

Information content of the differences in the charge radii of mirror nuclei

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Differences in the charge radii of mirror nuclei have been recently suggested to contain information on the slope of the symmetry energy L . To test this hypothesis, we perform statistical correlation analysis using quantified energy density functionals that are consistent with our previous knowledge on global nuclear observables such as binding energies and charge radii. We conclude that the difference in charge radii between a mirror pair, $\Delta R_{\text{ch}}^{\text{mir}}$, is an inferior isovector indicator compared to other observables, such as the neutron skin or electric dipole polarizability α_D . In particular, this quantity correlates poorly with both the neutron skin and L . We demonstrate that $\Delta R_{\text{ch}}^{\text{mir}}$ is influenced by pairing correlations in the presence of low-lying proton continuum in the proton-rich mirror-partner nucleus. Considering the large theoretical uncertainties on $\Delta R_{\text{ch}}^{\text{mir}}$, we conclude that the precise data on mirror charge radii cannot provide a stringent constraint on L .

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Introduction. A cursory information on properties of atomic nuclei is offered by the droplet model [1,2] whose key parameters are the nuclear matter (or bulk) characteristics such as volume energy E/A , equilibrium density ρ_{eq} , incompressibility K , symmetry energy J , and symmetry energy slope L . These quantities are widely used to characterize and compare nuclear models. The isoscalar parameters (E/A , ρ_{eq} , K) are well determined by empirical data because the chart of nuclei extends over a large range of mass numbers. However, the isovector parameters J and L are poorly constrained because available isotopic chains are fairly short. This is an uncomfortable situation because extrapolations to very neutron-rich isotopes and to neutron stars are crucial in nuclear astrophysics [3–6]. Consequently, there is a great demand for isovector-sensitive data that can be used for constraining the symmetry energy in the various nuclear models. The most promising isovector indicators [7] include neutron radii, neutron skins, dipole polarizability, and parity-violating asymmetry; see, e.g., Refs. [8–16].

Recently, it has been suggested [17] that a difference in the charge radii of mirror nuclei, $\Delta R_{\text{ch}}^{\text{mir}}(AX/Y) \equiv R_{\text{ch}}(AX_N) - R_{\text{ch}}(AY_Z)$, can serve as an isovector indicator that can be used to estimate the symmetry energy parameter L . Such estimates can be found in Refs. [17–21] for the $^{50}\text{Ni}/\text{Ti}$, $^{52}\text{Ni}/\text{Cr}$, $^{54}\text{Ni}/\text{Fe}$, and $^{36}\text{Ca}/\text{S}$ mirror pairs. It was concluded [17,18] that the precise data on mirror charge radii can provide a stringent constraint on L . In this study, we examine this finding.

Since the mirror nuclei considered in the previous work are all open-shell systems, pairing correlations, ignored in Refs. [17,18], are expected to play a role in the analysis of $\Delta R_{\text{ch}}^{\text{mir}}$. In this context, we first note that the proton-rich mirror partners considered in Refs. [17–20] are all weakly bound, which appreciably affects pairing correlations due to the pair

scattering into the continuum space. For such nuclei, to avoid the appearance of an unphysical particle gas that can impact radial behavior of nucleonic densities, nucleonic pairing must be handled within the full Hartree-Fock-Bogoliubov (HFB) scheme instead of the simpler Bardeen-Cooper-Schrieffer (BCS) approximation [22–24]. Recently, this has been discussed in relation to the ^{36}Ca charge radius measurement [25]. We also note that the effective pairing interaction in atomic nuclei has a strong isovector component; that is, a larger strength is required in the proton pairing channel than in the neutron pairing channel [26], and this can impact $\Delta R_{\text{ch}}^{\text{mir}}$.

To reduce theoretical uncertainties, a better understanding of model-dependent relationships between symmetry-energy parameters and nuclear observables is required. This can be achieved by means of the statistical correlation analysis [7,27] based on covariances obtained during the model calibration. Since the number of conceivable observables of interest is enormous, and the number of model parameters is significant (usually greater than 10), the covariance analysis is the least biased and most exhausting way to find out the correlations between all conceivable observables as it explores the whole parameter space of a given model.

In this work, we apply the HFB framework to assess the impact of weak binding and pairing on $\Delta R_{\text{ch}}^{\text{mir}}$. To study how strong and meaningful the suggested correlation between $\Delta R_{\text{ch}}^{\text{mir}}$ and L is, we use the statistical covariance framework.

Theoretical models. Our analysis has been carried out with nonrelativistic nuclear density-functional theory in its self-consistent nuclear energy density functional (EDF) variant [28]. In our applications, we employ the Skyrme parametrizations SV-min [29] and Fy(Δr , HFB) [25,30], which have been optimized to a large experimental calibration data sets by means of the standard linear regression, which provides

information on uncertainties and statistical correlations between observables.

Because we are dealing with weakly bound nuclei, pairing is treated at the HFB level using an iterative scheme in terms of canonical orbitals [31–33]. The pairing space is limited by a soft cutoff in single-particle space with a Woods-Saxon profile [33–35]. The pairing window involves canonical states in bands of 15 MeV above the Fermi level for $Fy(\Delta r, \text{HFB})$ and 5 MeV for SV-min(HFB) with a smoothing of a tenth of the pairing band width. The pairing window of SV-min(HFB) is chosen to comply with that of SV-min in the BCS approximation [29]. The EDF SV-min(HFB) has been calibrated using exactly the same strategy and dataset as for SV-min [29]. The changes in the model parameters when going from BCS to HFB are small but nonetheless important to maintain the high quality of the parametrization. The HFB calculations discussed in this study are compared to BCS results and to Hartree-Fock (HF) calculations without pairing applying the equal filling approximation (EFA) [36] which have been employed in Refs. [17,18].

Correlation analysis. Various kinds of correlations between observables and model parameters have been discussed in the context of symmetry energy. The most popular is the *intermodel analysis*, in which a set of models is used to make predictions and assess systematic uncertainties. Ideally, the models are supposed to be well calibrated to existing data and sufficiently different in terms of theoretical assumptions and optimization protocols. The implicit assumption here is that the biases introduced in different models are independent and that theoretical errors are randomized. Examples of such analyses can be found in, e.g., Refs. [8,12,13,37]. The intermodel analysis does not involve any statistical uncertainty quantification; hence, it does not explore the dependencies between the parameters of the model and the observables studied.

A more advanced approach to correlations is through the *trend analysis* within a given model. In this case, a selected model parameter or observable \mathcal{O} is systematically varied with other model parameters being calibrated to the optimization data set. In this way, one can study correlations related to the parameter variations in the direction of *one* variable \mathcal{O} . Studies involving such a trend analysis can be found in, e.g., Refs. [29,38,39].

In the *statistical correlation analysis*, one probes variations in a full space of model parameters \mathbf{p} . Key quantity is here the penalty function $\chi^2(\mathbf{p})$, the root-mean-square deviation of theoretical results from a given data set for the model parameters \mathbf{p} . It structures the space of parameters by the probability $\mathcal{P}(\mathbf{p})$ for a well-tuned set of parameters. The optimal parametrization is found at the minimum of χ^2 where \mathcal{P} has maximum. In its simplest variant, this can be done by probing the local structure of $\chi^2(\mathbf{p})$ surface around its minimum by studying the Hessian matrix (quadratic approximation) [7,27] or by means of the Monte Carlo sampling; see, e.g., Ref. [40]. A more general approach to correlations, which probes long-range parameter dependencies, involves Bayesian calibration. Here, the correlations are directly obtained from posterior distributions of model parameters [41–43].

The results of calculations presented in this study are analyzed using the tools of linear least square regression [44].

TABLE I. Proton Fermi levels (in MeV) of ^{36}Ca , ^{48}Ni , and ^{54}Ni isotopes computed in SV-min and $Fy(\Delta r, \text{HFB})$.

nucleus	SV-min	$Fy(\Delta r, \text{HFB})$
^{36}Ca	−1.0	−0.7
^{48}Ni	+1.9	+3.5
^{54}Ni	−2.6	−1.3

By computing the covariance $\text{cov}(x, y)$ of quantities x and y , as well as their respective variances σ_x and σ_y , we assess statistical x - y correlations in terms of the bivariate correlation coefficient

$$R_{x,y} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} \quad (1)$$

or its square R^2 , which is the coefficient of determination (CoD) [45]. We determine the CoDs as described in Refs. [46,47]. The CoDs contains information on how well one quantity is determined by another one.

The correlation between two observables can be visualized by the combined probability distribution $P(x, y)$. Each observable which can be described by the given model is also a function of the model parameters, i.e., $x = x(\mathbf{p})$ and $y = y(\mathbf{p})$. The probability to find certain values x and y is then derived from the probability distribution of \mathbf{p} as $P(x, y) = \int d\mathbf{p} (x - x(\mathbf{p}))(y - y(\mathbf{p}))\mathcal{P}(\mathbf{p})$. It becomes a two-dimensional Gaussian distribution in the quadratic approximation mentioned above. That can be well characterized by equiprobability line $P(x, y) = 1/e$, which forms the error ellipsoid in the plane of x and y .

The multiple correlation coefficient (MCC) [48] of observables with groups of parameters \mathbf{a} is

$$\text{MCC}(\mathbf{a}, x) = \mathbf{c}^T (R_{a,a})^{-1} \mathbf{c}, \quad (2)$$

where $R_{a,a}$ is the matrix of CoDs between the model parameters of group \mathbf{a} and the vector $\mathbf{c} = (R_{a_1,x}, R_{a_2,x}, \dots)$ contains the CoDs between the observables and the single group members. Values of R^2 range from 0 to 1, where 0 implies that those quantities are completely uncorrelated and 1 denotes that one quantity determines the other completely. An R^2 of, say, 0.60 means that 60% of the variance in x is predictable from \mathbf{a} ; see Refs. [49–51] for recent nuclear physics examples.

Results. To emphasize the need for using HFB in $\Delta R_{\text{ch}}^{\text{mir}}$ calculations, Table I lists the calculated proton Fermi levels for the proton-rich mirror partners considered in this work. These are all very weakly bound (^{36}Ca , ^{54}Ni) or unbound (^{48}Ni), which suggests that the proton continuum space can impact theoretical predictions, especially for radial properties [25].

To illustrate the relevance of the CoDs when analyzing isovector indicators, Fig. 1 compares the error ellipsoid of SV-min for $\Delta R_{\text{ch}}^{\text{mir}}(^{54}\text{Ni}/\text{Fe})$ and L with that for $R_{\text{skin}}(^{208}\text{Pb})$ and L . In the latter case, the error ellipsoid is very narrow with CoD = 0.99, which means that a given value of $R_{\text{skin}}(^{208}\text{Pb})$ uniquely determines L within this model. Similarly large is the correlation between α_{D} in ^{208}Pb and L with CoD = 0.98. The error ellipsoids associated with $\Delta R_{\text{ch}}^{\text{mir}}$ are significantly wider, and they appreciably depend on the

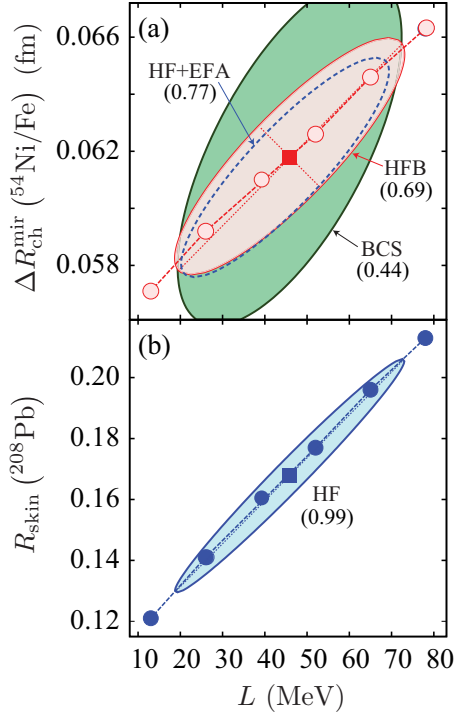


FIG. 1. The error ellipsoids in the planes of slope of symmetry energy L and (a) mirror radii $\Delta R_{\text{ch}}^{\text{mir}}(^{54}\text{Ni}/\text{Fe})$ and (b) R_{skin} in ^{208}Pb computed with SV-min. The principal axes of the ellipsoids are shown with thin dotted lines, and the corresponding CoDs are indicated by numbers in parentheses. The aspect ratio has been chosen such that a perfectly uncorrelated situation (CoD = 0) would show up as a circle. The results for $\Delta R_{\text{ch}}^{\text{mir}}(^{54}\text{Ni}/\text{Fe})$ were obtained with HFB, BCS, and HF + EFA. The R_{skin} results for ^{208}Pb were obtained with HF as the static pairing disappears in this case. The circles connected by a dotted line show the results from the SV forces with systematically varied symmetry energy for the values $L = 13, 26, 39, 52, 65,$ and 78 MeV.

treatment of pairing correlations. Namely, $\text{CoD}_{\text{HFB}} = 0.69$ while $\text{CoD}_{\text{BCS}} = 0.44$. The corresponding uncertainties are $\sigma_L(\text{BCS}) = 26.5$ MeV, $\sigma_L(\text{HFB}) = 27.2$ MeV, $\sigma_{\Delta R_{\text{ch}}^{\text{mir}}}(\text{BCS}) = 0.0065$ fm, and $\sigma_{\Delta R_{\text{ch}}^{\text{mir}}}(\text{HFB}) = 0.0041$ fm. That is, the uncertainties in L are very similar while $\sigma_{\Delta R_{\text{ch}}^{\text{mir}}}$ strongly depends on the way pairing correlations are treated. This is corroborated by considering also the HF+EFA variant. Here, the correlation (CoD = 0.77) is even higher than in the HFB case. This clearly demonstrates that pairing is responsible for weakening the correlation between $\Delta R_{\text{ch}}^{\text{mir}}$ and L .

Figure 1 shows also the trends from forces with systematically varied L (dots connected by a line). The trends are nearly linear for both R_{skin} and $\Delta R_{\text{ch}}^{\text{mir}}$. However, as illustrated in Fig. 1, the presence of a linear trend is not sufficient to assess the degree of correlation. That is, showing that results obtained with different models follow a regular trend is by no means a proof of a strong correlation. Indeed a CoD = 0.44 ($R = 0.66$) implies a moderate correlation while CoD = 0.69 ($R = 0.83$) is indicative of a stronger correlation. It is only in the case of R_{skin} in which the major axis of the correlation ellipsoid practically coincides with the systematic

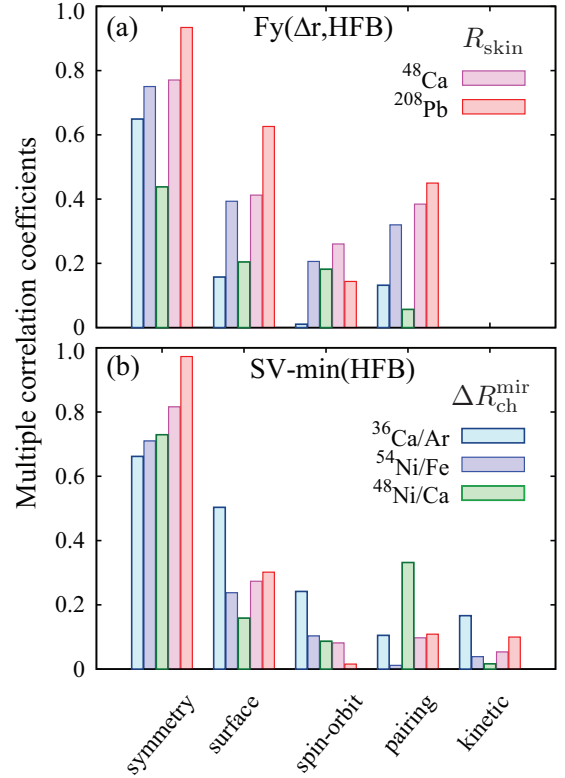


FIG. 2. The multiple correlation coefficients between various observables (R_{skin} and $\Delta R_{\text{ch}}^{\text{mir}}$ in different nuclei) and groups of selected model parameters for SV-min(HFB) and Fy(Δr , HFB): symmetry energy parameters J and L , isoscalar and isovector surface energy parameters, isoscalar and isovector spin-orbit parameter, pairing parameters, and the isoscalar and isovector effective masses as parameters of the kinetic energy.

trend line that a near-perfect correlation is obtained that can be used for the parameter reduction. It is interesting to note the large CoD values around 0.86–0.88 ($R_{L, \Delta R_{\text{ch}}^{\text{mir}}} = 0.93$ –0.94) that were obtained for $\Delta R_{\text{ch}}^{\text{mir}}$ in $^{54}\text{Ni}/\text{Fe}$ and other mirror pairs in Ref. [18]. In our opinion, the reason for the large correlations obtained in Ref. [18] is twofold. First, they ignore pairing correlations. Second, their relativistic energy density functionals have only two isovector parameters (instead of six for Skyrme functionals), which automatically implies strong correlations between all isovector observables [7].

The fact that mirror radii differences produce broader error ellipsoids as compared to those of excellent isovector indicators such as R_{skin} suggests that they are influenced also by other terms in the EDF parametrization such as, e.g., surface energy. This is analyzed systematically in Fig. 2 which shows the MCCs between R_{skin} and $\Delta R_{\text{ch}}^{\text{mir}}$ in different nuclei and groups of selected parameters characterizing SV-min(HFB) and Fy(Δr , HFB). As expected, R_{skin} in ^{208}Pb is practically determined by the symmetry energy parameters. This correlation is somehow reduced for R_{skin} in a lighter nucleus ^{48}Ca , because it is more impacted by shell effects. The symmetry energy still dominates MCCs with mirror radii differences, but its impact is not as pronounced as in the case of R_{skin} in ^{208}Pb .

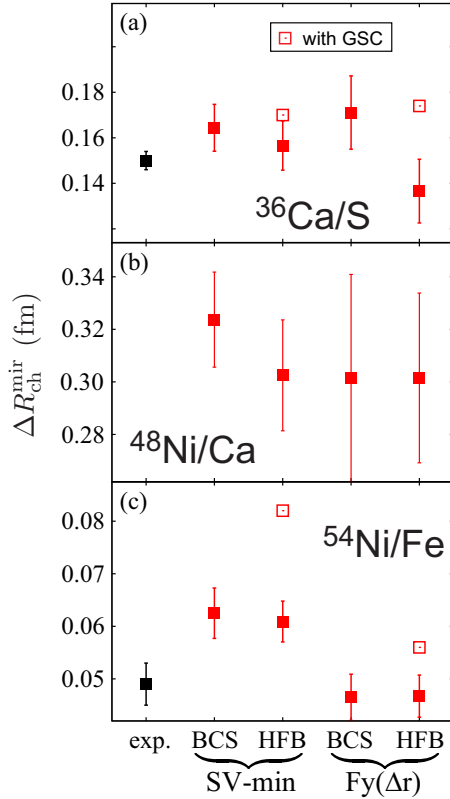


FIG. 3. The mirror radius differences for (a) $^{36}\text{Ca}/\text{S}$, (b) $^{48}\text{Ni}/\text{Ca}$, and (c) $^{54}\text{Ni}/\text{Fe}$ calculated with SV-min and FY(Δr) EDFs in BCS and HFB variants. The error bars indicate the statistical uncertainty associated with χ^2 optimization. For orientation, results HFB calculations including collective ground-state correlations in $^{36}\text{Ca}/\text{S}$ and $^{54}\text{Ni}/\text{Fe}$ are shown by open squares. The experimental $\Delta R_{\text{ch}}^{\text{mir}}$ values are shown for $^{36}\text{Ca}/\text{S}$ [19] and $^{54}\text{Ni}/\text{Fe}$ [20] (black squares).

The values of $\Delta R_{\text{ch}}^{\text{mir}}$ are in fact influenced by many terms of the functional; i.e., they are *distributed quantities* [49,50], which is indicative of shell effects. The corresponding MCCs still deliver useful information, but only when combined with other data in a consistent statistical analysis.

The values of $\Delta R_{\text{ch}}^{\text{mir}}$ for $^{36}\text{Ca}/\text{S}$, $^{48}\text{Ni}/\text{Ca}$, and $^{54}\text{Ni}/\text{Fe}$ mirror pairs calculated in BCS and HFB are shown in Fig. 3. In general, there is a good agreement with experiment considering the fact that the expected accuracy for precision calculation of charge radii is 0.015 fm [52]. The treatment of pairing does affect $\Delta R_{\text{ch}}^{\text{mir}}$, especially for weakly bound ^{36}Ca and unbound ^{48}Ni ; see Table I and Ref. [25]. To get some idea on the impact of zero-point correlations on mirror radii, we also show in Fig. 3 the effect of collective ground-state correlations on charge radii from low lying 2^+ states [53]. Those corrections are quite significant for $\Delta R_{\text{ch}}^{\text{mir}}$ ($^{54}\text{Ni}/\text{Fe}$). In this context, we note that the observed $B(E2)$ rates in mirror nuclei indicate the presence of strong isovector effects [54], which are likely to affect $\Delta R_{\text{ch}}^{\text{mir}}$. It has been suggested in Ref. [17] that differences in mirror charge radii $\Delta R_{\text{ch}}^{\text{mir}}$ are proportional to $|N - Z|L$. As seen in Fig. 4(a), this does not seem to be the case for the SV family of EDFs with systematically varied symmetry energy. On the other hand, the values of $\Delta R_{\text{ch}}^{\text{mir}}$ seem

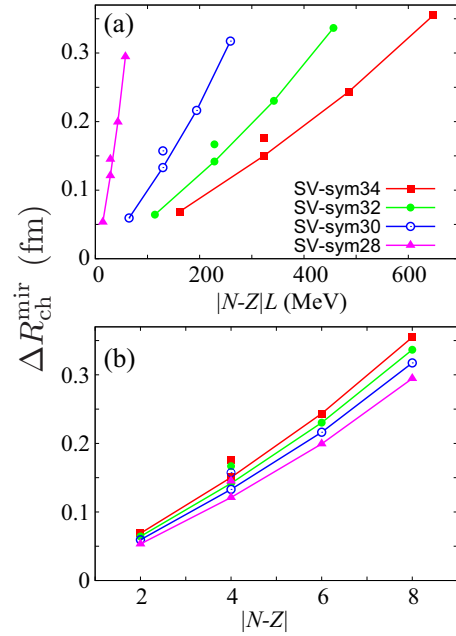


FIG. 4. The mirror radius differences for $^{54}\text{Ni}/\text{Fe}$, $^{52}\text{Cr}/\text{Ni}$, $^{50}\text{Ti}/\text{Ni}$, and $^{48}\text{Ca}/\text{Ni}$ mirror pairs, and for the single mirror pair $^{36}\text{Ca}/\text{S}$ as functions of (a) $|N - Z|L$ and (b) $|N - Z|$ calculated for four SV EDFs with constrained symmetry energy J .

to scale with $|N - Z|$ [55] or $|N - Z|/A$ [21] (we checked that the scaling with $|N - Z|/A$ produces very similar results).

Conclusions. In this study, we examine claims [17,18] that the precise data on mirror charge radii can provide a stringent constraint on L . While our statistical analysis confirms an appreciable correlation between L and $\Delta R_{\text{ch}}^{\text{mir}}$, this correlation is significantly weaker than that among L , R_{skin} , and α_D in ^{208}Pb . In other words, we find $\Delta R_{\text{ch}}^{\text{mir}}$ to be a much weaker isovector indicator than neutron skins in heavy nuclei, dipole polarizability, or parity-violating asymmetry [16].

Since the proton-rich nucleus in the mirror pair is usually weakly bound, special care should be taken when considering the impact of the proton continuum space. This can be done, e.g., by employing the HFB formalism.

Pairing correlations should always be considered for open-shell nuclei. As shown in our paper, by neglecting pairing [17–20] one artificially increases the correlation between $\Delta R_{\text{ch}}^{\text{mir}}$ and L .

The statistical errors on predicted $\Delta R_{\text{ch}}^{\text{mir}}$ vary between 0.005 and 0.02 fm, depending on $|N - Z|$. The systematic uncertainties, however, are significantly larger. Those are due to the choice of nuclear interaction, treatment of pairing, and evaluation of zero-point collective correlations in spherical and transitional nuclei.

Considering the large theoretical uncertainties on $\Delta R_{\text{ch}}^{\text{mir}}$ estimated in our study, we conclude that the precise data on mirror charge radii with an error of about 0.005 fm [17], while extremely valuable for studying isospin effects in nuclei and model developments, *cannot* provide a stringent constraint on L .

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