Λ*nn* bound state and resonance

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We perform the *ab initio* no-core shell model (NCSM) calculation to investigate the bound state problem of the three-body Λ*nn* system in chiral next-to-next-to-leading-order *NN* and chiral leading-order *YN* interactions. The calculations show that no Λ*nn* bound state exists, but predict a low-lying Λ*nn* resonant state near the threshold with an energy of $E_r = 0.124$ MeV and a width of about $\Gamma = 1.161$ MeV. In searching for Λ*nn* resonances, we extend the NCSM calculation to the continuum state by employing the *J*-matrix formalism in the scattering theory with the hyperspherical oscillator basis.

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I. INTRODUCTION

Microscopic calculations of few- and many-body systems with strangeness have been a focus in hypernuclear physics to explore the new dynamical features of the structure of hypernuclei and to improve understanding of hyperon-nucleon interactions. Indeed, hyperon-nucleon scattering data is very limited for a full determination of the *YN* interactions. The existing data of few-body hypernuclei could provide an important constraint on *YN* interaction. In hypernuclear physics, the hypertriton is used as the first testing ground for *YN* interaction. It is the simplest and most weakly bound hypernuclear system with Λ binding energy about ≈ 0.13 MeV [1]. It seems like a lambda is bound to a deuteron core in the study of the spin-triplet *NN* interaction [2]. In the Λ*nn* system, two neutrons interact in the spin-singlet state and its strength is weaker than that in the spin-triplet state. The strength of Λ*n* is also not sufficient to form a bound system. One expects that the existence of a neutral bound state of two neutrons and a hyperon is improbable. But, instead, three-body Λ*nn* resonance may exist and that could be used to constrain the *YN* interaction. If the Λ*nn* system were the lightest neutron-rich bound system, it would provide significant information of Λ*n* interaction and a better understanding of the nature of Λ*N*-Σ*N* coupling.

There have been a number of theoretical calculations of the Λ*nn* system as a very unlikely bound state. Nonexistence of Λ*nn* bound state was first revealed by Dalitz and Downs [3] using a variational approach. Garcilazo [4] investigated the Λ*nn* system by solving Faddeev equations using *YN* and *NN* interactions derived from a chiral constituent quark model and revealed that Λ*nn* bound system was not found. Later various approaches such as hyperspherical harmonics (HH) [5], Faddeev calculations [2,6–11], variational calculations [12], and pionless effective field theory [13–15] with various kinds of baryon-baryon interactions have been used to analyze the Λ*nn* system and all reported that it is highly unlikely to form a bound system in the theoretical analysis without a significant alteration of nuclear and hypernuclear forces.

The ${}^{3}_{\Lambda}n$ hypernucleus could not be produced in the earlier experiments due to absence of charge of its bound state. However, the HypHI Collaboration at GSI [16] reported the first evidence of the existence of the ${}^{3}_{\Lambda}n$ bound state from analysis of the observed two- and three-body decays mode without stating any value of binding energy. Their observation was inconsistent with the claim of the above theoretical analysis.

In this paper, we analyze the Λ*nn* bound state problem using the *ab initio* no-core shell model (NCSM) [17–19] technique. The calculation of the Λ*nn* system ($J^\pi = 1/2^+, T = 1$) is performed in Jacobi coordinate harmonic oscillator (HO) basis using the *NN* and *YN* interactions derived from the chiral effective field model. In the extension into the continuum state, we apply the SS-HORSE [20–25] formalism, which is a single-state harmonic oscillator representation of scattering equations, to calculate the low-energy phase shifts and scattering amplitudes at the NCSM eigenenergies by employing a hyperspherical harmonic oscillator basis. The low-lying Λ*nn* resonance energy and width are extracted from the scattering amplitude parametrization. The NCSM-SS-HORSE method [26] has been successfully applied to study a tetraneutron unbound system, considered a true four-body scattering. Here we first apply this method to study the three-body system with strangeness.

II. NCSM-SS-HORSE FORMALISM

The hypernuclear Hamiltonian for two nucleon and a hyperon system can be written as

$$H = -\sum_{i=1}^{3} \frac{\hbar^2}{2m_i}\vec{\nabla}_i^2 + V_{NN}(\vec{r}_1, \vec{r}_2) + \sum_{i=1}^{2} V_{YN}(\vec{r}_i, \vec{r}_3) + \Delta M, \tag{1}$$

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where the coordinates $\vec{r}_i$ and masses m_i are for the two nucleons with $i = 1, 2$ and the hyperon with $i = 3$. We work with nonrelativistic two-body *NN* and *YN* potentials, employing the leading-order chiral hyperon-nucleon interactions [27] and a family of 42 different nuclear interactions at next-to-next-to-leading order (also called the chiral $\mathrm{NNLO_{sim}}$ family of *NN* interactions) [28]. These nuclear interactions were constructed by varying the chiral regulator cutoff Λ_{NN} between 450 and 600 MeV in steps of 25 MeV and the truncation of the input *NN* scattering $T_{\mathrm{lab}} \leqslant T_{\mathrm{lab}}^{\max}$ between 125 and 290 MeV in six steps, which were obtained from a simultaneous optimization of all 26 low-energy constants (LECs) to different sets of *NN* and πN scattering plus bound state observables [28]. Unlike the *NN* interaction, it is hard to quantify the theoretical uncertainties in the *YN* interaction. As a result of poorly constrained *YN* interactions by the scarce *YN* scattering data, different *YN* interaction models give rather different results, causing large uncertainties in the predictions of the hypernuclear observables [18,29,30]. The *YN* scattering data are reasonably reproduced in the *YN* interaction with the regulator cutoff ranging from 550 and 700 MeV [27]. In this work, we use the *YN* interactions [27] with $\Lambda_{YN} = 600$ MeV and *NN* interactions with $\Lambda_{NN} = 500$ and $T_{\max} = 290$ MeV, which are the rather standard choice in most of the *ab initio* NCSM calculations [19,28,31]. The effect of ΛN-ΣN coupling is taken into account [19].

In NCSM, three active particles are considered in the three-dimensional harmonic oscillator (HO) basis. In the construction of HO basis states for such a few-body Λnn system, it is more effective to use the relative Jacobi coordinates where the center-of-mass (c.m.) coordinate $\vec{\xi}_0$ is separated, which allows us to perform NCSM calculations in a large model space. The relative Jacobi coordinates in terms of the rescaled version of the single-particle coordinates $\vec{x}_i = \sqrt{m_i}\vec{r}_i$ are defined as

$$\vec{\xi}_1 = \sqrt{1/2}(\vec{x}_1 - \vec{x}_2),$$
$$\vec{\xi}_2 = \sqrt{2m_N m_Y/2m_N + m_Y}[1/2\sqrt{m_N}(\vec{x}_1 + \vec{x}_2) - 1/\sqrt{m_Y}\vec{x}_3], \quad (2)$$

where m_N and m_Y are the masses of nucleon and hyperon. $\vec{\xi}_1$ is the relative coordinate of the two-nucleon pair and $\vec{\xi}_2$ is the relative coordinate of the hyperon with respect to the c.m. of the two-nucleon pair. Following the general Jacobi coordinate formulation in Ref. [19], we construct the JT-coupled HO basis states for the system of a two-nucleon pair and a hyperon,

$$|(n_{NN}(l_{NN}s_{NN})j_{NN}t_{NN}, N_Y\mathcal{L}_Y J_Y T_Y)JT\rangle\,, \quad (3)$$

depending on the coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$ respectively. n_{NN}, l_{NN}, s_{NN}, j_{NN}, t_{NN} (N_Y, $\mathcal{L}_Y$, J_Y, T_Y) are the HO radial quantum number, orbital angular momentum, spin, angular momentum, and isospin of the relative two-nucleon (hyperon) state. J and T are the total angular momentum and total isospin of the system. The basis (3) is antisymmetrized with respect to the exchange of two nucleons by restricting the two-nucleon relative quantum numbers with the condition $(-1)^{l_{NN}+s_{NN}+t_{NN}} = -1$. The basis (3) is suitable for evaluating two-body *NN* interaction matrix elements but not for evaluating two-body *YN* interaction matrix elements.

For a subsystem including a *YN* pair and a nucleon, another set of Jacobi coordinates is correspondingly introduced,

$$\vec{\eta}_1 = \sqrt{(m_N + m_Y)m_N/2m_N + m_Y}[1/\sqrt{m_N}\vec{x}_1 - 1/(m_N + m_Y)(\sqrt{m_N}\vec{x}_2 + \sqrt{m_Y}\vec{x}_3)],$$
$$\vec{\eta}_2 = \sqrt{m_N m_Y/m_N + m_Y}(1/\sqrt{m_N}\vec{x}_2 - 1/\sqrt{m_Y}\vec{x}_3), \quad (4)$$

where $\vec{\eta}_1$ is the relative coordinate of a nucleon with respect to the c.m. of the *YN* pair and $\vec{\eta}_2$ is the relative coordinate of the *YN* pair. By using orthogonal transformation, the antisymmetrized HO basis (3) can be expanded as

$$\begin{aligned}
&|(n_{NN}(l_{NN}s_{NN})j_{NN}t_{NN}, N_Y\mathcal{L}_Y J_Y T_Y)JT\rangle \\
&= \sum_{LS} \hat{L}^2\hat{S}^2\hat{j}_{NY}\hat{J}_N\hat{j}_{NN}\hat{J}_Y(-1)^{s_{NY}+1/2+s_{NN}+1/2+\mathcal{L}_N+\mathcal{L}_Y} \\
&\times \begin{Bmatrix} l_{NY} & s_{NY} & j_{NY} \\ \mathcal{L}_N & 1/2 & J_N \\ L & S & J \end{Bmatrix} \begin{Bmatrix} l_{NN} & s_{NN} & j_{NN} \\ \mathcal{L}_Y & 1/2 & J_Y \\ L & S & J \end{Bmatrix} \\
&\times \begin{Bmatrix} 1/2 & 1/2 & s_{NN} \\ 1/2 & S & s_{NY} \end{Bmatrix} \\
&\times (-1)^{t_{NY}+T_N+t_{NN}+T_Y}\hat{t}_{NY}\hat{t}_{NN} \begin{Bmatrix} 1/2 & 1/2 & t_{NN} \\ J_Y & T & t_{NY} \end{Bmatrix} \\
&\times \langle n_{NY}l_{NY}N_N\mathcal{L}_N|n_{NN}l_{NN}N_Y\mathcal{L}_Y\rangle_{d=\frac{2m_N+m_Y}{m_Y}} \\
&\times |(n_{NY}(l_{NY}s_{NY})j_{NY}t_{NY}, N_N\mathcal{L}_N J_N)JT\rangle\,,
\end{aligned} \quad (5)$$

in terms of HO basis states

$$|(n_{NY}(l_{NY}s_{NY})j_{NY}t_{NY}, N_N\mathcal{L}_N J_N)JT\rangle\,, \quad (6)$$

depending on the coordinates $\vec{\eta}_2$ and $\vec{\eta}_1$ respectively. The general HO bracket $\langle n_{NY}l_{NY}N_N\mathcal{L}_N|n_{NN}l_{NN}N_Y\mathcal{L}_Y\rangle_d$ follows Ref. [32]. *YN* interaction matrix elements involving Λ and Σ hyperons are evaluated in the antisymmmetrized basis (3) through its expansion in the basis (6) as

$$\left\langle \sum_{i=1}^{2} V_{YN}(\vec{r}_i, \vec{r}_3) \right\rangle = 2\,\langle V_{YN}(\vec{\eta}_2)\rangle\,, \quad (7)$$

where the matrix elements on the right-hand side are diagonal in all quantum numbers of the basis states (6), except for n_{NY} and l_{NY}. The lowest eigenstates of the Λnn system are calculated by the diagonalization of the truncated Hamiltonian matrix.

To look for resonances, we extend our study to the continuum state by employing J-matrix formalism, also known as the harmonic oscillator representation of scattering equation (HORSE) formalism, which arms one to study continuum spectrum using only positive energies obtained from a bound state approach like NCSM applying the HO basis. The HORSE method can be used to describe the open channels in the external subspace while the internal subspace is associated with the NCSM approach. For details of the HORSE formalism, refer to Refs. [22,33].

In the extension into the continuum, the three-body extension of the J-matrix formalism for all three-body decay channels is very complicated. We apply the democratic decay approximation (also known as true three-body scattering or $3 \to 3$ scattering) [34] which employs the hyperspherical harmonic (HH) basis to describe the Λnn system decaying through only the three-body breakup channel and it does not allow for other possible two-body channels associated with two-body sub-bound states.

The hyperspherical oscillator basis can be labeled as $|\kappa K \gamma\rangle$, where κ is the principal quantum number and K is the hypermomentum, and $\gamma \equiv \{l_1, l_2, L, s_1, s_2, S, t_1, t_2, T\}$ collects all possible quantum numbers corresponding to the Jacobi coordinates for a three-body system. The external subspace is spanned by hyperspherical oscillator functions with $N \equiv 2\kappa + K > N_{\max}$, where the Hamiltonian $H = T$ is used. Here $N_{\max}$ is the maximum number of excitation quanta defining the many-body NCSM basis space. Because of the high centrifugal barrier $\mathcal{L}(\mathcal{L}+1)/\rho^2$, the HH states with larger K can be neglected in the case of no sub-bound Λnn system (ρ is the hyperradius with the mass scaled Jacobi coordinates and $\mathcal{L} = K + 3/2$ is the effective momentum). It is adequate to consider a single hyperspherical channel with minimum hypermomentum $K_{\min} = 0$ to describe democratic three-body decays.

We follow the SS-HORSE approach [21,22,26] to compute the $3 \to 3$ scattering phase shifts at the eigenenergies $E_\nu > 0$ obtained directly from the NCSM calculation,

$$\tan \delta(E_\nu) = -\frac{S_{N_{\max}+2,\mathcal{L}}(E_\nu)}{C_{N_{\max}+2,\mathcal{L}}(E_\nu)}, \tag{8}$$

where $S_{N\mathcal{L}}$ and $C_{N\mathcal{L}}$ are regular and irregular solutions of the free Schrödinger equation in the hyperspherical oscillator representation, which can be applied in the case of arbitrary $\mathcal{L}$ (both integer and half integer), taking the simple analytical expressions [21,23,34]

$$S_{N\mathcal{L}}(E) = \sqrt{\frac{\left(N-\mathcal{L}+\frac{3}{2}\right)!}{\lambda\,\Gamma\left(\frac{N}{2}+\frac{\mathcal{L}}{2}+\frac{9}{4}\right)}}\, q^{\mathcal{L}+1}\, e^{-\frac{q^2}{2}} \times L^{\mathcal{L}+\frac{1}{2}}_{(N-\mathcal{L}+\frac{3}{2})/2}(q^2), \tag{9}$$

$$C^{(\pm)}_{N\mathcal{L}}(E) = \frac{1}{\pi\sqrt{\lambda}}\sqrt{\left(N-\mathcal{L}+\frac{3}{2}\right)!\,\Gamma\left(\frac{N}{2}+\frac{\mathcal{L}}{2}+\frac{9}{4}\right)} \times \Psi\left(\frac{N}{2}+\frac{\mathcal{L}}{2}+\frac{9}{4}, \mathcal{L}+\frac{3}{2}; e^{\mp i\pi} q^2\right) \times q^{\mathcal{L}+1}\, e^{\frac{q^2}{2}}\, e^{\mp i\pi(\mathcal{L}+\frac{1}{2})}, \tag{10}$$

$$C_{N\mathcal{L}}(E) = \tfrac{1}{2}\left(C^{(+)}_{N\mathcal{L}}(E) + C^{(-)}_{N\mathcal{L}}(E)\right), \tag{11}$$

where $q = \sqrt{\frac{2E}{\hbar\omega}}$ is dimensionless momentum, $L^{L+\frac{1}{2}}_{\kappa}(x)$ is the associated Laguerre polynomial, and $\lambda = \sqrt{\frac{m\omega}{\hbar}}$ is the oscillator radius at $\Psi(a, c; x)$ which is the Tricomi function.

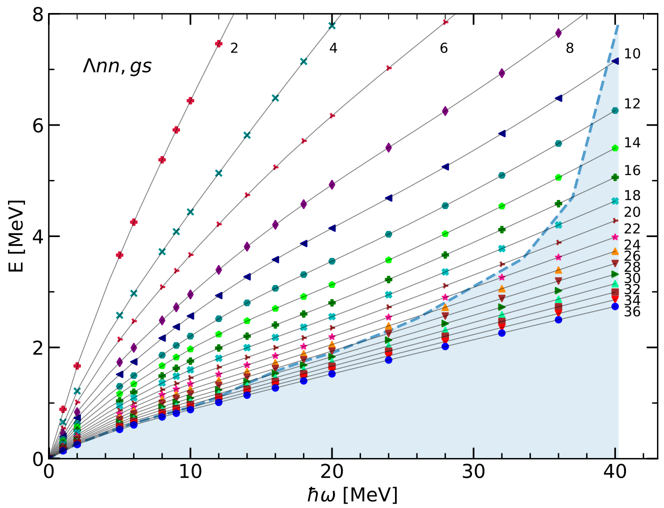

FIG. 1. The eigenenergies of the NCSM Hamiltonian with various model space sizes $N_{\max}$ as a function of oscillator frequency $\hbar\omega$. The numbers at the end of each line represent $N_{\max}$. The blue shaded area shows the selected energies for parametrization of the scattering amplitude.

The SS-HORSE scattering amplitude for neutral particles may be calculated in the standard way,

$$f(E_\nu)q = \frac{1}{\cot \delta(E_\nu) - i}. \tag{12}$$

We parametrize the scattering amplitude in the method proposed in [35] for the case in which a resonance is not sharp, but both the potential scattering (nonresonant background) and resonance contribution are not negligible. The scattering amplitude may be parametrized as

$$F(E)q = e^{i\delta_0(E)} \sin \delta_0(E) + \frac{-\Gamma/2}{E - E_r + i\Gamma/2} e^{2i\delta_0(E)}, \tag{13}$$

where $\delta_0(E)$ is the potential scattering phase shift, depending on the energy E. We will fit the SS-HORSE scattering amplitude by the complex-valued function $F(E)q$ in the next section to determine the form of $\delta_0(E)$ and derive the resonance energy E_r and width Γ.

III. RESULTS AND DISCUSSION

The Λnn system is analyzed using the NCSM approach with chiral NNLO$_{\rm sim}$ NN and LO YN interactions. The NCSM computational model space is characterized by a chosen maximal total HO quanta $N^{\rm tot}_{\max}$, that is,

$$2n_{NN} + l_{NN} + 2N_Y + L_Y \leqslant N^{\rm tot}_{\max} \equiv N_{\max} + N_0, \tag{14}$$

where the minimal possible number of HO quanta is $N_0 = 0$. In the Λnn case, $N^{\rm tot}_{\max} = N_{\max}$. We have computed the total energy of the Λnn system in the oscillator basis with model space truncations $N_{\max} \leqslant 36$, and in the range of the HO frequencies $1 \leqslant \hbar\omega \leqslant 40$ MeV. It is found that there is no Λnn bound system. The Λnn ground-state energy as a function of the model space truncation $N_{\max}$ and HO frequency $\hbar\omega$ is presented in Fig. 1. The NCSM energies decrease with

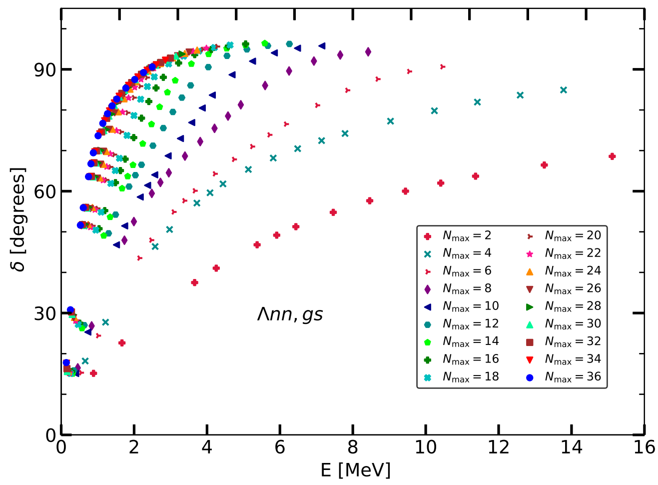

FIG. 2. $3 \to 3$ scattering phase shifts obtained directly from the NCSM eigenstates using Eq. (8).

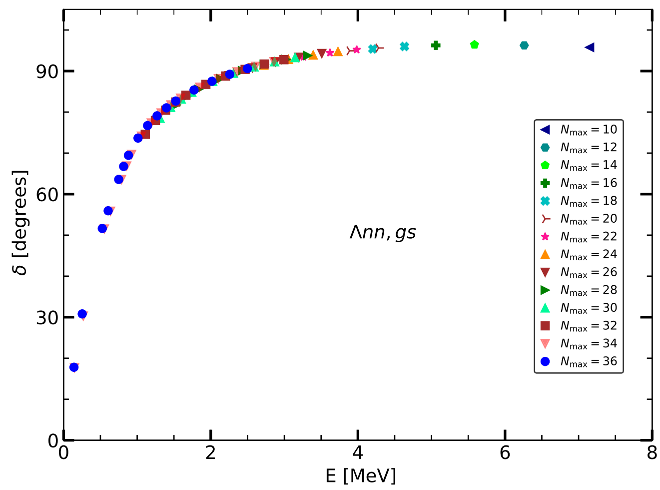

FIG. 3. $3 \to 3$ scattering phase shifts obtained from selected NCSM eigenstates with $N_{\max} \in [10, 36]$ for scattering amplitude parametrization.

increasing $N_{\max}$ and with decreasing $\hbar\omega$. Our model used here can reproduce well the binding energy of the hypertriton [36] and also those for s-shell hypernuclei, ${}^4_\Lambda$H and ${}^4_\Lambda$He, which will be in a future publication.

The SS-HORSE phase shifts covering all computed NCSM energies calculated by using Eq. (8) are shown in Fig. 2. The phase shifts obtained with smaller $N_{\max}$ lie in a wide energy region as the obtained Λnn ground-state energies spread widely. With $N_{\max}$ increasing, however, the obtained Λnn ground-state energies converge to lower values, as shown in Fig. 1, and hence the corresponding phase shifts shift to the lower energy region. The first convergence of phase shifts is achieved at smaller energies with larger $N_{\max}$. The phase shifts become almost the same results at $N_{\max} = 34$ and 36 MeV. We follow the selection procedure of Refs. [21,26,37] and select a set of eigenvalues E_ν from the $N_{\max} = 10$–36 model spaces, which is illustrated by the shaded area in Fig. 1, to produce a single smooth curve of phase shifts for parametrization. The SS-HORSE phase shifts corresponding to these selected smaller eigenvalues are plotted in Fig. 3.

We compute the SS-HORSE low-energy scattering amplitude for the purpose of extracting the resonance parameters from scattering amplitude parametrization. The function $|f(E_\nu)q|^2$ of the scattering amplitude given in Eq. (12) is shown by symbols in Fig. 4. The fitting to the SS-HORSE result $|f(E_\nu)q|^2$ by the function $|F(E)q|^2$ leads to $\delta_0(E)$ of the form

$$\delta_0(E) = a_0 + a_2(\sqrt{E})^2 + a_4(\sqrt{E})^4, \qquad (15)$$

with the adjustable parameters $a_0 = 1.856$, $a_2 = -0.014$ MeV^{-1}, $a_4 = 2.959 \times 10^{-4}$ MeV^{-2}. The resonance energy and width are derived as $E_r = 0.124$ MeV and $\Gamma = 1.161$ MeV. The result is not very consistent with those in Refs. [11,15] which, by employing simplified contact interactions, reveal that a resonant state of Λnn does not exist with the standard strength of the Λn interaction since the real part of the resonance energy turns negative. We look forward to the results of Λnn bound and resonance states from the ongoing experiment (E12-17-003) at Jefferson Lab (JLab) [38]. Such Λnn bound and resonance states, if any, are expected to provide a new perspective on Λn interactions.

IV. SUMMARY

We have performed *ab initio* no-core shell model calculations for the Λnn system ($J^\pi = 1/2^+, T = 1$) without tuning the strength of realistic NN and YN potentials at various $N_{\max}$ and $\hbar\omega$ values with full inclusion of ΛN-ΣN coupling, and found that no bound state exists. To look for resonance states of the Λnn, we have applied the NCSM-SS-HORSE technique to calculate the Λnn scattering phase shifts, which suggest a Λnn resonant state at energy $E_r = 0.124$ MeV and $\Gamma = 1.161$ MeV. Further theoretical studies and experimental searches for Λnn resonances would be of great benefit for constraining Λn interactions.

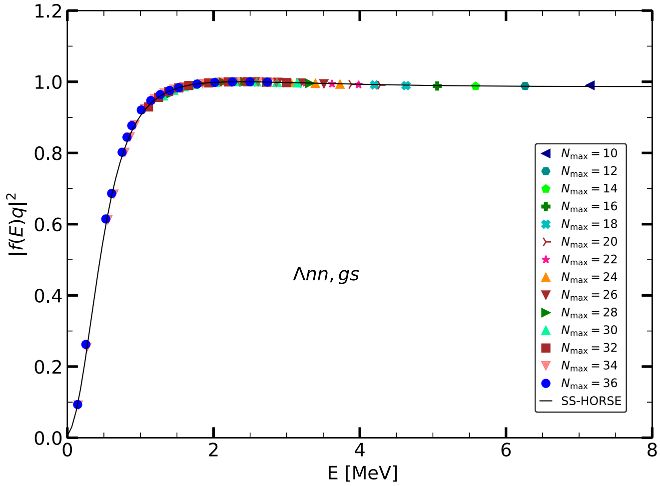

FIG. 4. The scattering amplitude $|f(E)q|^2$ using Eq. (12) obtained from NCSM eigenstates (symbol). The solid line shows the parametrization of scattering amplitude in Eq. (13).

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