Spin symmetry energy and equation of state of spin-polarized neutron star matter

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The equation of state (EOS) of spin-polarized nuclear matter (NM) is studied within the Hartree-Fock (HF) formalism using the realistic density-dependent nucleon-nucleon interaction. With a nonzero fraction Δ of spin-polarized baryons in NM, the spin- and spin-isospin dependent parts of the HF energy density give rise to the *spin symmetry* energy that behaves in about the same manner as the *isospin symmetry* energy, widely discussed in the literature as the nuclear symmetry energy. The present HF study shows a strong correlation between the spin symmetry energy and nuclear symmetry energy over the whole range of baryon densities. The important contribution of the spin symmetry energy to the EOS of the spin-polarized NM is found to be comparable with that of the nuclear symmetry energy to the EOS of the isospin-polarized or asymmetric (neutron-rich) NM. Based on the HF energy density, the EOS of the spin-polarized (β-stable) $npe\mu$ matter is obtained for the determination of the macroscopic properties of neutron stars (NS). A realistic density dependence of the spin-polarized fraction Δ has been suggested to explore the impact of the spin symmetry energy on the gravitational mass M and radius R, as well as the tidal deformability of NS. Based on the empirical constrains inferred from a coherent Bayesian analysis of gravitational wave signals of the NS merger GW170817 and the observed masses of the heaviest pulsars, the present study shows the strong impact of the spin symmetry energy W, nuclear symmetry energy S, and nuclear incompressibility K on the EOS of nucleonic matter in magnetar.

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I. INTRODUCTION

Rotating neutron stars are known to possess strong magnetic fields [1–3], with the field strength B on the order of 10^{14} to 10^{19} G, so that the effects of magnetic fields on the equation of state (EOS) of neutron star (NS) matter should not be negligible. In particular, a significant fraction of baryons in NS matter might have their spins polarized along the axis of the magnetic field. The full spin polarization of neutrons was shown by Broderick *et al.* [2] to likely occur at the high field strength of $B \gtrsim 4.41\times10^{18}$ G. It is commonly assumed that the magnetic field of NS is usually much weaker than the upper limit of $B \approx 10^{19}$ G, and the spin polarization of baryons is often neglected in the mean-field studies of the EOS of NS matter. Recently, the "blue" kilonova ejecta observed in the aftermath of the NS merger GW170817 [4–6] have been suggested by Metzger *et al.* [7] to be caused by both the γ decay of the r-process nuclei and magnetically accelerated wind from the strongly magnetized hypermassive NS remnant. A rapidly rotating hypermassive NS remnant with the magnetic field of $B \approx (1\text{–}3)\times10^{14}$ G at the surface has been assumed to explain the velocity, total mass, and enhanced electron fraction of the kilonova ejecta [7]. Such a scenario seems to agree with the prediction made by Fujisawa *et al.* [8] for the strength of magnetic field in the outer core of magnetar, and a partial spin polarization of baryons might well occur in the two merging neutron stars of GW170817. To investigate such effects, a nonrelativistic Hartree-Fock (HF) study of the spin-polarized nuclear matter (NM) has been done recently [9], assuming different (relative) strengths Δ of the spin polarization of baryons. The EOS obtained in the HF approach for NS matter consisting of strongly interacting baryons and leptons, i.e., the $npe\mu$ matter in β equilibrium was used as input to determine the gravitational mass M and radius R of NS [9,10] from the solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations [11].

Given a realistic EOS of NS matter, general relativity not only explains the compact shape of NS in the hydrostatic equilibrium but also predicts interesting behavior of NS in the strong gravitational field formed by two inspiralling neutron stars during their merger. In particular, the shape of each NS is tidally deformed by the mutual attraction of two coalescing neutron stars to gain nonzero multipole moments [12–14], with the energy being lost via the emission of gravitational waves (GW). The tidal deformation is usually expressed in terms of the tidal Love number k_2 of the second order, which has been inferred recently from the analysis of the observed GW signals from GW170817 and translated into the constraint for the gravitational mass M and radius R of NS [15]. Because this empirical constraint serves now as an important reference in validating different models of the EOS of NS matter, we have applied in the present work the HF approach suggested in Ref. [9] to study in more detail the impact by the spin polarization of baryons to the EOS of NS matter. Governed by the same SU(2) symmetry, the spin symmetry energy W is shown in Sec. II to behave in about the same manner as the isospin symmetry energy S, widely known as the *nuclear symmetry energy*. In particular, the parabolic approximation is valid also for the spin symmetry energy, so that the (repulsive) contribution from W to the total NM energy is directly proportional to Δ^2. An interesting correlation between the slope parameter L of the nuclear symmetry energy S and the slope L_s of the spin symmetry energy W has been found and discussed in detail.

The EOS of the β-stable spin-polarized NS matter obtained in Sec. III is used as the input to solve the linearized Einstein equation for the metric perturbation of the stress-energy tensor of NS to determine the tidal deformability Λ of NS matter and compare with the empirical Λ deduced from the analysis of the GW data of GW170817 in Sec. IV. The explicit treatment of the spin- and isospin variables in the CDM3Yn density-dependent interaction allows us to show explicitly the impact by the spin symmetry energy W, nuclear symmetry energy S, and nuclear incompressibility K on the EOS of nucleonic matter, the tidal deformability, and the mass and radius of NS.

II. HARTREE-FOCK CALCULATION OF SPIN-POLARIZED NUCLEAR MATTER

The nonrelativistic Hartree-Fock (HF) approach [16] has been extended recently [9] to study spin-polarized NM at zero temperature. In this case, NM is characterized by the neutron and proton number densities, n_n and n_p, or equivalently by the total baryon number density $n_b = n_n + n_p$ and neutron-proton asymmetry $\delta = (n_n - n_p)/n_b$. The spin polarization of baryons is treated explicitly for neutrons and protons by using the densities with baryon spin aligned up or down along the axis of magnetic field $\Delta_{n,p} = (n_{\uparrow n,p} - n_{\downarrow n,p})/n_{n,p}$. In general, the total HF energy density of NM is given by

$$\mathcal{E} = \mathcal{E}_{\rm kin} + \frac{1}{2}\sum_{k\sigma\tau}\sum_{k'\sigma'\tau'}[\langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau'|v_{\rm d}|\boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau'\rangle + \langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau'|v_{\rm ex}|\boldsymbol{k}'\sigma\tau, \boldsymbol{k}\sigma'\tau'\rangle], \quad (1)$$

where $|\boldsymbol{k}\sigma\tau\rangle$ are plane waves, and $v_{\rm d}$ and $v_{\rm ex}$ are the direct and exchange components of the (in-medium) density-dependent nucleon nucleon (NN) interaction. Although the neutron and proton magnetic moments are of different strengths and of opposite signs, in the presence of a strong magnetic field, $|\Delta_n|$ and $|\Delta_p|$ should be of the same order. Like the previous HF study [9], we also assume hereafter the baryon spin polarization $\Delta = \Delta_n \approx -\Delta_p$.

We have used in the present work several versions of the density-dependent CDM3Yn interaction which is based upon the (G matrix) M3Y-Paris interaction [17]. These interactions were well tested in the earlier HF studies of NM [9,10,16] as well as the folding model analyses of nucleus-nucleus scattering [18,19]. Explicitly, the CDM3Yn interaction is just the original M3Y-Paris interaction [17] supplemented by an empirical density dependence [9,18,19]

$$v_{\rm d(ex)}(n_b, r) = F_{00}(n_b)v_{00}^{\rm d(ex)}(r) + F_{10}(n_b)v_{10}^{\rm d(ex)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}') + F_{01}(n_b)v_{01}^{\rm d(ex)}(r)(\boldsymbol{\tau}\cdot\boldsymbol{\tau}') + F_{11}(n_b)v_{11}^{\rm d(ex)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')(\boldsymbol{\tau}\cdot\boldsymbol{\tau}'). \quad (2)$$

The radial dependence of the central interaction (2) is determined from the spin (isospin) singlet and triplet components of the M3Y-Paris interaction [17] and expressed terms of three Yukawa functions [20] as

$$v_{st}^{\rm d(ex)}(r) = \sum_{\kappa=1}^{3} Y_{st}^{\rm d(ex)}(\kappa)\frac{\exp(-R_\kappa r)}{R_\kappa r}. \quad (3)$$

The Yukawa strengths $Y_{st}^{\rm d(ex)}(\kappa)$ are given explicitly, e.g., in Table I of Ref. [9]. If the spin polarization is neglected ($\Delta = 0$) then the NM can be treated as *spin-saturated*, and the σ components of plane waves are averaged out in the HF calculation (1). As a result, only the $st = 00$ and $st = 01$ terms of the central interaction (2) are necessary for the determination of the energy density of NM. The situation becomes different when $\Delta \neq 0$, and the spin-dependent ($st = 10$) and spin-isospin-dependent ($st = 11$) terms of the interaction (2) need to be properly taken into account in the HF calculation. In this case, the total HF energy density (1) of the spin-polarized NM is obtained as

$$\mathcal{E} = \mathcal{E}_{\rm kin} + F_{00}(n_b)\mathcal{E}_{00} + F_{10}(n_b)\mathcal{E}_{10} + F_{01}(n_b)\mathcal{E}_{01} + F_{11}(n_b)\mathcal{E}_{11}. \quad (4)$$

The explicit expressions of $\mathcal{E}_{st}$ obtained with the density dependent CDM3Yn interaction are given in Ref. [9]. We note that the *isoscalar* density dependence $F_{00}(n_b)$ was first parametrized in Ref. [18] to properly saturate symmetric NM at the density $n_0 \approx 0.17$ fm^{-3}, while giving different values of the nuclear incompressibility K. The *isovector* density dependence $F_{01}(n_b)$ was parametrized later to reproduce the microscopic Brueckner-Hartree-Fock (BHF) results of asymmetric NM, with the total strength fine tuned by the folding model description of the charge exchange (p, n) reaction to the isobar analog states in finite nuclei [21,22]. Because the spin polarization of baryons gives rise to the nonzero contribution from $\mathcal{E}_{10}$ and $\mathcal{E}_{11}$ to the total NM energy density (4), the density dependencies $F_{10}(n_b)$ and $F_{11}(n_b)$ of the CDM3Yn interaction (2) need to be properly determined for the HF calculation of the spin-polarized NM. For the convenience in numerical calculations, the density-dependent functional $F_{st}(n_b)$ of the CDM3Yn interaction (2) has been assumed in the analytical form

$$F_{st}(n_b) = C_{st}[1 + \alpha_{st}\exp(-\beta_{st}n_b) + \gamma_{st}n_b]. \quad (5)$$

Such a procedure has been carried out for the CDM3Y8 version of the interaction in the HF calculation of the spin-polarized NM [9], where the parameters of $F_{10}(n_b)$ and $F_{11}(n_b)$ were adjusted to reproduce the BHF results for the spin-polarized symmetric NM and neutron matter by Vidaña *et al.* [23] using the Argonne V18 free NN potential added by the Urbana IX three-body force. In the present work, we apply the same procedure to four versions (CDM3Y4, CDM3Y5, CDM3Y6, and CDM3Y8) of the CDM3Yn interaction for a comparative HF study. Note that the isovector density dependence $F_{01}(n_b)$ of these four interactions were fine tuned

TABLE I. Parameters of the density dependence (5) of 4 versions of the CDM3Yn interaction used in the present HF calculation. K is the nuclear incompressibility (6) of symmetric NM determined at the saturation density $n_0 \approx 0.17$ fm^{-3}.

Interaction	st	C_{st}	α_{st}	β_{st} (fm^3)	γ_{st} (fm^3)	K (MeV)
CDM3Y4	00	0.3052	3.2998	2.3180	−2.0	228
	01	0.2129	6.3581	7.0584	5.6091	
	10	0.1494	6.7055	2.5766	116.5455	
	11	0.6830	0.6949	3.2104	1.0433	
CDM3Y5	00	0.2728	3.7367	1.8294	−3.0	241
	01	0.2204	6.6146	7.9910	6.0040	
	10	0.1607	3.3867	2.8341	106.7274	
	11	0.7016	0.6299	3.4552	1.0752	
CDM3Y6	00	0.2658	3.8033	1.4099	−4.0	252
	01	0.2313	6.6865	8.6775	6.0182	
	10	0.1887	−0.9998	3.1342	92.1075	
	11	0.7259	0.5452	3.6416	1.0775	
CDM3Y8	00	0.2658	3.8033	1.4099	−4.3	257
	01	0.2643	6.3836	9.8950	5.4249	
	10	0.2162	−2.3396	3.3397	77.3144	
	11	0.7573	0.4858	4.2011	1.0179	

recently [10] to reach a good agreement of the nuclear symmetry energy given by the HF calculation with that given by the *ab initio* calculations [24,25] at suprasaturation densities $n_b > n_0$. The isoscalar density dependence $F_{00}(n_b)$ of these interactions has been kept unchanged as suggested in Ref. [18]. All the parameters used in the present work are given explicitly in Table I. Dividing the total NM energy density (1) by the baryon number density n_b we obtain the NM energy per baryon E/A, which is shown for the spin-unpolarized and spin-polarized NM in Figs. 1 and 2, respectively. The energy of symmetric NM at high baryon densities correlates strongly with the nuclear incompressibility K determined at the saturation density as

$$K = 9n_b^2 \frac{\partial^2}{\partial n_b^2}\frac{E}{A}(\delta = 0)\Big|_{n_b\to n_0} = 9\frac{\partial P(\delta = 0)}{\partial n_b}\Big|_{n_b\to n_0}. \quad (6)$$

Note that a slight difference of the HF results for the energy of symmetric NM at high baryon densities (upper panel of Fig. 1) is due to the different values of the incompressibility K obtained with four versions of the interaction (see Table I). The K value is strongly sensitive to the EOS of NM, and K has been, therefore, a key research topic of numerous structure studies of nuclear monopole resonances (see, e.g., Ref. [26] and references therein) as well as studies of the refractive light heavy-ion (HI) scattering [27]. These studies have narrowed the empirical range to $K \approx 240 \pm 20$ MeV. While the four density-dependent versions of the CDM3Yn interaction (2) give $K \approx 228$–257 MeV, which is well within the empirical range, such a difference in K values was shown to affect significantly the gravitational mass of NS obtained with the EOS given by these interactions [10]. In the present work we explore this effect also for the EOS of the spin-polarized NM. One can see in Fig. 2 that the full spin polarization of baryons ($\Delta = 1$) substantially enhances the energy of both the

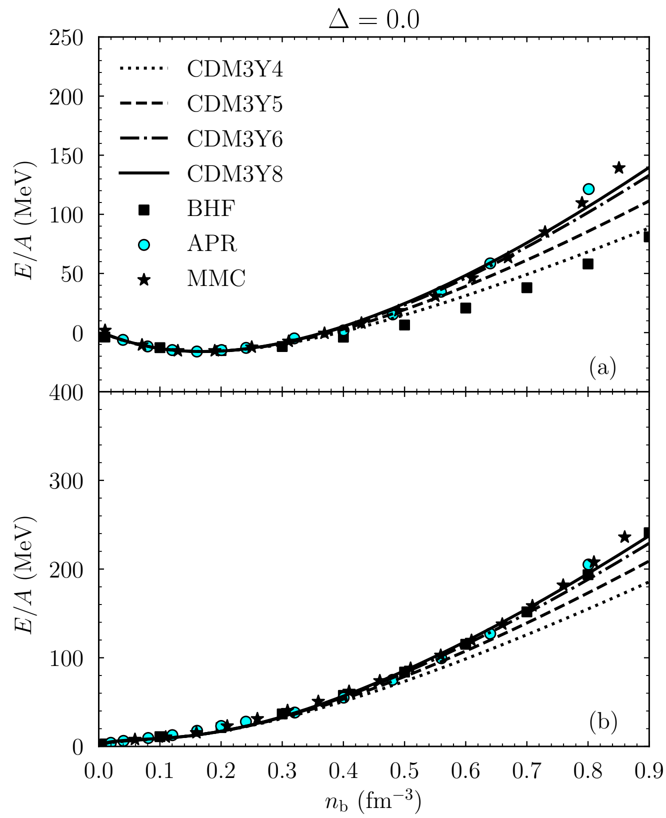

FIG. 1. Energy per baryon of the spin-unpolarized (a) symmetric NM and (b) pure neutron matter given by the HF calculation (1) using four versions of the CDM3Yn interaction, in comparison with the BHF results (squares) [23]. The circles and stars are results of the *ab initio* calculations by Akmal, Pandharipande, and Ravenhall (APR) [24] and the microscopic Monte Carlo (MMC) calculation by Gandolfi *et al.* [25], respectively.

symmetric NM and pure neutron matter over the whole range of densities, and the energy required per baryon to change the spin-unpolarized NM into the fully spin-polarized NM is the *spin symmetry* energy W [9]. Governed by the same SU(2) symmetry, the spin symmetry energy behaves in about the same manner as the *isospin symmetry* energy S, which is widely known as the nuclear symmetry energy S. Thus, the total energy of NM can be expressed alternatively as

$$\frac{E}{A} = \frac{E}{A}(n_b, \Delta, \delta = 0) + S(n_b, \Delta)\delta^2 + O(\delta^4) + \cdots \quad (7)$$

$$= \frac{E}{A}(n_b, \Delta = 0, \delta) + W(n_b, \delta)\Delta^2 + O(\Delta^4) + \cdots . \quad (8)$$

The contribution from both the higher-order terms $O(\delta^4)$ and $O(\Delta^4)$ were proven to be small and can be neglected in the well-known *parabolic* approximation [9,20]. In the present context, we find it illustrative to consider the pure neutron matter as the fully *isospin* polarized NM ($\delta = 1$), so that the nuclear symmetry energy $S(n_b)$ equals just the energy required per baryon to change the (isospin) symmetric NM into the fully isospin-polarized NM, i.e., the pure neutron matter. The (isospin) symmetry energy $S(n_b, \Delta)$, widely discussed in the literature as the *nuclear symmetry energy*, is the key

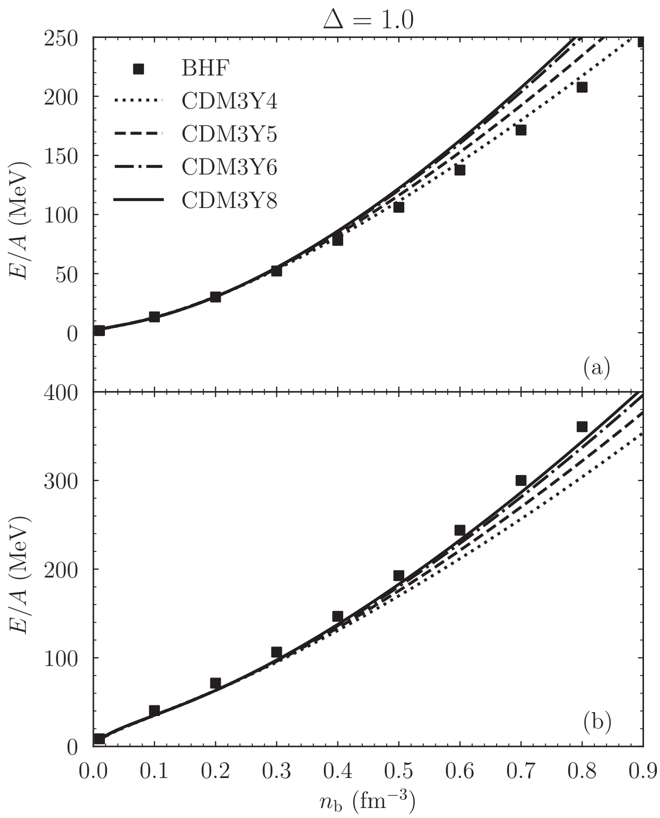

FIG. 2. The same as Fig. 1 but for the fully spin-polarized (a) symmetric NM and (b) pure neutron matter, in comparison with the BHF results (squares) [23].

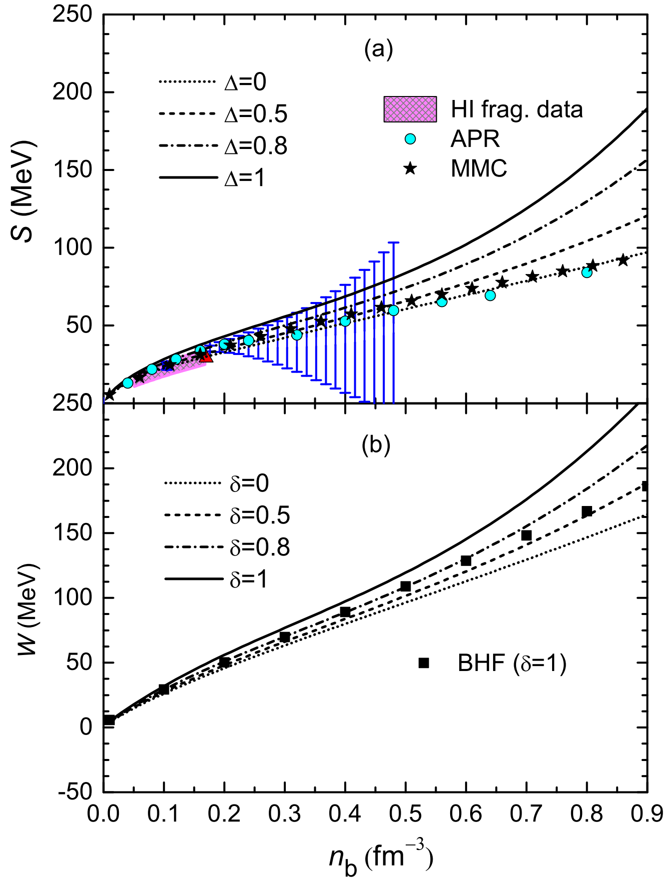

FIG. 3. (a) The HF results obtained with the CDM3Y8 interaction for the nuclear symmetry energy (7) at different spin polarizations of baryons, in comparison with the *ab initio* results [24,25] obtained with $\Delta = 0$; the shaded region is the constraint by the HI fragmentation data [31,32]; the vertical bars are given at 90% confidence level by the Bayesian analysis [33]. (b) The spin symmetry energy (8) given by the CDM3Y8 interaction at different isospin polarizations δ, in comparison with the BHF results for the fully spin-polarized neutron matter [23].

characteristics of the EOS of NS matter and is, therefore, a longstanding goal of numerous nuclear physics studies (see, e.g., Refs. [28–30]). However, these studies were done mainly for the spin-saturated NM and describe, therefore, $S(n_{\rm b}, \Delta = 0)$. The HF results obtained with the CDM3Y8 interaction for the nuclear symmetry energy (7) at different spin polarizations of baryons Δ are compared in the upper panel of Fig. 3 with the *ab initio* results obtained at $\Delta = 0$ [24,25] as well as the constraint implied by the analysis of the HI fragmentation data [31,32]. Using the parameters of $F_{01}(n_{\rm b})$ of the CDM3Yn interaction fine tuned recently [10], the HF results obtained with $\Delta = 0$ agree nicely with those of the *ab initio* calculations at high densities. The nuclear symmetry energy increases significantly with increasing spin polarization Δ, but the $S(n_{\rm b}, \Delta)$ values remain well within the empirical range inferred from a Bayesian analysis of the correlation of different EOS's of the $npe\mu$ matter with the GW170817 constraint on the radius $R_{1.4}$ of NS with mass $M = 1.4\,M_\odot$ [33] (the vertical bars in the upper panel of Fig. 3). It is natural to expect that this GW170817 constraint also has an imprint of the spin polarization of baryons in the two coalescing neutron stars. The density dependence of the nuclear symmetry energy is widely investigated in terms of the symmetry coefficient J, slope L, and curvature $K_{\rm sym}$ of an expansion of S around the saturation density n_0 [28–30]

$$S(n_{\rm b}) = J + \frac{L}{3}\left(\frac{n_{\rm b}-n_0}{n_0}\right) + \frac{K_{\rm sym}}{18}\left(\frac{n_{\rm b}-n_0}{n_0}\right)^2 + \cdots . \quad (9)$$

These quantities and the incompressibility K of symmetric NM are the main characteristics of the EOS of NM. For the *spin-unpolarized* asymmetric NM, the symmetry coefficient J is well established to be around 30 MeV, but the L and $K_{\rm sym}$ values remain much less certain. A recent systematic survey by Li *et al.* [34] quotes $L \approx 57.7 \pm 19$ MeV at the 68% confidence level. The relativistic mean-field studies have suggested [35] that the neutron skin thickness $R_{\rm skin} = R_n - R_p$ of the ^{208}Pb nucleus is a stringent laboratory constraint on the slope L of the symmetry energy $S(n_{\rm b})$. Given $R_{\rm skin} \approx 0.283 \pm 0.071$ fm deduced recently from the measurement of the parity-violating asymmetry in the elastic scattering of polarized electrons from ^{208}Pb by the PREX Collaboration [36], one obtains $L \approx 106 \pm 37$ MeV using different relativistic energy density functionals [35]. This L value is significantly

TABLE II. The symmetry coefficient, slope, and curvature of the nuclear symmetry energy (9) for the spin-unpolarized asymmetric NM ($\Delta = 0$), and those of the spin symmetry energy (10) for the spin-polarized symmetric NM ($\delta = 0$) given by the present HF calculation using four versions of the CDM3Yn interaction.

Interaction	J (MeV)	L (MeV)	$K_{\rm sym}$ (MeV)	$J_{\rm s}$ (MeV)	$L_{\rm s}$ (MeV)	$K_{\rm syms}$ (MeV)
CDM3Y4	30.0	50.0	−63.5	40.4	96.0	−70.7
CDM3Y5	30.0	50.0	−52.1	40.3	96.5	−67.7
CDM3Y6	30.0	50.0	−44.2	40.3	97.4	−64.0
CDM3Y8	29.9	49.5	−31.4	40.1	96.7	−63.3

higher than that predicted by most mean-field calculations and impacts strongly the calculated macroscopic properties of NS [35].

At variance with the nuclear symmetry energy (7), the spin symmetry energy (8) (see lower panel of Fig. 3) was much less studied, and we could compare the HF results for $W(n_{\rm b}, \delta)$ only with the BHF result obtained for the fully spin-polarized neutron matter [23]. With the quadratic dependence on δ and Δ of the NM energy, the stiffness of the EOS of spin-polarized NM increases significantly with the increasing polarization of spin (Δ) and isospin (δ) of baryons. Such effect is well seen in the behavior of $S(n_{\rm b}, \Delta)$ and $W(n_{\rm b}, \delta)$ with increasing Δ and δ, respectively. Given such a correlation of the S and W, it is obvious that the spin polarization of baryons should not be neglected in a mean-field study of NS matter. It is interesting that the density dependence of the spin symmetry energy can also be expressed in terms of the spin-symmetry coefficient $J_{\rm s}$, slope $L_{\rm s}$, and curvature $K_{\rm syms}$, in the same manner as the expansion (9),

$$W(n_{\rm b}) = J_{\rm s} + \frac{L_{\rm s}}{3}\left(\frac{n_{\rm b}-n_0}{n_0}\right) + \frac{K_{\rm syms}}{18}\left(\frac{n_{\rm b}-n_0}{n_0}\right)^2 + \cdots . \tag{10}$$

$J_{\rm s}$, $L_{\rm s}$, and $K_{\rm syms}$ have *not* been investigated so far in different mean-field models of the EOS of NM. In the present work these quantities are obtained with four versions of the CDM3Yn interaction, and the those determined at $\delta = 0$ are shown in Table II together with the symmetry coefficient, slope, and curvature of the nuclear symmetry energy (9) determined at $\Delta = 0$. Although the spin symmetry energy at the saturation density $W(n_0) = J_{\rm s} \approx 40$ MeV, which is only about 10 MeV larger than $S(n_0) = J \approx 30$ MeV. The slope $L_{\rm s}$ of the spin symmetry energy is nearly twice that of the nuclear symmetry energy L, and this makes the EOS of the spin-polarized NM much stiffer than that of the spin-unpolarized NM (see Figs. 1 and 2). In general, the parameters J, L, and $K_{\rm sym}$ of the nuclear symmetry energy must depend on the spin polarization Δ of baryons, and vice versa, $J_{\rm s}$, $L_{\rm s}$, and $K_{\rm syms}$ also depend on δ. Such a spin-isospin correlation of the two slope parameters is shown in Fig. 4 and one can see about the same increasing trend of $L(\Delta)$ and $L_{\rm s}(\delta)$ with the increasing spin and isospin polarization, respectively. Over the whole range $0 \lesssim \Delta \lesssim 1$, the obtained $L(\Delta)$ values remain well within the empirical range implied by the nuclear physics studies and astrophysical observations [34], but are still below

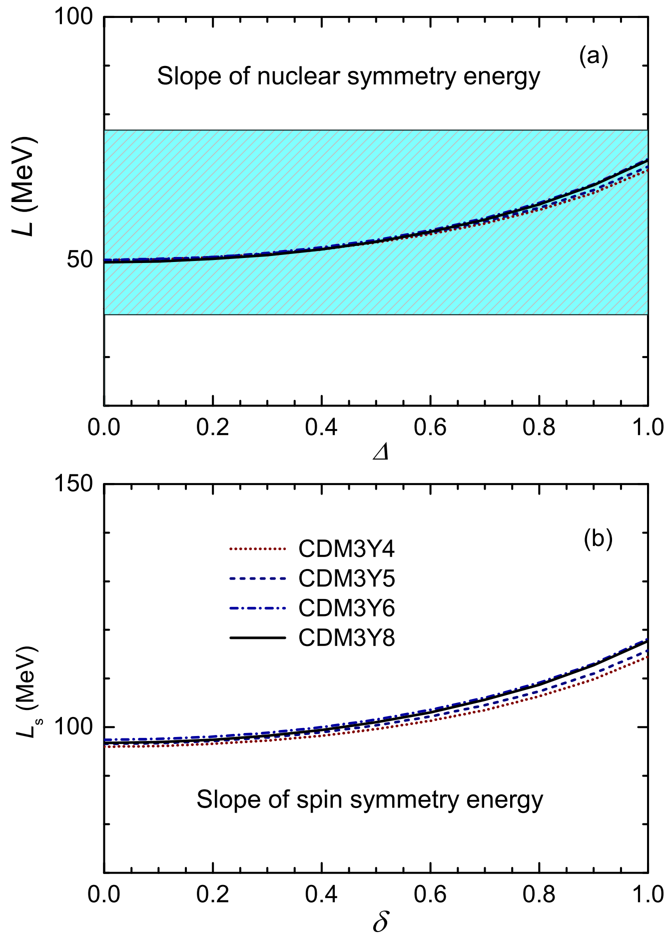

FIG. 4. (a) The HF results obtained with four versions of the CDM3Yn interaction for the slope L of the nuclear symmetry energy (9) at different spin polarizations Δ of baryons in comparison with the empirical range suggested by Li *et al.* [34] at the 68% confidence level (the shaded region). (b) The slope $L_{\rm s}$ of the spin symmetry energy (10) at different isospin asymmetries δ.

the lower limit of the L value implied by the neutron skin of ^{208}Pb measured in the PREX-2 experiment [35,36]. As discussed above, the energy of the spin-polarized asymmetric NM depends on both the spin polarization Δ of baryons and the neutron-proton asymmetry (or the isospin polarization) δ in about the same manner. As a result, it turns out to be possible to expand E/A simultaneously in the spin- and isospin polarizations, which enables the description of the energy of NM in terms of the nuclear symmetry (9) and spin symmetry energy (10) that depend on the baryon density only

$$\frac{E}{A} = E_0(n_{\rm b}) + S_0(n_{\rm b})\delta^2 + W_0(n_{\rm b})\Delta^2 + O(\delta^4) + O(\Delta^4) + \cdots ,$$

where $E_0(n_{\rm b}) = \frac{E}{A}(n_{\rm b}, \Delta = 0, \delta = 0)$,

$$S_0(n_{\rm b}) = S(n_{\rm b}, \Delta{=}0), \quad \text{and } W_0(n_{\rm b}){=}W(n_{\rm b}, \delta = 0). \tag{11}$$

By comparing the full HF result for E/A and that given by the parabolic approximation in the expansion (11), we found that the contributions from the quartic and higher orders in δ and

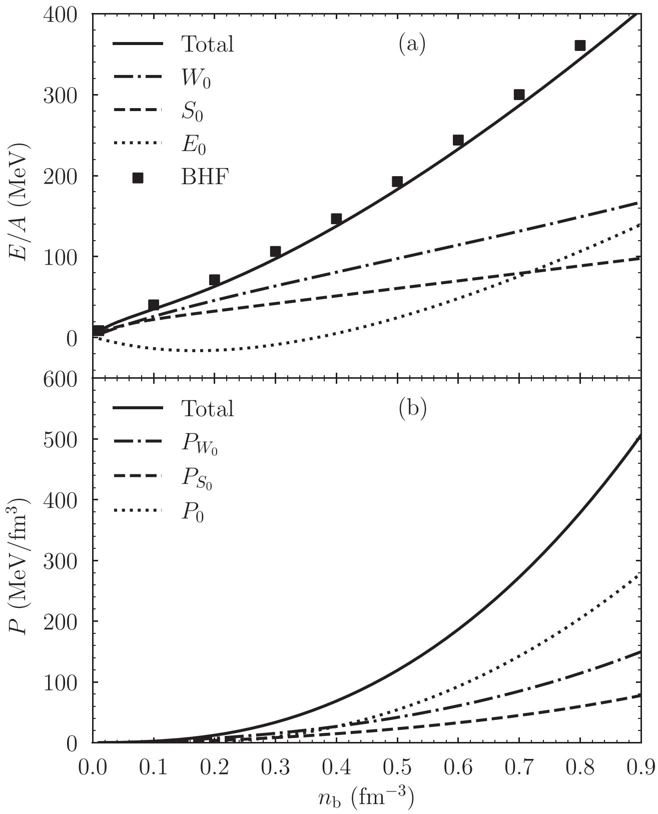

FIG. 5. (a) Energy per baryon (11) of the fully spin-polarized neutron matter given by the HF calculation using the CDM3Y8 interaction (solid line), and that given by the BHF calculation (squares) [23]. The energy per baryon E_0 of the spin-unpolarized symmetric NM, nuclear symmetry energy S_0 and spin symmetry energy W_0 are shown as dotted, dashed, and dash-dotted lines, respectively. (b) The total baryonic pressure of the fully spin-polarized neutron matter (solid line) in terms of the contributions obtained separately from E_0, S_0, and W_0.

Δ are negligible over baryon densities up to $n_\mathrm{b} \approx 0.9$ fm^{-3}. This important feature shows a close similarity between the spin symmetry and isospin symmetry in the mean-field study of the spin-polarized NS matter. The explicit contributions of the spin-symmetry and isospin-symmetry energies to the EOS of the fully spin-polarized neutron matter ($\Delta = \delta = 1$) are illustrated in Fig. 5, and one can see that $W_0(n_\mathrm{b})$ has about the same strength as that of $S_0(n_\mathrm{b})$ at low baryon densities $n_\mathrm{b} \lesssim 0.1$ fm^{-3}. However, the spin symmetry energy becomes much stronger with increasing n_b, and W_0 is nearly double the nuclear symmetry energy S_0 at high baryon densities, which significantly stiffens the EOS of neutron matter. It can be seen in the lower panel of Fig. 5 that the baryonic pressure of the fully spin-polarized neutron matter is also about twice that of the spin-unpolarized neutron matter. We note that the total energy per baryon (11) of the fully spin-polarized neutron matter given by the present HF calculation is quite close to that given by the microscopic BHF calculation [23] using the Argonne V18 potential supplemented by a realistic three-body force (see upper panel of Fig. 5). The nuclear symmetry energy $S_0(n_\mathrm{b})$ given by the HF calculation is also close to that given by the *ab initio* calculations [24,25] (see upper panel of Fig. 3). Consequently, the expansion (11) should be of interest for the mean-field studies of the spin-polarized NS matter, where some estimate of the spin symmetry energy $W_0(n_\mathrm{b})$ can be done based on the expansion (10) using parameters given in Table II.

III. EQUATION OF STATE OF SPIN-POLARIZED NEUTRON STAR MATTER IN β EQUILIBRIUM

The HF approach (1)–(4) describes NM that contains nucleons only. In fact, the NS matter contains significant lepton fraction in both the crust and uniform core, and a realistic EOS of NS matter should include the lepton contribution. For the inhomogeneous NS crust, we have adopted the EOS given by the nuclear energy density functional calculation [37,38] using the BSk24 Skyrme functional, with atoms being fully ionized and electrons forming a degenerate Fermi gas. At the edge density $n_\mathrm{edge} \approx 0.06$ fm^{-3}, a weak first-order phase transition takes place between the NS crust and the uniform core of the NS. At baryon densities $n_\mathrm{b} \gtrsim n_\mathrm{edge}$, the core of NS is described as a homogeneous matter of neutrons, protons, electrons, and negative muons (μ^- appear at n_b above the muon threshold density $\mu_e > m_\mu c^2 \approx 105.6$ MeV). The total mass-energy density $\mathcal{E}$ of the spin-polarized $npe\mu$ matter is determined as

$$\mathcal{E}(n_n, n_p, \Delta, n_e, n_\mu) = \mathcal{E}_\mathrm{HF}(n_n, n_p, \Delta) + n_n m_n c^2 + n_p m_p c^2 + \mathcal{E}_e(n_e) + \mathcal{E}_\mu(n_\mu), \quad (12)$$

where $\mathcal{E}_\mathrm{HF}(n_n, n_p, \Delta)$ is the HF energy density (4) of the spin-polarized baryonic matter, and $\mathcal{E}_e$ and $\mathcal{E}_\mu$ are the energy densities of electrons and muons given by the relativistic Fermi gas model [39]. In such a Fermi gas model, the spin polarization of leptons does not affect the total energy density $\mathcal{E}$, and the lepton number densities n_e and n_μ are determined from the charge neutrality condition ($n_p = n_e + n_\mu$), and the β-equilibrium of (neutrino-free) NS matter is sustained by the balance of the chemical potentials

$$\mu_n = \mu_p + \mu_e \text{ and } \mu_e = \mu_\mu, \quad \text{where } \mu_j = \frac{\partial \mathcal{E}_j}{\partial n_j}. \quad (13)$$

The fractions of the constituent particles in the spin-polarized $npe\mu$ matter are determined at the given baryon density n_b as $x_j = n_j/n_\mathrm{b}$. Below the muon threshold density ($\mu_e < m_\mu c^2 \approx 105.6$ MeV) the charge neutrality condition leads to the following relation [16]:

$$3\pi^2(\hbar c)^3 n_\mathrm{b} x_p - \hat{\mu}^3 = 0, \quad \hat{\mu} = \mu_n - \mu_p = 2\frac{\partial}{\partial \delta}\left(\frac{\mathcal{E}_\mathrm{HF}}{n_\mathrm{b}}\right). \quad (14)$$

The proton fraction in the β-stable (spin-polarized) $npe\mu$ matter, $x_p(n_\mathrm{b}, \Delta)$, can then be obtained from the solution of Eq. (14). If we assume the parabolic approximation and neglect the contribution from higher-order terms in (7), then $x_p(n_\mathrm{b}, \Delta)$ is given by the solution of the well-known equation

$$3\pi^2(\hbar c)^3 n_\mathrm{b} x_p - [4S(n_\mathrm{b}, \Delta)(1 - 2x_p)]^3 = 0, \quad (15)$$

which shows the direct link between the nuclear symmetry energy $S(n_b, \Delta)$ and the proton abundance in the NS matter. Above the muon threshold, $\mu_e > m_\mu c^2 \approx 105.6$ MeV, it is energetically favorable for electrons to convert to negative muons, and the charge neutrality condition results in the following equation for $x_p(n_b, \Delta)$:

$$3\pi^2(\hbar c)^3 n_b x_p - \hat{\mu}^3 - [\hat{\mu}^2 - (m_\mu c^2)^2]^{3/2}\theta(\hat{\mu} - m_\mu c^2) = 0, \tag{16}$$

where $\theta(x)$ is the Heaviside step function. The proton fraction $x_p(n_b, \Delta)$ is known to correlate with the NS cooling rate. In particular, the direct Urca (DU) process of NS cooling via neutrino emission is possible only if x_p is above the DU threshold x_{DU} [16]:

$$x_{DU}(n_b) = \frac{1}{1 + \left[1 + r_e^{1/3}(n_b)\right]^3}, \tag{17}$$

where $r_e(n_b) = n_e/(n_e + n_\mu)$ is the leptonic electron fraction at the given baryon number density. At low densities $r_e = 1$ and $x_{DU} \approx 11.1\%$, which is the muon-free threshold for the DU process. Since the lepton-baryon interaction is neglected in the present study, x_{DU} depends very weakly on the spin polarization Δ of baryons. Because the nuclear symmetry energy $S(n_b, \Delta)$ increases steadily with the increasing spin polarization of baryons Δ (see Fig. 3), from Eqs. (14)–(16) one can expect the same trend for the proton fraction x_p of the spin-polarized β-stable $npe\mu$ matter. As shown in the lower panel of Fig. 6, x_p increases significantly with the increasing spin polarization of baryons, and it exceeds the DU threshold at densities $n_b \gtrsim 2n_0$ for the fully spin-polarized NS matter with $\Delta = 1$. It is also natural that the neutron-proton asymmetry δ decreases with increasing x_p, as shown in the upper panel of Fig. 6. The charge neutrality implies also an increasing electron fraction with the increasing x_p, up to 20%–30% when $\Delta \lesssim 1$. Such a high electron fraction was found in the blue kilonova ejecta following GW170817 [4–6], and suggested by Metzger *et al.* [7] to be of the magnetar origin. The EOS of the spin-polarized $npe\mu$ matter in β equilibrium is determined entirely by the mass-energy density $\rho_m(n_b, \Delta) = \mathcal{E}(n_b, \Delta)/c^2$ and the total pressure

$$P(n_b, \Delta) = n_b^2 \frac{\partial}{\partial n_b}\left[\frac{\mathcal{E}_{HF}(n_b, \Delta)}{n_b}\right] + P_e + P_\mu. \tag{18}$$

We show in Fig. 7 the total pressure (18) of the spin-polarized NS matter $P(n_b, \Delta)$ obtained with different Δ values from the HF calculation using the CDM3Y8 interaction over baryon densities up to above $6n_0$, in comparison with the empirical pressure given by the "spectral" EOS inferred from the Bayesian analysis of the GW signals of GW170817 at the 50% and 90% confidence levels [15]. One can see the substantial impact by the spin symmetry energy (8) with the increasing Δ, which stiffens the EOS and compresses nucleonic matter inside the NS core to the higher pressure over the whole range of densities. It is noticeable that $P(n_b, \Delta)$ obtained with $0.8 \lesssim \Delta \lesssim 1$ overestimates the empirical constraint at the baryon densities within the range $0.05\, n_0 \lesssim n_b \lesssim 2n_0$.

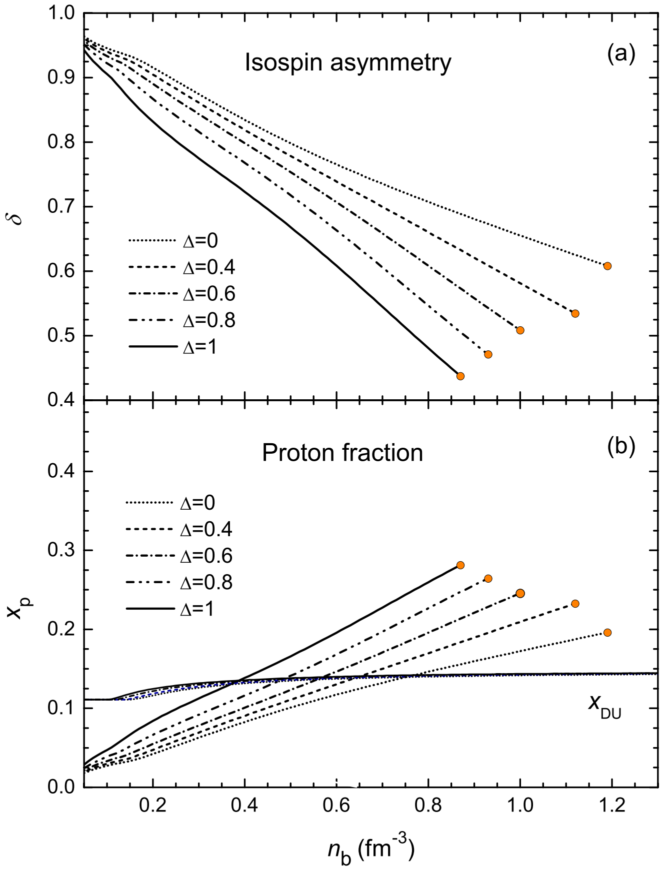

FIG. 6. (a) The isospin asymmetry δ and (b) proton fraction x_p of β-stable $npe\mu$ matter at different spin polarizations Δ of baryons given by the HF calculation (12)–(15) using the CDM3Y8 interaction. The circles are δ and x_p values obtained at the maximum central densities n_c, and the thin lines are the corresponding DU thresholds (17).

IV. TIDAL DEFORMABILITY, MASS, AND RADIUS OF NEUTRON STAR

The interesting effect inferred from the GW170817 observation is the tidal deformation of NS induced by the strong gravitational field which enhances the GW emission and accelerates the decay of the quasicircular inspiral [15]. We recall briefly the tidal deformability of a static spherical star being exposed to the gravitational field created by the attraction of the companion star in a binary system [12,13]. At the range close enough, this star is tidally deformed and gains a nonzero quadrupole moment Q_{ij} that is directly proportional to the strength E_{ij} of the gravitational field,

$$Q_{ij} = -\lambda E_{ij}. \tag{19}$$

λ characterizes the star response to the gravitational field and is dubbed as the *tidal deformability* or quadrupole polarizability of the star. In general relativity, λ is related to the $l = 2$ tidal Love number k_2 [12] as

$$\lambda = \frac{2}{3G} k_2 R^5, \tag{20}$$

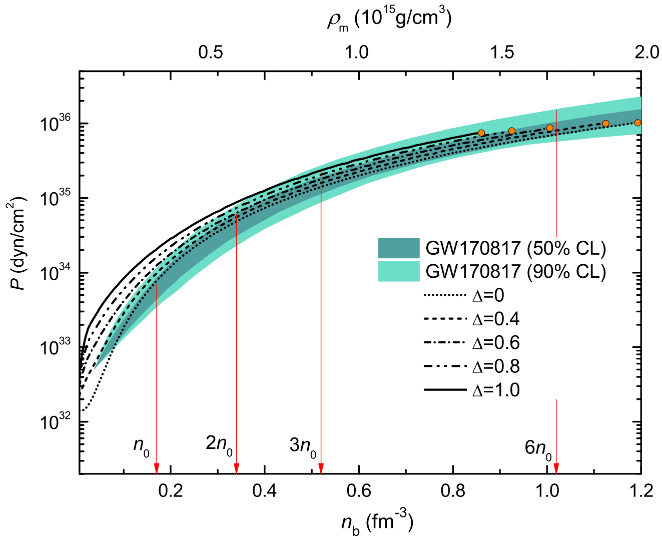

FIG. 7. The total pressure (18) inside the core of NS at different spin polarizations Δ given by the HF calculation using the CDM3Y8 interaction and over the range of the mass-energy (ρ_m) and baryon-number (n_b) densities. The dark and light shaded regions are the empirical constraints given by the "spectral" EOS inferred from the Bayesian analysis of the GW170817 data at the 50% and 90% confidence levels, respectively [15]. The circles are $P(n_c, \Delta)$ values at the corresponding maximum central densities n_c.

where R and G are the star radius and gravitational constant, respectively. It is convenient to consider the (dimensionless) tidal deformability parameter Λ [15] expressed in terms of the compactness C of star with mass M and radius R as

$$\Lambda = \frac{2}{3}k_2 C^{-5}, \quad \text{with } C = \frac{GM}{Rc^2}. \tag{21}$$

Using the linearized Einstein equation, the Love number k_2 can be expressed in terms of the nonzero metric perturbation of the stress-energy tensor $H(r)$ and its radial derivative $H'(r)$ [12,13], which are determined from the solution of a differential equation that is integrated together with the Tolman-Oppenheimer-Volkoff equations. More details on this computation can be found, e.g., in Ref. [10].

The tidal deformability and gravitational mass-radius of NS given by the EOS of the spin-polarized NS matter obtained with different Δ values from the HF calculation using the CDM3Y8 interaction are shown in Figs. 8 and 9, respectively. One can see that the heavier the NS, the smaller its tidal deformability, and the impact by the spin symmetry energy to Λ is very well revealed in Fig. 8, where only the Λ values given by the EOSs of the partially spin-polarized NS matter with $\Delta \lesssim 0.6$ are inside the empirical range implied by the GW170817 data at the 90% confidence level for NS with $M = 1.4\,M_\odot$ [15]. Like the tidal deformability, the NS mass and its radius comply with the GW170817 constraint for NS with mass $M = 1.4\,M_\odot$ [15] (the colored contours in Fig. 9) when the EOSs of the partially spin-polarized NS matter with $\Delta \lesssim 0.6$ are used for the input of the TOV equations. The stiffening of the EOS of the spin-polarized NS matter by the spin symmetry energy shown in Fig. 6 is well reflected in the

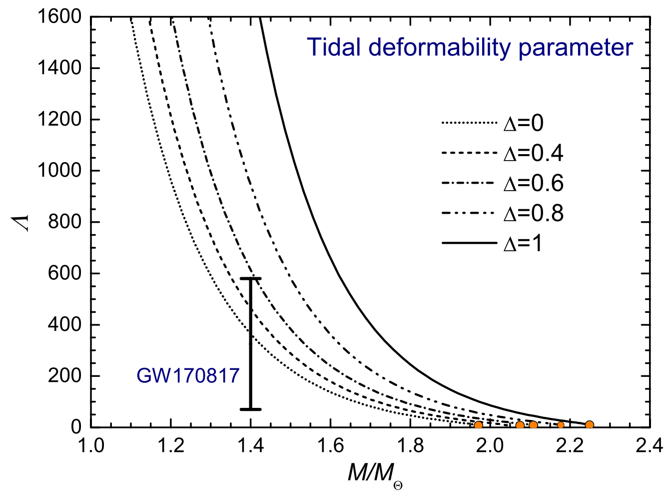

FIG. 8. The tidal deformability parameter (21) given by the EOS of the spin-polarized NS matter obtained with different Δ values from the HF calculation using the CDM3Y8 interaction. The vertical bar is the empirical Λ value for NS with $M = 1.4\,M_\odot$ inferred from the Bayesian analysis of the GW170817 data at the 90% confidence level [15], and the circles are Λ values obtained at the corresponding maximum central densities n_c.

calculated M-R values, and the obtained maximum masses M of NS (solid circles in Fig. 9) span the whole empirical range of the masses deduced for the second PSR J0348 + 0432 [40] and millisecond PSR J0740 + 6620 [41], the heaviest neutron

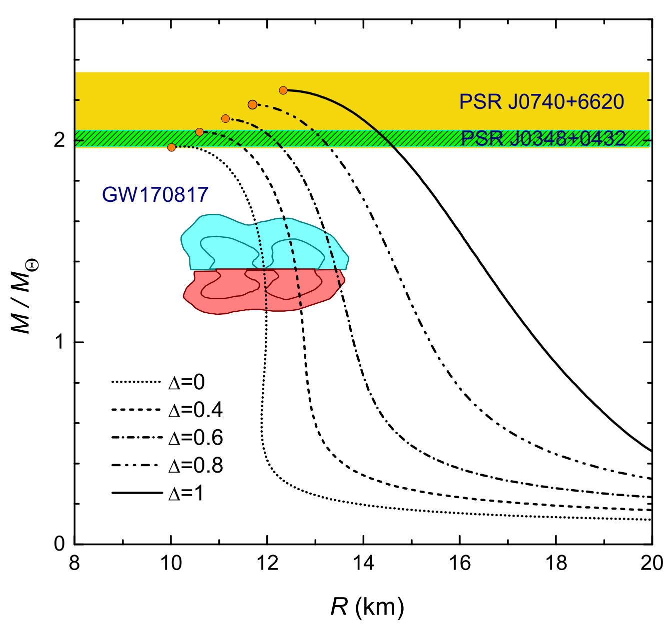

FIG. 9. The gravitational mass of NS versus its radius given by the EOS of the spin-polarized NS matter obtained with different Δ values from the HF calculation using the CDM3Y8 interaction. The colored contours are the GW170817 constraint for NS with mass $M = 1.4\,M_\odot$ [15], and the circles are M-R values calculated at the corresponding maximum central densities n_c. The shaded areas are the observed masses of the second PSR J0348 + 0432 [40] and millisecond PSR J0740 + 6620 [41].

stars observed so far. The results shown in Figs. 8 and 9 confirm that the GW170817 constraint excludes the full spin polarization of baryons ($\Delta = 0.8$–1) inside the core of NS, as pointed out earlier in Refs. [9,42].

A. Density dependence of spin polarization

The above results were obtained with the uniform spin polarization of baryons, which is independent of the baryon density n_b. However, nucleons inside the inner core of NS are known to be fully degenerate and occupy all possible quantum states allowed by the Pauli principle [43]. Such a full degeneracy exhausts all spin orientations of baryons and a spin polarization (or asymmetric spin orientation of baryons with $\Delta > 0$) is unlikely inside the inner core of NS. As a result, Δ inside the core of NS must depend on the baryon density. Moreover, the distribution of magnetic field inside magnetar was shown to be quite complex [8], and the spin polarization of baryons is expected to decrease gradually to $\Delta \approx 0$ in the central region of magnetar where the intensity of the magnetic field is diminishing to zero [8]. Although it is beyond the scope of the present mean-field approach to properly calculate the density profile $\Delta(n_b)$ of the spin polarization of baryons in magnetized NS, we try to explore this effect by assuming a realistic scenario for the density dependence of Δ based on the magnetic-field distribution in magnetar obtained by Fujisawa and Kisaka using the Green's function relaxation method [8].

Several simple scenarios of $\Delta(n_b)$ having its maximum at the surface and decreasing gradually to zero towards the center of NS were considered in Ref. [9], and it was shown that up to 60% of baryons might have their spins polarized during the NS merger GW170817. A more elaborate density dependence of $\Delta(n_b)$ is suggested in the present work, based on the spatial distribution of the magnetic flux Ψ derived in Ref. [8]. Namely, the radial distribution of Ψ from the star center to the surface (see Fig. 5 in Ref. [8]) has first been translated into the density profile of this quantity, then it is reasonable to assume the density profile of $\Delta(n_b)$ to have the same shape as that of $\Psi(n_b)$. In such a scenario, the spin polarization of baryons Δ and the intensity of magnetic field reach their maximum values at the same baryon density (around $2n_0$). The asymmetric spin orientation Δ is expected to decrease gradually to zero in the inner core of NS where baryons are believed to be fully degenerate [43]. To explore the impact of the spin symmetry energy, we have probed different maximum values of $\Delta(n_b)$ at its peak, as shown in Fig. 10. We show in Fig. 11 the gravitational mass and radius of NS given by the EOS of the β-stable spin-polarized NS matter obtained with different (density dependent) spin polarizations of baryons $\Delta(n_b)$ shown in Fig. 10. Following the trend of $\Delta(n_b)$, the strength of the spin symmetry energy (10) is also diminishing to zero in the inner core of NS, and the impact of the spin polarization of baryons on the maximum mass of magnetar becomes weaker compared with that shown in Fig. 9. Based on the suggested scenario for $\Delta(n_b)$, we found that the gravitational mass and radius of NS given by the EOS of the spin-polarized NS matter with $\Delta \lesssim 0.8$ are well within the empirical range implied for NS with $M = 1.4\,M_\odot$ [15]. In general, NM becomes less compressible [9] when the spin polarization of baryons is nonzero, and NS expands its size with the maximum mass M_{max} and radius R_{max} becoming larger with increasing Δ, as shown in Fig. 9. With the damping of the Δ strength in the inner core of NS shown in Fig. 10, the impact of the spin symmetry energy at high densities becomes less significant, and the enhancement of the NS mass and

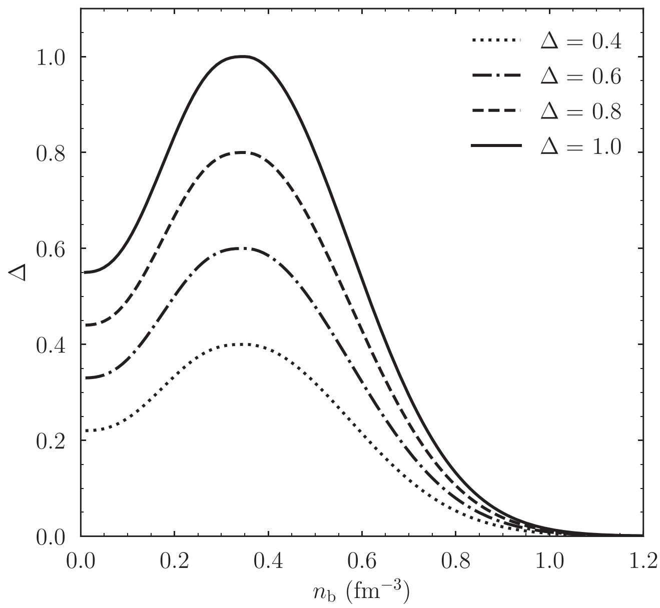

FIG. 10. Density dependence of the spin polarization Δ of baryons inside the core of NS that mimics the distribution of the magnetic field in NS obtained by Fujisawa and Kisaka using the Green's function relaxation method [8], with different maximum Δ values.

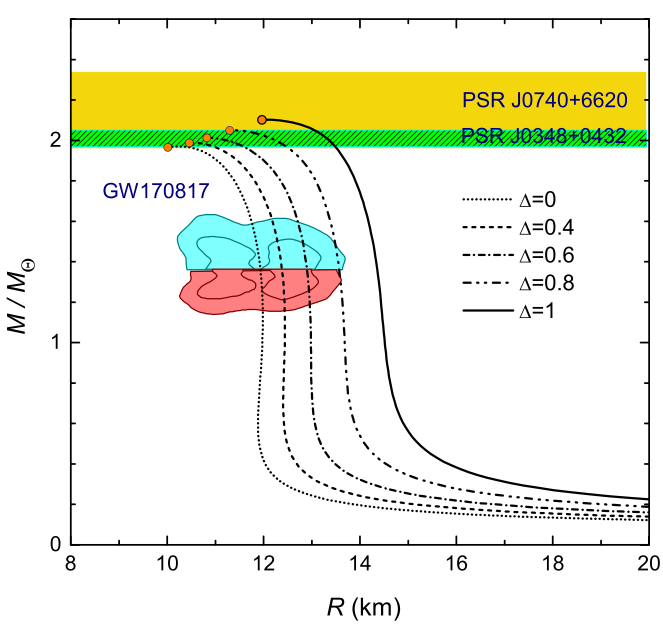

FIG. 11. The same as Fig. 9 but obtained with the density-dependent spin polarization $\Delta(n_b)$ of different strengths shown in Fig. 10.

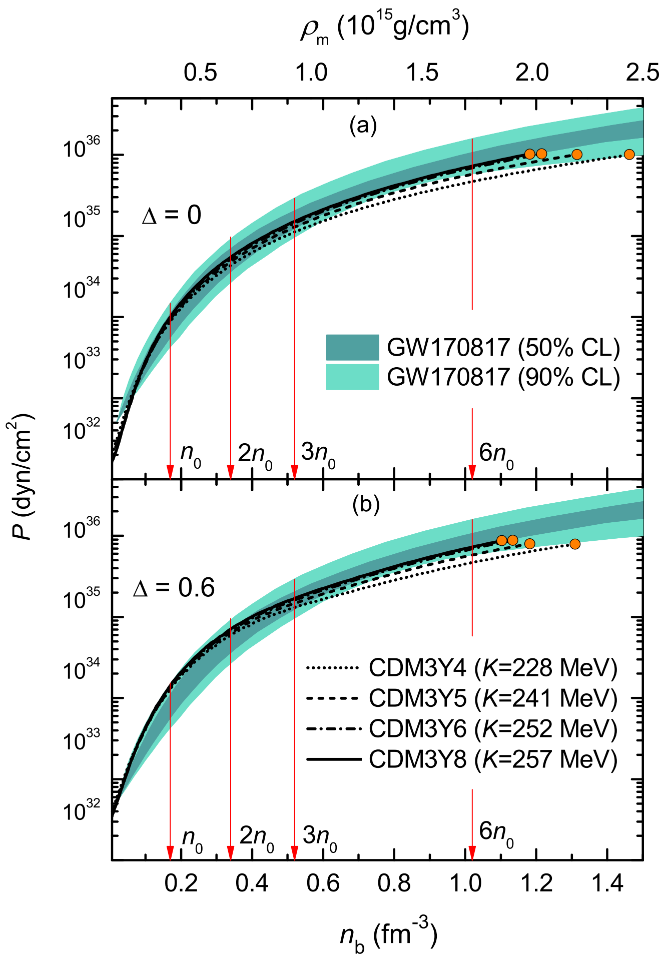

FIG. 12. The same as Fig. 7 but obtained with four versions of the CDM3Yn density dependent interaction (2)–(5) that are associated with four different values of the nuclear incompressibility K. (a) The results obtained for the spin-unpolarized NS matter. (b) The results obtained for the partially spin polarized NS matter with the maximum $\Delta = 0.6$ of the spin-polarization strength shown in Fig. 10.

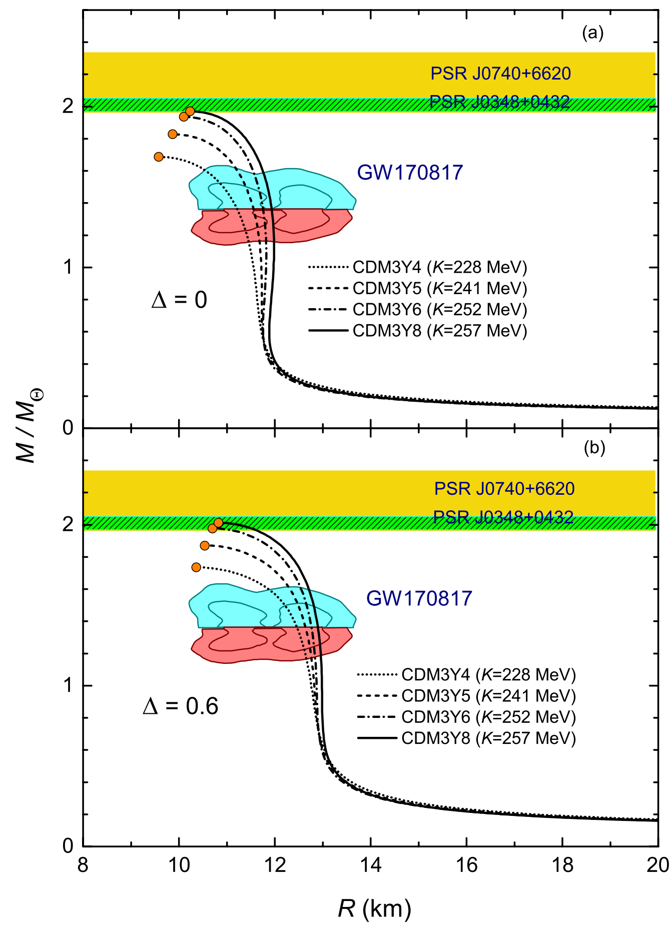

FIG. 13. The same as Fig. 9 but obtained with four versions of the CDM3Yn density dependent interaction (2)–(5) that are associated with four different values of the nuclear incompressibility K. (a) The results obtained for the spin-unpolarized NS matter. (b) The results obtained for the partially spin-polarized NS matter with the maximum $\Delta = 0.6$ of the spin-polarized fraction of baryons shown in Fig. 10.

radius (see Fig. 11) with the increasing maximum Δ value is not as drastic as shown in Fig. 9. At variance with the spin symmetry energy, the impact of the nuclear symmetry energy to the EOS of NS matter remains still strong at large baryon densities where the δ value sustained by the β equilibrium is up to 0.6 (see upper panel of Fig. 6). Thus, the nuclear symmetry energy $S(n_b)$ at high baryon densities in the inner core of NS, where the spin symmetry energy $W(n_b)$ decreases quickly to zero, is the most important input for the EOS of the β-stable NS matter.

B. Impact of nuclear incompressibility

Although the impact of the nuclear incompressibility on the EOS of NM is well known as shown in Fig. 1, it is of interest to explore explicitly this effect on the calculated macroscopic properties of NS. The total pressure (17) inside the uniform core of NS obtained with four versions of the CDM3Yn density-dependent interaction (2)–(5), associated with four different values of the nuclear incompressibility K, is shown in Fig. 12. One can see that the difference caused by different K values is significant at high baryon densities $n_b > 2n_0$ for both the spin-unpolarized and spin-polarized cores of NS. Such a difference in the pressure results in quite different maximum masses of NS, and the results shown in Fig. 13 suggest that a slightly stiffer EOS of NS matter associated with $K \approx 250$–260 MeV not only complies with the GW170817 constraints but also gives the maximum mass of NS close to $2M_\odot$, at the lower mass limit of the heaviest pulsars observed so far [40,41]. Although the partial spin polarization of baryons expands the size of NS and increases the radius $R_{1/4}$ up to about 1 km, the difference in the NS maximum masses up to $0.3\,M_\odot$ shown in the upper and lower panels of Fig. 13 is mainly due to the difference in the K values. In connection with these results, we note that Annala *et al.* have suggested recently [44] that the NS matter in the interior of massive NS with $M \approx 2\,M_\odot$ might contain a quark-matter core that contributes up to $0.25\,M_\odot$ to the total mass of NS. In any case, the present mean-field results are complementary to those of a joint analysis of the NICER and LIGO-Virgo data [45] that prefers a stiff EOS associated with the observed masses of the heaviest pulsars.

V. SUMMARY

The equation of state of the spin-polarized NM is studied within the HF formalism by using the realistic CDM3Yn density-dependent interaction. Given the nonzero spin polarization or asymmetric spin orientation of baryons, the spin- and spin-isospin dependent terms of the HF energy density give rise to the spin symmetry energy W, which behaves in a manner similar to that of the isospin- or nuclear symmetry energy S. The parabolic approximation is shown to be valid also for the spin symmetry energy, so that the (repulsive) contribution from the spin symmetry energy to the total NM energy is directly proportional to Δ^2. The EOS of NM becomes much stiffer with the increasing spin polarization of baryons, with the pressure given by the spin symmetry energy at high baryon densities being much larger than that given by the nuclear symmetry energy.

Like the nuclear symmetry energy (9), the density dependence of the spin symmetry energy can also be expressed (10) in terms of three quantities: the symmetry coefficient J_s, slope L_s, and curvature $K_{\rm syms}$. A close correlation of these characteristics with those of the nuclear symmetry energy S has been discussed; in particular, a very similar behavior of the slope parameters L and L_s of $S(n_b)$ and $W(n_b)$, respectively. The slope L of the nuclear symmetry energy depends strongly on the spin polarization Δ of baryons, and the slope L_s of the spin symmetry energy depends on the isospin polarization δ in exactly the same manner. The L values obtained at $0 \leqslant \Delta \leqslant 1$ comply well with the constraint inferred from the nuclear physics studies and astrophysical observations [34], but remain below the lower limit of L implied by the neutron skin of ^{208}Pb measured in the PREX-2 experiment [35,36].

With the EOS of the β-stable $npe\mu$ matter of NS obtained at different spin polarization of baryons, we found that the proton fraction x_p increases strongly with the increasing Δ, which should result in the larger probability of the direct Urca process in the cooling of the magnetar. The charge neutrality then implies an increasing electron fraction with the increasing x_p that might reach up to around 30%.

The stiffening of the EOS of the NS matter with the increasing spin polarization of baryons affects significantly the calculated tidal deformability as well as the gravitational mass and radius of NS. This effect of the spin symmetry energy is very strong if we assume a uniform (density-independent) spin polarization of baryons in both the outer and inner cores of NS. In such a scenario, the GW170817 constraint for the tidal deformability, mass and radius of NS with $M = 1.4 M_\odot$ [15] excludes the spin polarization of baryons inside the core of NS with $0.6 \lesssim \Delta \lesssim 1$.

A more realistic scenario of $\Delta(n_b)$ is further suggested, based on the distribution of magnetic field inside magnetar obtained in Ref. [8] and the full degeneracy of baryons in the inner core of NS [43], where Δ reaches its maximum in the outer core and decreases quickly to zero in the inner core of NS. By subjecting the mean-field results obtained at different spin polarizations of baryons in this scenario to the GW170817 constraint, we found that up to 80% of baryons in the outer core of NS might have their spins polarized during the NS merger.

The impact of the nuclear incompressibility K on the macroscopic properties of NS is shown clearly for both the spin saturated ($\Delta = 0$) and spin polarized ($\Delta \neq 0$) NS matter. While the M-R results given by four versions of the CDM3Yn interaction comply well with the GW170817 constraint deduced for NS with mass $M = 1.4 M_\odot$ [15], only the EOS associated with $K \approx 250$–260 MeV gives the maximum mass of NS close to $2M_\odot$, at the lower mass limit of the second PSR J0348 + 0432 [40] and millisecond PSR J0740 + 6620 [41], the heaviest neutron stars observed so far.

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