Electromagnetic transitions between low-lying nonrotational states of odd-neutron nuclei in α-decay chains starting from ^{265,267,269}Hs

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Background: Calculations of the structure of the low-lying states of nuclei with $Z \geqslant 100$ play an important role in understanding the properties of nuclei belonging to the new region of the nuclide chart, which is available now for experimental study.
Purpose: We calculate quasiparticle-phonon structure and the reduced γ-transition probabilities for the excited states with excitation energies up to 1 MeV for nuclei with $Z \geqslant 100$.
Methods: The quasiparticle-phonon model, which takes into account the quasiparticle-phonon interaction of different multipolarities, is used as a basis for the calculations.
Results: The quasiparticle-phonon structure and the γ-reduced transition probabilities of odd-neutron ^{265,267,269}Hs, ^{261,263,265}Sg, ^{257,259,261}Rf, ^{253,255,257}No, and ^{249,251,253}Fm are calculated. The α-decay chains starting from ^{265,267,269}Hs are analyzed.
Conclusion: It is shown that below 500 keV the structure of the nuclear states is mainly exhausted by the single-quasiparticle component. The quasiparticle-phonon interaction starts to play an important role at excitation energies above 500 keV. The nuclei before Fm in the α-decay chains starting from ^{265,267,269}Hs have at least two α-decay lines.

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I. INTRODUCTION

Recent experimental studies of superheavy nuclei [1–7] have led to the production of new nuclei and their isotopes, and have provided us valuable information about their single-particle excitations and equilibrium deformations. To support the experimental investigations, systematic calculations of single-particle spectra of the heaviest nuclei have been performed [8–12] within the model of an independent quasiparticle with quasiparticle-phonon coupling, which is important for correct description of the excitation spectra [13–15]. These calculations, along with experimental results, allowed us to assign quantum numbers to the states observed and analyze the α-decay chains.

The microscopic approaches, which are used to study the structure of the heaviest nuclei, are either the self-consistent ones based on some parametrizations of the energy-density functional [16–30] or the microscopic-macroscopic methods [10,11,31–37] and the quasiparticle-phonon model (QPM) [38–41]. In the present work the spectra and reduced γ-transition probabilities are calculated within the QPM using the microscopic-macroscopic approach to locate the ground state in the space of deformation parameters. This approach has been used already to describe the structure of well deformed rare-earth and actinide nuclei with $A > 228$ [42–50]. Also the shape coexistence in Fl isotopes was found [12]. Here, the coupling to phonons in the QPM is fixed to have a better description of known states. Then this coupling is used for all nuclei considered.

Any analysis of the properties of nuclei is based on the information of their equilibrium deformations. In many cases the low-lying excitations of nuclei are very sensitive to the deformation parameters of the ground state. As found experimentally [51], the deformations of nuclei even with close values of Z or A can differ, so this difference should be taken into account in the calculations. In the present work, the equilibrium deformations are defined within the microscopic-macroscopic approach [12]. In Ref. [12], the calculated low-lying spectra of the odd-neutron transcurie nuclei are presented. We are going to supplement these results by calculations of the $M1$, $E1$, $E2$, and $E3$ reduced transition probabilities between the low-lying states of the considered nuclei. These calculations will provide us a good basis for the investigation of the appearance of isomeric states in the heaviest nuclei. The lifetimes of found isomeric states will be estimated and possible α decays from these states will be discussed. The α-decay chains starting from ^{265,267,269}Hs will be analyzed to find possible α-decay energies.

II. MODEL HAMILTONIAN

The QPM Hamiltonian has the following structure:

$$H = H_{sp} + H_{\text{pair}} + H_M + H_{SM}, \tag{1}$$

where H_{sp} is the single-particle part of the total Hamiltonian, and H_{pair} describes the monopole pairing forces with strength set to reproduce the odd-even differences of the experimental nuclear masses. The terms H_M and H_{SM} in (1) take into account the multipole and spin-multipole residual forces. The mean field potential V_{sp} in H_{sp} contains the central potential V_{WS} in the Woods-Saxon form for neutrons and protons, the spin-orbit part V_{ls}, and the Coulomb field for protons, V_C:

$$V_{sp} = V(r) + V_{ls}(r) + V_C, \tag{2}$$

where

$$V_{WS}(r) = \frac{-V_0^\tau}{1 + \exp\{[r - R(\theta, \varphi)]/a\}} \tag{3}$$

and V_{ls} are defined as in Ref. [38] taking into account the isospin dependence of V_0^τ, where τ denotes neutron or proton. Here we assume axially deformed shape of nucleus defined as

$$R(\theta, \varphi) = R_0[1 + \beta_0 + \beta_2 Y_{20}(\theta, \varphi) + \beta_4 Y_{40}(\theta, \varphi)], \tag{4}$$

with $R_0 = r_0 A^{1/3}$. The parameter β_0 takes into account the volume conservation, and β_2, β_4 are the parameters of the quadrupole and hexadecapole axial deformations, respectively.

The higher multipoles could significantly influence the order of quasiparticle states in the case of a dense spectrum [52]. So, the uncertainty in the order of states should be noted for nuclei with a dense spectrum near the Fermi surface. However, the order of two nearest states with large ΔK does not affect the existence of low-lying isomeric one-quasiparticle states.

To describe the long-range particle-hole residual interaction, the effective separable forces are expressed through the operators of multipoles and spin-multipole moments:

$$\begin{aligned} H_M &= -\frac{1}{2} \sum_{\tau=p,n} \sum_{\rho=\pm 1} \sum_{l\mu} \left(\kappa_0^{l\mu} + \rho \kappa_1^{l\mu}\right) M_{l\mu}^+(\tau) M_{l\mu}(\rho\tau), \\ H_{SM} &= -\frac{1}{2} \sum_{\tau=p,n} \sum_{\rho=\pm 1} \sum_{l\lambda\mu} \left(\kappa_0^{l\lambda\mu} + \rho \kappa_1^{l\lambda\mu}\right) M_{l\mu}^{(\lambda)+}(\tau) M_{l\mu}^{(\lambda)}(\rho\tau). \end{aligned} \tag{5}$$

If τ corresponds to neutron then $-\tau$ denotes proton and vice versa. All other information on the Hamiltonian is given in [12].

After transformation to quasiparticle and phonon operators, the Hamiltonian takes the form

$$\begin{aligned} H &= \sum_q \epsilon_q \alpha_q^+ \alpha_q + \sum_{\mu\pi i} \omega_{\mu^\pi i} Q_{\mu^\pi i}^+ Q_{\mu^\pi i} \\ &\quad + \sum_{qq'\mu\pi i} \Gamma_{qq'\mu^\pi i} \alpha_q^+ \alpha_{q'} (Q_{\mu^\pi i}^+ + Q_{\mu^\pi i}), \end{aligned} \tag{6}$$

where α_q^+ is the creation operator of the quasiparticle in the state q with the energy ϵ_q, and $Q_{\mu^\pi i}^+$ is the creation operator of the ith phonon with the energy $\omega_{\mu^\pi i}$ in the state with given μ and parity π. Thus, in the random phase approximation the problem is reduced to determining the phonon energy in even-even nuclei and the excitation energies of odd-A nuclei. Note that the amplitudes $\Gamma_{qq'\mu^\pi i}$ of the quasiparticle-phonon interaction do not contain free parameters and are uniquely determined by the matrix elements of the residual interaction, phonon energy, and other phonon characteristics.

The Hamiltonian (6) is diagonalized in the configuration space including single-quasiparticle and quasiparticle $\otimes$ phonon states. The wave function of an odd-A nucleus has the structure

$$\Psi(K^\pi) = \left(\sum_\rho C_\rho \alpha_\rho^+ + \sum_{\nu\mu\pi' i} D_{\nu\mu^{\pi'} i} \alpha_\nu^+ Q_{\mu^{\pi'} i}^+ \right) \Psi_0 \tag{7}$$

with normalization

$$\sum_\rho C_\rho^2 + \sum_{\nu\mu\pi i} D_{\nu\mu^\pi i}^2 = 1. \tag{8}$$

The summation over single-particle state quantum number ρ includes all states with given K^π. Ψ_0 is the quasiparticle and phonon vacuum. We confine ourself in this paper to the three lowest phonon states ($i = 1, 2, 3$) with $\mu = 0, 1, 2, 3$. As shown in our calculations, the contribution of the phonons with higher i and μ to the wave functions of the states with energies lower than 1.2 MeV is small. The calculations include single-particle states with energies from the bottom of the potential up to +5 MeV.

The calculation of the equilibrium deformations are carried out using the microscopic-macroscopic two-center shell model [32,35,36] taking into account pairing and Strutinsky shell corrections [53,54]. Note that, with the parameters used, the energy spectra of the Woods-Saxon and the two-center potentials coincide with a good accuracy (see Fig. 1 in [12]).

The basic set of the Woods-Saxon parameters and the spin-orbit strength were suggested in our previous publications based on the numerous calculations of the one-quasiparticle spectra of well studied heavy nuclei. We use the same parameter set also for superheavy nuclei. To check the sensitivity of the results to the variation of the Woods-Saxon parameters, the calculations were performed not only with the basic set but also with other sets of the single-particle potential parameters. As shown, reasonable variations of the radius and diffuseness parameters do not cause large changes in the energy spectra. The variation of the spin-orbit strength produces larger deviations in energies. However, these variations do not exceed 300 keV, which is still acceptable to conclude on the stability of the results obtained.

As mentioned above, the calculations are performed taking into account phonons with $\mu \leqslant 3$. Inclusion of phonons with $3 < \mu \leqslant 7$ changes very little the results of calculations. Of course, a significant variation of the multipole strength constants, which leads to large changes of the phonon energies, influences strongly the excitation spectra, but only at energies larger than 400 keV where the admixture of the quasiparticle $\otimes$ phonon components can be essential. Other details of the calculation scheme are given in Ref. [12].

III. γ TRANSITIONS BETWEEN LOW-LYING STATES IN NUCLEI OF α-DECAY CHAINS

Using the theoretical approach formulated and the basic set of the parameters described above, we calculate the reduced γ-transition probabilities between the low-lying states in several superheavy nuclei. The structure of the nonrotational

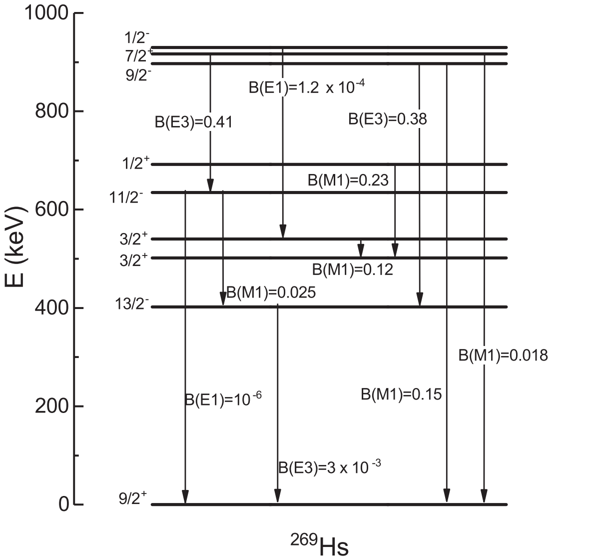

FIG. 1. Calculated one-quasiparticle spectrum of ^{269}Hs. The structure of all indicated states is exhausted by the one-quasiparticle component. The reduced transition probabilities are given in Weisskopf units.

low-lying excited states is determined by the single-particle levels lying near the Fermi surface and their coupling to phonons. We study the γ transitions in odd-neutron nuclei with Z from 100 to 108 and number of neutrons from 149 to 161. The nuclei in α-decay chains are considered to trace the evolution of the characteristics of γ transitions and find isomeric states whose influence on delay of α decay cannot be excluded.

A. α-decay chain starting from ^{269}Hs

Consider first the following α-decay chain: ^{269}Hs–^{265}Sg–^{261}Rf–^{257}No–^{253}Fm. The calculated ground state of ^{269}Hs has $K^{\pi} = 9/2^{+}$ (Fig. 1) and its structure is exhausted (99%) by the [615↓] one-quasiparticle component. The order of one-quasiparticle levels in Fig. 1 is similar to that presented in Ref. [12] because the spectrum of this nucleus is rather sparse and less sensitive to the parameters used than the dense spectrum near the ground state. The calculated first excited state of this nucleus is $K^{\pi} = 13/2^{-}$ and its structure is exhausted by the [716↑] (99%) one-quasiparticle state. The $E3$ reduced transition probability between these two states is equal to $B(E3; 13/2^{-} \rightarrow 9/2^{+}) = 3 \times 10^{-3}$ W.u. This corresponds to the γ-decay lifetime $\tau_{\gamma} = 0.08$ s. Thus, the $13/2^{-}$ (402 keV) excited state is an isomeric state. All states shown in Fig. 1 have a one-quasiparticle structure with the weight of this component larger than 96%.

Possible α-decay chains starting from ^{269}Hs are shown in Fig. 2. The ground state of ^{269}Hs decays by emitting an α particle to the $9/2^{+}$ excited state of ^{265}Sg with the calculated excitation energy 302 keV. This is a favored α decay since quantum numbers of the odd neutron are not changed. This state decays by $E1$ transition with $B(E1; 9/2^{+}(302\text{ keV}) \rightarrow 11/2^{-}(22\text{ keV})) = 0.12 \times 10^{-5}$ W.u. to the isomeric state of ^{265}Sg with $K^{\pi} = 11/2^{-}$ at energy 22 keV. Thus, in this case

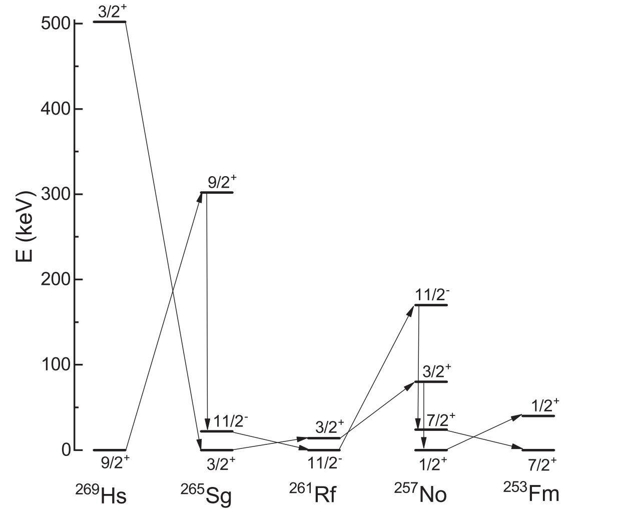

FIG. 2. Possible α-decay chains starting from ^{269}Hs.

the α decay of ^{265}Sg occurs from the isomeric state $11/2^{-}$ to the calculated ground state of ^{261}Rf (Fig. 2).

However, another scenario cannot be excluded. According to our calculations the ground state of ^{265}Sg is the one-quasiparticle state $3/2^{+}[622\downarrow]$ (Fig. 3). If the isomeric state $3/2^{+}[622\downarrow]$ at 502 keV is populated in ^{269}Hs, the favored α decay from it to the ground state of ^{265}Sg is possible. This α decay is about 200 times faster than that from the ground state because of larger value of Q_{α}. In ^{269}Hs, the γ transitions from the $3/2^{+}$ state are strongly hindered because of weak $M3$ transition to the ground state. So, the lifetime of the $3/2^{+}$ isomeric state with respect to γ transitions can be longer than $\tau_{\alpha} \approx 80$ ms estimated for α decay from this state. Thus, the ground and isomeric states of ^{265}Sg are populated in the α decay of ^{269}Hs. The α decays from these states populate the

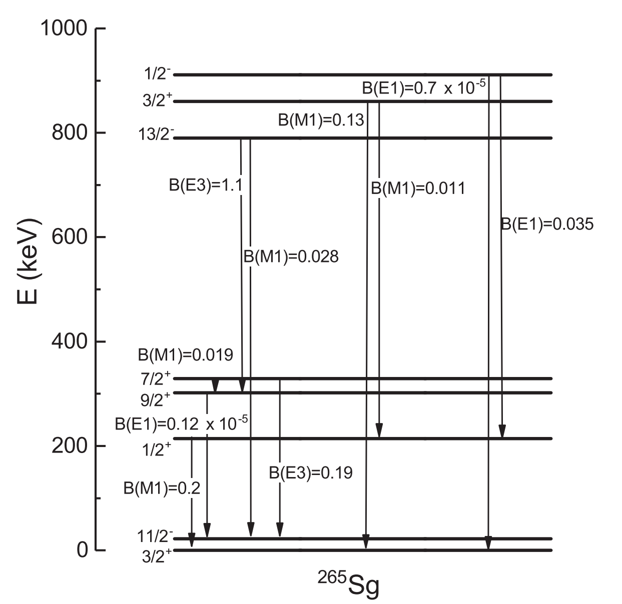

FIG. 3. The same as in Fig. 1, but for ^{265}Sg.

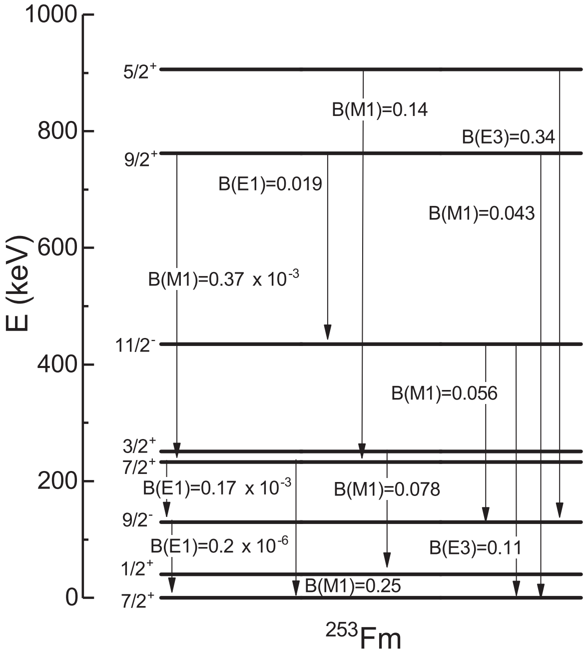

FIG. 4. The same as in Fig. 1, but for ^{253}Fm.

In this paper we do not consider unfavored α decays like $9/2^+(^{269}\mathrm{Hs}) \rightarrow 11/2^-(^{265}\mathrm{Sg})$ or α decays like $7/2^+(^{255}\mathrm{No}) \rightarrow 9/2^-(^{251}\mathrm{Fm})$. The reason is the following. Due to the effect of the pairing correlations, the α particles are formed from nucleon pairs occupying many states around the Fermi level, if the single-particle state of the odd-nucleon is not changed. This leads to the increase of the α-decay probability. In the opposite case, if in the α-decay odd-neutron transits from the single-particle state s_1 to the single-particle state s_2, and α particle is formed of proton pairs that occupy the orbitals near the Fermi surface but only from neutrons being in the states s_1 and s_2. Therefore, the unfavored α-decays are strongly hindered compared to the favored ones in even-odd nuclei.

We mention also that based on the formalism in [59] there is a hindrance factor of $(3\text{–}4)^l$ for α decays with nonzero angular momentum $l = \Delta K$ of the α particle. So, the α decays with $\Delta K = 1$ are at least three times weaker than those with $\Delta K = 0$. The weaker α decays which cannot be completely ruled out enrich the α-decay spectra, but do not change the population of isomeric states in the α-decay chains considered.

corresponding states in ^{261}Rf. So, the α decay of ^{261}Rf can occur from both the ground state $11/2^-$ and the isomeric state $3/2^+[622\downarrow]$ at energy 14 keV.

The favored α decay from the ground state of ^{261}Rf populates the excited state $11/2^-$ in ^{257}No at excitation energy 170 keV. This state decays by $E3$ transition to the isomeric state $7/2^+$ in ^{257}No at energy 24 keV. For this γ transition $B(E3; 11/2^-(170\text{ keV}) \rightarrow 7/2^+(24\text{ keV})) = 0.12$ W.u. In α decay of the isomeric state $7/2^+$ (24 keV) of ^{257}No the ground state $7/2^+$ of ^{253}Fm is populated. The α decay from the isomeric state $3/2^+$ of ^{261}Rf populates the state $3/2^+$ in ^{257}No from which $M1$ transition to the ground state $1/2^+$ is likely. So, in ^{257}No the ground and low-lying isomeric states can be populated in the α-decay chain of ^{269}Hs and α decays from these states to the corresponding states in ^{253}Fm are possible. Indeed, the isomeric state $7/2^+$ in ^{257}No is at small energy and can live for a time comparable to the time of α decay. The α decay of ^{253}Fm requires about 3 days and so a long lifetime of the isomeric $1/2^+$ state in this nucleus seems to be unlikely, but cannot be completely excluded. The calculated spectrum of ^{253}Fm is presented in Fig. 4. One can see that the γ-transitions from higher states populate either ground or isomeric $1/2^+$ states.

As seen in Fig. 2, the favored α-decay from the ground state of ^{269}Hs leads to the population of the states with large K in ^{265}Sg, ^{261}Rf, and ^{257}No, while the α decay from the isomeric $3/2^+$ state can only populate the states with small K in these nuclei. The larger the K value, the larger is the hindrance to spontaneous fission [55]. In Refs. [56–58], the spontaneous fission of ^{265}Sg, ^{261}Rf, and ^{257}No was observed along with α decays. This means that α decay takes place also from the isomeric $3/2^+$ state of ^{269}Hs. The appearance of low-lying isomeric states can influence the branches between spontaneous fission and α decay.

B. α-decay chain starting from ^{267}Hs

Let us consider the structure of nuclei belonging to the α-decay chain starting from ^{267}Hs. The expected ground state of ^{267}Hs has $K^\pi = 3/2^+$ with the structure $[622\downarrow]$ (98%). The calculated first excited state of ^{267}Hs with $K^\pi = 11/2^-$ and excitation energy of 95 keV is the isomeric one with the structure $[725\uparrow]$ (97%). The excitation spectrum of ^{267}Hs is shown in Fig. 5. All excited states of this nucleus with excitation energies below 800 keV have a one-quasiparticle nature (>95%). However, the structure of the states above 800 keV is characterized by the coupling of quasiparticles with octupole phonons.

Possible α-decay chains starting from ^{267}Hs are shown in Fig. 6. The ground state of ^{267}Hs decays by emission of an α particle to the calculated first excited state $3/2^+$ in ^{263}Sg at an excitation energy of 21 keV. This is the isomeric state since the γ transition from it to the ground state is characterized by the high value of $\Delta K = 4$. If the isomeric state $11/2^-$ in ^{267}Hs is populated (or if this state is the ground state, in contrast to the results of our calculations), then the ground state of ^{263}Sg is populated by the α decay from this isomeric state. The structures of all excited states of ^{263}Sg are exhausted by the one-quasiparticle component whose weight is larger than 88%.

The ground-state α decay of ^{263}Sg populates the state $11/2^-$ in ^{259}Rf, whose expected excitation energy is 88 keV. This state decays with low probability to the isomeric state $7/2^+$ at 24 keV. The reduced $E3$ γ-transition probability $B(E3; 11/2^-(983\text{ keV}) \rightarrow 7/2^+(24\text{ keV})) = 10^{-4}$ W.u.$^{-1}$. The corresponding γ-decay lifetime is, however, larger than the α-decay time from the isomeric state $11/2^-$ in ^{259}Rf to the excited state $11/2^-$ in ^{255}No at 383 keV. Thus, in the α decay of ^{259}Rf the $11/2^-$ state of ^{255}No can be populated at 383 keV. This state decays by $M1$ transition to the $9/2^-$ state

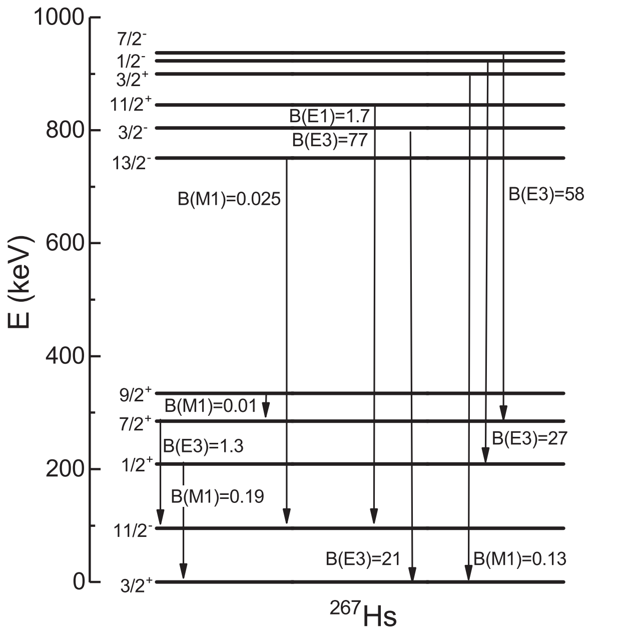

FIG. 5. Calculated one-quasiparticle spectrum of ^{267}Hs. The structure of all indicated states below 800 keV is exhausted by the one-quasiparticle component. The states above 800 keV have a quasiparticle ⊗ phonon structure excluding the $3/2^+$ state at 900 keV. The reduced transition probabilities are given in Weisskopf units.

(94 keV), which then decays by $E1$ transition to the ground state of ^{255}No with $K^\pi = 7/2^+$ (Fig. 6).

The α decay of ^{263}Sg from the lowest isomeric $3/2^+$ state populates the $3/2^+$ state in ^{259}Rf, which then decay by either $E2$ or $M1$ transitions to the lowest isomeric state $7/2^+$ or ground state, respectively. So, the α decay of ^{259}Rf could occur from three states (Fig. 6).

The α decay of ^{255}No (Fig. 7) can occur from the ground state and the lowest $1/2^+$ isomeric state at 72 keV. The estimated time for α decay of ^{251}Fm is long, about 130 days [60],

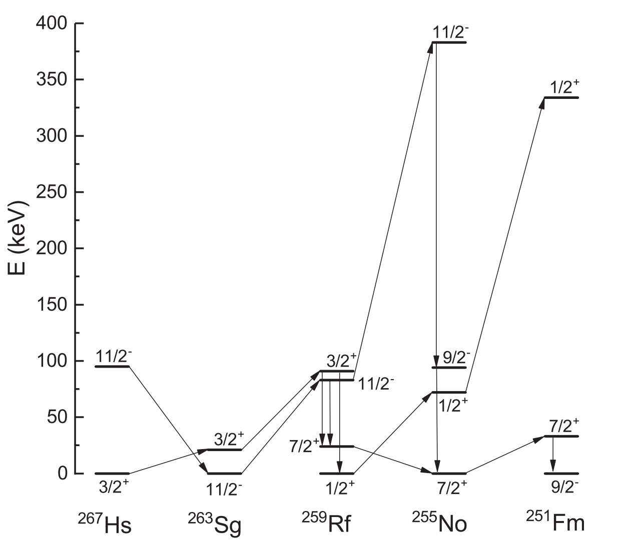

FIG. 6. Possible α-decay chains starting from ^{267}Hs.

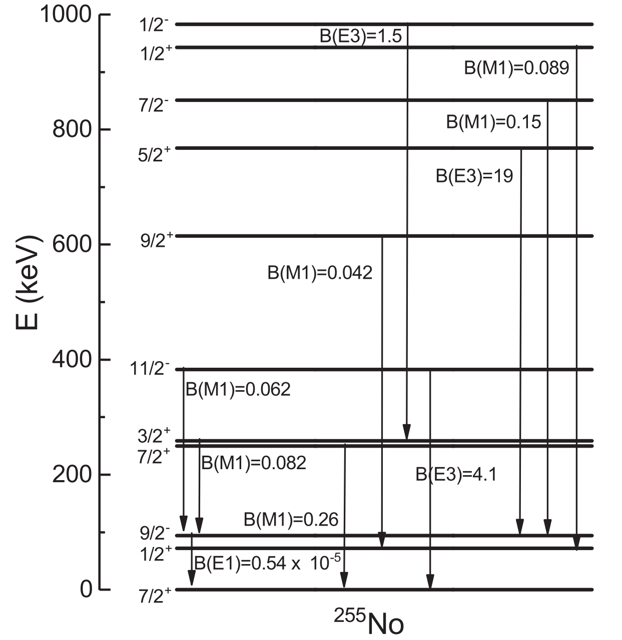

FIG. 7. The same as in Fig. 5, but for ^{255}No. The structure of states below 700 keV is exhausted by a one-quasiparticle component. The states above 800 keV have a quasiparticle ⊗ phonon structure.

and such a long lifetime of the isomeric state is unlikely. Thus, the α decay of ^{251}Fm is expected to occur only from the ground state.

As in the case of ^{269}Hs, in the nuclei of the α-decay chain of ^{267}Hs the states with small and large K are populated. This can influence the branching ratio between spontaneous fission and α decay in ^{263}Sg and ^{259}Rf [61,62].

C. α-decay chain starting from ^{265}Hs

Let us consider the structure of the low-lying states of nuclei belonging to the α-decay chain starting from ^{265}Hs [63,64]. The calculated ground state of ^{265}Hs is $11/2^-$ (Fig. 8). In the calculated spectrum there are a $3/2^+$ state with excitation energy 6 keV, a $1/2^+$ state with excitation energy 52 keV, and a $7/2^+$ state with excitation energy 171 keV. We cannot exclude that, in reality, other sequences of these close states can be realized in contrast to our calculations. The structure of all these states is almost exhausted by the one-quasiparticle component. The weight of this component in the structure of the states with excitation energies below 1000 keV exceeds 90%. The calculated $3/2^+$ (6 keV) state is the isomeric one from which α decay is possible to the corresponding state in ^{261}Sg. The $1/2^+$ (52 keV) state decays by M1 transition to the isomeric $3/2^+$ state with the $B(M1) = 0.21$ W.u., i.e., with the total transition probability $T(1/2^+ \to 3/2^+) = 6.5 \times 10^8$ s^{-1}. The excited state $7/2^+$ (171 keV) decays by $E3$ transition to the ground state with $B(E3; 7/2^+ \to 11/2^-) = 0.02$ W.u., i.e., with the total transition probability $T(7/2^+ \to 11/2^-) = 0.2$ s^{-1}. The corresponding lifetime is long compared to the α-decay lifetime 10^{-3} s for transition to the calculated $7/2^+$ state of ^{261}Sg. Thus, if the $7/2^+$ state is populated in ^{265}Hs, the α decay from this state will populate

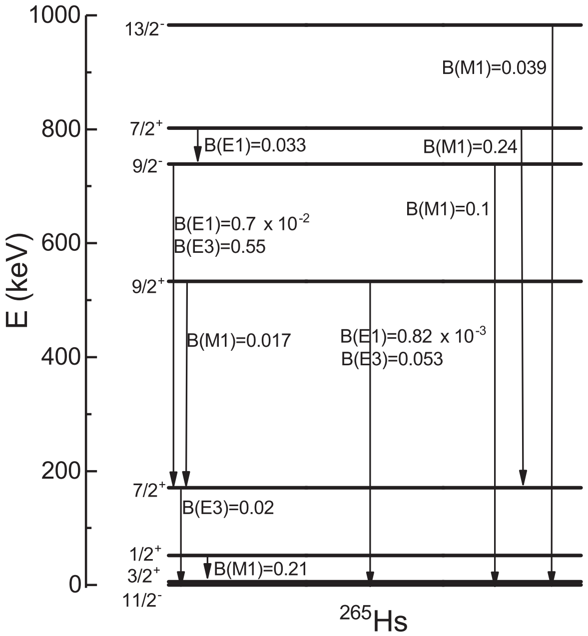

FIG. 8. Calculated one-quasiparticle spectrum of ^{265}Hs. The structure of all indicated states is exhausted by thge one-quasiparticle component. Only the strongest γ transitions are indicated by arrows. The reduced transition probabilities are given in Weisskopf units.

the $7/2^+$ state of ^{261}Sg whose calculated excitation energy is 4 keV. The α-decay lifetime of the ground state of ^{265}Hs is about 2.5×10^{-3} s. So, three α-decay lines can be found in ^{265}Hs (Fig. 9).

The calculated ground state of ^{261}Sg is $1/2^+$. Several excited states are concentrated near the ground state: $7/2^+$ (4 keV), $3/2^+$ (77 keV), and $11/2^-$ (88 keV) (Fig. 10). In reality, any of these states could be the ground state of ^{261}Sg because the firm definition of the sequence of close levels is beyond the ability of the model. In the calculated

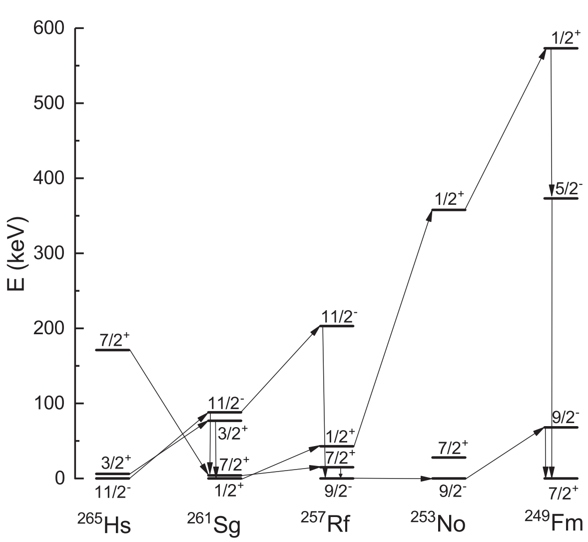

FIG. 9. Possible α-decay chains starting from ^{265}Hs.

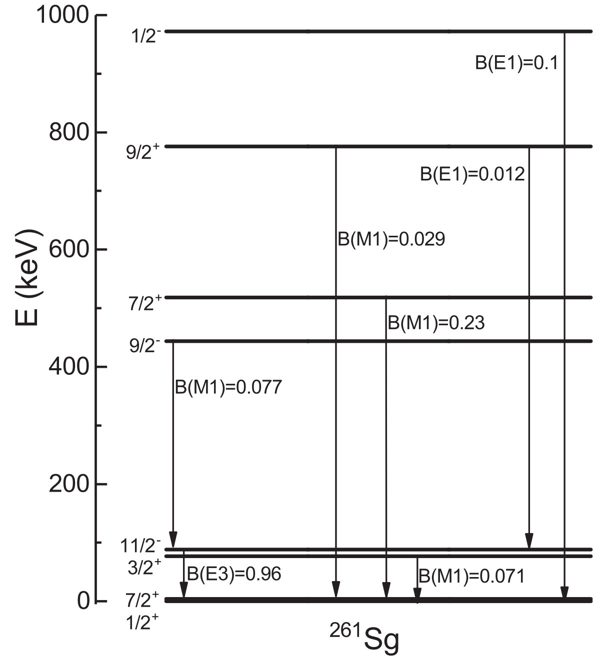

FIG. 10. The same as in Fig. 8, but for ^{261}Sg. The structure of states below 900 keV is exhausted by the one-quasiparticle component.

spectrum the $7/2^+$ (4 keV) state is the isomeric one. Thus, α decay of ^{261}Sg can occur from the ground state to the $1/2^+$ excited state of ^{257}Rf at excitation energy of 43 keV, and from the isomeric state $7/2^+$ to the $7/2^+$ excited state of ^{257}Rf at the calculated excitation energy of 15 keV. In both cases α-decay lifetime is close to $\tau_\alpha = 0.35$ s. In the case of the α decay of ^{265}Hs from its ground state the $11/2^-$ excited state at about 88 keV is populated in ^{261}Sg. This state decays by $E3$ γ transition to the $7/2^+$ isomeric state with $B(E3; 11/2^- \rightarrow 7/2^+) = 0.96$ W.u. The corresponding γ-decay lifetime is $\tau_\gamma = 15$ s, which is larger than the α-decay time from this state. The corresponding α-decay lifetime is about 0.7 s. Thus, the excited state $11/2^-$ (88 keV) of ^{261}Sg can decay directly to the $11/2^-$ (203 keV) excited state of ^{257}Rf. As one can see, there could be three α-decay lines in ^{261}Sg.

The calculated spectrum of low-lying states of ^{257}Rf is quite dense (Fig. 11). There are five excited states with excitation energies below 250 keV. The expected ground state is $9/2^-$. All states below 300 keV have a one-quasiparticle structure. The states above 500 keV are formed by the coupling of one quasiparticle with octupole phonon with $K^\pi = 1^-$. The first excited state $7/2^+$ of ^{257}Rf with calculated excitation energy 15 keV decays by $E1$ transition to the ground state in about $\tau_\gamma = 3 \times 10^{-7}$ s, which is much shorter than the α-decay lifetime for this state. In ^{257}Rf the lowest states, $9/2^-$ (ground state), $7/2^+$, and $1/2^+$, are very close in energy and it is difficulty to predict their real order. For example, $1/2^+$ is assigned to the ground state in Ref. [7]. The excited $7/2^+$ (28 keV) state decays fast to the ground state by $E1$ transition with $B(E1) = 0.7 \times 10^{-3}$ W.u. (Fig. 11). This value of $B(E1)$ corresponds to γ-decay lifetime $\tau_\gamma = 10^{-5}$ s.

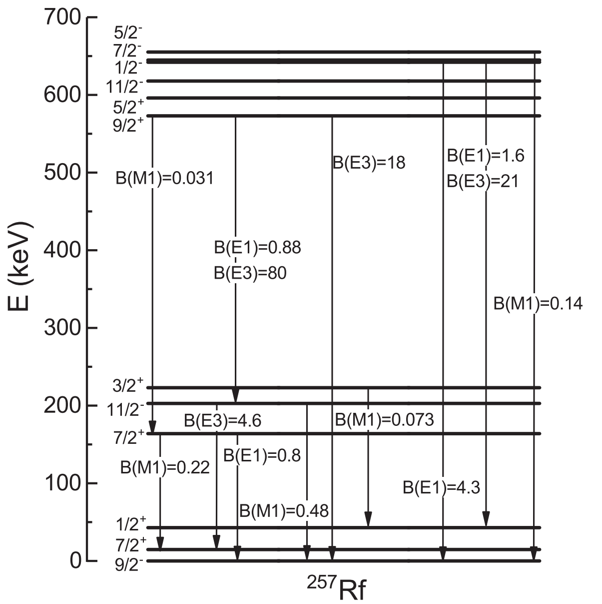

FIG. 11. The same as in Fig. 8, but for ^{257}Rf. The structure of states below 300 keV is exhausted by the one-quasiparticle component. The states above 500 keV contain a significant one-quasiparticle ⊗ octupole phonon component.

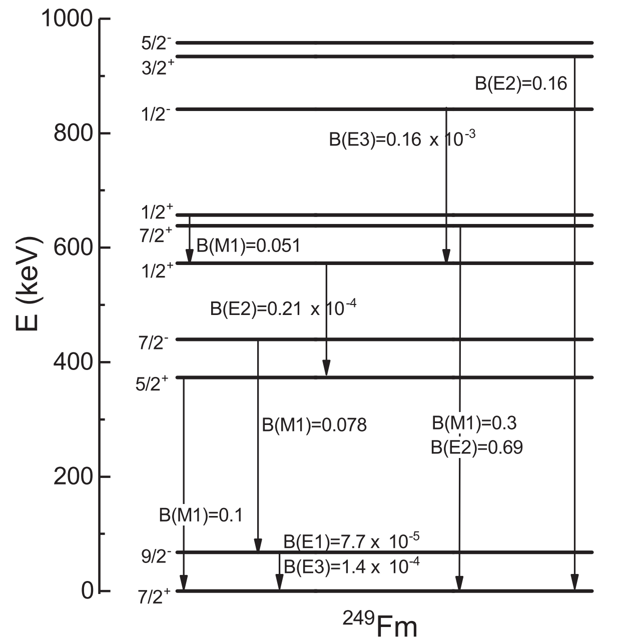

FIG. 12. The same as in Fig. 8, but for ^{249}Fm. The structure of states below 1000 keV is exhausted by the one-quasiparticle component.

The structure of the $9/2^-$ ground state is exhausted by the one-quasiparticle component. The wave function of the first excited $7/2^+$ state consists of one-quasiparticle components [624↓] (76%) and [613↑] (11%) and a one-quasiparticle ⊗ octupole phonon component. The states with excitation energies above 500 keV consist of one-quasiparticle ⊗ phonon components. As a rule, these are octupole phonons with $K^\pi = 1^-$ or 2^-. As in Ref. [7], in ^{257}Rf there are two low-lying states, $1/2^+$ and $9/2^-$, with a big difference in K from which α decays are possible. The α decay from the isomeric state $1/2^+$ in ^{257}Rf (Fig. 9) can populate the corresponding state at 358 keV in ^{253}No. So, there could be two α-decay lines in ^{257}Rf. The α decay from the ground state of ^{253}No requires about 1.6 min. So long lifetime of the isomeric state $1/2^+$ cannot be excluded. Indeed, all lower states have larger values of angular momentum that hinders the γ transitions. Thus, there could be two α-decay lines in ^{253}No.

The ground state of ^{249}Fm is the $7/2^+$ one-quasiparticle state (99%). The first excited state is $9/2^-$ at 68 keV which quickly decays into the ground state by $E1$ transition (Fig. 12). The structure of this state is almost exhausted by the one-quasiparticle component [734↑] (98%). The second excited state $5/2^+$ has also one-quasiparticle structure (97%). Its calculated excitation energy is 373 keV (Fig. 12). As one can see, ^{249}Fm can have only one α-decay line because the states $5/2^+$ and $9/2^-$ decay to the ground state by rather fast γ transitions.

IV. SUMMARY

The systematic calculations of the excitation spectra, structure of the wave functions, and the γ-transition probabilities of the group of odd-neutron nuclei with $Z = 100$, 102, 104, 106, and 108 were performed. The QPM was used to take into account the monopole pairing and the quasiparticle-phonon interaction. This model is improved by finding out the ground-state deformations for each nucleus using the microscopic-macroscopic approach. It is shown that the quasiparticle-phonon interaction influences the ordering of the levels at the excitation energies characterized by a quite dense excitation spectra. The quasiparticle-phonon interaction becomes important at the excitation energies above 500 keV. Note that for all considered nuclei the calculations are performed with fixed parameters of the Hamiltonian which seem to be reliable in a wide region of the nuclide chart including the heaviest nuclei. The calculated γ-transition probabilities allow us to find the isomeric states in the spectra of nuclear excitations and estimate the lifetimes of isomers.

The α-decay chains starting from ^{265,267,269}Hs were analyzed. In the α-decay chains of ^{267,269}Hs, the nuclei with $Z = 108$, 106, 104, and 102 can have two α-decay lines. The nuclei ^{265}Hs and ^{261}Sg likely have three α-decay lines. The appearance of the dense one-quasiparticle spectra enriches the possibilities of α-decay and existence of isomeric states. The difference in spontaneous fission from low-lying states with large difference in K is worth studying. The nuclei considered can be produced in the reactions ^{26}Mg + ^{248}Cm, ^{34}S + ^{238}U, and ^{48}Ca + ^{226}Ra.

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