Compatibility of the asymptotic normalization coefficient for the $^{14}C \rightarrow {}^{13}B + p$ overlap extracted from the $^{14}C(^{11}B, {}^{12}C)\,^{13}B$ reaction with $^{14}C(d, {}^{3}He)\,^{13}B$ data

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When the asymptotic normalization coefficient (ANC) for the $^{A+1}(Z+1) \rightarrow {}^{A}Z + p$ overlap is extracted from data for proton pickup from the $^{A+1}(Z+1)$ nucleus, the value obtained may strongly depend on the degree of completeness of the modeling of the reaction mechanism, particularly when heavy-ion probes are employed. Taking the specific case of the ANC for the $^{14}C \rightarrow {}^{13}B + p$ overlap, an analysis of data for the heavy-ion $^{14}C(^{11}B, {}^{12}C)\,^{13}B$ reaction showed that the distorted wave Born approximation (DWBA) yields a significantly larger ANC than the coupled reaction channel (CRC) technique, incorporating multistep reaction paths. In this paper we present an analysis of data from the literature for the light-ion $^{14}C(d, {}^{3}He)\,^{13}B$ pickup reaction and show that the ANC extracted from the heavy-ion data using the CRC technique is entirely compatible with the light-ion data when used in either DWBA or more complete coupled discretized continuum channel plus CRC calculations. The present results also show that the ANC extracted from the light-ion reaction is essentially unaffected by multistep reaction paths, in contrast with the heavy-ion case.

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I. INTRODUCTION

Analyses of direct reaction data are routinely used to extract nuclear structure information, the selectivity of the reaction process providing a useful tool for probing specific aspects such as single-particle or single-hole levels. Typically, the structure information is obtained in the form of a spectroscopic factor, and requires the use of a theoretical model of the reaction. A number of models of varying degrees of completeness have been developed, from the distorted wave Born approximation (DWBA), which treats the reaction mechanism as a single-step direct transfer process, weak enough to be treated adequately within first-order perturbation theory, to the coupled reaction channel (CRC) technique, which treats the transfer step to all orders and allows the incorporation of multistep paths leading to the same final state. See, for example, Refs. [1,2] for a full description of the various models.

However, independent of the reaction model used, the spectroscopic factors extracted from reaction data have been found to be subject to significant ambiguities due to the choice of binding potential parameters necessary to calculate the associated bound-state radial wave functions. Exploiting the apparent peripheral nature of many direct reactions, which means that the cross section is only sensitive to the tail of the bound-state radial wave function, the concept of the asymptotic normalization coefficient (ANC) was introduced [3,4]. The ANC is much less sensitive to the binding potential parameters and removes most of the ambiguities owing to this source suffered by the spectroscopic factors, although any sensitivity to the completeness of the reaction model or the choice of inputs such as entrance and exit channel optical potentials will remain. Reference [5] provides a comprehensive review of the ANC and its applications.

Nevertheless, many analyses of reaction data where the ANC is extracted employ the DWBA to model the reaction process, implicitly ignoring the possible influence of multistep paths and/or strong coupling effects. This can be particularly important in reactions involving heavy-ion probes where multistep contributions to the reaction process are frequently important, so that their omission from the reaction model could lead to erroneous results. Since the ANC is a nuclear structure quantity, the value for a given overlap should not depend either on the reaction used to extract it or the energy at which the reaction is performed. Thus, if consistent results are to be obtained it is important to verify the assumptions underlying the reaction model employed.

In a recent publication [6] the ANC for the $^{14}C \rightarrow {}^{13}B + p$ overlap was obtained from an analysis of data for the $^{14}C(^{11}B, {}^{12}C)\,^{13}B$ proton pickup reaction at an incident ^{11}B energy of 45 MeV. The availability of a relatively complete data set, angular distributions of the elastic and inelastic scattering as well as the pickup reaction, meant that multistep paths could be included in a CRC analysis in a consistent way, thus enabling the influence of these paths on the obtained value of the ANC to be tested reliably. It was found that the use of the DWBA yielded a value for the ANC significantly larger than that derived from the full CRC analysis.

While the CRC result is expected to be the more realistic, it is important to compare it with values obtained from data for

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the more usually employed light-ion reactions, normally considered to be less likely to involve multistep transfer paths. To this end, we use the ANC obtained from the full CRC analysis of Ref. [6] as input to calculations for existing ^{14}C$(d,{}^3$He$)^{13}$B proton pickup data [7] employing both standard DWBA and coupled discretized continuum channel (CDCC) plus CRC methods. We find that the ANC of Ref. [6] is fully compatible with the ^{14}C$(d,{}^3$He$)^{13}$B data in either approach, confirming the importance of correctly including the multistep transfer paths in the analysis of the heavy-ion data if a reliable value for the ANC is to be extracted. We also find that in this specific case the result obtained from the light-ion data is relatively insensitive to the details of the reaction model used, although this need not necessarily hold for other systems and/or incident energies.

II. DATA ANALYSIS

The ^{14}C$(d,{}^3$He$)^{13}$B data of Ref. [7] were obtained at an incident deuteron energy of 52 MeV. The corresponding deuteron elastic scattering was not measured. The original analysis used the zero-range DWBA and $d + {}^{12}$C and ^{3}He $+ {}^{16}$O optical potentials in the entrance and exit channels, respectively. In this work we first reanalyze these data within the full finite-range DWBA using a recent $\langle{}^3\text{He} \mid d + p\rangle$ overlap and global deuteron and ^{3}He optical potential parameter sets adapted to p-shell targets, then with a more complete CDCC + CRC formalism. All reaction calculations were performed with the code FRESCO [8].

A. Distorted wave Born approximation calculations

Since there are no data available for $d + {}^{14}$C elastic scattering at the appropriate energy the global deuteron optical potential parameters of Zhang *et al.* [9], specifically adapted to 1p-shell target nuclei, were used in the entrance channel. Similarly, the ht1p parameter set of Pang *et al.* [10] was employed to calculate the ^{3}He $+ {}^{13}$B exit channel optical potential. The $\langle{}^3\text{He} \mid d + p\rangle$ overlap was calculated according to the prescription given by Brida *et al.* [11]. The $\langle{}^{14}\text{C} \mid {}^{13}\text{B} + p\rangle$ overlaps were calculated using the values obtained from the full CRC analysis of Ref. [6], i.e., that including ^{13}B ground-state reorientation coupling in the exit channel. Calculations were performed for all values of the $p + {}^{13}$B binding potential radius parameter r_0 investigated in Ref. [6] in order to check for any possible residual dependence of the calculated pickup cross section on the specific value used. All calculations were performed using the prior form of the DWBA and included the full complex remnant term.

The results of DWBA calculations with $\langle{}^{14}\text{C} \mid {}^{13}\text{B} + p\rangle$ overlaps using r_0 values of 1.25 fm, 1.55 fm, and 1.75 fm are compared with the data of Ref. [7] in Fig. 1. It will be noted that while all three calculations adequately describe the forward angle data, for angles $\theta_{\text{c.m.}} > 17^\circ$ the result for $r_0 = 1.25$ fm significantly overpredicts the data while those for $r_0 = 1.55$ fm and 1.75 fm give a good description over the whole angular range. The forward angle cross section for $r_0 = 1.25$ fm is approximately 10% greater than those for

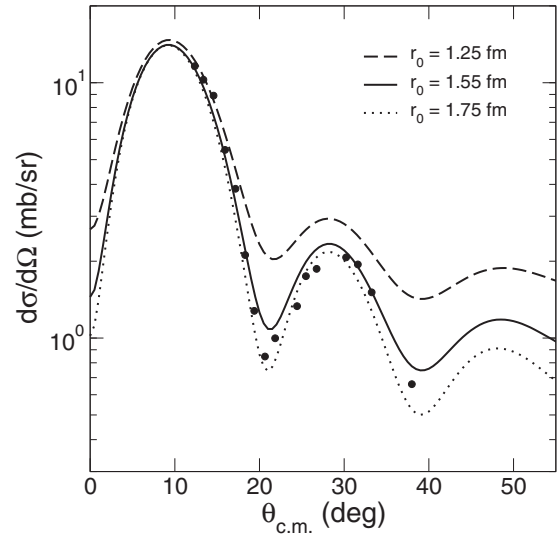

FIG. 1. The $E_d = 52$ MeV ^{14}C$(d,{}^3$He$)^{13}$B data of Ref. [7] (filled circles) compared with DWBA calculations using the $\langle{}^{14}\text{C} \mid {}^{13}\text{B} + p\rangle$ overlaps obtained from the full CRC analysis of Ref. [6] for values of the $p + {}^{13}$B binding potential radius parameter $r_0 = 1.25$ fm (dashed curve), 1.55 fm (solid curve), and 1.75 fm (dotted curve).

$r_0 = 1.55$ fm and 1.75 fm; it converges for values of $r_0 > 1.35$ fm although there remains some slight dependence of the shape of the angular distribution at larger angles on the choice of r_0. It is interesting to note that while the ^{14}C$({}^{11}$B, ^{12}C$)^{13}$B calculations of Ref. [6] did not show any significant variation in the shape of the pickup angular distribution as a function of r_0, the ANC values for $r_0 = 1.25$ fm and 1.35 fm were eliminated from the averaging procedure used there to obtain a final value as unphysical, on the grounds that the corresponding spectroscopic factors were greater than the theoretical maximum value of $2j + 1$ under the convention adopted by FRESCO.

In Fig. 2 we compare the result of a DWBA calculation using the ANC obtained from the full CRC calculation of Ref. [6] with $r_0 = 1.55$ fm (ANC$^2 = 1491^{+279}_{-264}$ fm^{-1}) with the ^{14}C$(d,{}^3$He$)^{13}$B data of Ref. [7]. The uncertainty band, denoted by the shaded area on Fig. 2, is that obtained in Ref. [6] by varying the ANC^2 to give a 10% increase in χ^2 from the minimum value of the CRC fit to the heavy-ion data. The description of the data is almost perfect, demonstrating that the ANC obtained from the full CRC analysis of the ^{14}C$({}^{11}$B, ^{12}C$)^{13}$B data of Ref. [6] is fully compatible with the ^{14}C$(d,{}^3$He$)^{13}$B data of Ref. [7] when used within a standard DWBA analysis.

B. Coupled reaction channel calculations

A further series of calculations was performed using a more complete reaction model involving the use of the CDCC technique explicitly to model the deuteron breakup in the entrance partition plus the CRC formalism for the pickup step. The ^{13}B ground-state reorientation coupling was also included in the exit channel using the same coupling parameters as in Ref. [6]. The $\langle{}^3\text{He} \mid d + p\rangle$ and $\langle{}^{14}\text{C} \mid {}^{13}\text{B} + p\rangle$ overlaps were

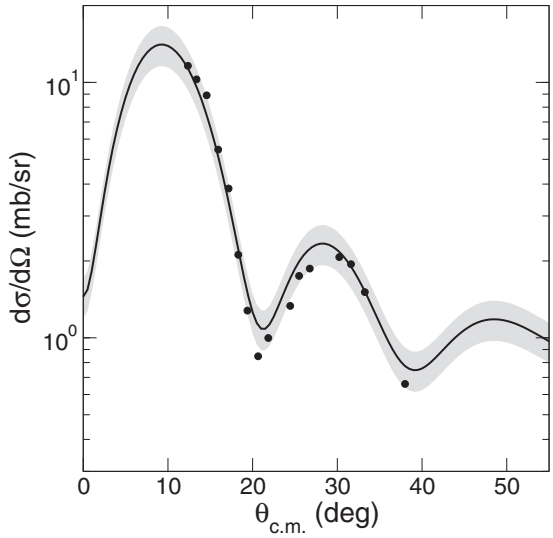

FIG. 2. The $E_d = 52$ MeV ^{14}C$(d, ^3$He$)^{13}$B data of Ref. [7] (filled circles) compared with a DWBA calculation using the $\langle ^{14}\text{C} \mid ^{13}\text{B} + p\rangle$ overlap obtained from the full CRC analysis of Ref. [6] with a $p + ^{13}$B binding potential radius parameter $r_0 = 1.55$ fm (solid curve). The shaded area represents the uncertainty band on the ANC, as in Ref. [6].

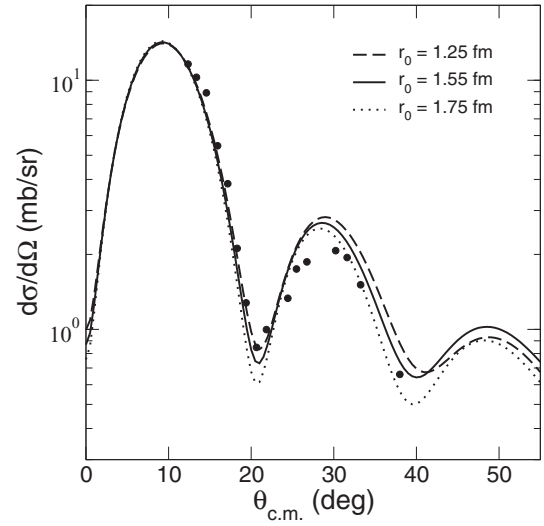

FIG. 3. The $E_d = 52$ MeV ^{14}C$(d, ^3$He$)^{13}$B data of Ref. [7] (filled circles) compared with CRC calculations using the $\langle ^{14}\text{C} \mid ^{13}\text{B} + p\rangle$ overlaps obtained from the full CRC analysis of Ref. [6] for values of the $p + ^{13}$B binding potential radius parameter $r_0 = 1.25$ fm (dashed curve), 1.55 fm (solid curve), and 1.75 fm (dotted curve).

as used in the DWBA calculations described in the previous section. The CDCC part of the calculation was similar to those described in Ref. [12]. The $n + p$ continuum was divided into bins in momentum (k) space of width $\Delta k = 0.125$ fm^{-1} up to a maximum value $k_{\max} = 1.0$ fm^{-1}. The $n + p$ angular momentum was limited to $\ell = 0, 2$. The deuteron diagonal and coupling potentials were obtained by Watanabe-type folding of n and $p + ^{14}$C optical model potentials over the deuteron internal wave function, calculated using the Reid soft-core potential [13]. The nucleon optical potentials were calculated using the central parts of the global parameter set of Koning and Delaroche [14]. The exit channel optical potential was as used in the DWBA calculations; it was found that the effect of including the ^{13}B ground-state reorientation on the predicted ^{3}He $+ ^{13}$B elastic scattering at the appropriate energy was small, being confined to reducing the depth of some of the minima in the angular distribution. It was therefore not considered necessary to adjust the potential parameters. The calculations were again performed using the prior form and included both the full complex remnant term and nonorthogonality correction.

Calculations were performed using the $\langle ^{14}\text{C} \mid ^{13}\text{B} + p\rangle$ overlaps from Ref. [6] for all values of r_0 from 1.25–1.75 fm to check for any residual dependence of the pickup cross section on this parameter, as was done for the DWBA calculations. The results for $r_0 = 1.25$ fm, 1.55 fm, and 1.75 fm are compared with the data of Ref. [7] in Fig. 3. The description of the data by all the calculations is good; in particular, it is seen that the shapes of the angular distributions predicted by the CRC calculations are much less sensitive to the choice of r_0 than the corresponding DWBA results. The calculations for all the values of r_0 tested show no significant difference in the shape of the pickup angular distribution for angles $\theta_{\text{c.m.}} < 30^\circ$ and even at larger angles the differences are relatively small compared to the DWBA results.

In Fig. 4 we compare the result of a CRC calculation using the ANC obtained from the full CRC calculation of Ref. [6] with $r_0 = 1.55$ fm, including the uncertainty band in the ANC, with the ^{14}C$(d, ^3$He$)^{13}$B data of Ref. [7]. The reproduction of the data is near perfect, even better than for the DWBA calculation presented in Fig. 2. We note that omission of the

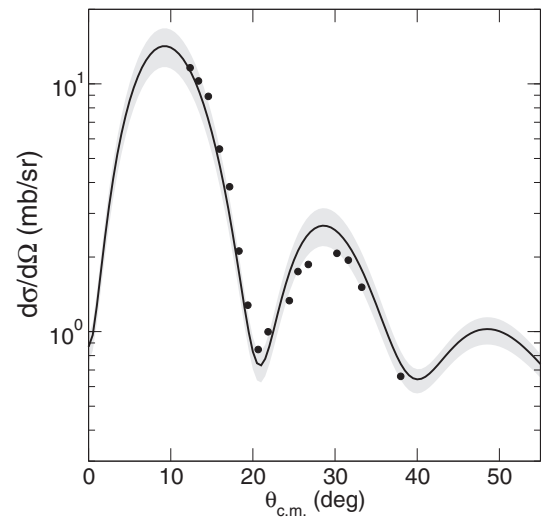

FIG. 4. The $E_d = 52$ MeV ^{14}C$(d, ^3$He$)^{13}$B data of Ref. [7] (filled circles) compared with a CRC calculation using the $\langle ^{14}\text{C} \mid ^{13}\text{B} + p\rangle$ overlap obtained from the full CRC analysis of Ref. [6] with a $p + ^{13}$B binding potential radius parameter $r_0 = 1.55$ fm (solid curve). The shaded area represents the uncertainty band on the ANC, as in Ref. [6].

^{13}B ground-state reorientation coupling in the exit channel reduces the pickup cross section by approximately 10% without materially affecting the shape of the angular distribution, i.e., if one were fitting the ^{14}C(d, ^{3}He) data omission of this coupling would lead to a 10% larger $\langle ^{14}\text{C} \mid ^{13}\text{B} + p\rangle$ ANC^2, exactly as was found in Ref. [6] for the ^{14}C(^{11}B, ^{12}C) data.

III. SUMMARY AND CONCLUSIONS

In summary, we have analyzed existing data [7] for the ^{14}C(d, ^{3}He) ^{13}B proton pickup reaction using both DWBA and CRC formalisms employing the $^{14}\text{C} \rightarrow {}^{13}\text{B} + p$ ANCs extracted from a CRC analysis of ^{14}C(^{11}B, ^{12}C) ^{13}B data [6]. The results were in excellent agreement with the data, confirming that a reliable ANC value can be obtained from heavy-ion transfer data provided a sufficiently complete data set is available to allow an adequate coupling scheme involving multistep transfer paths to be employed consistently. The equivalence of the DWBA and CRC results also suggests that the expectation that the light-ion transfer reaction should not involve significant contributions from multistep paths is justified, at least in the present case. We note, however, that the more sophisticated CRC model gives significantly more stable results than the DWBA, the shape of the pickup angular distribution being essentially independent of the choice of $p + {}^{13}$B binding potential radius parameter r_0 (however, the magnitude of the cross section, and thus the value obtained for the ANC, does show some minor dependence on the choice of r_0, as noted in Ref. [6]).

The complete consistency of the present results with the previous analysis of the heavy-ion data also suggests that the $\langle ^{3}\text{He} \mid d + p\rangle$ overlap of Brida *et al.* [11] provides a realistic normalization for (d, ^{3}He) reactions. In connection with this, in Fig. 5 we investigate the sensitivity of the DWBA calculations to the choice of entrance and exit channel optical potentials. Figure 5 shows that repeating the full finite-range DWBA calculations with the entrance and exit channel distorting potentials used in the original analysis of Ref. [7] but retaining all other inputs as in Sec. II A gives a cross section that underpredicts the data by approximately 20% (the calculations shown on Fig. 5 are for $r_0 = 1.55$ fm but the results are similar for other values of r_0). Replacing the entrance channel deuteron optical potential with the global parameters of Zhang *et al.* [9] has little influence on the predicted cross section, cf. the dotted curve on Fig. 5, reducing the magnitude by a further 10% and deepening the minima of the angular distribution. However, if we retain the deuteron optical potential of Ref. [7] but replace the exit channel ^{3}He potential with the ht1p parameter set of Pang *et al.* [10], the magnitude of the cross section increases to match the data reasonably well: it now overpredicts it by about 10% and the shape of the angular distribution is not as well reproduced as in the DWBA calculations presented in Sec. II A. We therefore conclude that the good agreement between the ANC obtained from the heavy-ion data and the present analysis of the ^{14}C(d, ^{3}He) ^{13}B data of Ref. [7] depends to some extent on the choice of distorting potentials in the entrance and exit channels. The most important influence is from the choice of

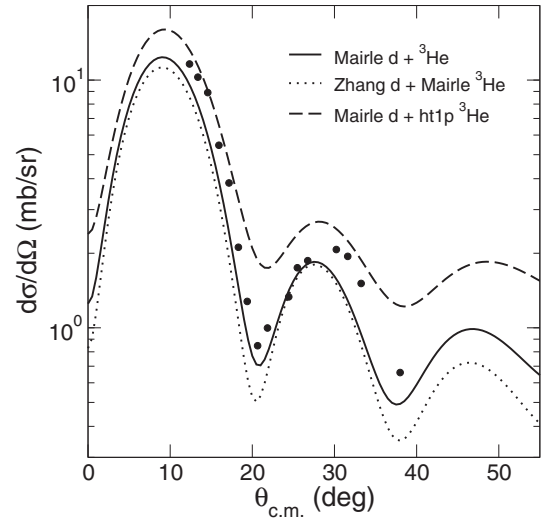

FIG. 5. The $E_d = 52$ MeV ^{14}C(d, ^{3}He) ^{13}B data of Ref. [7] (filled circles) compared with DWBA calculations using the $\langle ^{14}\text{C} \mid ^{13}\text{B} + p\rangle$ overlap obtained from the full CRC analysis of Ref. [6] with a $p + {}^{13}$B binding potential radius parameter $r_0 = 1.55$ fm and deuteron and ^{3}He optical potentials from Ref. [7] (solid curve), the deuteron potential of Ref. [9] plus the ^{3}He potential used in Ref. [7] (dotted curve), and the deuteron potential used in Ref. [7] plus the ht1p ^{3}He potential [10] (dashed curve).

exit channel $^{3}\text{He} + {}^{13}$B optical potential. Ideally one would prefer to be able to use optical potential parameters that describe the appropriate elastic scattering in both entrance and exit channels. However, in this case the necessary data for the elastic scattering in the exit channel at least are unlikely to be forthcoming in the immediate future since they would require a ^{13}B radioactive beam incident on a ^{3}He target, both significant complications. Nevertheless, the most appropriate available global parameter sets, when used in conjunction with a modern $\langle ^{3}\text{He} \mid d + p\rangle$ overlap [11], give a consistent picture.

Finally, we may summarize our conclusions as follows:

(i) Consistent results for the ANC for the $^{14}\text{C} \rightarrow {}^{13}\text{B} + p$ overlap (ANC^2 $= 1532 \pm 138$ fm^{-1}) were obtained from analyses of both ^{14}C(d, ^{3}He) ^{13}B and ^{14}C(^{11}B, ^{12}C) ^{13}B data, confirming that a relatively complete CRC calculation including multistep reaction paths in a consistent way can be used reliably to extract the ANC from heavy-ion reaction data.

(ii) At least in the present case, a DWBA analysis of the heavy-ion data is shown to be inadequate, since it gives a result for the ANC that is not consistent with that obtained from the light-ion data.

(iii) Both DWBA and more complete CRC analyses of the light-ion data yield similar results for the ANC, confirming the usual expectation that multistep effects are not as important for light-ion reactions, at least at this relatively higher incident energy. However, the CRC result is more stable, in that the shapes of the resulting pickup angular distributions are much less sensitive to

the choice of r_0 used to calculate the $\langle {}^{14}\mathrm{C} \mid {}^{13}\mathrm{B} + p\rangle$ overlap.

(iv) The $\langle {}^{3}\mathrm{He} \mid d + p\rangle$ overlap of Brida *et al.* [11], based on state-of-the-art shell model calculations, appears to provide a realistic normalization for the light-ion vertex.

(v) The light-ion DWBA results are sensitive to the choice of both entrance and exit channel optical potentials, although the main influence is from the exit-channel ${}^{3}\mathrm{He} + {}^{13}\mathrm{B}$ parameters. In the absence of the necessary elastic scattering data the most appropriate available global parameter sets give consistent results.

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