

Photoproduction of pion pairs at high energy and small angles

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We explore the photoproduction mechanisms for charged and neutral pion pairs off a heavy nucleus at threshold. We calculate the production of charged pairs in the Coulomb field of the nucleus in the Born approximation using explicit expressions for the differential cross sections and their connection with the total cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$. The production of σ mesons at threshold and their subsequent decay into pions is an important mechanism and is investigated quantitatively in a framework that can be used for both charged and neutral pions. σ mesons can be produced via electromagnetic (photon exchange) or strong (ω exchange) interactions. For neutral pions, another production mechanism consists of the two-step process where charged pions are produced first and then charge exchange into two neutral pions. Differential and total cross sections are computed for each production mechanism.

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I. INTRODUCTION

The photoproduction of charged pion pairs on the proton, $\gamma p \rightarrow \pi^+\pi^- p$, has been widely investigated beginning from the seminal experiments at SLAC [1]. At high energies $E_\gamma \gtrsim 3$ GeV, the pairs are primarily produced diffractively (Pomeron exchange) in the P wave (orbital momenta $L = 1$) producing $\rho(770)$ vector mesons and the $f_2(1270)$ tensor mesons (D wave), which are easily observable in the $\pi^+\pi^-$ invariant mass spectrum. In addition, the charged pion pairs may be produced by photon or vector (ω, ρ) meson exchanges with the target nucleus (Fig. 1). Such pairs are produced in the S wave ($L = 0$), resulting in scalar mesons: $f_0(500)$ and $f_0(980)$. The threshold region is dominated by the $f_0(500)$ meson, which we will refer to as the σ meson in this paper. The production of neutral pion pairs is a result of scalar meson production decaying to $\pi^0\pi^0$ [Figs. 2(a) and 2(b)] or the production of charged meson pairs with subsequent charge exchange [$\pi^+\pi^- \rightarrow \pi^0\pi^0$, Fig. 2(c)].

Understanding double pion photoproduction at threshold is crucial for experiments whose goal is to measure the pion polarizability using the Primakoff production of two pions [2,3]. In addition, it can also be an important background to single meson Primakoff production experiments of η, η' with the goal of determining their lifetimes with high precision [4]. Therefore, this paper presents the formalism and calculations for electromagnetic and strong production of two pions in the field of a nucleus. We start by estimating the contribution to the photoproduction cross section of charged pions in the Coulomb field and then present the relevant expressions that describe the production of neutral pion pairs.

II. PHOTOPRODUCTION OF CHARGED PION PAIRS IN THE COULOMB FIELD

The reaction of interest can be expressed as

$$\gamma(k) + A(p) \rightarrow \pi^+(q_+) + \pi^-(q_-) + A(p'), \quad (1)$$

where the four-vectors of each particle are given in parentheses. The differential cross section for the photoproduction of charged pion pairs in the Coulomb field of a nucleus (Fig. 1) can be obtained using the techniques from Ref. [5] for $M \gg 2m$, where m and M are the single pion and pion pair invariant masses:

$$d\sigma = \frac{8\alpha^3 Z^2}{(2\pi)^2} \frac{d^2q}{Q^4} (\bar{q}_-(\bar{q}_+^2 + m^2) + \bar{q}_+(\bar{q}_-^2 + m^2))^2 \times \frac{\beta_-(1 - \beta_-)d\beta_- d^2q_-}{(\bar{q}_-^2 + m^2)^2(\bar{q}_+^2 + m^2)^2} |F_{\text{em}}(q, q_L)|^2. \quad (2)$$

Here and below we are using the following notation:

$k = (E, \vec{k})$ are the incident photon four-momentum, energy, and three-momentum;

$k' = (E', \vec{k}')$ are the pion pair four-momentum, energy and three-momentum;

Ω, θ are the solid and polar angles of the produced pion pair;

$\vec{q}_{+,-}$ are the transverse parts of the positive and negative pion three-momenta relative to the incident photon direction;

$\vec{q} = \vec{q}_- + \vec{q}_+$ is the transverse part of the transfer momentum;

$q_L = \frac{M^2}{2k}$ is the longitudinal transfer momentum;

$F_{\text{em}}(q, q_L)$ is the nucleus electromagnetic form factor;

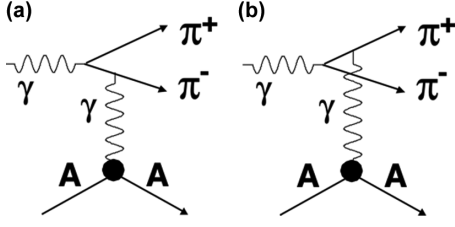


FIG. 1. Diagrams for the Primakoff production of charged pion pairs.

β_- is the fraction of photon energy taken by the π^- ;

Z is the charge of the nucleus;

$Q^2 = \vec{q}^2 + q_L^2$;

and $t = (k - k')^2$ is the Mandelstam variable.

In the Primakoff region, where the transverse nuclear recoil can be safely neglected, \vec{q}^2 is small and one can approximate $\vec{q}^2 \approx q_+^2$, the expression simplifies to

$$d\sigma = \frac{8\alpha^3 Z^2 \vec{q}^2 d^2 q \beta_- (1 - \beta_-) d\beta_- d^2 q_-}{(2\pi)^2 Q^4 (\vec{q}_-^2 + m^2)^2} |F_{\text{em}}|^2. \quad (3)$$

Recalling that the invariant mass of the pair $M^2 = \frac{\vec{q}_-^2 + m^2}{\beta_- (1 - \beta_-)}$ and replacing $d^2 q$ with πdt we obtain

$$\frac{d\sigma}{dt} = 2\alpha^3 Z^2 \frac{\vec{q}^2}{Q^4} \frac{dM^2}{M^4} |F_{\text{em}}|^2. \quad (4)$$

This expression was derived under the assumption of large pion pair mass M , where the asymptotic value for $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-) = 2\pi\alpha^2/M^2$. A more accurate calculation is obtained by using the full expression for the cross section as a function of pair invariant mass [6]:

$$\sigma(\gamma\gamma) = \frac{2\pi\alpha^2}{M^2} \left[\sqrt{1 - \frac{4m^2}{M^2}} \left(1 + \frac{4m^2}{M^2}\right) - \frac{4m^2}{M^2} \left(2 - \frac{4m^2}{M^2}\right) \right] \times \ln \left(\frac{M}{2m} + \sqrt{\frac{M^2}{4m^2} - 1} \right). \quad (5)$$

We note that the cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$ is affected by higher order corrections due to the polarizability (see for example Ref. [2]). This calculation is compared to data in Fig. 3 where the calculation is scaled by a factor of 0.6.¹ Equation 4 can be rewritten as

$$\frac{d\sigma}{dt} = \frac{2\alpha Z^2 \vec{q}^2}{\pi Q^4} \frac{dM}{M} \sigma(\gamma\gamma) |F_{\text{em}}|^2. \quad (6)$$

Equations (2)–(4) and (6) were obtained for $M \gg 2m$, but the differential cross section of the pair production can be expressed via the $\sigma(\gamma\gamma \rightarrow \pi\pi)$ as in Eq. (6) for any M values [6,7]. Equation (6) coincides with Eq. (5.49) in Ref. [6]. Finally, substituting dt with $\frac{|\vec{k}||\vec{k}'|}{\pi} d\Omega$, we obtain

$$\frac{d\sigma}{d\Omega dM} = \frac{2\alpha Z^2}{\pi^2} \frac{|\vec{k}|^2 |\vec{k}'|^2 \sin^2 \theta}{Q^4} \frac{1}{M} \sigma(\gamma\gamma) |F_{\text{em}}|^2. \quad (7)$$

¹This naive scaling factor accounts for the first-order correction due to the limited kinematic coverage of the experimental data.

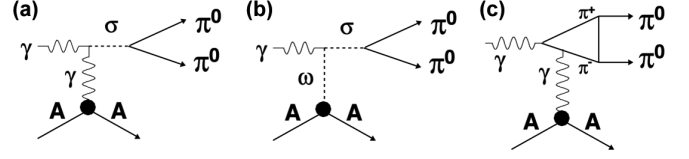


FIG. 2. Diagrams for the production of neutral pion pairs through (a) Primakoff σ -meson production and decay, (b) strong σ -meson production and decay, and (c) production via charge exchange.

Thus, the photoproduction of charged pion pairs in the Primakoff region has the same angular dependence as in the case of single π^0 meson photoproduction [8], while dependence on invariant mass of the pair M is determined by the cross section of the process $\gamma\gamma \rightarrow \pi^+\pi^-$.

III. NEUTRAL PION PAIR PHOTOPRODUCTION IN THE COULOMB FIELD

Neutral pion pairs can be produced via a two-step process: the photoproduction of scalar resonance f , followed by its decay to two pions [Fig. 2(a)]. The cross section of this process

$$\gamma(k) + A(p) \rightarrow f + A \rightarrow \pi^0 + \pi^0 + A(p') \quad (8)$$

can be written in the factorized form as

$$\frac{d\sigma}{dt dM^2} = \sum_f \frac{1}{\pi} \frac{d\sigma_f(t)}{dt} \frac{m_f \Gamma(f \rightarrow \pi^0\pi^0)}{(M^2 - m_f^2)^2 + m_f^2 \Gamma_{\text{tot}}^2}. \quad (9)$$

Here $\frac{d\sigma_f(t)}{dt}$ is the differential cross section of the scalar meson f off the nucleus, $\gamma + A \rightarrow f + A$; $\Gamma(f \rightarrow \pi^0\pi^0)$ and Γ_{tot} are its partial and total decay widths, respectively; M is the two-pion invariant mass, and m_f is f -meson mass. We adopt cross section normalization as $\int \frac{1}{\pi} \frac{d\sigma}{dt} dM^2 = B_r \frac{d\sigma}{dt}$, where B_r is the branching fraction of the f meson decay to $\pi^0\pi^0$. The use of the Breit-Wigner formula in Eq. (9) is a poor approximation for the $f_0(500)$ meson because of its large

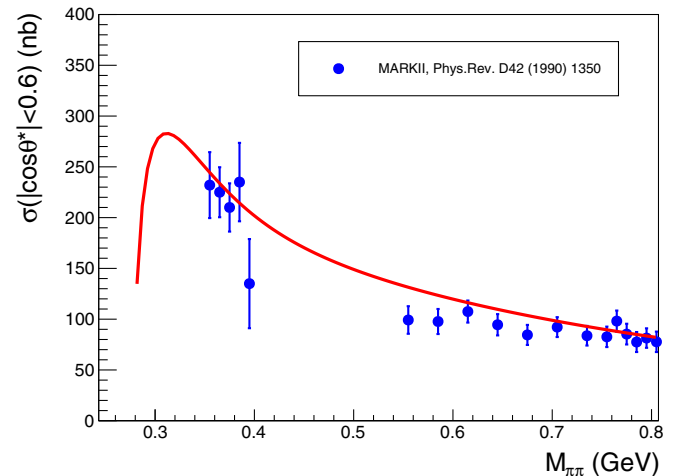


FIG. 3. The total cross section of the process $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$ [Eq. (5)], scaled by factor of 0.6. The points with errors are the experimental data for the solid angle range $|\cos(\theta^*)| < 0.6$ from Table III in [9].

width ($\Gamma_{\text{tot}} \approx 0.500$ GeV). However, this representation is illustrative and can be easily substituted with a more realistic description without affecting the basic results of the present work. We note that the $f_0(500)$, which is also referred to as the σ meson, dominates the cross section at small invariant masses [10].

The differential cross section of σ meson photoproduction in the nuclei Coulomb field is similar to π^0 photoproduction and reads

$$\frac{d\sigma_f}{dt} = \Gamma(f \rightarrow \gamma\gamma) \frac{8\pi\alpha Z^2\beta}{m_f^3} \frac{\vec{q}^2}{Q^4} |F_{\text{em}}|^2, \quad (10)$$

where β is the σ meson velocity [5,11]. Thus the photoproduction cross section for the σ meson has the same angular dependence as the well known [8] photoproduction cross section of the π^0 with replacement of relevant masses and decay widths. Substituting this expression in Eq. (9) we get the differential cross section of two- π^0 photoproduction in the Coulomb field of the nucleus:

$$\frac{d\sigma}{dt dM^2} = \frac{8\alpha Z^2\beta}{m_\sigma^2} \frac{\vec{q}^2}{Q^4} \frac{\Gamma(\sigma \rightarrow \gamma\gamma)\Gamma(\sigma \rightarrow \pi^0\pi^0)}{(M^2 - m_\sigma^2)^2 + m_\sigma^2\Gamma_{\sigma\text{tot}}^2} |F_{\text{em}}|^2. \quad (11)$$

This expression allows one to calculate the neutral pion pair photoproduction in the Coulomb field of the nucleus for any target, invariant mass of the pair, and transfer momentum values. The meson radiative width can be written as

$$\Gamma(f \rightarrow \gamma\gamma) = \left(\frac{m_f}{4\pi}\right)^3 \left(\frac{\alpha}{F_f}\right)^2 N_c^2 \langle e_f^2 \rangle^2, \quad (12)$$

where N_c is the number of colors, $\langle e_f^2 \rangle$ the average of relevant quark charges, and $F_\sigma = F_\pi = 92.4$ MeV is the charge meson decay constant. For the π^0 meson $N_c \langle e_\pi^2 \rangle = 3[(2/3)^2 - (1/3)^2] = 1$, whereas for the σ meson $N_c \langle e_\sigma^2 \rangle = 3[(2/3)^2 + (1/3)^2] = \frac{5}{3}$. The radiative decay width $\Gamma(\sigma \rightarrow \gamma\gamma)$ calculated by this expression yields $\Gamma(\sigma \rightarrow \gamma\gamma) = 1.1$ keV. However, for our calculations we have adopted the more reliable estimate of $\Gamma(\sigma \rightarrow \gamma\gamma) = 2.05$ keV [12]. For the σ decay to $\pi^0\pi^0$ partial width, we use the value of $\Gamma(\sigma \rightarrow \pi^0\pi^0) = 0.210$ GeV [13].

IV. NEUTRAL PION PAIRS VIA THE CHARGE-EXCHANGE REACTION

As it was mentioned above, the $\pi^0\pi^0$ pair can be produced as a result of charged pion pair photoproduction followed by charge-exchange reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$. The expression in Eq. (6) for the differential cross section of charged pion pair photoproduction is also valid for the neutral pion pair photoproduction if one replaces the $\sigma(\gamma\gamma)$ with the total cross section $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ of the two-step process $\gamma\gamma \rightarrow \pi^+\pi^- \rightarrow \pi^0\pi^0$. This cross section has been calculated in the framework of the Chiral theory [14] and gives

$$\begin{aligned} \sigma(\gamma\gamma \rightarrow \pi^0\pi^0) &= \frac{\alpha^2}{8\pi^2} \left(1 - \frac{4m^2}{M^2}\right) \left(1 + \frac{m^2}{M^2} f(M^2)\right) \\ &\times \sigma(\pi^+\pi^- \rightarrow \pi^0\pi^0) \end{aligned}$$

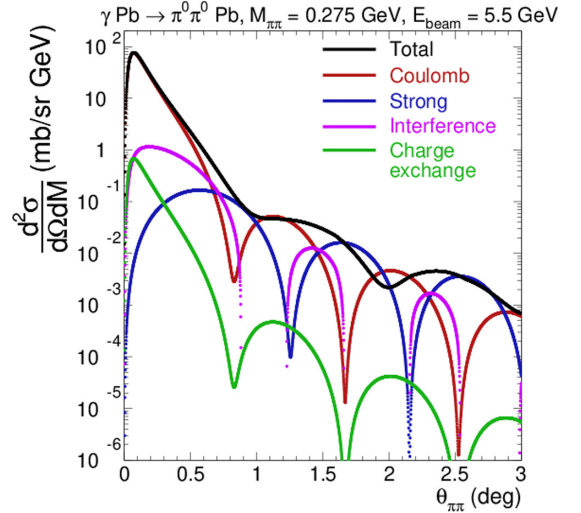


FIG. 4. Double differential cross section $\frac{d^2\sigma}{d\Omega dM}$ of two-neutral-pion photoproduction and its components as a function of two-pion production angle for the lead-208 nucleus, 0.275 GeV pion pair mass, and 5.5 GeV photon beam energy. Red: Coulomb cross section for the amplitude shown in Fig. 2(a); blue: strong cross section [Fig. 2(b)]; magenta: cross section from the interference of Coulomb and strong amplitudes for 1 radian interference angle; green: cross section for the charge exchange amplitude shown in Fig. 2(c); black: the total cross section, which is the sum of all contributions.

$$\sigma(\pi^+\pi^- \rightarrow \pi^0\pi^0) = \frac{1}{32\pi M^2 F_\pi^4} (M^2 - m^2)^2,$$

$$\text{where } f(M^2) = 2[\ln^2(z_+/z_-) - \pi^2]$$

$$+ \frac{m^2}{M^2} [\ln^2(z_+/z_-) + \pi^2]^2,$$

$$z_\pm = \frac{1 \pm (1 - 4m^2/M^2)^{1/2}}{2}. \quad (13)$$

Substituting this expression in Eq. (6), one obtains the differential photoproduction cross section of two neutral pions generated by the charge-exchange reaction. We note that this contribution is about 2 orders of magnitude smaller than the Coulomb production in the threshold region (see Fig. 4).

V. VECTOR MESON EXCHANGES IN $\pi^0\pi^0$ PHOTOPRODUCTION

The photoproduction of neutral pion pairs in S wave can also proceed through vector-meson exchange in addition to photon exchange. However, we only consider the ω -meson exchange contribution to the coherent photoproduction on nuclei because the contribution from isospin-1 exchange (ρ meson) is small and can be safely neglected. To estimate the contribution of ω exchange, we could proceed as in the case of π^0 meson photoproduction [15], by substituting $\Gamma(\sigma \rightarrow \gamma\gamma)$ with $\Gamma(\sigma \rightarrow \omega\gamma)$ and replacing the photon propagator with the ω propagator in Eq. (11). An alternative approach, which we implement here, is to use the simple parametrization from Ref. [16], which was used previously to calculate the strong

amplitude for the ω exchange contribution to π^0 photoproduction on the nucleon with $f_N(\gamma N \rightarrow \pi^0 N) = L(\frac{E}{k_0})^{1.2} \sin \theta$, where $L = 10\sqrt{\mu b}$, $k_0 = 1$ GeV [17]. For the strong amplitude of σ -meson photoproduction, one can follow the same procedure in Ref. [16] by scaling the amplitude with the coefficient of 5/3 from Eq. (12). With this modification, the strong part of the differential cross section of σ -meson photoproduction off a nucleus with atomic mass A in the Primakoff region reads as follows:

$$\frac{d\sigma_{\text{str}}}{dt} = A^2 \frac{25\pi}{9E^2} f_N^2 |F_{\text{str}}(q, q_L)|^2 \quad (14)$$

The full differential cross section for neutral pion pair photoproduction can be written similarly to the case of π^0 photoproduction in the Primakoff region:

$$\begin{aligned} \frac{d\sigma}{dt dM^2} &= \frac{1}{\pi} \left(\frac{d\sigma_{\text{em}}}{dt} + \frac{d\sigma_{\text{str}}}{dt} + \sqrt{\frac{d\sigma_{\text{em}}}{dt} \frac{d\sigma_{\text{str}}}{dt}} \cos \varphi \right) \\ &\times \frac{m_\sigma \Gamma(\sigma \rightarrow \pi^0 \pi^0)}{(M^2 - m_\sigma^2)^2 + m_\sigma^2 \Gamma_{\text{tot}}^2}, \end{aligned} \quad (15)$$

where $\frac{d\sigma_{\text{em}}}{dt}$ and $\frac{d\sigma_{\text{str}}}{dt}$ are given by the expressions in Eqs. (10) and (14) and φ is the relative phase between the Coulomb and strong amplitudes. The difference of the strong form factor $F_{\text{str}}(q, q_L)$ from the π^0 case [16] is minor. One has to replace the pion mass in the longitudinal momenta transfer q_L with the σ meson mass and also replace the total cross section on the nucleon, $\sigma(\pi N) \rightarrow \sigma(\sigma N)$.

We briefly digress to elaborate on some of our assumptions and approximations, which are at the level of about 10% for the strong cross section. First, we do not take into account the shadowing of the initial photon, noting that the effect is very small for the Primakoff cross section. For the strong cross section it is still a small correction; for example in the case of single-pion photoproduction [16] its effect is less than 10%. The final-state interactions of the produced meson are treated assuming that the σ meson travels through the nucleus with a cross section of $\sigma = 2\sigma(\pi N)$. However, a proper calculation will depend on the decay location, since the interaction of a single σ meson with the nucleon is different than that of two pions. Changes arise from different total cross sections and also due to differences in kinematics. Within the quark model, $\sigma(\sigma N) = \frac{5}{3}\sigma(\pi N)$, which is close to $2\sigma(\pi N)$. Given the limits of our knowledge of most cross sections, incorporating these details will not lead to a significantly more reliable result. However, we can estimate the size of these approximations by recalculating with the assumption that the σ meson decays outside the nucleus and $\sigma(\sigma N) = \frac{5}{3}\sigma(\pi N)$; see Table I. The difference coming from these two input values results in changes to the strong cross section of about 15%, depending on the dipion mass. The Primakoff cross section changes by less than 3%. The full effect of absorption on the Coulomb and strong form factors can be seen in Figs. 8 and 10, respectively.

Within this context, the electromagnetic and strong form factors of the nucleus read as follows [16]:

$$F_{\text{em}}(q, q_L) = 2\pi \frac{q^2 + q_L^2}{q} \int_0^\infty J_1(qb) b^2 db$$

TABLE I. Percent changes to the cross section of $\sigma(\gamma\text{Pb} \rightarrow \pi\pi\text{Pb})$ for two assumptions: (1) The σ meson decays at the interaction point and the cross section $\sigma(\sigma N) = 2\sigma(\pi N)$ and (2) the σ meson decays outside the nucleus and the cross section $\sigma(\sigma N) = (5/3)\sigma(\pi N)$. The calculations were completed for two different values of the dipion mass.

Cross section affected	$M_{\pi\pi} = 270$ MeV	$M_{\pi\pi} = 770$ MeV
Coulomb	1.3%	3%
Strong	17%	14%
Interference	10%	7%

$$\begin{aligned} &\times \int_{-\infty}^{+\infty} \frac{e^{iq_L z}}{(b^2 + z^2)^{3/2}} dz \\ &\times \exp\left(-\frac{\sigma'A}{2} \int_z^\infty \rho(b, z') dz'\right) \\ &\times \int_0^{\sqrt{b^2+z^2}} x^2 \rho(x) dx, \end{aligned} \quad (16)$$

$$\begin{aligned} F_{\text{str}}(q, q_L) &= -\frac{2\pi}{q} \int_0^\infty J_1(qb) b db \int_{-\infty}^{+\infty} \frac{\partial \rho(b, z)}{\partial b} e^{iq_L z} dz \\ &\times \exp\left(-\frac{\sigma'A}{2} \int_z^\infty \rho(b, z') dz'\right). \end{aligned} \quad (17)$$

Here $\rho(b, z)$ is the nucleus density; $J_1(x)$ is the first order Bessel function; $\sigma' = \sigma_{\text{tot}}(\sigma N)(1 - i\frac{\text{Re}f(0)}{\text{Im}f(0)})$; $f(0)$ is the forward amplitude of $\sigma + N \rightarrow \sigma + N$; the invariant transfer momenta $t = -q^2 - q_L^2 = -4|\mathbf{k}| |\mathbf{k}'| \sin^2(\frac{\theta}{2}) - (\frac{m_\sigma^2}{2E})^2$. Substituting expressions from Eqs. (10) and (14) for the electromagnetic and strong cross sections into Eq. (15) one can calculate the momentum transfer and invariant mass dependence of the differential cross section for double π^0 production off a nucleus.

Figures 4–6 show differential cross sections as a function of the pion pair production angle and invariant mass obtained with the presented calculations for the lead-208 nucleus and a 5.5 GeV photon beam energy. For numerical calculations, we take the interference angle between the Coulomb and strong mechanisms to be $\phi = 1$ rad, which is close to the value measured for single π^0 production [18].

VI. SUMMARY

In this paper we presented explicit calculations for the most important production mechanism for pion pairs at forward angles and at threshold. The production of $\pi^+\pi^-$ pairs was calculated in the Born approximation. The production of $\pi^0\pi^0$ pairs at threshold is dominated by the production and decay of the σ meson, which has been calculated for electromagnetic (photon exchange) and strong (ω exchange) mechanisms. An additional production mechanism for neutral pion pairs proceeds via the two-step process where two charged pions are produced and then rescatter into two neutral pions, which is found to be approximately 2 orders of magnitude smaller than the Coulomb contribution in the threshold region. Differential

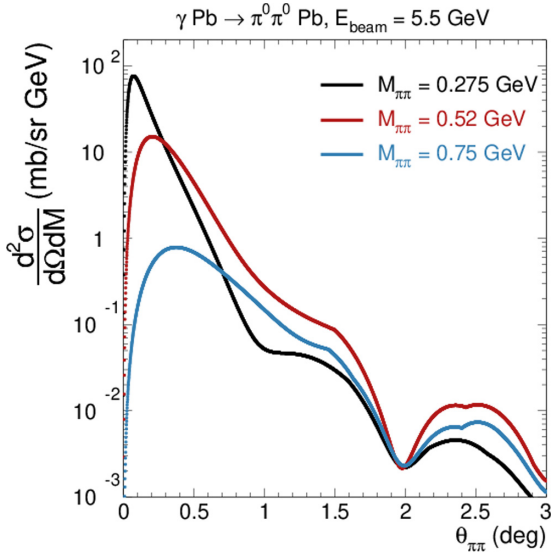


FIG. 5. Total double differential cross section $\frac{d^2\sigma}{d\Omega dM}$ of neutral pair pion photoproduction as a function of the pion pair production angle for the lead-208 nucleus, 5.5 GeV photon beam energy, and interference angle between Coulomb and strong mechanisms of 1 rad. Black curve: 0.275 GeV pion pair mass; red: 0.52 GeV; blue: 0.75 GeV.

and total cross sections are computed for each production mechanism using realistic form factors in all cases.

Recently there have been several experimental programs aimed at measuring mesons at very small production angles in photoproduction in Hall D at Jefferson Lab. An ongoing ex-

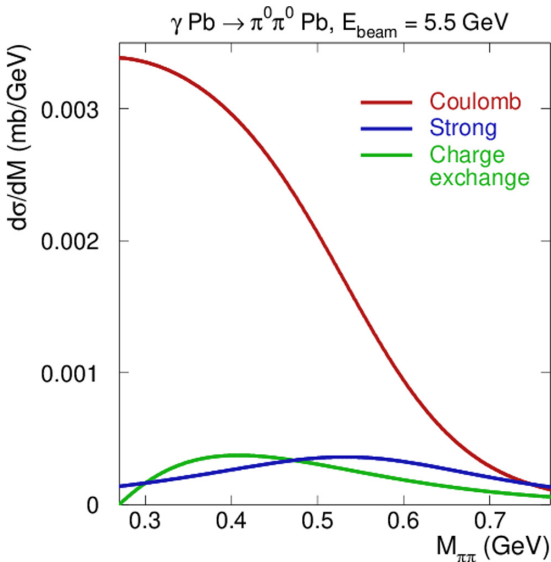


FIG. 6. Components of the differential cross section $\frac{d\sigma}{dM}$ of neutral pion pair photoproduction as a function of the two-pion invariant mass for the lead-208 nucleus, 5.5 GeV photon beam energy (integrated over production angle). Red: Coulomb cross section for the amplitude shown in Fig. 2(a); blue: strong cross section [Fig. 2(b)]; green: cross section for the charge exchange amplitude shown in Fig. 2(c).

periment is measuring the Primakoff production of η mesons with the goal of determining their lifetime to high precision [4]. Experiments to measure the Primakoff production of charged [2] and neutral [3] pion pairs are scheduled to run at Jefferson Lab in the near future. Therefore, quantitative estimates of the production of pion pairs at threshold and small angles will be important for the experimental program. In the first case these pairs represent background and in the second case they are the signal.

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APPENDIX: NUMERICAL CALCULATIONS AND VISUALIZATION

1. Primakoff cross section

The Primakoff cross section proceeds via photon exchange for the process $\gamma(k) + A(p) \rightarrow f + A \rightarrow \pi^0 + \pi^0 + A(p')$. Let us calculate the two-dimensional distribution of \vec{q}^2, M^2 using the expression

$$\frac{d\sigma}{d\Omega dM} = \frac{2M|\vec{k}||\vec{k}'|}{\pi} \frac{d\sigma}{dt dM^2}, \quad (\text{A1})$$

where $d\sigma/dt dM^2$ is given by Eq. (11). We have used the following numerical values in the calculations:

$$\beta = 1, \quad m_\sigma = 0.5 \text{ GeV}, \quad Q^2 = \vec{q}^2 + q_L^2,$$

$$\vec{q}^2 = 4|\vec{k}|^2|\vec{k}'|^2 \sin^2 \theta/2 \approx (E\theta)^2, \quad q_L = \frac{m_\sigma^2}{2k},$$

$$0.275 \leq M \leq 0.675 \text{ GeV}, \quad 0 \leq q^2 \leq 0.01 \text{ GeV}^2,$$

$$\Gamma_{\sigma\pi\alpha} = 0.5 \text{ GeV}, \quad \Gamma(\sigma \rightarrow \gamma\gamma) = 2.05 \times 10^{-6} \text{ GeV},$$

$$\Gamma(\sigma \rightarrow \pi^0\pi^0) = 0.21 \text{ GeV}.$$

The Coulomb double differential cross section $d^2\sigma/d\Omega dM$ is plotted in Fig. 7 and the electromagnetic form factor is given by Eq. (16). The upper limit of integration goes to infinity since the Coulomb field covers all space. For numerical calculations we note that outside of the nucleus the absorption exponent is a constant. Thus we can take the exponent term outside the integral and this allows us to use the following analytical formulas to integrate the remaining function:

$$\int_0^\infty \frac{\cos(ax)}{(b^2 + x^2)^{3/2}} dx = \frac{a}{b} K_1(ab),$$

$$\int_0^\infty \frac{\sin(ax)}{(b^2 + x^2)^{3/2}} dx = \frac{\pi}{2} \frac{a}{|b|} \{L_{-1}(ab) - I_1(|ab|)\},$$

where L_{-1} is the modified Struve function, and K_1 and I_1 are the corresponding Bessel functions. In this approach, we will need to subtract the difference between this constant term and the actual value of the absorption exponent within the

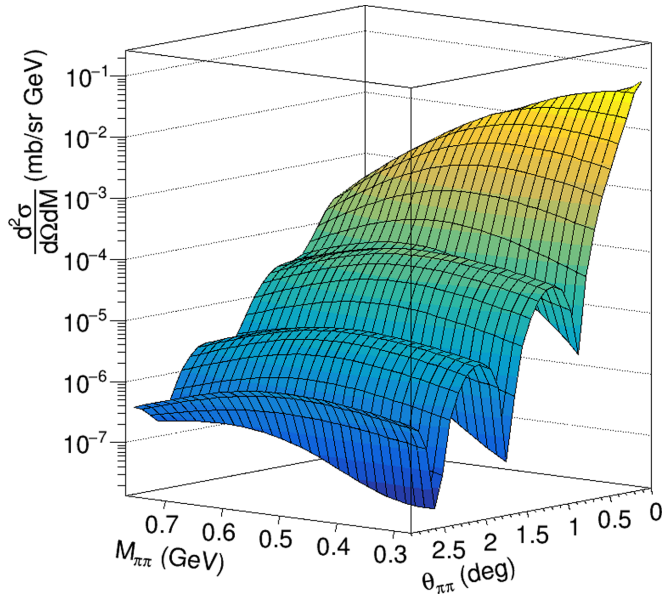


FIG. 7. Double differential cross section $\frac{d^2\sigma}{d\Omega dM}$ of the Coulomb neutral pion pair photoproduction as a function of the pair production angle and invariant mass for the lead-208 nucleus, and 5.5 GeV photon beam energy.

nucleus, which has a limited size and thus is convenient for the purpose of numerical calculations. Using this method it is also easy to estimate the necessary region of integration for the variable b to get precise form factor values. For large pion

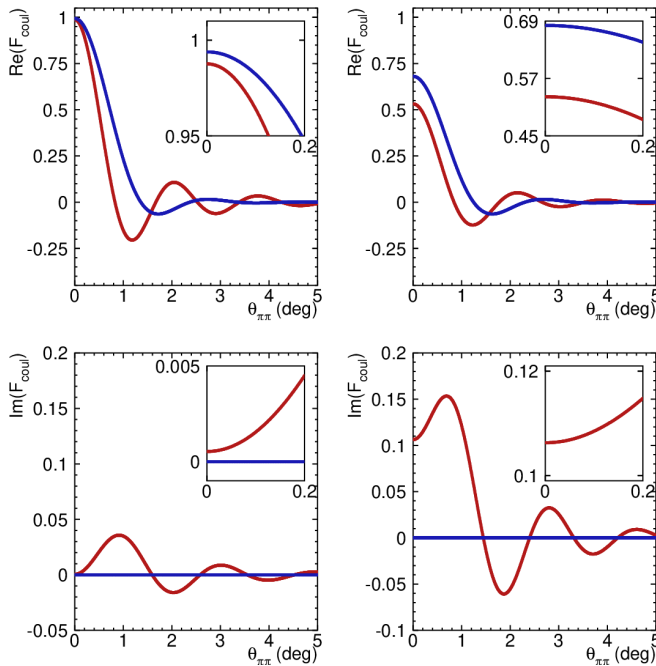


FIG. 8. Real (top plots) and imaginary (bottom plots) part of the Coulomb form factor as a function of production angle for the invariant mass values 0.275 GeV (left plots) and 0.765 GeV (right plots). Red curves; full calculation; blue curves; no absorption. Insets: The function behavior near the zero is zoomed in.

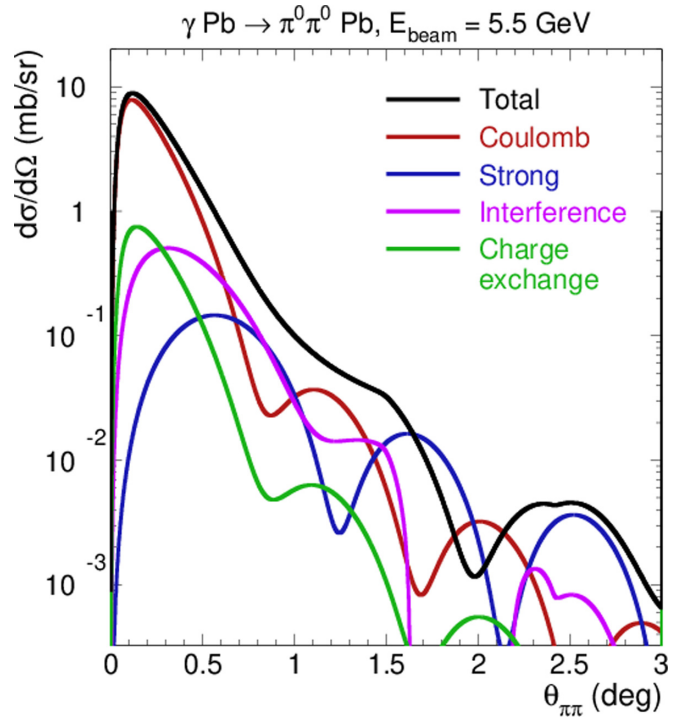


FIG. 9. Differential photoproduction cross section $\frac{d\sigma}{d\Omega}$ of neutral pion pairs for each component as a function of two-pion production angle for a lead-208 nucleus, 5.5 GeV photon beam energy, and integrated over the pion pair mass range between 0.27 and 0.77 GeV. Red: Coulomb part; blue: strong; magenta: their interference for 1 radian interference angle (negative values are omitted on the logarithmic scale); green: charge exchange; black: total sum.

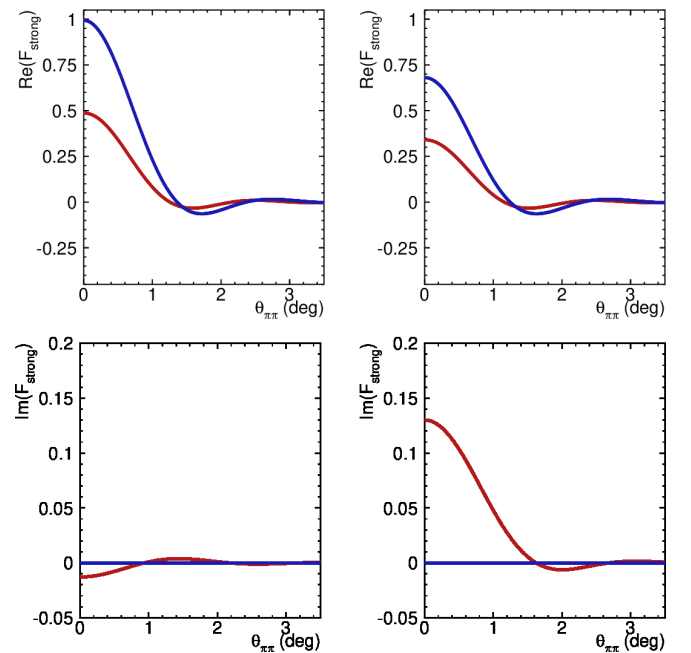


FIG. 10. Real (top plots) and imaginary (bottom plots) part of the strong form factor as a function of production angle for the invariant mass values: 0.275 GeV (left plots) and 0.765 GeV (right plots). Red curves: full calculation; blue curves: no absorption.

pair masses, we need to integrate b up to about 50–80 fm for convergence. For the pion pair masses near threshold, we need to integrate to about 700–1000 fm to achieve a similar level of convergence.²

The plots in Fig. 8 show the real and imaginary parts of the Coulomb form factor as a function of the production angle for $M_{\pi\pi} = 0.275$ and 0.765 GeV with and without including absorption. As seen from the plots, the presence of the absorption not only decreases the value of the real part of the form factor values but also enhances the oscillatory behavior. We note that the interaction cross section in the absorption term is actually a function of the pion energy and depends on the energy split between the pions in the pair. Including this dependence properly could change our calculations by more than a percent.

²Electron screening may play a role at the subpercent level at such large distances from the nucleus.

2. Electromagnetic and strong cross sections

The electromagnetic and strong cross sections integrated over invariant mass are

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{|\vec{k}||\vec{k}'|}{\pi} \frac{d\sigma}{dt}, \\ \frac{d\sigma}{dt} &= \frac{d\sigma_{\text{em}}}{dt} + \frac{d\sigma_{\text{str}}}{dt} + \sqrt{\frac{d\sigma_{\text{em}}}{dt} \frac{d\sigma_{\text{str}}}{dt}} \cos\varphi, \end{aligned} \quad (\text{A2})$$

where the expressions for the two components can be found in Eqs. (10) and (14). They are computed for $E = 5.5$ GeV, and the strong form factor is given by Eq. (17). For the pion pair invariant mass integration range of 0.27–0.77 GeV, the resulting cross section $\frac{d\sigma}{d\Omega} = \int_{0.27 \text{ GeV}}^{0.77 \text{ GeV}} \frac{d^2\sigma}{dM d\Omega}$ shown in Fig. 9 for each term separately as well as for the total. The strong form factor as a function of production angle value is plotted in Fig. 10.

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