$ channels in opposite directions, but no satisfactory theoretical argument was given to motivate this difference. Below, we show that these findings are consequences of SU(3)$_f$.

It was noted by DvH that Λ-Σ^0 mixing induces a long-range, one-pion exchange (OPE), CSB ΛN potential. This result was generalized in Ref. [17] relating the CSB ΛN potential to the charge-symmetric (CS) $\Lambda N \leftrightarrow \Sigma N$ transition potential,

$$\langle \Lambda N | V_{\rm CSB} | \Lambda N \rangle = -\frac{2}{\sqrt{3}}\, \mathcal{A}^{(0)}_{I=1}\, \langle \Sigma N | V_{\rm CS} | \Lambda N \rangle\, \tau_{Nz}. \qquad (3)$$

A schematic illustration of this CSB ansatz is given in Fig. 2, diagram (c). The factor 2 emerges from applying $\Lambda - \Sigma^0$ mixing to either incoming or outgoing Λ states. Projecting the $I_{YN} = 1/2$ ΣN state on the right-hand side onto its $\Sigma^0 N$ component produces the factor $-\tau_{Nz}/\sqrt{3}$, with $\tau_{Nz} = \pm 1$ for p, n respectively.

Here we explore whether the CSB ansatz (3) is satisfied in the ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He mirror hypernuclei, i.e., to what extent the value of $\mathcal{A}^{(0)}_{I=1}$ is renormalized in matter. To this end we introduce CSB into pionless effective field theory ($\not\pi$EFT) studies of few-body hypernuclei at LO [18,19]. Since ΛN one-pion exchange (OPE) is forbidden for the dominantly $I = 0$ Λ hyperon, a $\not\pi$EFT breakup scale $2m_\pi$ is assumed, remarkably close to the threshold value $p^{\rm th}_{\Lambda N} \approx 283$ MeV/c for exciting a ΣN pair. Although Σ hyperon degrees of freedom are generally excluded at LO, they may be entered implicitly for $p_{\Lambda N} \ll 2m_\pi$ through a CS $\Lambda N \leftrightarrow \Sigma N$ transition contact term, such as in Eq. (3), which we relate within a partially conserved SU(3)$_f$ to CS NN and ΛN diagonal contact terms. Doing so, we find that the free-space value $\mathcal{A}^{(0)}_{I=1} \approx -0.015$ persists also in the ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He mirror hypernuclei.

Model. The LO $\not\pi$EFT interaction for nucleons and Λ hyperons consists of two-baryon BB and three-baryon BBB s-wave contact terms shown schematically in Figs. 2(a) and 2(b). These contact terms are given by CS potentials of the form

$$V^S_{B_1B_2} = C^S_{B_1B_2}(\lambda)\, \mathcal{P}_S \delta_\lambda(\boldsymbol{r}_{12}), \qquad (4)$$

and

$$V^{IS}_{B_1B_2B_3} = D^{IS}_{B_1B_2B_3}(\lambda)\, \mathcal{Q}_{IS} \sum_{\rm cyc} \delta_\lambda(\boldsymbol{r}_{12}) \delta_\lambda(\boldsymbol{r}_{23}). \qquad (5)$$

Here, the λ (fm^{-1}) dependence attached to the low energy constants (LECs) $C^S_{B_1B_2}$ and $D^{IS}_{B_1B_2B_3}$ stands for momentum cutoff values, introduced in a Gaussian form to regularize the zero-range contact terms:

$$\delta_\lambda(\boldsymbol{r}) = \left(\frac{\lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\lambda^2}{4}\boldsymbol{r}^2\right), \qquad (6)$$

thereby smearing a zero-range (in the limit $\lambda \to \infty$) Dirac $\delta^{(3)}(\boldsymbol{r})$ contact term over distances $\sim\lambda^{-1}$. The cutoff parameter λ may be viewed as a scale parameter with respect to typical values of momenta Q. To make observables cutoff independent, LECs must be properly renormalized. Truncating $\not\pi$EFT at LO and using values of λ higher than the breakup scale $2m_\pi$, observables acquire a residual dependence $O(Q/\lambda)$ which diminishes with increasing λ. Finally, following the coupled-channel study of Ref. [20], we estimate that excluding explicit Σ hyperon degrees of freedom incurs a few percent error.

In Eq. (4), $\mathcal{P}_S$ projects on s-wave B_1B_2 pairs (NN or ΛN) with spin S associated, for a given cutoff λ, with four two-body LECs $C^S_{B_1B_2}$ fitted to low-energy two-body observables, e.g., to the corresponding CS NN and ΛN scattering lengths. Similarly, in Eq. (5), $\mathcal{Q}_{IS}$ project on NNN or ΛNN s-wave triplets with isospin I and spin S associated with four three-body LECs $D^{IS}_{B_1B_2B_3}$ fitted to given CS averages of binding energies: $A = 3$ without Λ, $A = 3, 4$ with Λ, thereby making the procedure applied in Ref. [18] explicitly CS. To calculate these binding energies, the A-body Schrödinger equation is solved variationally by expanding the wave function Ψ in a correlated Gaussian basis within a stochastic variational method [21]. Convergence to a level of below 1 keV was verified by increasing the number of basis states. The predictive power of CS $\not\pi$EFT(LO) in the s shell was already tested in Ref. [18] by calculating binding energies of ^{4}He and ${}^{5}_{\Lambda}$He and may soon be tested by comparing the calculated binding energy of ${}^{5}_{\Lambda\Lambda}$H-${}^{5}_{\Lambda\Lambda}$He [19], constrained by the ${}^{6}_{\Lambda\Lambda}$He binding energy datum, with a forthcoming measurement at J-PARC. A $\not\pi$EFT(LO) approach has been used recently in discussions of the ${}^{3}_{\Lambda}$H (hypertriton) lifetime [22] and of a likely Λnn continuum state [23].

Introducing CSB, the two-body ΛN contact terms in $V_{\Lambda N}$, Eq. (4), are modified by specifying nucleons as protons or neutrons:

$$C^S_{\Lambda N}\mathcal{P}_S \to \left(C^S_{\Lambda p}\frac{1+\tau_{Nz}}{2} + C^S_{\Lambda n}\frac{1-\tau_{Nz}}{2}\right)\mathcal{P}_S, \qquad (7)$$

This suggests to define CS and CSB LECs $C^S_{\Lambda N}$ and $\delta C^S_{\Lambda N}$, respectively, as

$$C^S_{\Lambda N} = \frac{1}{2}\left(C^S_{\Lambda p} + C^S_{\Lambda n}\right), \quad \delta C^S_{\Lambda N} = \frac{1}{2}\left(C^S_{\Lambda p} - C^S_{\Lambda n}\right). \qquad (8)$$

The ΛN interaction assumes then the form

$$V_{\Lambda N} = \sum_S \left(C^S_{\Lambda N} + \delta C^S_{\Lambda N}\,\tau_z\right)\mathcal{P}_S\delta_\lambda(\boldsymbol{r}_{\Lambda N}). \tag{9}$$

The CSB part of this potential, given in terms of two-body LECs $\delta C^S_{\Lambda N}$, is then treated perturbatively with respect to the LO CS wave function. No three-body CSB LECs are necessary at this order. The systematic accuracy of the present CS model calculations is about $(Q/2m_\pi)^2 \approx 6\%$, where $Q \approx \sqrt{2M_\Lambda B_\Lambda} \approx 66$ MeV/c is a typical momentum scale in ${}^4_\Lambda$H-${}^4_\Lambda$He. The suppression of OPE with respect to the dominant contact-terms contribution found in χEFT CSB calculations—see, e.g., Ref. [16]—leads to a similar error estimate ≈6% also for the present CSB calculations. Nevertheless, if judged by our residual cutoff dependence, a slightly larger error estimate is suggested. The precise determination of LO error requires consideration of higher order terms.

Results and discussion. The ΛN CSB LECs $\delta C^S_{\Lambda N}$, $S = 0, 1$, were fitted perturbatively to the two $A = 4$ binding-energy differences $\Delta B_\Lambda(0^+_{\rm g.s.})$ and $\Delta B_\Lambda(1^+_{\rm exc.})$ shown on the right of Fig. 1. Here we solved the resulting two linear equations for $\delta C^S_{\Lambda N}$, $S = 0, 1$, using LO ${}^4_\Lambda$H$(0^+, 1^+)$ wave functions generated by using CS LECs exclusively. Using ${}^4_\Lambda$He$(0^+, 1^+)$ wave functions instead leads to essentially the same result. Readjusting the CS three-body LECs $D^{IS}_{\Lambda NN}$ within given experimental errors of B_Λ's in the $A = 3, 4$ hypernuclear systems incurs only a few percent uncertainties in the fitted CSB LECs. The derived CSB LECs $\delta C^S_{\Lambda N}$, of order 1% of the respective ΛN CS LECs $C^S_{\Lambda N}$, were used in a distorted-wave Born approximation to produce ΛN scattering length differences $\delta a_{\Lambda N} = \frac{1}{2}(a_{\Lambda p} - a_{\Lambda n})$. Since there is no direct experimental extraction of ΛN scattering lengths, we used several model estimates for $a_S(\Lambda N)$ as input to our calculations. These models, including χEFT(LO) [26] and χEFT(NLO) [27] used in recent $A = 4$ CSB calculations [15,16], respectively, are listed in an inset to Fig. 3 and cited in its caption. The figure shows calculated values of $2\delta a_S$ as function of the cutoff momentum λ in these ΛN interaction models. In agreement with Ref. [16] we find that CSB hardly affects the spin triplet $a_{S=1}$, whereas the singlet $a_{S=0}$ of order $\approx (-2.5 \pm 0.5)$ fm is affected strongly, making $|a_0(\Lambda n)|$ larger by about 0.5 fm than $|a_0(\Lambda p)|$, roughly in proportion to $|a_0|$. The dominance of $S = 0$ CSB is shown below to arise naturally from SU(3)$_f$ considerations. Interestingly, going to pure neutron matter, the size of the (attractive) spin averaged ΛN scattering length $(3a_1 + a_0)/4$ increases by only ≈10% from its approximately 2 fm value in symmetric nuclear matter, thereby somewhat aggravating the "hyperon puzzle" [28–30] in neutron star matter.

To check the present extraction of $a_S(\Lambda p) - a_S(\Lambda n)$ we also applied a similar procedure to the nuclear NN case. We verified, successfully, that the experimentally derived CSB difference of $a_0(pp) - a_0(nn) = 1.6 \pm 0.6$ fm in the $(NN)_{S=0}$ sector can be obtained in $\not\pi$EFT(LO) from the $A = 3$ nuclear "datum" $\Delta B_{\rm CSB}({}^3$H-^{3}He$) = 67 \pm 9$ keV [14]. Details will be given elsewhere. Proceeding to the main point of this work, the ansatz Eq. (3), we identify $\langle \Lambda N|V_{\rm CSB}|\Lambda N\rangle$ for a given spin value $S = 0, 1$ with the CSB LEC $\delta C^S_{\Lambda N}$ extracted directly from the ${}^4_\Lambda$H-${}^4_\Lambda$He spectrum. Similarly, working within the framework of $\not\pi$EFT, we identify the spin dependent matrix element $\langle \Sigma N|V_{\rm CS}|\Lambda N\rangle$ with a new $\Sigma N \leftrightarrow \Lambda N$ LEC $C^S_{\Sigma N,\Lambda N}$. Following Dover and Feshbach [31], we use SU(3)$_f$ to relate $C^S_{\Sigma N,\Lambda N}$ to the NN and ΛN CS LECs established in the present application of $\not\pi$EFT(LO):

$$C^0_{\Sigma N,\Lambda N} = -3\left(C^0_{NN} - C^0_{\Lambda N}\right),$$
$$C^1_{\Sigma N,\Lambda N} = \left(C^1_{NN} - C^1_{\Lambda N}\right). \tag{10}$$

In the next step, inspired by Eq. (3), we replace the two CSB LECs $\delta C^S_{\Lambda N}$ derived from the two binding energy differences $\Delta B_\Lambda(0^+_{\rm g.s.})$ and $\Delta B_\Lambda(1^+_{\rm exc.})$ by two in-medium amplitudes $\mathcal{A}^S_{I=1}$, $S = 0, 1$, defined by

$$-\mathcal{A}^S_{I=1} = (\sqrt{3}/2)\,\delta C^S_{\Lambda N}/C^S_{\Sigma N,\Lambda N}, \tag{11}$$

where the CS LECs $C^S_{\Sigma N,\Lambda N}$ are expressed through Eq. (10), thereby eliminating any explicit reference to Σ hyperon degrees of freedom. The amplitudes $\mathcal{A}^S_{I=1}$, $S = 0, 1$, are shown on the left-hand side of Fig. 4 as function of the cutoff λ, for both $S = 0$ (upper) and $S = 1$ (lower) two-body spin states, using the same ΛN interaction models cited in the inset of Fig. 3. These amplitudes exhibit a rather weak dependence on λ, with a common value consistent with (and for $S = 0$ close to) the DvH value $-\mathcal{A}^{(0)}_{I=1} = 0.0148$ from Eq. (2).

The results exhibited for $\mathcal{A}^S_{I=1}$, $S = 0, 1$, in the left panel of Fig. 4 were derived using the relatively imprecise binding energy differences $\Delta B_\Lambda(0^+_{\rm g.s.}) = 233 \pm 92$ keV and $\Delta B_\Lambda(1^+_{\rm exc.}) = -83 \pm 94$ keV; see Fig. 1. Using instead the considerably more precise single value $\Delta E_\gamma = 316 \pm 20$ keV obtained from the difference between the two γ ray energies marked in the figure is, however, insufficient to determine both

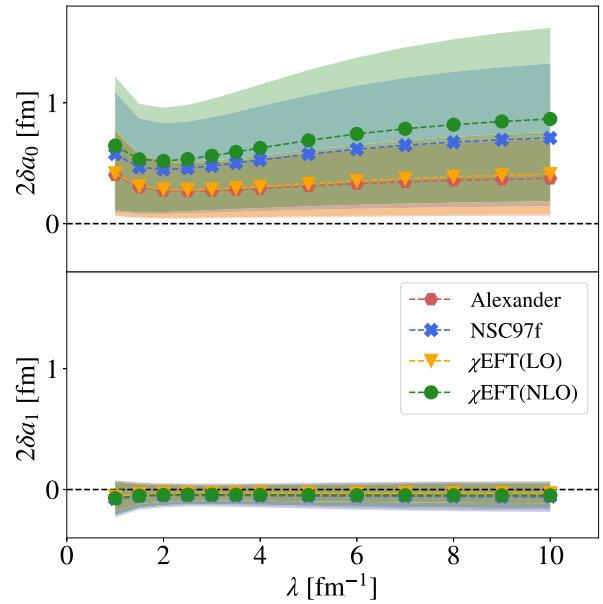

FIG. 3. Scattering length differences $2\delta a_S = a_S(\Lambda p) - a_S(\Lambda n)$ for $S = 0$ (upper) and $S = 1$ (lower) as a function of cutoff momentum λ, derived from $\delta C^S_{\Lambda N}$ LECs extracted here in $A = 4$ CSB calculations using ΛN potential models [24–27]; see text. The colored bands represent uncertainties induced by experimental errors in $\Delta B_\Lambda(0^+_{\rm g.s.})$ and $\Delta B_\Lambda(1^+_{\rm exc.})$ values.

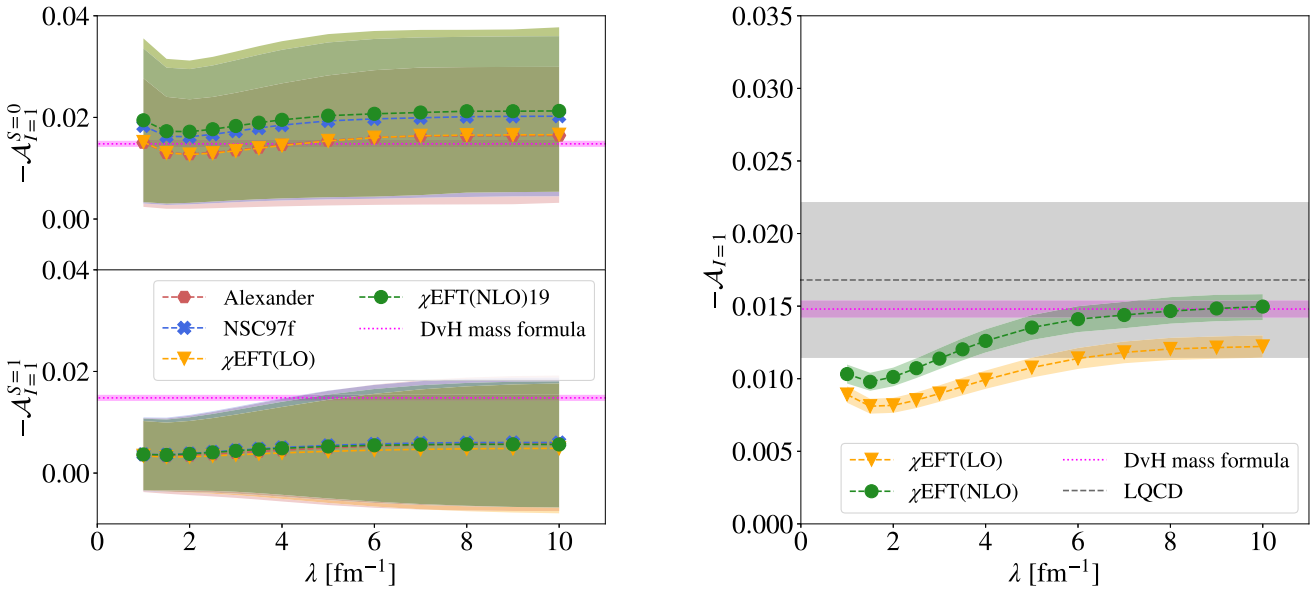

FIG. 4. Left: $\not\pi$EFT estimates for the in-medium DvH amplitudes $-\mathcal{A}^S_{I=1} = (\sqrt{3}/2)\delta C^S_{\Lambda N}/C^S_{\Sigma N,\Lambda N}$ ($S = 0$, upper; $S = 1$, lower), with $\delta C^S_{\Lambda N}$ derived by fitting $\Delta B_\Lambda(0^+_{\rm g.s.})$ and $\Delta B_\Lambda(1^+_{\rm exc.})$, plotted as a function of the cutoff momentum λ and specified in an inset by the ΛN interaction model [24–27] input. Colored bands provide uncertainties caused by ΔB_Λ input values. Right: $-\mathcal{A}_{I=1}$ values from $\Delta E_\gamma = E_\gamma(^4_\Lambda\mathrm{He}) - E_\gamma(^4_\Lambda\mathrm{H})$; see text. Colored bands provide uncertainties caused by that of ΔE_γ. The horizontal colored intervals mark, in pink, the DvH [8] value of $-\mathcal{A}^{(0)}_{I=1} = 0.0148 \pm 0.0006$ from Eq. (2), and in grey the LQCD [10] value 0.0168 ± 0.0054; see also Table I. Note the expanded vertical scale on the right with respect to that on the left.

$\mathcal{A}^{S=0}_{I=1}$ and $\mathcal{A}^{S=1}_{I=1}$. Assuming spin independence to start with, a precise value $\mathcal{A}_{I=1}$ is derived from ΔE_γ as shown on the right-hand side of Fig. 4. For all four ΛN potential models considered on the left-hand side, the derived value (shown for the χEFT models) is within the LQCD horizontal band for cutoff $\lambda \gtrsim 6$ fm^{-1}, while for NSC97f [25] and χEFT(NLO) [27] (and also its 2019 version [32]) the derived values do enter the considerably narrower ($\pm 4\%$) DvH SU(3)$_f$ horizontal band for $-\mathcal{A}^{(0)}_{I=1}$. All in-medium isospin $I = 1$ admixture amplitudes $-\mathcal{A}_{I=1}$ calculated for $\lambda \gg 2m_\pi$ exhibit a $\sim\lambda^{-1}$ dependence, and the corresponding values extrapolated to the renormalization scale-invariance limit $\lambda \to \infty$ are listed in Table I.

In view of its success, the above procedure can also be applied in reverse. That is, substituting the DvH value $\mathcal{A}^{(0)}_{I=1}$ from Eq. (2) and the SU(3)$_f$ relations Eq. (10) in Eq. (11), one extracts the CSB LECs $\delta C^S_{\Lambda N}$ and evaluates the $A = 4$ CSB splittings $\Delta B_\Lambda(0^+_{\rm g.s.})$ and $\Delta B_\Lambda(1^+_{\rm exc.})$. Doing so, we find that the ΔB_Λ values marked in Fig. 1 are reproduced within their experimental error. We also find that (i) the resulting LECs $\delta C^0_{\Lambda N}$, $\delta C^1_{\Lambda N}$ have opposite signs, and (ii) $|\delta C^0_{\Lambda N}| \gg |\delta C^1_{\Lambda N}|$, implying that CSB acts predominantly on the spin $S = 0$ channel.

TABLE I. Estimates of the $I = 1$ admixture amplitude $-\mathcal{A}_{I=1}$ in the Λ hyperon from (i) baryon-number $B = 1$ free-space studies of DvH and LQCD [8,10] and (ii) the present $\not\pi$EFT(LO) $B = 4$ $^4_\Lambda$H-$^4_\Lambda$He CSB study, using Eq. (3) and input from ΛN χEFT models. The $B = 4$ values are extrapolations to the renormalization scale invariance limit $\lambda \to \infty$ and their listed uncertainties reflect primarily input data uncertainties.

Method/Input	B	$-\mathcal{A}_{I=1}$
SU(3)$_f$ [8]	1	0.0148 ± 0.0006
LQCD [10]	1	0.0168 ± 0.0054
$\not\pi$EFT(LO)/χEFT(LO) [26]	4	0.0139 ± 0.0013
$\not\pi$EFT(LO)/χEFT(NLO) [27]	4	0.0168 ± 0.0014

Our results suggest strongly that the Λ's $I = 1$ isospin impurity of magnitude $\approx 1.5\%$ in free space is upheld in the $A = 4$ Λ mirror hypernuclei. This conclusion appears natural in the context of partial restoration of chiral symmetry in dense matter, since there is no direct link known to us between Λ-Σ^0 mixing and chiral symmetry breaking; for example, the nonstrange quark-mass difference $m_d - m_u$ does not enter the SU(3)$_f$ mass mixing matrix element $M_{\Sigma^0\Lambda}$, Eq. (1).

Closing remarks. $\not\pi$EFT(LO) applications to few-body hypernuclei are normally limited to N and Λ degrees of freedom. To consider hypernuclear CSB, the set of two-body and three-body s-wave CS LECs shown schematically in (a) and (b) of Fig. 2 was extended, adding two $S = 0, 1$ ΛN CSB LECs which were then fitted to the two experimentally available CSB $A = 4$ ΔB_Λ values. The resulting CS broken values of the ΛN scattering lengths shown in Fig. 3 come out then practically the same as those derived within a more involved χEFT(NLO) approach [16] that includes additional Σ hyperon and pseudoscalar octet meson degrees of freedom.

Apart from suggesting an economical way to evaluate CSB in strange matter, we were able to show that CSB is linked uniquely to the $I = 1$ isospin impurity $\mathcal{A}^{(0)}_{I=1}$ of the dominantly $I = 0$ Λ hyperon, provided (i) the ansatz (3) is adopted and (ii)

the $\langle \Sigma N|V_{\rm CS}|\Lambda N\rangle$ transition matrix element in (3) is related by SU(3)$_f$ to the $\langle NN|V_{\rm CS}|NN\rangle$ and $\langle \Lambda N|V_{\rm CS}|\Lambda N\rangle$ matrix elements, each near its own threshold. Having managed to avoid introducing explicitly Σ hyperon degrees of freedom into the employed $\not\pi$EFT(LO) scheme, we are spared of trying to impose SU(3)$_f$ symmetry simultaneously on all ΛN, ΣN, and $\Lambda N \leftrightarrow \Sigma N$ LECs fitted to low-energy scattering and reaction data, proven impossible at LO in χEFT [26] as discussed recently [32] upon introducing a new χEFT(NLO)19 version. It is worth mentioning that two ΣN channels, specifically ($I = \frac{3}{2}$, $J^\pi = 1^+$) and ($I = \frac{1}{2}$, $J^\pi = 0^+$), are plagued by "Pauli forbidden" six-quark configurations, say *uuuuds* for $I_z = \frac{3}{2}$ of the former channel [33], providing thereby a specific mechanism beyond the scope of SU(3)$_f$.

The usefulness of the CSB approach outlined here for the $A = 4$ mirror hypernuclei, where precise γ ray data exist, should be tested in heavier hypernuclei when similarly precise CSB data become available. However, phenomenological arguments regarding Σ hyperon admixtures in Λ hypernuclei [17,34] lead us to believe that CSB splittings of $1s_\Lambda$ mirror levels decrease quickly with A, making such tests more difficult but nonetheless challenging.

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Correction: Missing superscripts in Eqs. (4) and (5) and a missing summation in Eq. (9) have been inserted.