$\psi^{(*)}p$ scattering length based on near-threshold charmonium photoproduction

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Under the framework of vector meson dominance model, the value of scattering length can be expressed as a function of the ratio between total cross section $\sigma(W)$ and $R(W)$, where $R(W)$ is the ratio between final momentum $|\mathbf{p}_3|$ and initial momentum $|\mathbf{p}_1|$ and positively correlated with the center-of-mass energy. Based on the theoretical study of charmonium's photoproduction within two gluon exchange model and effective Pomeron model, we research the scattering lengths of vector mesons and proton interaction in this work. Results show that the scattering length $|\alpha_{J/\psi p}|$ obtained from the two models are close and basically in agreement with the theoretical prediction of Strakovsky and co-workers. Additionally, we first calculate the scattering length of the $\psi(2S)$-proton interaction in the two gluon exchange model and effective Pomeron model as 1.31 ± 0.92 am $(1 \text{ am} = 10^{-3} \text{ fm})$ and 3.24 ± 0.63 am, respectively. This is a little bit different from the two models and requires precise measurements from subsequent experiments. In short, our results will provide a theoretical reference for future studies on characterizing the vector meson-proton scattering length.

I. INTRODUCTION

The evaluation of the scattering lengths may serve as a unique input for QCD-motivated models of vector meson-nucleon interactions [1–4]. The behavior of the near-threshold cross section is related to vector meson-proton (Vp) scattering length [5]. Recently, scattering lengths for ωp, ϕp, $J/\psi p$, and Υp reactions have been reported using the recent photoproduction experiment data or quasidata [1–4]. However, there are also several Vp scattering lengths that have not been studied for various reasons, such as one narrow vector meson: $D_0^*(2007)$, since there is no charm conservation, the reaction $\gamma p \to D_0^*(2007)p$ is impossible. Moreover, we find that there are no good threshold measurements for ρ, K^* and D^* meson photoproduction. As the excited states of J/ψ, it is of great interest to estimate the scattering length of the vector meson $\psi(2S)$-proton interaction.

On the experimental side, the J/ψ photoproduction off the proton was conducted with increasing precision over a large energy range [6–13], while the measurement of $\psi(2S)$ photoproduction data is very meager [14–16]. Considering $\psi(2S)$ and J/ψ have close mass, both have $c\bar{c}$ structure, the same quantum numbers, spin, and parity as a photon which is $I^G(J^{PC}) = 0^-(1^{--})$, it is reasonable to study them with the same physical models and parameters. In our previous works [17], the two gluon exchange model is applied to systematically analyze the J/ψ and $\psi(2S)$ photoproduction data from threshold to medium energy (near 400 GeV) [6–13]. In the literatures [18,19], the Pomeron exchange process is considered to explain the photoproduction of charmonium by the JPAC collaboration. The numerical results from the two gluon exchange model and effective Pomeron model are both in agreement with J/ψ experimental data, while the predicted photoproduction of $\psi(2S)$ in the two models at the threshold is basically consistent [17]. In this paper, we will use the predicted $\psi(2S)$ cross section data under the framework of the two models to extract the scattering lengths $|\alpha_{\psi(2S)p}|$.

Many literatures give their scattering length results for proton interactions with a vector meson by different methods. From a global fit to both differential and total cross section data, one work [20] extracted the scattering length $|\alpha_{J/\psi p}|$ as 0.046 fm from $d\sigma/dt(s_{\text{thr}}, t = 0)$. Reference [21] discussed the charmonium bound states in nuclei from the method of multipole expansion and low-energy QCD theorems. Provided in the vector meson dominance (VMD) model [22,23], Strakovsky and co-workers estimated the J/ψ-nucleon scattering length $|\alpha_{J/\psi p}| = 3.08$ am $(1 \text{ am} = 10^{-3}$ fm) by fitting the recent GlueX total cross section data [4]. To avoid additional uncertainty when extrapolating the

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FIG. 1. The schematic Feynman diagram of the effective Pomeron model for J/ψ or $\psi(2S)$ production.
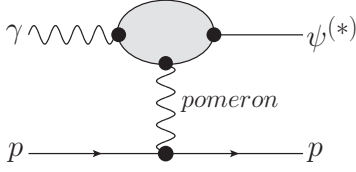

FIG. 2. The schematic Feynman diagrams of the two gluon exchange model for J/ψ or $\psi(2S)$ production.

differential cross section to the nonphysical point $t = 0$, the approach of Ref. [20] is not adopted. Compared to other methods, the VMD model does not contain free parameters in the process from γp to $V p$ reaction. It is superior for us to obtain qualitative estimates when extracting scattering lengths $|\alpha_{Vp}|$. The scattering length $|\alpha_{J/\psi p}|$ obtained by the VMD model is smaller than other references. And this small value can be attributed to the size of the "young" vector meson is smaller than that of the "old" one participating in the elastic $Vp \rightarrow Vp$ scattering, because $c\bar{c}$ pair lacks sufficient time to form the complete wave function of the vector meson [3].

The measurement of near-threshold $\psi(2S)$ photoproduction will give access to a variety of interesting physics aspects, e.g., trace anomaly, pentaquarks, cusp effects, vector-meson-nucleon scattering length, and so on [22,24]. Nowadays, the electron ion colliders (EIC) are proposed to be built for probing the deepest structure inside the hadron [25]. Meanwhile, the opportunities for Chinese EIC are now under discussion and will be an important and interesting future machine to collect $\psi^{(*)}$ data [26]. Relevant measurements will advance our understanding of QCD which governs the properties of hadrons and the interactions involving hadrons.

The paper is organized as follows. The formulas of the effective Pomeron model, two gluon exchange model, and an expression for scattering length $|\alpha_{Vp}|$ are provided in Sec. II. Next section, we show the numerical result on the explanations of the current experimental data of J/ψ. Through the study of scattering length $|\alpha_{J/\psi p}|$, the reliability of the two models and VMD method are determined. Then the results and discussion of $|\alpha_{\psi(2S)p}|$ are obtained in Sec. III. A summary is given in Sec. IV.

II. FORMALISM

In quark-interchange mechanisms, some light quarks (such as π, ρ, Δ, etc.) are strongly suppressed in the heavy quarkonium photoproduction. So the channels are always dominated by two gluon exchange mechanisms or Pomeron exchange contributions. Actually, the two gluon exchange and Pomeron contributions both reproduce well the existing data [27].

A. Effective Pomeron model

The effective t-channel Pomeron exchange contribution is shown in Fig. 1. Proposed by the JPAC collaboration, an effective Pomeron exchange model is used to have a systematical analysis for J/ψ and $\psi(2S)$ photoproduction [18]. The differential cross section for $\gamma p \rightarrow \psi^{(*)} p$ reaction takes the form [19]
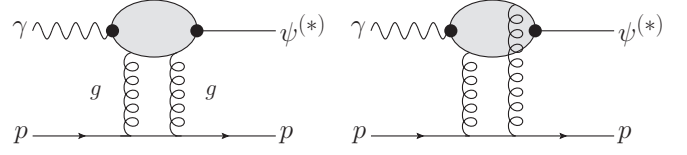
$$\frac{d\sigma}{dt} = \frac{4\pi\alpha}{64\pi W^2 p_i^2} \sum_{\lambda_\gamma, \lambda_p, \lambda_{\psi^{(*)}}, \lambda_{p'}} \frac{1}{4} |\langle \lambda_{\psi^{(*)}} \lambda_{p'} | T | \lambda_\gamma \lambda_p \rangle|^2. \quad (1)$$

The amplitude in low energy regions is given by [19,28]

$$\begin{aligned} \langle \lambda_{\psi^{(*)}} \lambda_{p'} | T_P | \lambda_\gamma \lambda_p \rangle &= F(W^2, t) \bar{u}(p_f, \lambda_{p'}) \gamma_\mu u(p_i, \lambda_p) \\ &\quad \times [\varepsilon^\mu(p_\gamma, \lambda_\gamma) q^\nu - \varepsilon^\nu(p_\gamma, \lambda_\gamma) q^\mu] \\ &\quad \times \varepsilon_\nu^*(p_{\psi^{(*)}}, \lambda_{\psi^{(*)}}). \end{aligned} \quad (2)$$

Here,

$$F(W^2, t) = iA_{\psi^{(*)}} \left(\frac{W^2 - W_{\text{thr}}^2}{W_0^2} \right)^{\alpha(t)} \frac{e^{b_0(t - t_{\min})}}{W^2}, \quad (3)$$

$\alpha(t) = \alpha_0 + \alpha' t$ is the Pomeron trajectory [29]. $W_0 = 1$ GeV is the energy scale parameter. $W_{\text{thr}} = M_p + M_{\psi^{(*)}}$ is the energy threshold.

B. Two gluon exchange model

The picture of the double gluon exchange between the nucleon state and the quark-antiquark pair is illustrated in Fig. 2. The differential cross section of $\psi^{(*)}$ photoproduction is given as [30,31]

$$\frac{d\sigma}{dt} = \frac{\pi^3 \Gamma_{e^+e^-}^{\psi^{(*)}} \alpha_s}{6\alpha m_q^5} [xg(x, m_{\psi^{(*)}}^2)]^2 \exp(b^{\psi^{(*)}} t), \quad (4)$$

where $x = m_{\psi^{(*)}}^2 / W^2$; $\alpha_s = 0.5$ is the strong coupling constant [32]; α is the fine-structure constant; m_q is the mass of charm quark; $\Gamma_{e^+e^-}^{\psi^{(*)}}$ is the radiative decay [33]. $xg(x, m_{\psi^{(*)}}^2)$ defines the gluon distribution function at $Q^2 = m_{\psi^{(*)}}^2$, which is parametrized using a simple function form $xg(x, m_{\psi^{(*)}}^2) = A_0 x^{A_1} (1 - x)^{A_2}$ [34]. The exponential slope parameter b can use the standard form based on the Regge phenomenology [16,35] for $\psi(2S)$ and similar form [30] for J/ψ.

The total cross section is obtained by integrating the differential cross section [Eq. (1) or (4)] over the allowed kinematical range from $t_{\min}$ to $t_{\max}$, the total cross section can be written as

$$\sigma = \int_{t_{\min}}^{t_{\max}} dt \frac{d\sigma}{dt}. \quad (5)$$

Here, the limiting values $t_{\min}$ and $t_{\max}$ are

$$t_{\max}(t_{\min}) = m_1^2 + m_3^2 - 2E_1 E_3 \pm 2|\mathbf{p}_1||\mathbf{p}_3|. \quad (6)$$

The energies and momenta of the photon and meson in the center-of-mass (c.m.) frame are

$$|\mathbf{p_1}| = \frac{1}{2W}\sqrt{W^4 - 2(m_1^2 + m_2^2)W^2 + (m_1^2 - m_2^2)^2}; \quad (7)$$

$$|\mathbf{p_3}| = \frac{1}{2W}\sqrt{W^4 - 2(m_3^2 + m_4^2)W^2 + (m_3^2 - m_4^2)^2}; \quad (8)$$

$$E_1 = \sqrt{|\mathbf{p_1}|^2 + m_1^2}, \quad E_3 = \sqrt{|\mathbf{p_3}|^2 + m_3^2}. \quad (9)$$

C. VMD model and scattering length $|\alpha_{Vp}|$

The ratio between the initial c.m. momentum and the final momentum $R(W)$ is used as

$$R(W) = \frac{|\mathbf{p_3}|}{|\mathbf{p_1}|}. \quad (10)$$

Note that, $R(W)$ has a range of $R(W) \in [0, 1)$ and is positively correlated with the c.m. energy. $R(W) \to 0$ respects $W \to W_{\text{thr}}$ while $R(W) \to 1$ respects $W \to \infty$. The ratio of total cross section $\sigma(R)$ and R is given by

$$\frac{\sigma(R)}{R} = a_1(R), \quad (11)$$

where the function $a_1(R)$ can be described by Eqs. (1), (4), and (5).

The Vp scattering length is related to the near-threshold photoproduction of vector mesons. In this paper, the VMD model is used to connect the reaction $\gamma p \to V p$ and $V p \to V p$. Applying the effective VMD approach, the near-threshold cross section during the elastic scattering processes becomes [22]

$$\sigma|_{\text{thr}}(R) = \frac{|\mathbf{p_3}|}{|\mathbf{p_1}|}\frac{4\alpha\pi^2}{g_V^2}\frac{d\sigma^{Vp \to Vp}}{d\Omega}\bigg|_{\text{thr}} = R\frac{4\alpha\pi^2}{g_V^2}|\alpha_{Vp}|^2. \quad (12)$$

Here, the VMD coupling constant g_V is deduced from the leptonic decay width $\Gamma_{e^+e^-}^V$ as [4]

$$g_V = \sqrt{\frac{\pi\alpha^2 M_V}{3\Gamma_{e^+e^-}^V}}. \quad (13)$$

Combining Eqs. (11), (12), and (13), scattering length $|\alpha_{Vp}|$ is given as

$$|\alpha_{Vp}| = \frac{g_V}{2\pi}\sqrt{\frac{a_1(R)}{\alpha}}. \quad (14)$$

The scattering length $|\alpha_{Vp}|$ can also be expressed by differential photoproduction cross section. When the c.m. energy W approaches the threshold, the total cross section is related to the differential cross section as [36]

$$\sigma|_{\text{thr}} = |t_{\max} - t_{\min}|\frac{d\sigma}{dt}\bigg|_{\text{thr}} = 4|\mathbf{p_1}||\mathbf{p_3}|\frac{d\sigma}{dt}\bigg|_{\text{thr}}. \quad (15)$$

TABLE I. The fitted values of the parameters A_0, A_1, and A_2 in the two gluon exchange model [17].

A_0	A_1	A_2
0.228 ± 0.045	-0.218 ± 0.006	1.221 ± 0.055

TABLE II. The fitted values of the parameters $A_{J/\psi}$, α_0, α', and b_0 in the effective Pomeron model [18].

$A_{J/\psi}$	α_0	$\alpha'(\text{GeV}^{-2})$	$b_0(\text{GeV}^{-2})$
0.38	0.94	0.36	0.12

Combining Eqs. (12) and (15), we can obtain the relation between the differential cross section of $\gamma p \to V p$ reaction and scattering length as

$$\frac{d\sigma}{dt}\bigg|_{\text{thr}} = \frac{\alpha\pi^2}{g_V^2|\mathbf{p_1}|^2}|\alpha_{Vp}|^2. \quad (16)$$

If setting $\frac{d\sigma}{dt}|_{\text{thr}} = b_1$, the scattering length $|\alpha_{Vp}|$ is given as

$$|\alpha_{Vp}| = \frac{g_V|\mathbf{p_1}|}{\pi}\sqrt{\frac{b_1}{\alpha}}. \quad (17)$$

Note that, b_1 and $|\mathbf{p_1}|$ must satisfy the conditions that obtained from the threshold.

III. RESULTS AND DISCUSSIONS

In our previous work, the free parameters A_0, A_1, and A_2 in the two gluon exchange model were obtained by a global analysis of both the total cross section data below medium energy [6–13] and the near-threshold ($W = 4.58$ GeV) differential cross section data of J/ψ [6]. JPAC collaboration determined the free parameters $A_{J/\psi}$, α_0, α', and b_0 by fitting the total cross section of J/ψ experimental data [6,9]. The fitted parameters from the two models are listed in Tables I and II. The numerical results of $\gamma p \to J/\psi p$ from the two gluon exchange model and effective Pomeron model are shown in Figs. 3 and 4, compared with GlueX, SLAC,
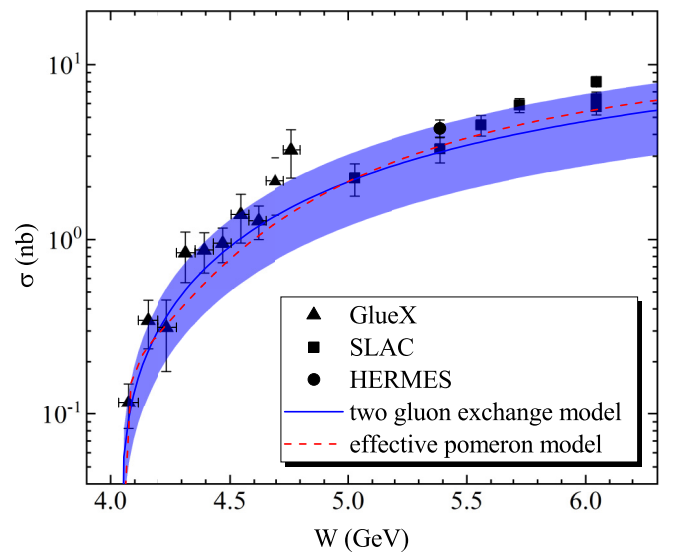

FIG. 3. The total cross section of $\gamma p \to J/\psi p$ as a function of W. Blue-solid curve is obtained from the two gluon exchange model. Red-dashed curve is obtained from the effective Pomeron model. Data are from Refs. [6,9,37].
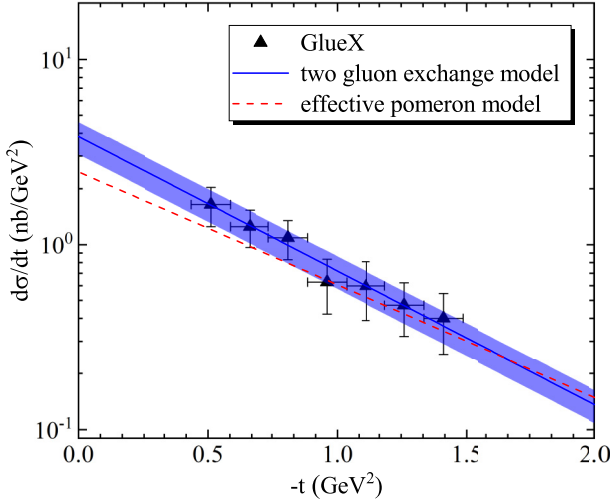
FIG. 4. The differential cross section of $\gamma p \to J/\psi p$ as a function of $-t$. The curves have the same meaning as in Fig. 3. Data are from Ref. [6].

and HERMES experiments [6,9,37]. We perceive that the two models are reliable to explain $J/\psi p$ photoproduction.

According to the obtained total cross section from models, Eqs. (11) and (14), the $J/\psi p$ scattering length as a function of R is shown in Fig. 5 (blue-solid curve). Listed in Table III, the average value of scattering length $|\alpha_{J/\psi p}|$ is 3.85 ± 0.96 am from the two gluon exchange model and 3.75 ± 0.84 am from the effective Pomeron model. Here, the range of R is selected as [0,0.5]. Note that $R = 0.5$ corresponds to c.m. energy $W = 4.79$ GeV, which represents a near-threshold energy. We also compared our results with the phenomenological result [36] and odd-polynomial fitted result [4]. Our results are in agreement with the above phenomenological results.
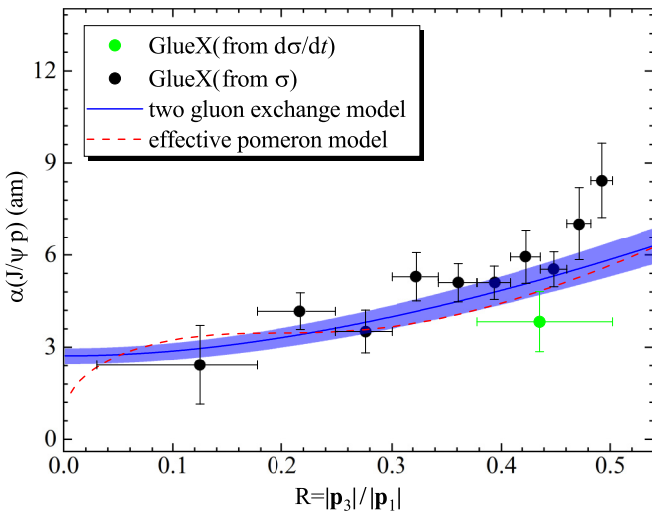

FIG. 5. The obtained scattering length $|\alpha_{J/\psi p}|$ as a function of R. The black circles show the results derived from the GlueX total cross section data. The green circle shows the results derived from the GlueX differential cross section data [36]. The curves have the same meaning as in Fig. 3.

TABLE III. Comparison of different determinations of $|\alpha_{J/\psi p}|$ from different methods.

Method	$|\alpha_{J/\psi p}|$ (am)
odd-polynomial fit [4]	3.08 ± 0.55
phenomenological result [36]	3.64 ± 0.26
two gluon exchange model (this work)	3.85 ± 0.96
effective Pomeron model (this work)	3.75 ± 0.84

We also extract the scattering lengths from the total cross section GlueX data, as shown in the black circles in Fig. 5. Reference [36] obtained $|\alpha_{J/\psi p}| = 3.83 \pm 0.98$ am (the green circle in Fig. 5) derived from the differential cross section GlueX data. Although this energy ($W = 4.59$ GeV) is a little far from the threshold, the value of scattering length is close to our estimation from two models and other works [4,36]. Therefore, the scattering lengths extracted from the differential cross section experimental data may be more advantageous, compared with the instability extracted from total cross section data.

In the above study, we found that the scattering lengths $|\alpha_{J/\psi p}|$ given by the two models are approximate and consistent with other literatures [4,36]. The reliability of the two models and VMD method was determined. Next, we will extend the study to $\psi(2S)p$ scattering length based on the above study. The photoproduction of $\psi(2S)$ is obtained by using the same parameters listed in Tables I and II. Note that, the parameter $A_{\psi(2S)}$ in Eq. (3) can be written as $A_{\psi(2S)} = R_{\psi(2S)}A_{J/\psi}$, and the relative strength $R_{\psi(2S)} = 0.55$ is obtained from the extraction by CLEO [18,38]. Then the $\psi(2S)p$ scattering length as a function of R is shown in Fig. 6 (red-dashed curve). We obtained calculation results from the two gluon exchange model and effective Pomeron model to be 1.31 ± 0.92 am and 3.24 ± 0.63 am, respectively (Table IV). Note that, the value of $|\alpha_{J/\psi p}|$ is bigger than $|\alpha_{\psi(2S)p}|$ in both models. Because of the difference in the size of the total cross section predicted
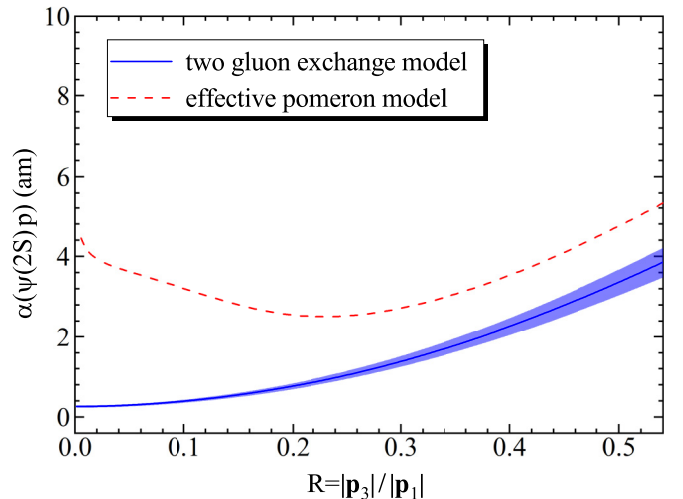

FIG. 6. The obtained scattering length $|\alpha_{\psi(2S)p}|$ as a function of R. The curves have the same meaning as in Fig. 3.

TABLE IV. The average value of scattering length $|\alpha_{\psi(2S)p}|$ obtained from two gluon exchange model and effective Pomeron model.

Method	Two gluon exchange model	Effective Pomeron model
$|\alpha_{\psi(2S)p}|$ (am)	1.31 ± 0.92	3.24 ± 0.63

by the two models, the $|\alpha_{\psi(2S)p}|$ scattering length is a little bit different. Precise results require more measurement from subsequent experiments.

Based on the recent threshold measurements of the photoproduction of ω and ϕ mesons off the proton by the A2 (MAMI) [1] and CLAS (JLab) [39], one can determine vector meson proton scattering lengths $|\alpha_{Vp}|$ using the VMD model [1,2]. What is more, the absolute value of the Υp scattering length is studied using quasidata generated from the QCD model [3,40]. The corresponding results for the scattering lengths are shown in Table V and Fig. 7 as a function of the inverse vector meson mass. Concretely, the relationship including $\omega, \phi, J/\psi, \psi(2S)$, and Υ can be determined as

$$|\alpha_{\Upsilon p}| < |\alpha_{\psi(2S)p}| < |\alpha_{J/\psi p}| < |\alpha_{\phi p}| < |\alpha_{\omega p}|. \qquad (18)$$

Actually, the binding energy E_b in nuclear matter can be determined by scattering length $|\alpha_{Vp}|$. In a linear density approximation, the binding energy E_b can be written as [21]

$$E_b \simeq \frac{2\pi (M_N + M_{\psi(2S)})\alpha_{\psi(2S)p}}{M_N M_{\psi(2S)}} \rho_{nm}, \qquad (19)$$

in which the nuclear matter density $\rho_{nm} \simeq 0.17$ fm^{-3}. Usually, we define the initial stage of meson formation as the "young" stage of meson, and there is also a binding energy between meson and nucleon. The $\psi(2S)$ binding energy in nuclear matter is calculated as 0.073 MeV. Therefore, the smallness of the binding energy may be related to the "young" age of the vector mesons participating in the interaction with the proton. As a primitive meson, its properties show some differences. The weak combination between $\psi(2S)$ and proton can promote the reaction of $\psi(2S)p \to \psi(2S)p$. Moreover, if it is assumed that the interaction between vector meson and nucleon needs more time to reach equilibrium specifically for slow heavy quarkoniums [$J/\psi, \psi(2S)$, and Υ], this means that the "young" vector meson effect is more pronounced for heavy quarkoniums. For light vector mesons (such as ω, ϕ, etc.), this "young" vector meson effect may be relatively weak.

TABLE V. The values of scattering length $|\alpha_{Vp}|$ from ω, ϕ, and Υ mesons.

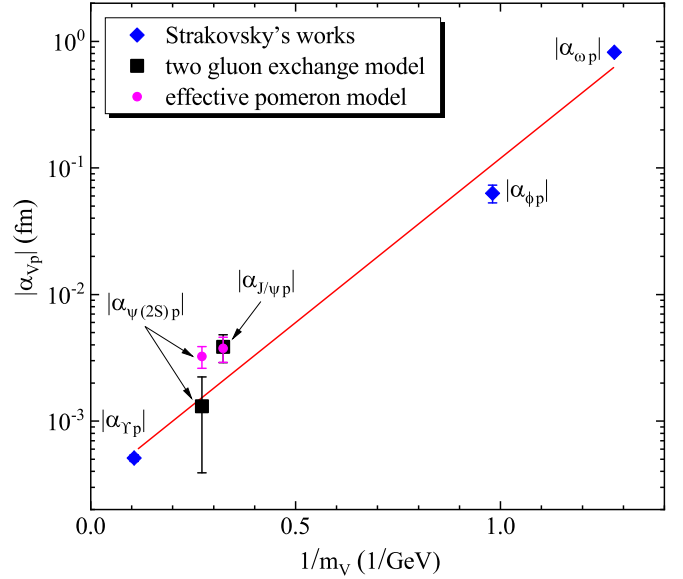| Vector meson	m_V (GeV)	$|\alpha_{Vp}|$ (fm)
ω	0.87265	0.82 ± 0.03 [1]
ϕ	1.01946	0.063 ± 0.010 [2]
Υ	9.4603	$(0.51 \pm 0.03) \times 10^{-3}$ [3]

FIG. 7. Comparison of the scattering lengths $|\alpha_{Vp}|$ as a function of the inverse mass of vector mesons, including $\omega, \phi, J/\psi, \psi(2S)$, and Υ. The blue-rhombus show the analysis of ω (Strakovsky 2015: [1]), ϕ (Strakovsky 2020: [2]), and Υ (Strakovsky 2021: [3]). The magenta circles show the analysis from the two gluon exchange model, black-squares from the effective Pomeron model. The red-solid line is hypothetical [3].

IV. SUMMARY

In this paper, the value of the scattering length is expressed as a function of the ratio between $\sigma(W)$ and $R(W)$ within the VMD model. This description can avoid delivering unnecessary inaccuracy in the numerical calculation. It is not only suitable for extracting the scattering length from the experimental data directly, but also convenient for observing the near-threshold overall situation for theoretical model. In this paper, we research the J/ψ and $\psi(2S)$ scattering lengths according to the theoretical study of charmonium photoproduction within the two gluon exchange model and effective Pomeron model. The scattering lengths $|\alpha_{J/\psi p}|$ from the two models are basically consistent with several other theoretical predictions. Moreover, the scattering length of the $\psi(2S)$-proton interaction extracted from the two gluon exchange model is 1.31 ± 0.92 am. Additionally, $|\alpha_{\psi(2S)p}| = 3.24 \pm 0.63$ am extracted from effective Pomeron model. The value of $|\alpha_{J/\psi p}|$ is bigger than $|\alpha_{\psi(2S)p}|$ in both models. And these results satisfy the nonlinear exponential increase $|\alpha_{Vp}| \propto \exp(1/m_V)$ basically. According to the present results, it can be roughly concluded that one of the main factors affecting the scattering length $|\alpha_{Vp}|$ is the size of the corresponding cross section of vector meson photoproduction. For example, the cross section of the ϕ meson photoproduction is more than two orders of magnitude higher than the cross section of J/ψ photoproduction, and correspondingly, the scattering length $|\alpha_{\phi p}|$ is nearly 20 times higher than the $|\alpha_{J/\psi p}|$. In addition, the cross section at the threshold is generally more complicated, and the results given by the two models also show differences. However, due to the lack of

experimental data, especially the experimental data of $\psi(2S)$ photoproduction, the scattering length of the $\psi(2S)$ proton cannot be determined very accurately. Therefore, to better determine the scattering lengths of vector mesons and the proton interaction, more high-precision experimental measurements for the photo/electroproduction of charmoniums are highly needed, which cannot only be realized in the JLab experiment [6], but also within the capabilities of EicC and US-EIC facility [25,26].

[1] I. I. Strakovsky, S. Prakhov, Y. I. Azimov, P. Aguar-Bartolomé, J. R. M. Annand, H. J. Arends, K. Bantawa, R. Beck, V. Bekrenev, H. Berghäuser *et al.*, Photoproduction of the ω meson on the proton near threshold, Phys. Rev. C **91**, 045207 (2015).

[2] I. I. Strakovsky, L. Pentchev, and A. I. Titov, Comparative analysis of ωp, ϕp, and $J/\psi p$ scattering lengths from A2, CLAS, and GlueX threshold measurements, Phys. Rev. C **101**, 045201 (2020).

[3] I. I. Strakovsky, W. J. Briscoe, L. Pentchev, and A. Schmidt, Threshold Upsilon-meson photoproduction at the EIC and EicC, Phys. Rev. D **104**, 074028 (2021).

[4] I. I. Strakovsky, D. Epifanov, and L. Pentchev, $J/\psi p$ scattering length from GlueX threshold measurements, Phys. Rev. C **101**, 042201(R) (2020).

[5] M. Gell-Mann and F. Zachariasen, Form-FZactors and vector mesons, Phys. Rev. **124**, 953 (1961).

[6] A. Ali *et al.* (GlueX Collaboration), First Measurement of Near-Threshold J/ψ Exclusive Photoproduction off the Proton, Phys. Rev. Lett. **123**, 072001 (2019).

[7] S. Chekanov *et al.* (ZEUS Collaboration), Exclusive photoproduction of J / psi mesons at HERA, Eur. Phys. J. C **24**, 345 (2002).

[8] M. E. Binkley, C. Bohler, J. Butler, J. P. Cumalat, I. Gaines, M. Gormley, D. Harding, R. L. Loveless, J. Peoples, P. Callahan, G. Gladding, C. Olszewski, and A. Wattenberg, J/ψ Photoproduction from 60-GeV/c to 300-GeV/c, Phys. Rev. Lett. **48**, 73 (1982).

[9] P. L. Frabetti (E687 Collaboration), A Measurement of elastic J / psi photoproduction cross-section at Fermilab E687, Phys. Lett. B **316**, 197 (1993).

[10] C. Alexa *et al.* (H1 Collaboration), Elastic and proton-dissociative photoproduction of J/psi mesons at HERA, Eur. Phys. J. C **73**, 2466 (2013).

[11] B. B. Abelev *et al.* (ALICE Collaboration), Exclusive J/ψ Photoproduction Off Protons in Ultra-peripheral p-Pb Collisions at $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV, Phys. Rev. Lett. **113**, 232504 (2014).

[12] R. Aaij *et al.* (LHCb Collaboration), Exclusive J/ψ and $\psi(2S)$ production in pp collisions at $\sqrt{s} = 7$ TeV, J. Phys. G: Nucl. Part. Phys. **40**, 045001 (2013).

[13] S. Acharya *et al.* (ALICE Collaboration), Energy dependence of exclusive J/ψ photoproduction off protons in ultra-peripheral p–Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV, Eur. Phys. J. C **79**, 402 (2019).

[14] C. Adloff *et al.* (H1 Collaboration), Diffractive photoproduction of psi(2S) mesons at HERA, Phys. Lett. B **541**, 251 (2002).

[15] R. Aaij *et al.* (LHCb Collaboration), Central exclusive production of J/ψ and $\psi(2S)$ mesons in pp collisions at $\sqrt{s} = 13$ TeV, J. High Energy Phys. 10 (2018) 167.

[16] M. Hentschinski and E. Padrón Molina, Exclusive J/Ψ and $\Psi(2s)$ photo-production as a probe of QCD low x evolution equations, Phys. Rev. D **103**, 074008 (2021).

[17] X. Y. Wang, F. Zeng and Q. Wang, Systematic analysis of the proton mass radius based on photoproduction of vector charmoniums, Phys. Rev. D **105**, 096033 (2022).

[18] M. Albaladejo *et al.* (JPAC Collaboration), XYZ spectroscopy at electron-hadron facilities: Exclusive processes, Phys. Rev. D **102**, 114010 (2020).

[19] D. Winney, C. Fanelli, A. Pilloni, A. N. H. Blin, C. Fernandez-Ramirez, M. Albaladejo, V. Mathieu, V. I. Mokeev, and A. P. Szczepaniak (JPAC Collaboration), Double polarization observables in pentaquark photoproduction, Phys. Rev. D **100**, 034019 (2019).

[20] O. Gryniuk and M. Vanderhaeghen, Accessing the real part of the forward J/ψ-p scattering amplitude from J/ψ photoproduction on protons around threshold, Phys. Rev. D **94**, 074001 (2016).

[21] A. B. Kaidalov and P. E. Volkovitsky, Heavy-Quarkonia Interactions with Nucleons and Nuclei, Phys. Rev. Lett. **69**, 3155 (1992).

[22] A. I. Titov, T. Nakano, S. Date, and Y. Ohashi, Comments on differential cross-section of phi-meson photoproduction at threshold, Phys. Rev. C **76**, 048202 (2007).

[23] N. M. Kroll, T. D. Lee, and B. Zumino, Neutral vector mesons and the hadronic electromagnetic current, Phys. Rev. **157**, 1376 (1967).

[24] M. F. M. Lutz, G. Wolf, and B. Friman, Scattering of vector mesons off nucleons, Nucl. Phys. A **706**, 431 (2002); Erratum: **765**, 495 (2006).

[25] A. Accardi, J. L. Albacete, M. Anselmino, N. Armesto, E. C. Aschenauer, A. Bacchetta, D. Boer, W. K. Brooks, T. Burton, N. B. Chang *et al.*, Electron ion collider: The next QCD frontier: Understanding the glue that binds us all, Eur. Phys. J. A **52**, 268 (2016).

[26] D. P. Anderle, V. Bertone, X. Cao, L. Chang, N. Chang, G. Chen, X. Chen, Z. Chen, Z. Cui and L. Dai *et al.*, Electron-ion collider in China, Front. Phys. **16**, 64701 (2021).

[27] J. M. Laget and R. Mendez-Galain, Exclusive photoproduction and electroproduction of vector mesons at large momentum transfer, Nucl. Phys. A **581**, 397 (1995).

[28] F. E. Close and G. A. Schuler, Evidence that the Pomeron transforms as a nonconserved vector current, Phys. Lett. B **464**, 279 (1999).

[29] V. N. Gribov, in *Strong Interactions of Hadrons at High Energies: Gribov Lectures on Theoretical Physics*, edited by Y. L. Dokshitzer and J. Nyiri (Cambridge University Press, Cambridge, 2012).

[30] F. Zeng, X. Y. Wang, L. Zhang, Y. P. Xie, R. Wang, and X. Chen, Near-threshold photoproduction of J/ψ in two-gluon exchange model, Eur. Phys. J. C **80**, 1027 (2020).

[31] A. Sibirtsev, S. Krewald, and A. W. Thomas, Systematic analysis of charmonium photoproduction, J. Phys. G: Nucl. Part. Phys. **30**, 1427 (2004).

[32] Y. Xu, Y. Xie, R. Wang, and X. Chen, Estimation of $\Upsilon(1S)$ production in ep process near threshold, Eur. Phys. J. C **80**, 283 (2020).

[33] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).

[34] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky, and W. K. Tung, New generation of parton distributions with uncertainties from global QCD analysis, J. High Energy Phys. 07 (2002) 012.

[35] J. Cepila, J. Nemchik, M. Krelina, and R. Pasechnik, Theoretical uncertainties in exclusive electroproduction of S-wave heavy quarkonia, Eur. Phys. J. C **79**, 495 (2019).

[36] L. Pentchev and I. I. Strakovsky, J/ψ-p scattering length from the total and differential photoproduction cross sections, Eur. Phys. J. A **57**, 56 (2021).

[37] M. J. Amarian, Strangeness and charm production with HERMES, Few Body Syst. Suppl. **11**, 359 (1999).

[38] J. Libby *et al.* (CLEO Collaboration), Inclusive radiative psi(2S) decays, Phys. Rev. D **80**, 072002 (2009).

[39] B. Dey *et al.* (CLAS Collaboration), Data analysis techniques, differential cross sections, and spin density matrix elements for the reaction $\gamma p \to \phi p$, Phys. Rev. C **89**, 055208 (2014).

[40] Y. Guo, X. Ji and Y. Liu, QCD analysis of near-threshold photon-proton production of heavy quarkonium, Phys. Rev. D **103**, 096010 (2021).