Comment on "Effect of density and nucleon-nucleon potential on the fusion cross section within the relativistic mean field formalism"

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We discuss critical approximations making the calculated results of the article by Bhuyan *et al.* [Phys. Rev. C **101**, 044603 (2020)] unreliable. Namely, all target nuclei used for the calculations are strongly deformed, whereas, in the article they are considered to be spherical. In addition, we indicate other misleading points and missing/incorrect references.

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It is said in Ref. [1] "Here, we use spherical densities for the target (t) and projectile (p) as the input to estimate the nucleus-nucleus interaction potential." However, all target nuclei used for the calculations in Ref. [1] are strongly deformed (see, e.g., Refs. [2,3]). The deformations are known to be of crucial importance for the capture cross section near and below the barrier. To demonstrate this, we evaluated the capture cross sections within the single parabolic barrier penetration model using the code of Ref. [4]. In these calculations for the nucleus-nucleus potential, the double-folding model is used. For the proton densities, the $2pF$ formula is applied with the charge density parameters from Ref. [5] [the diffuseness has been corrected for the finite charge distributions in the proton and neutron (see Eq. (19) in Ref. [4])]. The neutron densities were scaled using the N/Z ratio. For the effective *NN* forces, the M3Y interactions with the zero-range exchange term are utilized. It is well known that the M3Y interaction is developed in two versions, Paris and Reid (see, e.g., Refs. [6–8]), therefore, we performed calculations using these two options. Results are presented in Table I in comparison with the experimental data of Ref. [9]. Note that in Ref. [10] the data referred to as data from Ref. [9] do not coincide with those from Ref. [9]; the same happens with data from Ref. [11] presented in Ref. [10]. We prefer to use the data from the original paper [9]. From the results presented in Table I, one sees that ignoring the deformation makes the calculated cross sections vanishingly small in comparison with the data except for the highest energy. Yet accounting for the deformations of uranium brings the Paris cross sections in reasonable agreement with the data. Note that in Ref. [1] the Reid version has been used which results in poorer agreement with the data for the calculations presented in Table I.

In addition, in Ref. [1], the M3Y *NN* forces are called "phenomenological." Meanwhile, stressing the phenomenological character of these forces seems not to be in accord with literature: in Refs. [4,6–8,12–17], M3Y *NN* forces are referred to as microscopic or semimicroscopic.

Furthermore, there are several incorrect or missing references in Ref. [1]. Namely, the following references concerning the relativistic mean-field (RMF) and M3Y effective *NN* interactions are not relevant:

(i) Reference [69] of Ref. [1], Schiff, Phys. Rev. **83**, 252 (1951), is about the electron scattering and is not related to Eqs. (7) and (8);

(ii) Reference [69] of Ref. [1], Schiff, Phys. Rev. **84**, 10 (1951), and Ref. [70] of Ref. [1], Schiff, Phys. Rev. **84**, 1 (1951), contain neither Eqs. (7) and (8) nor any details about these equations.

Meanwhile, the following works, indeed including discussions and applications of the M3Y *NN* forces, are ignored [4,6–8,12–19]. Also, the work by Walecka [20] where an equation similar to Eq. (8) for the RMF *NN* forces seems to have been written for the first time is ignored.

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TABLE I. Experimental σ_{exp} [9] and calculated capture cross sections for the reaction $^{48}Ca + ^{238}U$. These calculations were performed for spherical (σ_{sph}) and deformed (σ_{def}) target nucleus with Paris and Reid M3Y *NN* forces.

E_{cm} (MeV)		181	186	193	201
σ_{exp} (mb)		2.48×10^{-2}	3.04	4.02×10^{1}	1.07×10^{2}
σ_{sph} (mb)	Paris	1.57×10^{-9}	2.30×10^{-6}	4.18×10^{-2}	8.58×10^{1}
	Reid	3.00×10^{-10}	4.40×10^{-7}	8.08×10^{-3}	5.67×10^{1}
σ_{def} (mb)	Paris	1.48×10^{-2}	3.64	3.17×10^{1}	1.11×10^{2}
	Reid	2.97×10^{-3}	1.88	2.52×10^{1}	8.91×10^{1}
$\frac{\sigma_{sph}}{\sigma_{exp}}$	Paris	6.4×10^{-8}	7.6×10^{-7}	1.0×10^{-3}	0.80
	Reid	1.2×10^{-8}	1.5×10^{-7}	2.0×10^{-4}	0.53
$\frac{\sigma_{def}}{\sigma_{exp}}$	Paris	0.60	1.20	0.79	1.03
	Reid	0.12	0.62	0.63	0.83

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