Effect of Coriolis mixing on lifetime of isomeric states in heavy nuclei

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(Received 16 May 2022; accepted 7 July 2022; published 18 July 2022)

The effect of Coriolis K mixing on the γ decay of one-quasiparticle isomeric states is examined in transfermium nuclei with neutron number $N = 153$. We consider a core-plus-quasiparticle coupling in which the K mixing sharply increases if the isomeric quasiparticle state closely approaches the rotational state with the same angular momentum I, but built on another quasiparticle state which is either directly coupled to the isomeric ($\Delta K = 1$) or connected with it through the intermediate mixing quasiparticle orbitals ($\Delta K > 1$). This mechanism is likely to explain the 38-ns $K^\pi = 7/2^+$ isomer in ^{251}Cf which is quasidegenerated with an $I^\pi = 7/2^+$ rotation state built on $K^\pi = 1/2^+$. Our model explains the enhancement of its decay by three orders of magnitude compared to the $K^\pi = 7/2^+$ isomer in ^{249}Cm. We suggest that the same mechanism could be of general importance for the isomer decay properties in a wide range of heavy odd-mass nuclei.

DOI: 10.1103/PhysRevC.106.014310

I. INTRODUCTION

Nuclear isomers, though having a long history, still pose a number of challenging problems especially in less studied mass regions such as heavy and superheavy nuclei, where the analysis of new data requires a deep understanding of the mechanism which governs the formation and decay properties of metastable excited states [1–7]. The K isomers with relatively long lifetime, even longer than the ground state, are of special interest and considered the result of the K trapping (or forbiddenness) effect [8]. This is a situation in which the third projection K of the total angular momentum of the nucleus on its symmetry axis essentially differs from the projections of neighboring lower states. As a result, the γ decay through "fast" low-multipolarity ($E2$ or $M1$) transitions appears to be forbidden. However, the situation is more complicated and this prohibition can be softened. The collective dynamics, the deformation, and the intrinsic shell structure of a nucleus offer a variety of conditions in which the pureness of the quantum number K is violated, which leads to a weakening of the selection rules and to an enhancement of the isomer decays. This effect is essentially related to the Coriolis mixing interaction and/or the presence of triaxial deformation. As the latter strongly depends on the particular nuclear characteristics, the K isomers appear to be sensitive indicators for both the intrinsic structure and collective dynamics of the nucleus that is of utmost importance in the exploration of new or less studied mass regions.

K-isomeric states have been studied with the cranked mean-field model [9–12], the quasiparticle random-phase approximation model [13–15], the configuration-constrained nuclear potential energy calculations [16], projected shell model [17–21], and in the energy-density functional approach [22]. The last one is particularly useful to calculate the rotational bands built on the multiquasiparticle high-K states.

The purpose of this work is to examine the effect of Coriolis K mixing on the γ decay of low-lying one-quasiparticle isomeric states in the region of transfermium odd-mass nuclei with neutron number $N = 153$. Among all isotonic chains considered in Ref. [7], the $N = 153$ isotonic chain contains the nuclei in which the lifetimes of the lowest $7/2^+$ isomeric states differ significantly. So, the $N = 153$ isotones provides us a good test for the model proposed to describe the lifetimes of isomers that differ by several orders of magnitude. We find an interesting situation in ^{251}Cf where the presence of a quasidegenerate rotation counterpart state for the 38-ns $K^\pi = 7/2^+$ isomer [7] provides a unique chance to examine how the proximity of two states affects the K-mixing process and, thus, the isomer decay rate. Moreover, since the partner state is built on the $K^\pi = 1/2^+$ ground state, we have the chance to study in detail the mechanism of mixing through the states possessing intermediate K values. We apply a simple core-plus-particle scheme to study the overall K-mixing mechanism involving two intermediate quasiparticle states and its impact on the isomer lifetime. As it will be shown below, this approach provides a quantitative explanation of the three orders in magnitude shorter lifetime, 38 ns, of the ^{251}Cf isomer compared to the 23-μs $K^\pi = 7/2^+$ isomer in the neighboring ^{249}Cm. We will demonstrate that based on the quasiparticle spectra obtained by the two-center deformed shell model [23] one can estimate the capability of the $N = 153$ isotones to form similar K isomers and, moreover, to predict the corresponding isomer decay rates and lifetimes.

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II. CORIOLIS *K* MIXING

A. Hamiltonian and matrix elements

The Hamiltonian of a core-plus-particle system is taken as in Refs. [14,24],

$$H = H_{\rm intr} + H_{\rm rot} + H_{\rm coupl}, \tag{1}$$

where $H_{\rm intr}$ describes the single-particle (SP) motion of nucleons in the deformed potential taking into account the BCS pairing, $H_{\rm intr} = H_{\rm sp} + H_{\rm pair}$, while

$$H_{\rm rot} = \frac{\hbar^2}{2\Im}\left[\mathbf{I}^2 - I_3^2\right] \tag{2}$$

stands for the rotational motion of a nucleus with a moment of inertia $\Im$. The operators of total angular momentum and its third projection are denoted as $\mathbf{I}$ and I_3, respectively. The term $H_{\rm coupl} = H_{\rm cor} + H_{\rm vib}$ takes into account the coupling of quasiparticle motion with collective rotation ($H_{\rm cor}$) and vibrational motion of the core ($H_{\rm vib}$). The Coriolis interaction is given as

$$H_{\rm cor} = -\frac{\hbar^2}{2\Im}[I_- j_+ + I_+ j_-], \tag{3}$$

where $I_\pm = I_1 \pm iI_2$ and $j_\pm = j_1 \pm i j_2$ are the spherical components of the total nuclear and SP angular momenta, respectively.

Assuming that the core has a quadrupole deformation with an equilibrium value β_0 corresponding to the axially symmetric shape, the coupling of the SP motion with the vibrational motion of the core is written as

$$H_{\rm vib} = -\kappa_2 \frac{\beta_0 \gamma}{\sqrt{2}}(q_{2,+2}(\mathbf{r}) + q_{2,-2}(\mathbf{r})), \tag{4}$$

where $q_{2,\pm2}(\mathbf{r}) = r^2 Y_{2\pm2}(\theta,\phi)$ are the quadrupole operators acting on the SP degrees of freedom, and γ is the collective coordinate describing the γ vibration of the core. The interaction strength κ_2 is obtained by taking into account that the SP motion is described by a Nilsson-type model. Therefore, one has $\kappa_2 = m_0\omega_0^2$, where m_0 is the nucleon mass and ω_0 is the frequency determining the nuclear mean field [14]. The β-vibrational motion is not considered as it does not create mixing of the SP state with different K and leads only to a shift of the energy scale.

The wave function of the core-plus-particle system without coupling is

$$|n(K_p, K_\gamma)IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2}} g_{K_\gamma}(\gamma)\big(D^{I\,*}_{M,K}\alpha^+_{nK_p} + (-1)^I D^{I\,*}_{M,-K}\alpha^+_{n-K_p}\big)|0\rangle, \tag{5}$$

where $D^I_{M,K}$ are the Wigner functions, with M and K being the projection of $\mathbf{I}$ in the laboratory and intrinsic frames, respectively. The operator $\alpha^+_{nK_p}$ acting on the quasiparticle vacuum $|0\rangle$ creates the quasiparticle at the state ϕ_{nK_p}, while the operator $\alpha^+_{n-K_p}$ creates the quasiparticle at the state $\phi_{n-K_p} \equiv e^{-i\pi j_1}\phi_{nK_p}$. Here, j_1 is the SP angular momentum operator. The index n enumerates the orbitals (or the quasiparticle states) considered for the odd nucleon, typically several orbitals below and above the Fermi level. The wave function $g_{K_\gamma}(\gamma)$ describes γ vibrations of the core [25]. We consider only the vibrational modes with the lowest energy, i.e., $K_\gamma = 0$ or 2 and $n_\gamma = 0$. The projection $K = K_p + K_\gamma$ of the total angular momentum is a sum of projections of the angular momenta generated by the quasiparticle motion and γ vibrations, respectively.

Hamiltonian (1) is diagonalized for given I in the space of states (5) providing the spectrum of the core-plus-particle system. The nonzero matrix elements of H in this space are as follows:

$$\begin{aligned}
E_{n_iK} &\equiv \langle n_i(K_p,K_\gamma)IMK|H|n_i(K_p,K_\gamma)IMK\rangle = \epsilon_{n_iK_p} + \frac{\hbar^2}{2\Im}(I(I+1)-K^2) + \hbar\omega_\gamma\frac{|K_\gamma|}{2} + \delta_{K,\frac12}\frac{\hbar^2}{2\Im}a_{ii}(-1)^{I+\frac12}\left(I+\frac12\right),\\
H^{\rm cor}_{K,K} &\equiv \langle n_f(K_p,K_\gamma)IMK|H|n_i(K_p,K_\gamma)IMK\rangle = \delta_{K_p,\frac12}\frac{\hbar^2}{2\Im}a_{fi}(-1)^{I+\frac12}\left(I+\frac12\right)(u_fu_i + v_fv_i),\\
H^{\rm cor}_{K,K\pm1} &\equiv \langle n_f(K_p\pm1,K_\gamma)IMK\pm1|H|n_i(K_\gamma,K_p)IMK\rangle = -\frac{\hbar^2}{\Im}\sqrt{I(I+1)-K(K\pm1)}\langle\phi_{n_f(K\pm1)}|j_x|\phi_{n_iK}\rangle(u_fu_i + v_fv_i),\\
H^{\rm vib}_{K,K} &\equiv \langle n_f(K_p\pm2,K_\gamma\mp2)IMK|H|n_i(K_\gamma,K_p)IMK\rangle = -\kappa_2\beta_0\frac{\gamma_0}{2}\langle\phi_{n_f(K_p\pm2)}|q_{2\pm2}|\phi_{n_iK_p}\rangle(u_fu_i - v_fv_i),\\
H^{\rm vib}_{-K,K} &\equiv \langle n_f(-K_p+2,-K_\gamma-2)IM-K|H|n_i(K_p,K_\gamma)IMK\rangle = -\kappa_2\beta_0\frac{\gamma_0}{2}\langle\phi_{n_f(K_p+2)}|q_{2+2}e^{-i\pi j_1}|\phi_{n_iK_p}\rangle(u_fu_i - v_fv_i).
\end{aligned} \tag{6}$$

Here, $a_{fi} = i\langle\phi_{n_fK=\frac12}|j_+e^{-i\pi j_1}|\phi_{n_iK=\frac12}\rangle$ is the usual decoupling factor for the states with $K = 1/2$, $\epsilon_{n_i,K}$ is the quasiparticle energy, and the quantities u and v are related to the standard BCS occupation values. The frequency of γ vibrations is denoted as ω_γ. The values of $H^{\rm vib}_{-K,K}$ are nonzero only at $K_p = 1/2$ or $K_p = 3/2$. Superscripts "cor" or "vib" in the notation of the matrix elements mark which part of the coupling Hamiltonian generates the corresponding matrix element.

The coupling with γ vibrations leads to the mixture of quasiparticle states with different K_p quantum numbers, while the total K remains unchanged. Therefore, the only K-mixing mechanism in the model is provided by the Coriolis interaction. The nonzero off-diagonal matrix elements in Eqs. (6) cause a mixing of states with $\Delta K = \pm1$. Usually this mixing is rather weak and can be neglected or taken into account as a perturbation. However, if the levels of the same angular momentum I built on the intrinsic states with $\Delta K = \pm1$ are

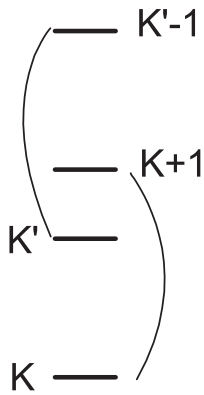

FIG. 1. Schematic presentation of two-level (K and K') mixing through two intermediate states produced by the couplings of K and $K+1$ states and K' and $K'-1$ states. These intermediate states have the components with $\delta K = 1$ and are coupled by the Coriolis interaction.

close in energy, the Coriolis coupling becomes strong and should be treated exactly. This situation is well known in ^{183}W [26].

The electric quadrupole operator employed in this work consists of two parts:

$$Q_{2\mu} = Q_{2\mu}^{(p)} + Q_{2\mu}^{\rm coll}. \tag{7}$$

Here, the part describing the valence nucleon has the form

$$Q_{2\mu}^{(p)} = e_{\rm eff} r^2 \sum_{\mu'} Y_{2\mu'}(\theta,\phi) D^2_{\mu\mu'}(\Omega). \tag{8}$$

The effective charge is taken as $e_{\rm eff} = e$, since the coupling between SP motion and collective vibration is treated explicitly by Hamiltonian (1). Angles Ω are the Euler angles describing the orientation of the core (mean field) with respect to the laboratory system. Assuming that β deformation of the core is constant, $\beta = \beta_0$, and the amplitude of γ vibrations is small, for the core quadrupole operator we use [27]

$$Q_{2\mu}^{\rm coll}(\beta_0,\gamma) = \frac{3Ze}{4\pi}\left(\beta_0 D^2_{\mu,0}(\Omega) + \frac{1}{\sqrt{2}}\beta_0\gamma[D^2_{\mu,2}(\Omega) + D^2_{\mu,-2}(\Omega)]\right). \tag{9}$$

For the magnetic dipole operator, we use the standard expression of a particle-plus-rotor model:

$$M_{1\mu} = \sqrt{\frac{3}{4\pi}}\mu_N\left[g_R(I_\mu - j_\mu) + M_{1\mu}^{(p)}\right],$$
$$M_{1\mu}^{(p)} = g_l l_\mu + g_s s_\mu. \tag{10}$$

For the gyromagnetic factors, we use $g_R = Z/A$, $g_l = 0$, $g_s = 0.7 g_s^{\rm free}$, and $g_s^{\rm free} = -3.82$ [24].

To assess the role of proximity of two levels in a more general aspect, we will illustrate schematically the case of a direct two-level mixing and mixing through two intermediate states. The Coriolis interaction mixes two levels with K and $K+1$, and K' and $K'-1$ (Fig. 1). The resulting states contain admixtures with $\Delta K = K' - K - 2$. So, the value of ΔK decreased by 2 units for the intermediate states.

B. Two-level mixing

Let us consider the two-level mixing Hamiltonian

$$H = \begin{pmatrix} E_K & H_{K,K+1} \\ H_{K,K+1} & E_{K+1} \end{pmatrix}, \tag{11}$$

where $H_{K,K+1} = H^{\rm cor}_{K,K+1}$ and E_K and E_{K+1} are the original unmixed levels corresponding to the diagonal part of Hamiltonian (1) determined in Eqs. (6) (the other quantum numbers are omitted for simplicity). After the diagonalization (solving a quadratic equation) the distance between already mixed levels with energies $E^{\rm up}$ and $E^{\rm low}$ is

$$E^{\rm up} - E^{\rm low} = \Delta = \sqrt{4H^2_{K,K+1} + \delta^2}, \tag{12}$$

where $\delta = |E_{K+1} - E_K|$. In the case of a degenerated pair of original (basis) states (5), $\delta = 0$, and the mixed-levels difference

$$\Delta = \Delta_0 = 2|H_{K,K+1}| \tag{13}$$

is entirely determined by the off-diagonal Hamiltonian term. Further, we consider the wave functions $\psi_i(I)$ $(i = 1, 2)$ obtained after the diagonalization in the form

$$\psi_i = c_{i,K}\phi_K + c_{i,K+1}\phi_{K+1} \quad (i = 1, 2 \Leftrightarrow \text{up, low}), \tag{14}$$

where ϕ_K shortly denote the basis states (5). The nonzero expansion coefficients in Eq. (14) now make possible $E2$ and $M1$ transitions between the states determined by the K-mixed wave functions ψ_1 and ψ_2. These expansion coefficients are expressed as

$$c_{1,K} = -\frac{1}{\sqrt{2}}\left(1 + \sqrt{1 - \frac{\Delta_0^2}{\Delta^2}}\right)^{1/2},$$
$$c_{1,K+1} = \frac{1}{\sqrt{2}}\frac{\Delta_0}{\Delta}\left(1 + \sqrt{1 - \frac{\Delta_0^2}{\Delta^2}}\right)^{-1/2},$$
$$c_{2,K} = \frac{1}{\sqrt{2}}\left(1 - \sqrt{1 - \frac{\Delta_0^2}{\Delta^2}}\right)^{1/2},$$
$$c_{2,K+1} = \frac{1}{\sqrt{2}}\frac{\Delta_0}{\Delta}\left(1 - \sqrt{1 - \frac{\Delta_0^2}{\Delta^2}}\right)^{-1/2}. \tag{15}$$

If $\Delta = \Delta_0$, the eigenstates of Hamiltonian (11) are fully mixed and each original quasiparticle component contributes 50% to the wave functions. This situation obviously leads to maximal values of the interband transition matrix elements connecting the states ψ_1 and ψ_2.

C. Mixing through intermediate states

Before considering the detailed numerical diagonalization we qualitatively examine whether the Coriolis interaction can be responsible for a strong mixing through the intermediate

states. First, we take the intermediate couplings between the K and $K+1$ and between the K' and $K'-1$ states within the perturbation theory. Denoting the intermediate states by the K values of their main components, we obtain

$$\psi_K = \phi_K + \frac{H_{K,K+1}}{E_K(I) - E_{K+1}(I)}\phi_{K+1},$$
$$\psi_{K'} = \phi_{K'} + \frac{H_{K',K'-1}}{E_{K'}(I) - E_{K'-1}(I)}\phi_{K'-1}. \quad (16)$$

The energies of these states are

$$E'_K(I) = E_K(I) + \frac{|H_{K,K+1}|^2}{E_K(I) - E_{K+1}(I)},$$
$$E'_{K'}(I) = E_{K'}(I) + \frac{|H_{K',K'-1}|^2}{E_{K'}(I) - E_{K'-1}(I)}. \quad (17)$$

The resulting two states can be again mixed as in the two-level case. The corresponding Hamiltonian has the form (11) with E'_K and $E'_{K'}$ staying on the diagonal and the off-diagonal (mixing) matrix element as

$$\tilde{V}_{K,K'} = \tilde{\Delta}_0 = \frac{H_{K,K+1}}{E_K(I) - E_{K+1}(I)} H_{K+1,K'-1} \frac{H_{K',K'-1}}{E_{K'}(I) - E_{K'-1}(I)}. \quad (18)$$

After its diagonalization the final wave functions are obtained as in Eqs. (14):

$$\tilde{\psi}_i = c_{i,K}\psi_K + c_{i,K'}\psi_{K'} \quad (i = 1, 2), \quad (19)$$

where the expansion coefficients are defined as in Eq. (15) with $\Delta_0 = \tilde{\Delta}_0$ and $\Delta = \sqrt{\tilde{\Delta}_0^2 + \tilde{\delta}^2}$ ($\tilde{\delta} = |E'_{K'} - E'_K|$). From Eq. (18) we deduce that the final (observed) states should be close in energy in order to feel the strong mixing effect.

Expression (13) or (18) can be used to check if the interaction determined by the matrix elements $H_{i,j}$ can cause a strong mixing of states separated by energy interval Δ. As mentioned below, in Eq. (15) the strongest mixing is achieved when the condition $\Delta \sim \Delta_0$ (or $\tilde{\Delta}_0$) is valid.

III. CALCULATED RESULTS

A. Energies of one-quasiparticle states

As shown above, the coupling may occur between states whose K values differ by more than a unit. Such a situation seems to take place in a number of $N = 153$ isotones, for example, in ^{251}Cf where there are two $I = 7/2^+$ states with energies 105.7 and 106.3 keV [28]. The first one is a rotational state built on the $1/2^+[620]$ ground (quasiparticle) state, while the state at 106.3 keV is the isomer assigned to the $7/2^+[613]$ quasiparticle state. These states are not mixed directly by the Coriolis interaction due to the large $\Delta K = 3$ difference, but can be coupled through the corresponding rotational states built on the $3/2^+[622]$ and $5/2^+[622]$ quasiparticle states. The observed sudden increase of the decay rate of the $7/2^+[613]$ quasiparticle state in ^{251}Cf with respect to the same state in the neighboring nucleus ^{249}Cm supports the consideration of a strong mixing generated as described above.

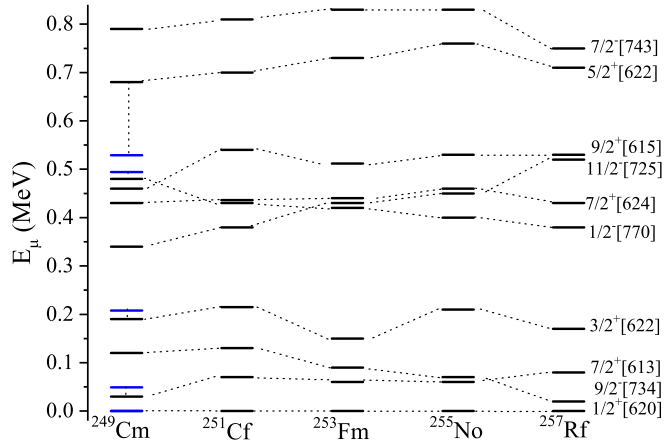

FIG. 2. The calculated one-quasiparticle spectra for indicated nuclei of the $N = 153$ isotonic chain. The calculations were performed with the TCSM. The experimental values [28] for ^{249}Cm are shown by blue lines.

In the following, Hamiltonian (1) is numerically diagonalized for the nuclei ^{249}Cm, ^{251}Cf, and ^{253}Fm belonging to the isotonic chain $N = 153$. We use the single-particle spectra obtained from the two-center shell model (TCSM) [23] and axially symmetric deformed shell model [29]. In the TCSM, the nucleus is assumed to be reflection and axially symmetric. Its shape is described by the elongation parameter λ and the β deformation corresponding to its ellipsoid ends. The ground state of the nucleus results from the minimization of the potential energy surface as a function of λ and β [30]. Such a parametrization effectively includes all even multipolarities. By calculating the quadrupole and hexadecapole moments, a relation between the TCSM parameters λ and β and the conventional parameters of quadrupole β_{20} and hexadecapole β_{40} deformations is established. As shown in Ref. [31], the microscopic shell corrections and quadrupole deformation parameters calculated with the TCSM are close to those obtained in Ref. [32]. The energies of quasiparticle states are determined as $E_{n,K} = \sqrt{(\epsilon_{n,K} - \epsilon_F)^2 + \Delta^2}$, where $\epsilon_{n,K}$ are the energies of the single-particle neutron states. The Fermi energy ϵ_F and the gap parameter Δ are calculated in the BCS approximation with the pairing interaction of the monopole type with the strength parameter $G_n = (19.2 - 7.4\frac{N-Z}{A})A^{-1}$ MeV [33,34]. The blocking effect is effectively taken into account by reducing the calculated pairing gap by factor of 0.85 [14,30,35]. The resulting quasiparticle spectra for nuclei of $N = 153$ isotonic chain are presented in Fig. 2. Note that the quasiparticle energies in Fig. 2 differ from the energies of states obtained by the diagonalization of Hamiltonian (1) in which the coupling of quasiparticles with collective degrees of freedom is taken into account.

The calculations of quasiparticle spectra are also performed in the framework of the axially symmetric deformed shell model [29] with the deformation parameters from Ref. [32]. The quasiparticle energies and occupation numbers are calculated within the BCS approach with the pairing constant $G_n = 0.0645$ MeV, resulting in the neutron gap of about 0.5 MeV. The comparison with experimental data for ^{251}Cf

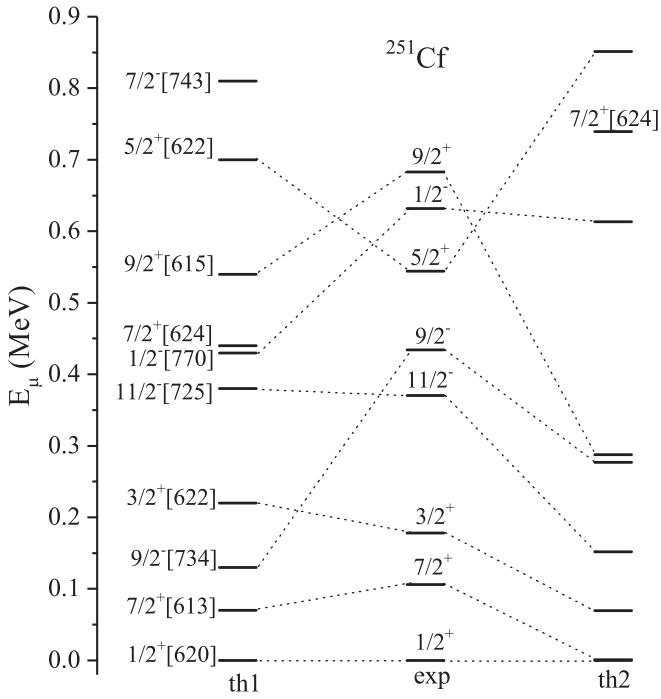

FIG. 3. Energies of one-quasiparticle states in ^{251}Cf calculated by the TCSM (th1) and axially symmetric deformed shell model (th2) compared with available experimental data [28].

(Fig. 3) demonstrates a similar quality of the description of low-lying quasiparticle states for two models considered. The $7/2^+$, $5/2^+$, and $3/2^+$ states, which are most important for the present study, agree with the experimental levels within 0.25 MeV. However, as seen from Eq. (18), if one wants to describe well the effects of mixing of one-quasiparticle states by the Coriolis interaction, a more precise adjustment of the one-quasiparticle levels is required. Such a tuning is obviously outside of the accuracy limits of the present nuclear models. So, the experimental lifetimes are compared with those obtained with the experimental quasiparticle spectra and calculated wave functions.

B. Half-lives of isomeric states

In addition to the Coriolis interaction in Eqs. (6) we take into account the coupling of SP motion with collective vibration of the core, which is used with the following set of parameters. The value of $\hbar\omega_0 = 6.5$ MeV corresponds to $\kappa_2 = 1.018$ MeV. The frequency of γ vibrations is estimated as $\hbar\omega_\gamma = 1$ MeV from the energy of the γ-vibrational 2^+ state in ^{248}Cm. Using the value of mass for the collective γ vibrations by assuming its equality to the rotational mass, we obtain $\gamma_0 \approx 0.2$. The moment of inertia of the core is fixed to reproduce the $7/2^+$ rotational state build on the $1/2^+$ bandhead.

The accounting for the coupling between quasiparticle motion and γ vibrations of the core given by Eq. (4) is necessary to describe the probabilities of electromagnetic transitions from "purely" isomeric states, when the condition for the resonant mixture (see Sec. II C) is not fulfilled. Indeed, in the absence of degeneracy between the isomeric $7/2^+$ state and the rotational $7/2^+_{K=1/2}$ state and if $H^{\rm vib}$ in the Hamiltonian is disregarded, the reduced probability $B(E2, 7/2^+ \to 3/2^+_{K=1/2})$ is almost equal to zero, since in this case the Coriolis coupling is too weak to provide the mixing between states differing by $\Delta K = 3$. However, if $H^{\rm vib}$ is considered, then the state $3/2^+_{K=1/2}$ in addition to the component with $(K_p = 1/2, K_\gamma = 0, K = 1/2)$ contains the contribution $(K_p = 5/2, K_\gamma = 2, K = 1/2)$ with an amplitude $a_{\rm vib} \sim H^{\rm vib}_{1/2,1/2}/(\hbar\omega_\gamma)$. At the same time, the Coriolis mixing leads to the admixture of the $(K_p = 5/2, K_\gamma = 0, K = 5/2)$ component to the wave function of the $7/2^+_{K=1/2}$ state. The amplitude of this admixture is proportional to $a_{\rm cor} \sim H^{\rm cor}_{7/2,5/2}/(E_{7/2} - E_{5/2})$. The accounting for these two components in the corresponding states leads to nonzero probability of the quadrupole transition in the nonresonant case:

$$B(E2, 7/2^+ \to 3/2^+_{K=1/2}) \sim a^2_{\rm cor} a^2_{\rm vib} |\langle K_p = 5/2, K_\gamma = 2, K = 1/2 || \left(Q_2^{\rm coll}(\beta_0, \gamma)\right) || K_p = 5/2, K_\gamma = 0, K = 5/2\rangle|^2. \tag{20}$$

In complete analogy, the contribution of $(K_p = 3/2, K_\gamma = 0, K = 3/2)$ to the state $3/2^+_{K=1/2}$ as provided by the Coriolis coupling and contribution of $(K_p = 3/2, K_\gamma = 2, K = 7/2)$ to the isomeric $7/2^+$ state due to vibrational mixing again lead to the nonzero transition probability of the isomeric state decay. Our calculations show that the value of $B(E2, 7/2^+ \to 3/2^+_{K=1/2})$ from Eq. (20) is still too small to provide the description of the transition probabilities in the vicinity of the resonance (i.e., for ^{251}Cf), but sufficient to describe the experimental value for ^{249}Fm.

Diagonalizing Hamiltonian (1) with the matrix elements given by Eqs. (6) numerically, we obtain the following reduced transition probabilities for the decay of the $7/2^+$ isomeric state: $B(E2, 7/2^+ \to 3/2^+_{K=1/2})$=0.93×10^{-3} W.u. for ^{249}Cm and $B(E2, 7/2^+ \to 3/2^+_{K=1/2}) = 0.124$ W.u. for ^{251}Cf. The isomeric state in ^{251}Cf can also decay via $E2$ and $M1$ transitions to the $5/2^+_{K=1/2}$ state. For these transitions our calculations yield $B(E2, 7/2^+ \to 5/2^+_{K=1/2}) = 0.017$ W.u and $B(M1, 7/2^+ \to 5/2^+_{K=1/2}) = 4.66\times10^{-5}$ W.u.

As seen our calculations reproduce the large difference in half-lives of the isomeric states in ^{249}Cm and ^{251}Cf. This difference cannot be attributed to the changes of the SP wave functions involved in the calculation, as their structures change smoothly along the $N = 153$ isotonic chain. As an example, the matrix elements used to calculate $H^{\rm cor}_{K,K}$ and $H^{\rm cor}_{K,K\pm1}$ between the most important SP states are listed in Table I. The absolute values of these matrix elements vary only slightly for ^{249}Cm, ^{251}Cf, and ^{253}Fm. Therefore, the only source for the enhancement of the electromagnetic decay rates in ^{251}Cf can be the accidental proximity of the isomeric state to the member of the ground-state rotational band.

To investigate and exemplify how the change of the distance between adjacent rotational and isomeric states influences the Coriolis mixing effect and, subsequently, the decay rates, we performed the calculations of $B(E2, 7/2^+ \to 3/2^+_{K=1/2})$ for different values of the moment of inertia $\Im$ of the nucleus. The dependence of the distance Δ between

TABLE I. Calculated matrix elements determining the Coriolis interaction between different quasiparticle states. The asymptotic Nilsson numbers are given for one-quasiparticle states.

	^{249}Cm	^{251}Cf	^{253}Fm
$i\langle\frac{1}{2}[620]\|j_+e^{-i\pi j_1}\|\frac{1}{2}[620]\rangle(u_fu_i+v_fv_i)$	0.196	0.227	0.227
$\langle\frac{1}{2}[620]\|j_x\|\frac{3}{2}[622]\rangle(u_fu_i+v_fv_i)$	0.238	0.293	−0.287
$\langle\frac{3}{2}[622]\|j_x\|\frac{5}{2}[633]\rangle(u_fu_i+v_fv_i)$	−0.078	−0.091	0.098
$\langle\frac{5}{2}[633]\|j_x\|\frac{7}{2}[624]\rangle(u_fu_i+v_fv_i)$	0.214	0.225	0.261

the lowest $7/2^+$ states on $x = \Im/\Im_{r.b.}$ is presented in Fig. 4. Here, $\Im_{r.b.} = 0.4m_0r_0^2A^{5/3}$ is the rigid body moment of inertia. The radius parameter is taken as $r_0 = 1.2$ fm. The sharp minimum appears at $\hbar^2/\Im = 12.44$ keV ($x_0 \approx 0.5589$) that corresponds to the closest approach of levels $\Delta_0 \approx 0.021$ keV. In the vicinity of x_0, the sudden rearrangement occurs in the structure of wave functions of the lowest $7/2^+$ states. Indeed, the weights of components with $K_\gamma = 0$ and $K_p = 1/2$, $3/2$, $5/2$, and $7/2$ contributing to the wave function of the second $7/2_2^+$ state in the energy spectrum of ^{251}Cf are shown in Fig. 5 as functions of x. As seen, the structure of this state is almost solely determined by the $K_p = 1/2$ quasiparticle component for $x < x_0$, and by the $K_p = 7/2$ component for $x > x_0$. In the vicinity of x_0, the Coriolis mixing effect is strongest and both quasiparticle components $K_p = 1/2$ and $K_p = 7/2$ contribute almost equally to the structure of the $7/2_2^+$ state. It is worth noting that, even for $x = x_0$, the weight of the $K_p = 3/2$ and $5/2$ components, which mediate the Coriolis mixing, remains relatively small.

To the left of x_0 the $7/2_2^+$ state can be ascribed to the $1/2^+[620]$ ground-state rotational band, while the lower-lying state $7/2^+[613]$ is an isomeric state. To the right of x_0, the situation is opposite, with the lower state being a rotational and the upper being almost a one-quasiparticle state. In ^{251}Cf, the

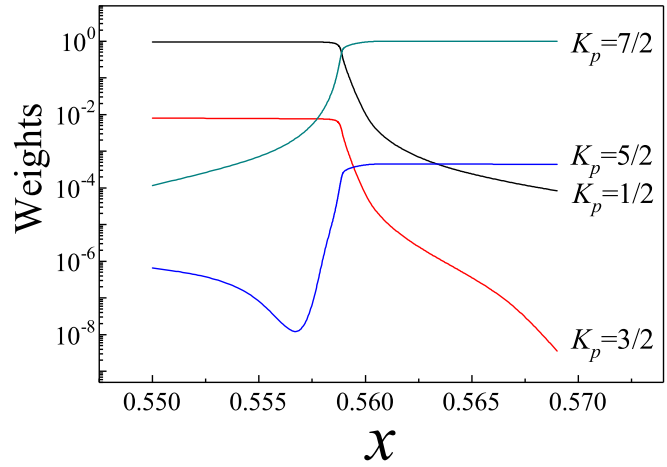

FIG. 5. Weights of components contributing to the wave function of the $7/2_2^+$ state. Only components with $K_\gamma = 0$ and $K_p = 1/2$, $3/2$, $5/2$, and $7/2$ are shown.

experiment gives $\Delta_{\rm exp} = 0.571$ keV, which corresponds to the latter case and is marked by an arrow in Fig. 4. In contrast to the neighboring isotopes, this value is small enough to ensure the nonzero contribution of the $K_p = 1/2$ one-quasiparticle component to the structure of the $7/2_2^+$ isomeric state in ^{251}Cf. In turn, this component gives rise to the $\Delta K = 0$ collective quadrupole transition from the isomeric state to the members of the ground-state rotational band, thus enhancing its decay rate. As an example, the reduced transition probability $B(E2, 7/2_2^+ \to 3/2^+_{K=1/2})$ is shown in Fig. 6. Initially the $B(E2)$ rate remains almost constant with a value typical for the transitions between the members of the rotational band. In the vicinity of x_0, it starts to decrease rapidly with increasing x in conjunction with the drastic change of the weight of the $K_p = 1/2$ component in the $7/2_2^+$ state. For the value of $x = x_{\rm exp}$ corresponding to the experiment, the admixture

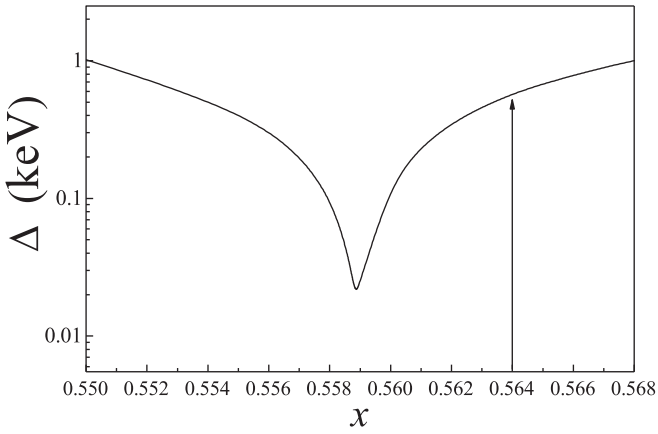

FIG. 4. The energy interval between the two lowest $7/2^+$ states: $\Delta = E(7/2_2^+) - E(7/2_1^+)$ as a function of the moment of inertia of the core. The closest approach, $\Delta \approx 0.02$ keV, occurs at $\hbar^2/\Im = 12.44$ keV ($x \approx 0.559$). The value $x_{\rm exp} = 0.564$ which corresponds to the experimentally measured energy distance between these states is marked by arrow.

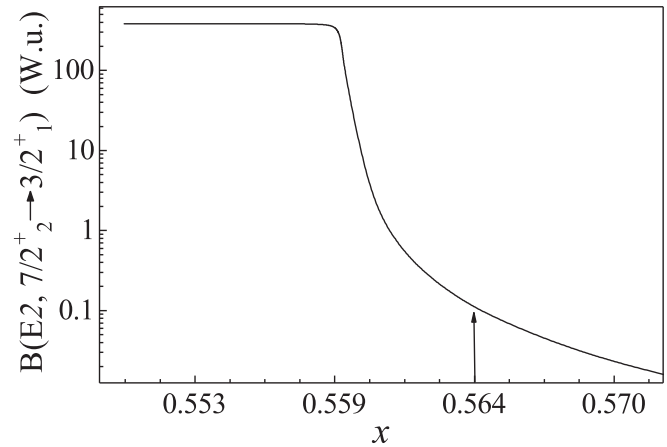

FIG. 6. Reduced transition probability $B(E2, 7/2_2^+ \to 3/2_1^+)$ is shown as a function of $x = \Im/\Im_{r.b.}$. The value $x_{\rm exp} = 0.564$, which corresponds to the experimentally measured energy distance between the levels, is marked by arrow.

TABLE II. Calculated energies, reduced transition probabilities for allowed decay modes, and half-lives of $7/2^+$ isomeric states in ^{249}Cm, ^{251}Cf, and ^{253}Cm. Experimental data [28], if available, are given in brackets.

Nucleus	^{249}Cm	^{251}Cf	^{253}Fm
$E_{7/2^+}$ (keV)	39.0 (58.8)	105.9 (106.3)	118.2
$B(E2, 7/2^+ \to 3/2^+_{K=1/2})$ (W.u.)	0.93×10^{-3} (2.7×10^{-3})	0.124 (0.47)	0.368×10^{-3}
$B(E2, 7/2^+ \to 5/2^+_{K=1/2})$ (W.u.)		0.017 (<0.032)	0.762×10^{-3}
$B(M1, 7/2^+ \to 5/2^+_{K=1/2})$ (W.u.)		4.66×10^{-5} ($>2.1\times10^{-5}$)	4.91×10^{-7}
$T_{1/2}$	73.4 μs (23 μs)	45 ns (38 ns)	3.65 μs

of the $K_p = 1/2$ component to the isomeric state $7/2^+[613]$ results in $B(E2, 7/2^+_2 \to 3/2^+_{K=1/2}) \approx 0.124$ W.u.

Therefore, it seems reasonable to assume that the Coriolis interaction can produce the strong enhancement of the decay rates of isomeric states due to the accidental degeneracy with the rotational states built on nonisomeric structures. Similar arguments can also be applied for the case of $M1$ transitions. Indeed, away from the resonance, the $\Delta K = 3$ difference between the initial and final states almost nullifies the magnetic transition probability. The only nonzero contribution comes from the transition between the $\Delta K = 1$ components admixtured to the corresponding wave functions by the Coriolis interaction, i.e., between the $K = 3/2^+$ admixture to the $3/2^+_{K=1/2}$ state and the $K = 5/2^+$ admixture to the $7/2^+$ state. This contribution is very small as it is proportional to the product of weights of corresponding components. In the case of the resonance, however, both states contain a significant contribution of the $K = 1/2^+$ component which leads to the sharp increase of $B(M1, 7/2^+_2 \to 5/2^+_{K=1/2})$ in the vicinity of x_0.

Taking into account possible decay modes, namely, $E2$ transition to the levels $3/2^+_{K=1/2}$, and $E2$ and $M1$ transitions to the level $5/2^+_{K=1/2}$ (see Table II), we obtain the half-life of 45 ns for the $7/2^+$ isomeric state in ^{251}Cf. In ^{249}Cm the isomeric state can only decay to $3/2^+_{K=1/2}$ via $E2$ transition which leads to the half-life of 73.4 μs. The electron conversion coefficients were taken from Ref. [28]. The experimental half-life values are 23 μs and 38 ns for ^{249}Cm and ^{251}Cf, respectively.

Our calculation for ^{253}Fm suggests that the two lowest $7/2^+$ levels lie at 111.4 keV (the $7/2^+$ state built predominantly on the $K_p = 1/2$ one-quasiparticle component) and at 118.3 keV (the isomeric state built upon $K_p = 7/2$). The distance between these states is $\Delta \approx 6.9$ keV which, as seen from the analysis given above, is too large to provide the strong Coriolis mixing effect. As a result, the reduced transition probability for the isomeric state $B(E2, 7/2^+ \to 3/2^+_{K=1/2}) = 0.368\times10^{-3}$ W.u. is close to the experimental one for ^{249}Fm. The reduced transition probabilities for two other allowed decay modes are listed in Table II, where the results for ^{249}Cm and ^{251}Cf are also summarized. Taking predicted electron conversion coefficients from Ref. [36], we obtain the half-life $T_{1/2} = 3.65$ μs for the $7/2^+$ isomeric state in ^{253}Fm. In comparison to ^{249}Cm, two additional decay modes in ^{253}Fm reduce the half-life of the isomeric state.

IV. SUMMARY

We examined the properties of one-quasineutron isomeric states $K_p = 7/2^+$ in nuclei with $N = 153$. The reduced transition probabilities obtained for the quadrupole decay of these isomeric states are $B(E2, 7/2_1 \to 3/2^+_{K=1/2}) = 0.93 \times 10^{-3}$ W.u. and $B(E2, 7/2_2 \to 3/2^+_{K=1/2}) = 0.124$ W.u. in ^{249}Cm and ^{251}Cf, respectively. For the decay of the isomeric state in ^{251}Cf, we calculated the probabilities of two additional decay modes, $B(E2, 7/2^+_2 \to 5/2^+_{K=1/2}) = 0.017$ W.u. and $B(M1, 7/2^+_2 \to 5/2^+_{K=1/2}) = 4.66\times10^{-5}$ W.u. These values lead to the half-lives of $T_{1/2} = 73.4$ μs and 45 ns for ^{249}Cm and ^{251}Cf, respectively. The obtained results are in good agreement with the experimental data. Our calculations predict $T_{1/2} = 3.65$ μs for the isomeric state in ^{253}Fm. Two additional decay modes in ^{253}Fm reduce the half-life of the isomeric state in comparison to ^{249}Cm. The sudden decrease of the lifetime of the $K^\pi = 7/2^+$ isomeric state in ^{251}Cf is explained by the exclusive proximity in energy of this state to the rotational $7/2^+$ state built on the $K = 1/2^+$ ground state. It was shown that such a proximity gives rise to a strong Coriolis mixing which, in turn, leads to the enhanced quadrupole transition rate from the isomeric state in ^{251}Cf. So, the Coriolis mixing is the main reason for the reduced half-life of the lowest isomeric state of ^{251}Cf. To analyze this effect we derived an analytical expression for the distance of closest proximity of levels Δ_0. For the Coriolis mixing with $\Delta K = 3$, this distance was estimated as $\Delta_0 \approx 0.02$ keV. The experimentally measured distance between these states is $\Delta = 0.571$ keV.

ACKNOWLEDGMENTS

T.M.S, G.G.A., and N.V.A. were supported by the Ministry of Science and Higher Education of the Russian Federation (Moscow, Contract No. 075-10-2020-117). N.M. gratefully acknowledges the support by the Bulgarian National Science Fund (BNSF) under Contract No. KP-06-N48/1.

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