Evolution of the statistical disintegration of finite nuclei toward high energy

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We develop a statistical approach for the description of complex nuclei formation from dynamically produced baryons in high energy heavy-ion reactions. We consider a finite highly excited expanding nuclear system formed after central nucleus-nucleus collisions. This system is subdivided into primary equilibrated nucleon clusters. The final nuclei are produced after the decay of these excited clusters. By the successful comparison with the FOPI experimental data we prove the possibility of such a local equilibrium in nuclear matter with the temperature corresponding to the phase coexistence region. The regularities obtained in this new nuclei production mechanism are shown.

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I. INTRODUCTION

Statistical models have a long and very successful history in nuclear physics. Most prominently they have been used for the description of nuclear decay when an equilibrated source can be identified in the reaction. The most famous example of such a source is the 'compound nucleus' introduced by Niels Bohr in 1936 [1]. Such a compound structure was clearly seen in low-energy nuclear reactions leading to excitation energies of a few tens of MeV. It is remarkable that this concept is also applicable for nuclear reactions induced by particles and ions of intermediate and high energies, when nuclei break up into many fragments (multifragmentation) [2,3]. Generally, this process is associated with the manifestation of the nuclear liquid-gas type phase transition at subnuclear densities. In these reactions one can extend the statistical approach towards the finite systems, and demonstrate that it works when these systems decay rapidly. The multifragmentation process has been considered as the statistical one and the first physical models were developed nearly 40 years ago [4–11]. In the following the multifragmentation was under intensive experimental and theoretical investigation as a decay of a single excited nuclear system produced in nucleus collisions. Some examples of the theoretical development of this approach one can find in Refs. [2,3,12–18]. The statistical models have successfully described a large amount of multifragmentation experimental data obtained both at Fermi energies (mostly in central collisions) [19–26] and at relativistic energies in peripheral collisions [27–37]. As was found in previous theoretical analyses of relativistic reactions the system excitation energy reaches about 10 MeV per nucleon and one can consistently describe all experimental data with relatively low temperatures ($T \lesssim$ 6–8 MeV) of the thermal sources. At very high excitation energies observed in finite systems there were usually problems to describe the kinetic energies of produced fragments. Phenomenologically, this was resolved by introducing the regular (hydrodynamical-like) flows, and by assuming only chemical equilibrium in the systems [38]. In this paper we propose a method to extend the statistical approach to highly excited finite nuclear systems. To this aim we consider local statistical equilibrium for the complex nuclei formation in separate parts (clusters) of nuclear matter. We show that our method is fully consistent with all observables concerning the production of nuclei. Therefore, it provides new insight on the fragment formation in high energy reactions.

II. FORMATION OF EXCITED NUCLEAR STATISTICAL SYSTEMS

According to the statistical hypothesis, the initial dynamical interactions between nucleons lead to a redistribution of the available energy among many degrees of freedom, therefore, the nuclear system evolves towards equilibrium. In the most general consideration the reaction may be subdivided into several stages: (1) a dynamical stage leading to formation of an equilibrated nuclear system, (2) the statistical fragmentation of the system into individual primary fragments, which can be accompanied by the de-excitation of hot primary fragments if they are in the excited states. Many transport models are used for the description of the dynamical stage of the nuclear reaction at high energies. They take into account the hadron-hadron interactions including the secondary interactions and the decay of hadron resonances. For this reason they may preserve some correlations between hadrons originated from the primary interactions in each event, which are ignored when we consider the final inclusive particle spectra only. Within dynamical models it is established that many particles are involved in this process via the intensive rescattering and the collective interaction during the primary collisions. In peripheral collisions the produced high energy particles leave the system and the remaining nucleons form an excited system (a residue). This system can collectively expand, and the residual interaction drives it towards an equilibrium at a low density. We may expect that the system evolves toward a state which is mostly determined by the statistical properties of the excited nuclear matter. Generally this equilibration may not be a complete one. However, as shown by comparison with experiment, in many cases it is sufficient to apply statistical theory for the nuclear fragment formation. Usually, the statistical approach has been applied to the excited residual nuclei formed from the spectator parts of the colliding nuclei, as well as for a nuclear system produced after their fusion (partial or full one). It was treated via the compound nucleus concept at low excitation energy [1]. This concept was generalized to the multifragmentation of a single nuclear source with a high energy which is sufficient for its fast thermal expansion before the disintegration [2,3,36].

However, there is another interesting possibility for the application of the statistical approach to describe the evolution of the produced diluted nuclear matter: At the end of the dynamical stage (at a time around $\approx$10–30 fm/c after the beginning of the nucleus-nucleus collision) many newborn baryons and nucleons are escaped from the colliding nuclei remnants. Some of these baryons may be located in the vicinity of each other with local subnuclear densities around $\approx 0.1\rho_0$ ($\rho_0 \approx 0.15$ fm^{-3} being the ground state nuclear density). This nuclear matter density is very similar to the densities expected in the freeze-out volume which is assumed in the statistical approach as the proper place of the nuclei formation. Namely, at this place the interaction between nucleon can still lead to fragment formation from these nucleons. Such nucleation processes will be improbable both at too low and too high densities. The system has to pass the above-mentioned density during its expansion, which allows to use the statistical models at this local space-time region.

To investigate this process, in the first approximation, it is instructive to consider a general situation of baryonic nuclear matter expanding as a result of the previous dynamical process within a simple controlled model. For example, we can simulate an expanded nuclear matter state with stochastically distributed baryons. We call our first method the phase space generation (PSG) method: Here, we perform an isotropic generation of all baryons of the excited nuclear system according to the microcanonical momentum phase space distribution with total momentum and energy conservation. It is assumed in the one-particle approximation that all particles are in a large volume (at subnuclear densities) where they can still interact with others to populate the phase space uniformly. Technically, this is done using the Monte Carlo method applied previously in the microcanonical SMM (statistical multifragmentation model) and Fermi break-up model [2], and taking into account the relativistic effects according to the relativistic connection between momentum $\vec{p}$, mass m, and kinetic energy of particles E_0, see Eq. (1). In Eq. (1) the sum is over all particles and we use units with $\hbar = c = 1$:

$$\sum \sqrt{\vec{p}^2 + m^2} = E_0 + \sum m. \tag{1}$$

The total kinetic energy available for the motion of baryons E_0 (we call it the source energy) is the important parameter which can be adjusted to describe the energy accumulated in the system after the dynamical stage. We believe that the PSG method is a reasonable assumption due to the very intensive interactions between the colliding nucleons of the target and projectile, which take place in some extended volume during the reaction, leading to the equilibration of the one-particle degrees of freedom. Note, that this is not an equilibrium with respect to the nucleation process. In this case we do not take directly into account the coordinates of the baryons but we assume they are proportional to their velocities and strictly correlate with them. This is also consistent with the results of dynamical models.

In the second method, we assume the momentum generation similar to the explosive hydrodynamical process when all nucleons fly out from the center of the system with the velocities exactly proportional to their coordinate distance to the center of mass. We call it the hydrodynamic generation (HYG) method. In this method we pace randomly (Monte Carlo) all nucleons uniformly inside a sphere with the radius $FR_nA_0^{1/3}$ without overlapping. Here, A_0 is the nucleon number, and $R_n \approx 1.2$ fm is the nucleon radius. The scaling factor $F \approx 3$ is assumed to describe the expanded volume in which the nucleon can still strongly interact with each other. At intermediate collision energies this volume corresponds approximately to the average expansion of the system in line with the transport model simulations, when the baryon interaction rate drastically decreases. Finally, we attribute to each nucleon a velocity by taking into account the momentum and energy conservation for the relativistic case [Eq. (1)]. Obviously, the velocities and coordinates of baryons are strongly correlated with each other.

For illustration, in Fig. 1 we demonstrate the energy distribution of nucleons generated using the PSG and HYG methods. Here we assume an intermediate source with $A_0 = 116$ and charge $Z_0 = 56$. However, we have found that the general trends do not depend on the system sizes.

It is obvious that both the PSG and HYG methods generate the baryonic matter which expands in each coordinate point. All parts of this matter do certainly pass through the ‘freeze-out’ density where nuclei can be still formed as supposed in the statistical models. However, because of the different momenta and locations of the nucleons, the different parts of the system pass this density at different times. Therefore, one may not claim that the whole nuclear system is in the same statistical freeze-out volume concerning the nuclei formation. However, it is possible to assume local equilibrium. Let us stress that it is important to consider the PSG and HYG methods as complementary descriptions of the finite expanding system, corresponding to different limits of the dynamical description.

III. SUBDIVISION OF THE NUCLEAR SYSTEM INTO EXCITED CLUSTERS

The idea is to divide the low-density nucleon matter into small parts (clusters) with nucleons which are in equilibrium respective to the nucleation process [39]. These clusters are

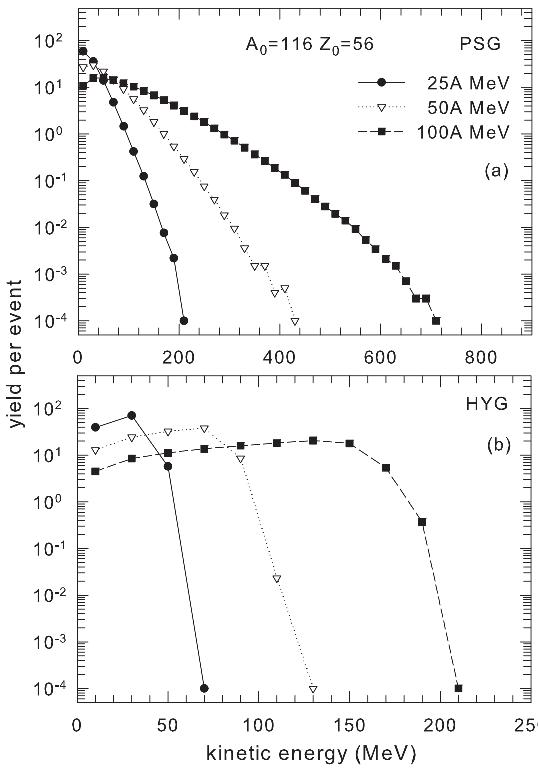

FIG. 1. Energy spectra for initial nucleons of the hot expanding nuclear system according to the microcanonical phase space distribution—PSG (a) and according to the hydrodynamical-like explosion—HYG (b). The assumed total kinetic energies are 25, 50, and 100 MeV per nucleon. The nucleon source size and composition are shown in the top panel.

analogous to the local freeze-out states for the liquid-gas type phase coexistence adopted in statistical models. Since the nucleons are moving with respect to each other these clusters are excited objects and distributed over the whole nuclear system. The subsequent evolution of such clusters, including the formation of nuclei from these baryons, can be described in the statistical way. Within our procedure we can claim that these hot clusters decay into nuclei. It may look like that the formation of the clusters is similar to the standard coalescence procedure [40]. However, it is assumed in the simplistic coalescence picture that only nucleons which combine a bound nucleus can interact in the final state. All other nucleons will not interact with this nucleus, or interact very slightly by taking extra energy to conserve the momentum/energy balance. In our case all nucleons of the primary clusters are fully involved in the interaction leading to final nuclei. Nevertheless, not all of these nucleons will be bound in the nuclei in the end.

Since the matter expands, a crucial question is, if the interactions between the baryons inside these clusters are sufficiently strong to lead to local equilibrium. In this case they could be considered as statistical subsystems where the phase-space dominates the nuclei formation. We remind the reader that the lifetime of finite nuclear species is related to the energy accumulated into these species. We know from the extensive studies of nuclear multifragmentation reactions [2,28–32] that the excitation energies of the excited nuclear residue systems can reach up to 8–10 MeV per nucleon, and the statistical models describe their disintegration very well. We have also learned from the analysis of nuclei production in multifragmentation that the densities before the breakup of these systems are around $0.1–0.3\rho_0$, and their lifetime is 50–100 fm/c [34,36]. We suggest that the difference between the multifragmentation of excited projectile- and target-like residues and the formation of the baryon clusters in the expanded matter is just due to the dynamical mechanisms leading to these diluted finite systems. In the spectator multifragmentation the systems are prepared after the dynamical knocked-out of many nucleons and thermal (or dynamical) expansion of the remaining nuclei. Our new baryon clusters can be formed as a result of the local interaction of the stochastically produced primary baryons. Therefore, we can estimate that an energy around ≈10 MeV per nucleon is a reasonable value which can be reached in such hot stochastic clusters, similar to the standard multifragmentation case. If the excitation energy is much higher, then the existence of such clusters as intermediate finite systems, including their following evolution in the statistical way, become problematic. We think that the final conclusion on the excitation energy can only be done after a detailed comparison with experimental data.

To describe the cluster formation we suggest the coalescence prescription, and apply the coalescence of baryon (CB) model [41,42]. In the PSG and HYG cases the coalescence criterion is the proximity of the velocities (or momenta) of the nucleons. In the both cases we do not need to include explicitly the coordinate of nucleons, because in the PSG and HYG approaches the velocities and space coordinates are already correlated. In particular, the coordinate vectors are directly proportional to the velocities vectors. So the velocity coalescence parameter is sufficient for the cluster identification in these models. Such a strong space-momentum correlation exists in many explosive processes and it influences the original clusterization. For the following evaluation of the cluster properties we assume that such clusters with nucleons inside have the density of $\rho_c \approx \frac{1}{6}\rho_0$ as it was established in the previous studies of statistical multifragmentation process [2,3,34,36]. This corresponds to the average distance of around 2 fm between neighbor nucleons, and the interaction between these nucleons can lead to the formation of nuclei. Within the CB model we assume that baryons (both nucleons and hyperons) can produce a cluster with mass number A if their velocities relative to the center-of-mass velocity of the cluster is less than v_c. Accordingly we require $|\vec{v}_i - \vec{v}_{\rm c.m.}| < v_c$ for all $i = 1, \ldots, A$, where $\vec{v}_{\rm c.m.} = \frac{1}{E_A}\sum_{i=1}^{A} \vec{p}_i$ ($\vec{p}_i$ are momenta and E_A is the sum energy of the baryons in the cluster). This is evaluated by sequential comparison of the velocities of all baryons. To avoid problems related to the sequence of nucleons within the algorithm, we apply the iterative coalescence procedure [41,42], starting from a small coalescence parameters for clusters and increasing it step-by-step up to v_c.

We show in Fig. 2 the distributions of clusters as a function of mass number A after the coalescence of initial nucleons of the primary source $A_0 = 116$, $Z_0 = 56$, for $E_0 = 10A$ MeV (top panel), $E_0 = 25A$ MeV (middle panel), and $E_0 = 100A$ MeV (bottom panel), for the velocity coalescence parameters

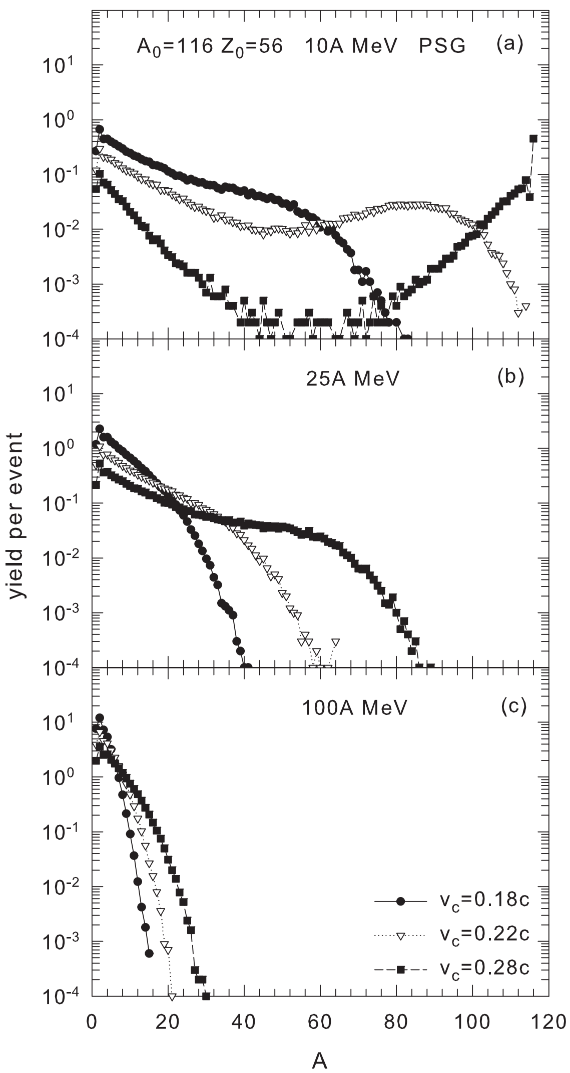

FIG. 2. Yield of coalescent-like clusters versus their mass number A after the CB calculations at the source energy of 10, 25, and 100 MeV per nucleon. Composition and sizes of sources, nucleon generator (PSG), as well as coalescence parameters (v_c) are indicated in the panels.

$v_c = 0.18$, 0.22, and 0.28 c. We only show the results for the PSG model since the HYG model leads to a qualitatively similar picture. In our case v_c is the maximum velocity deviation and all baryons with lower relative velocities do compose a cluster. The middle $v_c = 0.22\ c$ is approximately of the order of the Fermi velocity which is expected in such nuclei. It is obvious that the smaller v_c will lead to small clusters without excitation energies and these values are consistent with the coalescence parameters extracted from the analyses of experimental data for the production of lightest nuclei in previous years [40,43]. One can see that the large primary clusters can indeed be produced within this mechanism even at the high source energy. At low energies we are naturally transiting to the compound-like state with one big excited nucleus.

Such a clusterization procedure is fully consistent with the consideration of the low-energy compound nucleus processes and the multifragmentation processes which take place in one source at low and moderate excitation energies. Apparently, in our case we shall deal with several local statistical sources (clusters) in one events. This procedure can also be suggested as a generalization for the statistical description of the disintegration of highly excited finite nuclear systems which are produced in intermediate and high energy nucleus collisions.

IV. DISINTEGRATION OF EXCITED CLUSTERS INTO NUCLEI

The excited primary nuclear clusters will disintegrate into small pieces. As mentioned above, this disintegration can be considered as a result of the residual nuclear interaction at the subnuclear density between baryons of these clusters leading to the formation of final nuclear species. In the end the cold and stable nuclei are produced. The energy accumulated in such low-density finite clusters is the main ingredient which determines their following evolution. In the lowest limit we can estimate this excitation as a relative motion of the nucleons initially captured into a cluster respective to the center of mass of this cluster. In this case the excitation energy E^* of the clusters (j) with mass number A and charge Z is calculated as

$$E^* = \sum_{i=1}^{A} \sqrt{\vec{p}_{ri}^{\,2} + m_i^2} - M_A, \tag{2}$$

where M_A is the sum of the masses of the nucleons in this nuclear cluster, $i = 1, \dots, A$ enumerates the nucleons in the cluster, m_i are the masses of the individual nucleons in the cluster, $\vec{p}_{ri}$ are their relative momenta (respective to the center of mass of the cluster). However, in the cluster volume the nucleons can interact with each other and the binding interaction energy δE^* should be added to the E^*. As an upper limit we can take the ground state binding energy of normal nuclei with A and Z. However, since our clusters present pieces of nuclear matter expanded already during the previous dynamical reaction stage, we suggest that this energy should be lower. Therefore, as first approximation we use the following recipe for the evaluation of δE^*: It is known the ground state binding energy of nuclei can be written as the sum of short range contributions ($E_{\rm sr}$, which naturally includes volume, symmetry, surface energies), and the long-range Coulomb energy ($E_{\rm col}$), see, e.g., Ref. [2]. Since a cluster is extended, its Coulomb energy contribution will be smaller and we can recalculate it proportional to $(\frac{\rho_c}{\rho_0})^{1/3}$ (in the Wigner-Seitz approximation [2]). For the short range energies, it is assumed that all contributions do also decrease proportional to $(\frac{\rho_c}{\rho_0})^{2/3}$ as it follows from the decreasing of the Fermi energy of nuclear systems. This prescription can be used for any description of the initial dynamical expansion of the system, for example, with the transport models. This was worked out in our previous paper, Ref. [39], where we have suggested

$$\delta E^* = E_{\rm col}\left(\frac{\rho_c}{\rho_0}\right)^{1/3} + E_{\rm sr}\left(\frac{\rho_c}{\rho_0}\right)^{2/3}. \tag{3}$$

It provides a reasonable estimate in between the mentioned limits. In the following we call this energy the cluster excitation energy, or the cluster internal energy.

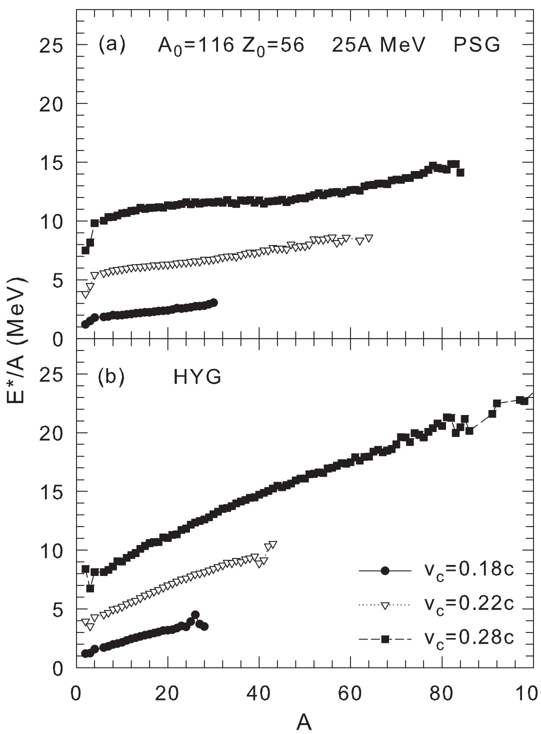

FIG. 3. Average internal energy of coalescent clusters versus their mass number A produced as a result of the coalescence (CB) in the sources with $A_0 = 116$ and $Z_0 = 56$ after PSG (a) and HYG (b). The source energy and coalescence parameters are shown in the panels.

In the present work we use only the PSG and HYG methods to generate nucleon distributions. In this case we are able to take into account the energy conservation in the expanded system. Since after the subdivision of nucleons we consider the isolated hot clusters it is reasonably to scale the cluster's internal excitation to fit the total energy balance. After the nucleon momentum generation the excitation of clusters is the only quantity which can correctly be used for this purpose. Therefore, finally for the cluster j we take

$$E_j^* = \beta(E^* + \delta E^*), \tag{4}$$

where β is found from the energy balance in the system

$$\sum_{j=1}^{N} \left(\sqrt{\vec{p}_j^{\,2} + m_j^2} + E_j^*\right) = E_t + M_t. \tag{5}$$

Here, N is the number of clusters in the system, $\vec{p}_j$ and m_j the cluster momenta and masses. The right part contains the initial total energy E_t deposited into the system and the initial mass of the system M_t. Further we take this new excitation energy E_j^* for the statistical calculations of the nucleation process, i.e., for the decays of all excited clusters.

For illustration, in Fig. 3 we present the average internal energies of such clusters versus their mass number for the big systems $A_0 = 116$, $Z_0 = 56$, and $E_0 = 25A$ MeV, with the coalescence parameters v_c from 0.18, to 0.28 c. One can see that the internal energy per nucleon increases with the parameter v_c. This is because more nucleons with large relative velocities are captured into the same cluster. By comparing the panels of Fig. 3 we see the effect of the source generator on these distributions: The internal energies are not very different, since they are determined by the relative nucleon motion inside the clusters. Nevertheless the HYG provides a general increase of the internal energy with the mass number since the large clusters are consisting of baryons having initially higher velocities.

As was done previously in the analyses of heavy-ion collisions at low and intermediate energies we use the statistical multifragmentation model (SMM) [2] to describe the break-up of normal nuclear clusters. This approach includes (consistently connected) multifragmentation, evaporation, fission (for large nuclear systems), and Fermi break-up (for small systems) models. Therefore, it can be used as an universal model to describe the decay of single statistical sources from very low to rather high excitation energies. At the same time it reflects properties of nuclear matter resulting in a phase transition. We remind the reader that SMM is very successful in the description of the disintegration of highly excited nuclear systems, as was demonstrated by numerous comparisons with multifragmentation data in peripheral relativistic collisions and in central nucleus-nucleus collisions around the Fermi energy [2,19,20,22,24,27,28,30–37].

In Fig. 4 we demonstrate the fragment yields obtained after the de-excitation of primary clusters. We have taken the same cases as were shown in Fig. 2. We obtain a well known general regularity that at high initial excitation energy the yield of nuclei decreases exponentially with their masses. And it becomes even steeper at the highest source energies. One may naively conclude from this observation that the source's temperature becomes higher. However, in the case of finite systems the situation is different. Actually, two effects contribute to this kind of behavior: (1) The cluster's de-excitation leads to small fragments, and (2) the size of the clusters decreases with increasing the initial energy, because the clusters can accumulate the limited amount of energy and the temperature of clusters may not change. It is interesting, however, that after this decay a small v_c may provide even larger fragment yields than a larger v_c. Because in our cases of relatively low initial source energy, a small v_c can still lead to the formation of sufficiently big clusters but with smaller cluster excitation energies.

To investigate the influence of the initial nucleon distribution on the final fragment yields we show in Fig. 5 the calculations as in Fig. 4, however, using the HYG nucleon generation method. We see qualitatively similar results concerning the yield evolution despite very different initial distributions (see Fig. 1). This gives us some confidence that the conclusions presented here are robust and will also not change, if more elaborated initial calculations, e.g., from transport simulations are used.

Besides the yields, the kinetic energy of produced nuclei is also a very important characteristic of the nucleation process. In Fig. 6 we demonstrate the average kinetic energy of the final nuclei for the same initial nuclear system in the cases of PSG and HYG nucleon generations. Usually, the kinetic energy is associated with the nuclear fragment flow. For clarity we use only one coalescence parameter $v_c = 0.22c$ for

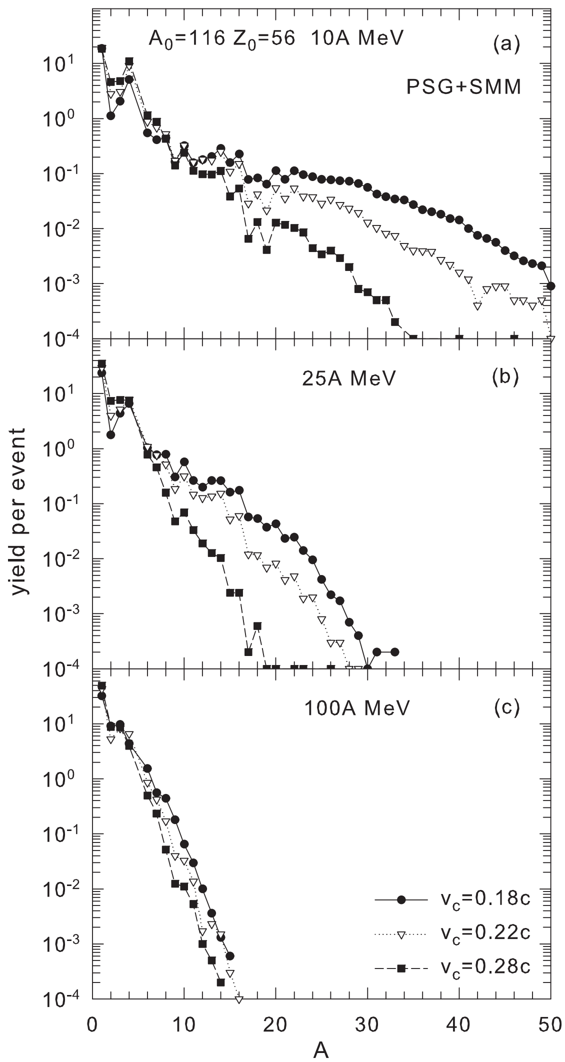

FIG. 4. Yield of final nuclei versus their mass number A after the de-excitation of primary clusters shown in Fig. 2. The notations are as in Fig. 2.

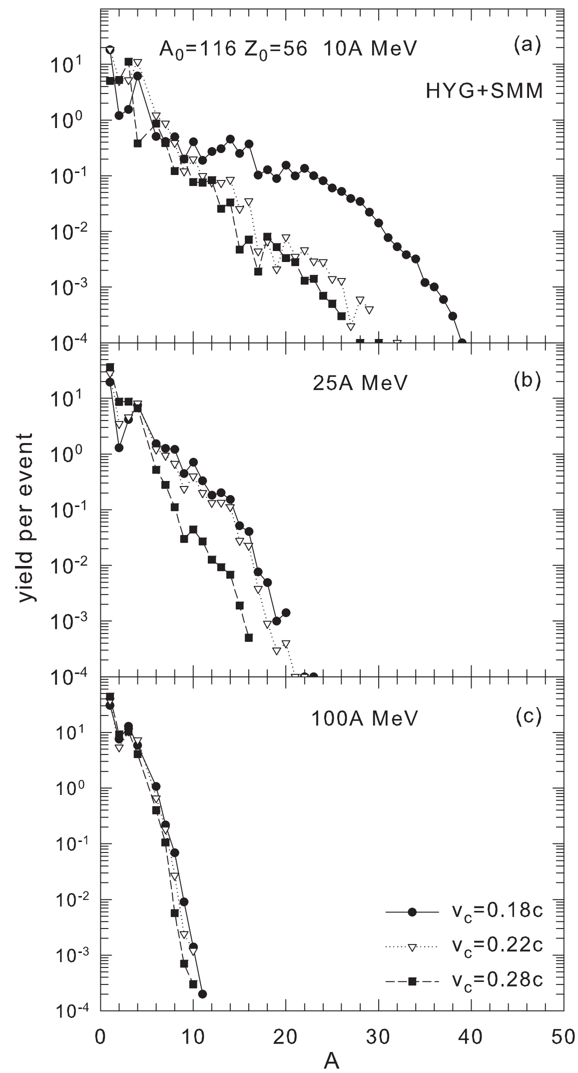

FIG. 5. Yield of final nuclei versus their mass number A after the de-excitation of primary clusters, by using the HYG initial nucleon distributions. Other notations are as in Fig. 4.

the primary cluster formation. One can see that substantial flows can be reached in those cases. Especially when we use HYG primary nucleons (bottom panel) at a high initial source energy. It is an obvious result, since many nucleons are concentrated at the high kinetic energy region in this case (see Fig. 1). A moderate decreasing of the flow energy with the nuclei charges (consequently, with mass numbers) in the PSG case (top panel) is also understandable, since most nucleons have low kinetic energies according to the PSG generation method.

V. COMPARISON WITH EXPERIMENTAL DATA

In the previous section we have investigated the general regularities for the production of fragments which can be obtained within the proposed mechanism. Below we demonstrate the results of our hybrid approach which include the initial generation of the nucleon momenta (PSG and HYG), the selection of the primary excited clusters (CB), and the statistical description of the nucleation inside the clusters (SMM). We provide a comparison with the extensive high-quality experimental data obtained by the FOPI collaboration [44]. They are obtained in Au + Au and Ni + Ni central collisions. Previously these fragment production data could not be consistently analyzed with neither dynamical models nor with statistical ones.

One remark on the experimental data obtained by different groups should be made. The published data depends essentially on the selection of the central events. In particular, the FOPI group has used the ERAT (energy ratio) criterion which includes the ratio of total transverse to the longitudinal kinetic energies of particles in the center-of-mass (c.m.) system [44,45]. While in Ref. [46] a simple criterion related to the light particle multiplicity is employed. As a result the extracted yields of the nuclei are slightly different. For example, the ratio of the intermediate mass fragment (with $Z \geqslant 3$) yields to the yield of $Z = 3$ fragments in Au + Au

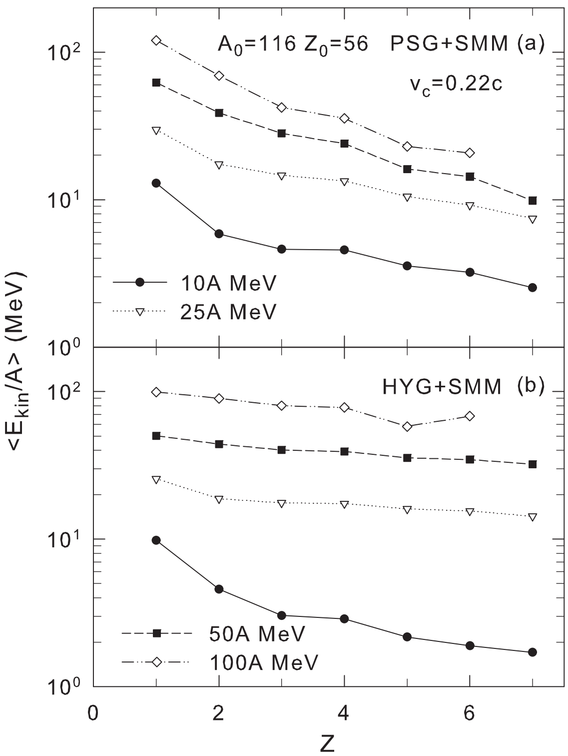

FIG. 6. Average kinetic energies (per nucleon) of final nuclei versus their charges Z. The calculations are performed for the initial source with $A_0 = 116$ and $Z_0 = 56$ at the initial excitation energies of 10, 25, 50, and 100 MeV per nucleon. Notations with the parameters are in the figure. (a) is for the PSG initial nucleon generation, and (b) is for the HYG one.

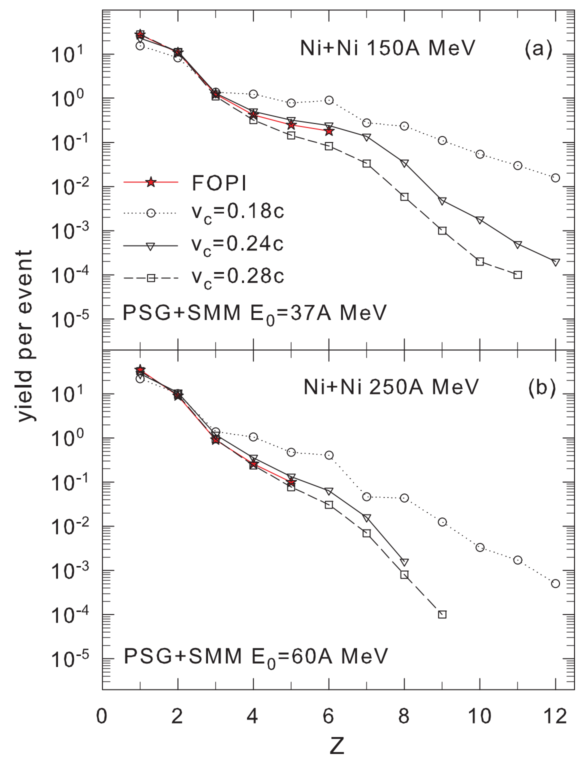

FIG. 7. Comparison of our calculations with the FOPI experimental data on the nuclei production in central Ni+Ni collisions at 150A MeV (a) and 250A MeV (b). The parameters of the initial source are given in the figure. The nucleon distributions are after PSG, and parameters $v_c = 0.18c$, $0.24c$, and $0.28c$ are used in the calculations.

central collisions at 100A MeV/nucleon obtained in Ref. [46] is nearly the same as the one obtained in the FOPI experiment, but at 120A MeV/nucleon. We believe that the FOPI criterion is more sophisticated and corresponds better to the thermalization condition for the one-particle distribution functions. However, we should keep in mind the possible deviations caused by the event selection in the analysis of the data.

In Fig. 7 we show the comparison of our calculations of the charge yields of nuclei (which include the formation of primary nucleon clusters and their decay according to SMM) with the experimental data measured in Ni + Ni central collisions at 150 and 250 Mev per nucleon. We assume the formation of an initial system with $A_0 = 116, Z_0 = 56$, and total excitation energies of $E_0 = 37A$ MeV and $E_0 = 60A$ MeV, which correspond to the kinetic energy available in the center of mass (including the relativistic corrections). The proper energy/momentum balance was taken into account in the calculations. In order to show the dependence on the internal excitation energy and size of the primary coalescent-like clusters we present results for $v_c = 0.18c$, $0.24c$, and $0.28c$. We take the PSG method for the generation of the nucleons because it better suits to the FOPI event selection. One can observe a quite good agreement with the experimental data using the middle v_c, when the excitation of clusters are around 6–10 MeV per nucleon.

It is important to involve larger initial systems in the analysis. Figures 8 and 9 present the comparison of our hybrid model calculations with the FOPI data on nuclei yields obtained in central Au + Au collisions at 90, 120, 150, and 250 A MeV (the corresponding center-of-mass energies are shown in the figures).

One can also see for this selection a very good agreement with the experimental data. The small difference for the middle v_c parameter is related to the fact that at low energies we need a slightly lower v_c in order to construct the clusters with the appropriate internal excitation energy (6–10 MeV/nucleon), that is necessary for the successful description of the data. To illustrate this conclusion we provide in Fig. 10 the average excitation energy per nucleon (E^*/A) distributions for the equilibrated clusters which give the best description of the data. By comparing with Fig. 3 one can see that addressing E^* as a better parameter is quite justified since the analyzed range of v_c can lead to very different values of E^*. Namely the excitation E^*, but not a v_c parameter, has a physical meaning in our approach, since it gives defining information on the local equilibrated sources. It is instructive that previously similar maximum excitation energies of single sources (around 10 MeV per nucleon) were extracted from analyses of peripheral collisions in relativistic reactions [28–32]. As we know from the previous investigations of the statistical disintegration of excited finite nuclear

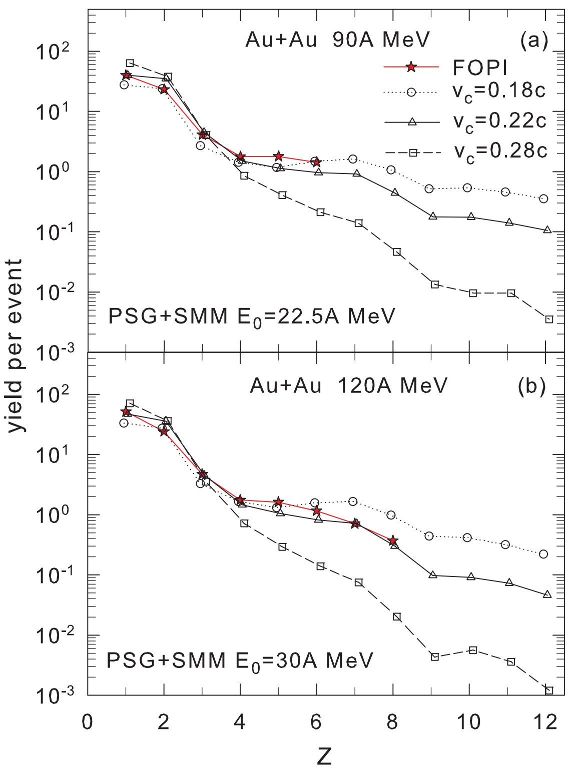

FIG. 8. Comparison of our calculations with the FOPI experimental data on the nuclei production in central Au+Au collisions at 90*A* MeV (a) and 120*A* MeV (b). The parameters of the initial source are given in the figure. The nucleon distributions are after PSG, and parameters $v_c = 0.18c$, $0.22c$, and $0.28c$ are used in the calculations.

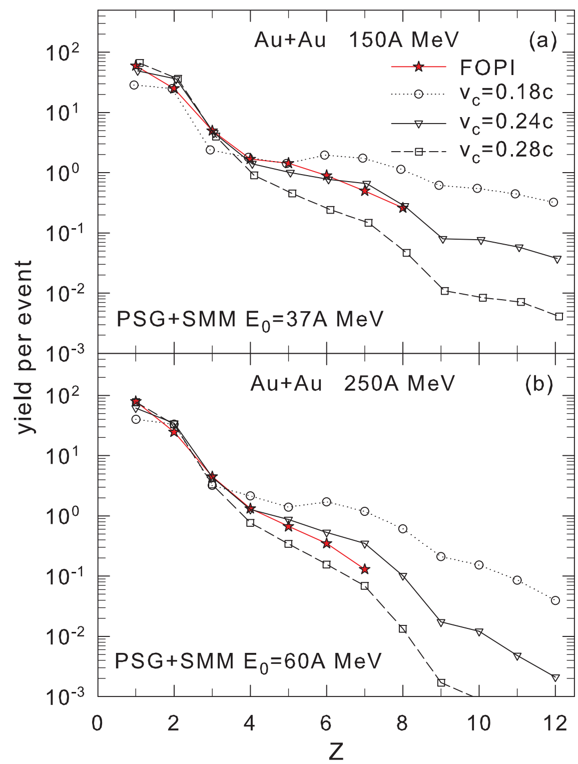

FIG. 9. The same as in Fig. 8, but for central Au+Au collisions at 150*A* MeV (a) and 250*A* MeV (b). The parameters $v_c = 0.18c$, $0.24c$, and $0.28c$ are used in the calculations.

systems (in multifragmentation reactions) these E^*/A values correspond approximately to the temperatures $T \approx 6$–8 MeV during the nuclear liquid-gas type phase transition in the phase co-existence region [2,32]. One can see also that at low beam energy we obtain a rather massive clusters. Though, at high beam energy the cluster sizes become smaller, their excitation does not change. This saturation of the cluster excitation energy gives evidence that we are dealing with the local equilibrium phenomena.

Another important experimental observable is the kinetic energy of the produced nuclei. Since the nuclei are formed from the nucleons belonging to the local clusters this kinetic energy depends on the initial energy of nucleons after the dynamical stage. When there is a correlation between the size of clusters and their positions in the expanding nuclear system we may infer that the correct description of the nuclei yields will lead also to their correct kinetic energies. In Figs. 11 and 12 we compare with the experimental data the kinetic energies of the nuclei per nucleon (E_{kin}/A) after their production. Sometimes this characteristic is associated with the flow energy. For the comparison we have selected the c.m. energies in the forward direction since they are better covered by the FOPI acceptance [44]. We have taken the same calculations as for the yields presented in Figs. 8 and 9. One can see that the agreement is quite satisfactory for the best v_c parameters (middle lines in the figures). The results on yields and energies are correlated with each other and support our conclusion.

One can see in Fig. 12 a slight difference between the predicted trends and the experimental data for the kinetic energies of the largest nuclei at the Au+Au collisions of the highest energy. As discussed already in Ref. [39] the reason could be a slightly different initial energy distribution of nucleons than the PSG one. For example, with increasing beam energy the HYG distribution can contribute more to the ERAT event

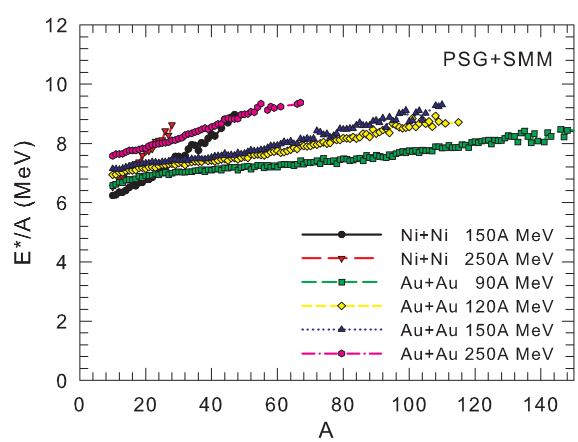

FIG. 10. Average excitation energy per nucleon (E^*/A) of local clusters of nuclear matter versus their mass number A corresponding to the v_c parameters which lead to the best description of the FOPI experimental data. The lines correspond to different reactions of central collisions, they are noted in the figure.

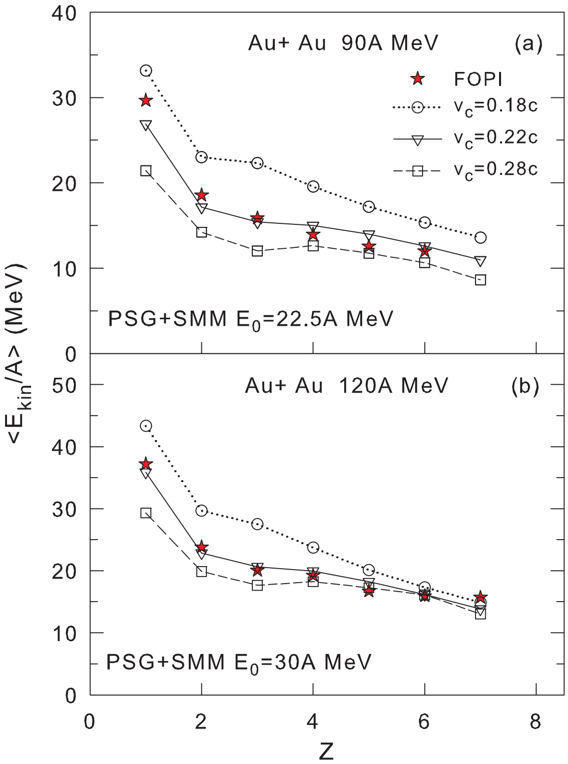

FIG. 11. Comparison of average kinetic energies per nucleon of produced nuclei versus their charges Z with the FOPI experimental data in Au+Au central collisions. (a) is for 90A MeV collisions, and (b) is for 120A MeV. The parameters of the initial source are $A_0 = 394$, $Z_0 = 158$, and the energy E_0 is given in the figure. The nucleon distributions are after PSG. The parameters $v_c = 0.18c$, $0.22c$, and $0.28c$ are used in the calculations.

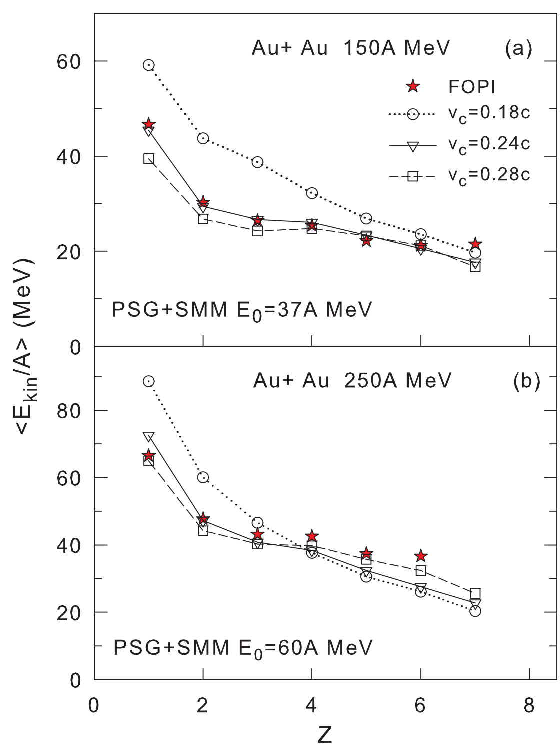

FIG. 12. The same as in Fig. 11, but for central Au + Au collisions at 150A MeV (a) and 250A MeV (b). The parameters $v_c = 0.18c$, $0.24c$, and $0.28c$ are used in the calculations.

selection. This would automatically lead to high kinetic energies of large fragments (see Fig. 6 and Ref. [39]). However, as we have verified, the HYG nucleon distribution does not change our conclusion on the mechanism of the nuclei formation and on the internal excitation of primary clusters of nuclear matter under the equilibrium respective to the nucleation process.

VI. DISCUSSION OF THE RESULTS

We have systematically analyzed the experimental production of light and intermediate mass nuclei obtained in central nucleus-nucleus collisions at high energy. We found that complicated many-body processes are responsible for the production of the nuclei. Specifically, it is not possible to describe them with one leading reaction channel. There are many transport models available, however, many of them are not equipped to treat the low-energy interaction of nucleons responsible for the nuclei formation in sufficient details. Because usual, they do not involve sufficiently realistic wave functions of nucleons, neglect details of many-body forces, collective interactions, and other processes important in this case. We have suggested a hybrid approach including dynamical and statistical reaction stages. Special attention is paid to the statistical description, and its generalization for the highly excited expanding nuclear systems. Since we believe that the formation of nuclei from nucleons can naturally take place at low nuclear densities in the later stage of the reaction process.

Two phenomenological methods (PSG and HYG) are used to simulate the dynamical part of these high-energy reactions. These methods lead to quite different energy distributions of nucleons covering the most important limits expected after the initial dynamical stage. This stage is mostly determined by the high-energy interaction of individual nucleons. The nucleon system expands and low-energy interactions between neighbor nucleons result in the nuclei formation. Our hypothesis is that the nuclei are produced from the nucleons at a low density state of these expanding nuclear systems. If some dense nuclear clusters would be formed at large densities of nuclear matter, they would be destroyed by the subsequent interaction with other nuclear species during the expansion. Only when the system is sufficiently diluted one can expect the production of final nuclei. This situation is typical for the statistical freeze-out state. Therefore, we expect that the statistical approach should effectively work in this case. However, we are dealing with the finite nuclear systems. For this reason we must take into account that the interaction within such systems should be sufficient to apply the statistical laws. Usually, it is related to the excitation energy accumulated in the sources. We obtain from our analysis that one can use the statistical models at moderate excitations of the nuclear sources, around maximum 6–10 MeV per nucleon, to describe the data obtained in central collisions. There will be several such sources (we construct them as coalescent-like clusters) in one highly excited expanding nuclear system. This is an essential difference from the previous statistical description which has considered in reactions only one a such source. We suggest that local equilibrium is reached with a limited temperature respective to the nuclei formation in pieces (clusters) of expanding nuclear matter. One can consider it as a generalization of the statistical methods for nuclear systems formed in high energy reactions.

It is interesting that similar maximum excitation energies were extracted from the analyses of the multifragmentation data in relativistic peripheral nucleus collisions [30–32]. However, it was done for single projectile-like sources remaining after the dynamical stage. In those collisions a source can expand to the freeze-out under thermal pressure or under the dynamical re-compression before the disintegration. Also low-excited compound nuclei are possible to produce from the projectile/target residues. Therefore, in those cases we have a very broad distribution of the source excitation energy from 0 up to ≈10 MeV per nucleon. The situation in central collisions is different. The whole nucleon system has already expanded after the dynamical stage and it can expand further entering the freeze-out state. It is concluded that nucleon clusters accumulating the excitations of 6–10 MeV per nucleon respective to their ground states may be considered as statistical sources. They have a transition temperature ($T \approx$ 6–8 MeV) and can be used for the statistical description of the fragment formation. In the considered finite systems with very high energy such a limitation of the temperature is related to selecting the clusters of smaller sizes. However, as is clear from our analysis, it will be inconsistent to involve very small clusters with low excitation energies. Since the interaction between nucleons leading to the fragmentation can already take place in the clusters of the intermediate size at the transition temperature.

The obtained excitation energy (6–10 MeV per nucleon) is surprisingly close to the binding energy of the corresponding nuclear clusters. We assume that the binding energy can serve a natural energy to characterize a collective interaction necessary for the application of the statistical theory in finite systems. Also these excitations are sufficiently high in order to most of nuclear fragment formation processes in the clusters be well above their threshold. Therefore, the phase space can dominate over other kinematic restrictions for these processes and it can determine the final formation of nuclei. The statistical models are very effective for such many-body processes. On the other hand, when the clusters excitation energy is much higher than the binding energy we may expect that the one-particle scattering dominates and we cannot effectively consider the nuclei formation.

It is important for the statistical description, to confirm the local equilibration in finite expanding systems. The best experimental confirmation of these phenomena would be to measure particle correlations coming from the decay of the primary clusters during the nuclei production. Such correlations can bring direct evidences of the many-body character of the fragmentation process and the phase coexistence. Following our results, we predict the decreasing of the size of the local equilibrated clusters of nuclear matter with increasing total energy in the system, in order for the temperatures of the clusters not to change. This could clearly be seen in the correlations. Also, the differences from other nuclei formation mechanisms can be easily determined. For example, if we try to describe the cluster decay with the help of a transport model, which includes only one-particle distribution functions, in the end we can obtain many free nucleons and a large residue. While by applying the statistical model we obtain a lot of small nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He) in addition to a large residue. These different physical products can be observed in event-by-event correlation measurement. It is also important for the extension of the method to new phenomena: As was discussed in Ref. [39] the correlation measurements would be specially instructive for the hypernuclei production to investigate the appropriate channels of their yield. As was also discussed in Ref. [39] this new mechanism influences essentially the light particle production: We were able to explain the main regularities of their production, as well as the crossover behavior of the ^{3}He and ^{4}He yields in central heavy-ion collisions versus the beam energy. It would be interesting to study additional details of the light particle yields observed in experiments (see, e.g., Refs. [44,47,48]).

It is instructive to emphasize once more the theoretical difference of our approach from the standard coalescence results. As in any phenomenology the coalescence parameter extracted from the comparison with experiment may depend on the initial stage description, and, therefore, it is entangled with this description. For example, the coalescence parameters for light nuclei obtained after the integration of nucleon spectra over all events [49,50] and the ones extracted from the event-by-event analysis [40,43] may be different by a factor 2–3. In our case we avoid this uncertainty. Independent of the dynamical stage the statistical clusters must have the excitation energy dependent on the nuclear liquid-gas phase transition properties. It has a clear physical meaning related to the nuclear matter.

According to our analysis we can give a practical recipe for the calculation of the nuclei production in high energy nucleus collisions. This hybrid approach consists of several steps: (1) The calculation of the nucleon distributions after the dynamical stage. One can use transport models, e.g., see Refs. [39,42,43,50,51]. (2) The selection of nucleon clusters at the low density of nuclear matter ($\approx$0.1–0.3ρ_0) and calculating their internal excitation energy. This should be done step by step, starting from large clusters and decreasing the cluster's sizes: The excitation energy decreases with the number of nucleons in the clusters. 3) When the excitation energy is around 6–10 MeV per nucleon (close to the cluster binding energy) one can apply a statistical model to describe the cluster decay leading to the nuclei formation at the low density matter. We think that the uncertainty related to the cluster size will be small if their excitation is within the suggested range. Since at so high excitations the statistical models (see, e.g., [2,3,16]) lead to the scaling properties in the nuclei yields respective to the source size.

VII. CONCLUSIONS

In the previous years the analyses of experimental data on disintegration of excited nuclear systems into nuclei (see, e.g., Refs. [2,18–20,22–37]) result in the conclusion that such fragmentation is of the statistical nature in many reactions. Also it was discussed that these processes can be the manifestation of the liquid-gas type phase transition in finite nuclei systems [2]. As was obtained in the theoretical analyses of data on multifragmentation of relativistic projectiles [28–32] there is an upper limit for the excitation energy for finite thermalized nuclear systems, around 10 MeV per nucleon, with the values close to the binding energies of normal nuclear systems. These systems decay in time about ≈ 100 fm/c [34–36] after the beginning of the reaction that is several times longer than the initial dynamical reaction stage. We believe that it is a general property of finite nuclear systems: Independent on the way how the primary excited systems are formed, they can manifest the same properties of interacting nucleons in the region of the nuclear-liquid gas coexistence.

In the present work we extend the statistical approach by considering the fragment production in central high energy nucleus collisions. We demonstrate that after the initial dynamical stage we can separate in each collision several excited statistical sources which decay producing the fragments. We call these small sources "coalescent-like" clusters, in order to emphasize the primary dynamics leading to the formation of such diluted nuclear systems. To simulate the dynamical stage we use the phenomenological phase space and hydrodynamical-inspired nucleon generators which provide the one-particle distributions, and which cover the most important limits of the nucleon momenta. The subsequent statistical decay of these clusters is the second part of our hybrid model. To verify our approach we have used the high quality FOPI data on the production of nuclei in central collisions [44]. Within our approach we have shown that it is possible to describe the fragment production with charges greater than one, including the yield and the kinetic energies, that was impossible with the previous methods. We believe it is also a result important for all statistical approach describing the disintegration of finite nuclear systems: Namely, the maximum excitation energy of such systems should be moderate, in the range of 6–10 MeV per nucleon. This is similar to the binding energies of the corresponding nuclei. Higher excitations are excluded as a result of our analysis. The lower excitations are possible in reactions of low energies at the formation of the single compound-like sources. However, in our case of high energy collisions the above mentioned cluster excitation provides the best description of the data. A limited temperature of the clusters is naturally caused by decreasing their sizes when more energy is deposited in the initial system during the nucleus collision. Actually, such clusters are locally equilibrated sub-systems within a very excited expanding nuclear system. In this respect the evolution of the nucleation process toward a high energy consists of a natural fragmentation of a low-density matter into such clusters. The statistical models are suitable for the decay of the clusters into nuclei at these energies because the nuclei production is essentially a many-body process and the phase space dominates in the nuclei formation process.

Also we have pointed out that the correlations of the produced particles can be an important consequence of this kind of the fragment formation. We believe that this approach, with adequate dynamical models for the first reaction stage, should be used in future at high energies. It will give us a possibility to analyze new nuclear species formed from various baryons, e.g., hypernuclei [39], which can be abundantly produced in central collisions.

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