Drag of heavy quarks in an anisotropic QCD medium beyond the static limit

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Heavy quark dynamics in an anisotropic QCD medium have been analyzed within the Fokker-Planck approach. Heavy quark drag force and the momentum diffusion tensor have been decomposed by employing a general tensor basis for an anisotropic medium. Depending upon the relative orientation of the direction of the momentum anisotropy of the medium and heavy quark motion, two drag and four diffusion coefficients have been estimated in the anisotropic QCD medium. The relative significance of different components of drag and momentum diffusion coefficients has been explored. The dependence of the angle between the anisotropic vector and heavy quark motion to the drag and diffusion coefficients has also been studied. Furthermore, the energy loss of heavy quarks due to the elastic collisional process in an anisotropic medium has been studied. It is seen that the anisotropic contributions to heavy quark transport coefficients and its collisional energy loss have a strong dependence on the direction and strength of momentum anisotropy in the QCD medium.

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I. INTRODUCTION

The heavy-ion collision experiments pursued at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) have confirmed the existence of strongly interacting matter, the quark-gluon plasma (QGP) [1–6]. Among various signatures from experimental observables, heavy quarks (HQs), mainly charm and bottom quarks, are identified as excellent experimental probes to study the properties of the hot QCD medium [7–17]. HQs undergo random motion and witness the QCD medium expansion. This is attributed to the fact that HQs are mostly created in the early stages of heavy-ion collisions, and their thermalization time is greater than the lifetime of the QGP. Several theoretical efforts have been made to explore HQ dynamics and the associated experimental observables such as the nuclear suppression factor R_{AA} and flow coefficients [18–37]. A few attempts have been made to explore the impact of momentum anisotropic aspects of the QCD medium on HQ transport. However, a systematic study of HQ transport by constructing the drag force and diffusion tensor using a general tensor basis in an anisotropic medium is essential for the proper understanding of HQ observables.

Momentum anisotropy arises due to the rapid expansion of the created QCD medium in the longitudinal direction compared with the transverse directions and may sustain in the entire evolution of the medium. This anisotropy may induce instability in the Yang-Mills fields (Chromo-Weibel instability) and may have a vital role in the evolution of the QCD medium [38–41]. It has been argued that the QCD medium has an anomalous viscosity that arises from Chromo-Weibel instabilities, which may provide a possible explanation for the near-perfect liquidity of the QGP without considering the strongly coupled state assumption [42,43]. The momentum anisotropic aspects have been explored in the context of electromagnetic probes [44–46], collective modes of QCD [47–49], momentum broadening of energetic partons [50], and in the hydrodynamical expansion of the medium [51–53]. Medium anisotropy will affect the dynamics of HQ and can be quantified in terms of its transport coefficients, dragm and momentum diffusion in the medium. A magnetic-field-induced anisotropy to the HQ momentum diffusion has been recently explored in Refs. [54,55] and generates huge attention toward the recent RHIC and LHC observations [56,57]. Nonequilibrium effects of the QCD medium to the HQ transport have been studied in Refs. [58–65].

The focus of the present study is to set up a general framework to explore HQ dynamics in an anisotropic QCD medium for arbitrary relative orientation of the direction of anisotropy and HQ motion. To that end, we have performed a decomposition of HQ drag force and momentum diffusion tensor by using a tensor basis for an anisotropic medium. This gives rise to two drag and four diffusion coefficients of HQs in the QCD medium. The anisotropic effects are entering through the nonequilibrium part of the distribution and are obtained by solving the transport equation. We have analyzed the impact of anisotropy on the temperature and momentum dependence

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of the HQ transport coefficients in the medium. In addition, we have explored the dependence of the orientation of HQ motion with the direction of anisotropy on HQ dynamics in the medium. These anisotropic transport coefficients may have a significant role in the estimation of HQ experimental observables in the heavy-ion collision experiments by treating it as input parameters in the Langevin dynamics.

The paper is organized as follows: Section II is devoted to the theoretical formulation of HQ transport along with the general decomposition of drag and momentum diffusion tensors in an anisotropic QCD medium. In Sec. III, we present the results of HQ transport coefficients and its collisional energy loss in the anisotropic medium. We summarize the analysis with an outlook in Sec. IV.

Notations and conventions. In this paper, the subscript k represents the particle species of the medium, i.e., $k = (g, \tilde{q})$ with g and $\tilde{q}$ denoting the gluons and quarks. The HQ energy is defined by $E_p = (|\mathbf{p}|^2 + m_{\rm HQ}^2)^{1/2}$ where $\mathbf{p}$ and $m_{\rm HQ}$ respectively denote the momentum and mass of HQs. The energy of constituent particles (in the massless limit) is represented as $E_q = |\mathbf{q}|$ with $\mathbf{q}$ as the momentum. The quantity $a_k = 1, -1, 0$ for Bose-Einstein, Fermi-Dirac, and Maxwell-Boltzmann distributions, respectively.

II. HEAVY QUARK DRAG AND DIFFUSION

The dynamics of HQs in the hot QCD medium is considered as Brownian motion and can be described within the Fokker-Planck equation as follows [59,66]:

$$\frac{\partial f_{\rm HQ}}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p}) f_{\rm HQ} + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p}) f_{\rm HQ}]\right], \quad (1)$$

where $f_{\rm HQ}$ denotes the HQ distribution in the medium. The interactions of HQ with the light quarks and gluons are quantified in terms of drag force A_i and momentum diffusion B_{ij} in the QGP medium. In the current analysis, we consider the two-body elastic collisional process $\mathrm{HQ}(P) + l(Q) \rightarrow \mathrm{HQ}(P') + l(Q')$, where l denotes quarks and antiquarks, and gluons. Here, $P = (E_p, \mathbf{p})$ and $Q = (E_q, \mathbf{q})$ define the four-momentum of HQ and medium constituent particle before the interaction. The matrix element $|\mathcal{M}_{2\rightarrow 2}|$ for the elastic collisions of HQs with medium particles has been investigated in Ref. [66,67]. The drag force of HQ describes the thermal average of the momentum transfer due to the interaction, whereas the momentum diffusion quantifies the average of the square of the momentum transfer. The HQ drag and momentum diffusion in the QGP medium take the following forms:

$$\begin{aligned} A_i =& \frac{1}{2E_p}\int\frac{d^3\mathbf{q}}{(2\pi)^3 2E_q}\int\frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{q'}}\int\frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_{\rm HQ}} \\ &\times \sum |\mathcal{M}_{2\rightarrow 2}|^2 (2\pi)^4 \delta^4(P + Q - P' - Q') f_k(\mathbf{q}) \\ &\times [1 + a_k f_k(\mathbf{q}')][(\mathbf{p} - \mathbf{p}')_i] = \langle\langle(\mathbf{p} - \mathbf{p}')_i\rangle\rangle, \end{aligned} \quad (2)$$

$$B_{ij} = \frac{1}{2}\langle\langle(\mathbf{p} - \mathbf{p}')_i(\mathbf{p} - \mathbf{p}')_j\rangle\rangle, \quad (3)$$

where $\gamma_{\rm HQ}$ is the statistical degeneracy factor of the HQ, f_k represents the near-equilibrium distribution function of quark and antiquark and gluon. In general, HQ drag and diffusion coefficients can be schematically described as

$$X_c = \int \text{phase space} \times \text{interaction} \times \text{transport part.}$$

HQ transport coefficients can be obtained with the proper decomposition of drag force and momentum diffusion matrix in the background QGP medium. We proceed with the decomposition of A_i and B_{ij} in the isotropic QCD medium.

A. For isotropic QCD medium

In an isotropic medium, the drag force depends on the HQ momentum and A_i can be decomposed as

$$A_i = p_i A_0(p^2), \quad (4)$$

where $p^2 = |\mathbf{p}|^2$ and A_0 is the drag coefficient of the HQ in the isotropic QGP medium. The drag coefficient can be obtained from Eqs. (2) and (4) as

$$A_0 = p_i A_i / p^2 = \langle\langle 1\rangle\rangle - \frac{\langle\langle \mathbf{p}\cdot\mathbf{p}'\rangle\rangle}{p^2}. \quad (5)$$

Similarly, B_{ij} can be decomposed into longitudinal and transverse components in the isotropic QCD medium as

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_0(p^2) + \frac{p_i p_j}{p^2} B_1(p^2), \quad (6)$$

where the transverse and longitudinal diffusion coefficients can be defined as follows:

$$B_0 = \frac{1}{4}\left[\langle\langle p'^2\rangle\rangle - \frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2}\right], \quad (7)$$

$$B_1 = \frac{1}{2}\left[\frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2} - 2\langle\langle(\mathbf{p}'\cdot\mathbf{p})\rangle\rangle + p^2\langle\langle 1\rangle\rangle\right]. \quad (8)$$

The kinematics of $2 \rightarrow 2$ processes can be simplified in the center-of-momentum (COM) frame of the system, and the average of a function $F(\mathbf{p})$ in the COM frame for the isotropic medium can be described as follows:

$$\begin{aligned} \langle\langle F(\mathbf{p})\rangle\rangle =& \frac{1}{(512\pi^4) E_p \gamma_{\rm HQ}}\int_0^\infty dq \left(\frac{s - m_{\rm HQ}^2}{s}\right) f_k^0(E_q) \\ &\times \int_0^\pi d\chi \sin\chi \int_0^\pi d\theta_{cm} \sin\theta_{cm} \sum |\mathcal{M}_{2\rightarrow 2}|^2 \\ &\times \int_0^{2\pi} d\phi_{cm}[1 + a_k f_k^0(E_{q'})] F(\mathbf{p}), \end{aligned} \quad (9)$$

where f_k^0 is the isotropic distribution function, and χ quantifies the angle between the incident HQ and medium constituent particles in the laboratory frame. The quantities θ_{cm} and ϕ_{cm} respectively describe the zenith and azimuthal angle in the COM frame. Here, the Mandelstam variables s, t, u are defined as follows:

$$s = (E_p + E_q)^2 - (p^2 + q^2 + 2pq\cos\chi), \quad (10)$$

$$t = 2p_{cm}^2(\cos\theta_{cm} - 1), \quad (11)$$

$$u = 2m_{\rm HQ}^2 - s - t, \quad (12)$$

with $p_{cm} = |\mathbf{p}_{cm}|$ as the magnitude of initial momentum of HQ in the COM frame.

B. For an anisotropic QCD medium

Momentum anisotropies arise due to the rapid expansion of the hot QCD medium in the early stages of the relativistic heavy-ion collisions. In the present analysis, the impact of momentum anisotropy is entering through the distribution function of the medium constituent particles. The anisotropic momentum distribution can be described in terms of isotropic distribution function by rescaling one direction in momentum space as follows [47,48]:

$$f_k^{(\text{aniso})}(\mathbf{q}) = \sqrt{1+\xi}\, f_k^0\big(\sqrt{q^2+\xi(\mathbf{q}\cdot\mathbf{n})^2}\big), \tag{13}$$

where ξ is the anisotropic parameter that quantifies the stretching or squeezing of the momentum distribution in the prescribed direction $\mathbf{n}$ where $\mathbf{n}$ is the unit vector that indicates the direction of momentum anisotropy in the medium. The present focus is on a weakly anisotropic medium such that $\xi \ll 1$ and the distribution function reduces to the form $f_k^{(\text{aniso})}(\mathbf{q}) = f_k^0 + \delta f_k$ with [68]

$$\delta f_k = -\frac{\xi}{2E_qT}(\mathbf{q}\cdot\mathbf{n})^2\big(f_k^0\big)^2 \exp\left(\frac{E_q}{T}\right). \tag{14}$$

By defining $\tilde{n}^i = (\delta_{ij} - \frac{p_ip_j}{p^2})n^j$ such that $\mathbf{p}\cdot\tilde{\mathbf{n}} = 0$, the drag force in the anisotropic medium can be decomposed on the orthogonal basis as follows:

$$A_i = p_i A_0^{(\text{aniso})} + \tilde{n}_i A_1^{(\text{aniso})}. \tag{15}$$

The components of the drag force in the anisotropic medium can be obtained as follows,

$$A_0^{(\text{aniso})} = p_iA_i/p^2 = \langle\langle 1\rangle\rangle - \frac{\langle\langle \mathbf{p}\cdot\mathbf{p}'\rangle\rangle}{p^2}, \tag{16}$$

$$A_1^{(\text{aniso})} = \tilde{n}_iA_i/\tilde{n}^2 = -\frac{1}{\tilde{n}^2}\langle\langle \tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle, \tag{17}$$

where $\tilde{n}^2 = 1 - \frac{(\mathbf{p}\cdot\hat{\mathbf{n}})^2}{p^2} = 1-\cos^2\theta_n$. Employing the definition of the near-equilibrium distribution function as described in Eq. (14), the average of a function $F(p')$ in the anisotropic medium can be defined as

$$\langle\langle F(p')\rangle\rangle = \langle\langle F(p')\rangle\rangle_0 + \langle\langle F(p')\rangle\rangle_{\text{a}}, \tag{18}$$

where the isotropic part $\langle\langle F(p')\rangle\rangle$ is defined in Eq. (9). Following the same prescription as in the case of isotropic case, we can represent $\langle\langle F(p')\rangle\rangle_a$ in the COM frame as

$$\begin{aligned}\langle\langle F(\mathbf{p})\rangle\rangle_a &= \frac{1}{(1024\pi^5)E_p\gamma_{\text{HQ}}}\int_0^\infty dqq\left(\frac{s-m_{\text{HQ}}^2}{s}\right)\\ &\times\int_0^\pi d\chi\sin\chi\int_0^{2\pi}d\phi\int_0^\pi d\theta_{cm}\sin\theta_{cm}\\ &\times\sum|\mathcal{M}_{2\to2}|^2\\ &\times\int_0^{2\pi}d\phi_{cm}\big\{\delta f_k(\mathbf{q})\big[1+a_kf_k^0(\mathbf{q}')\big]\\ &+a_kf_k^0(\mathbf{q})\delta f_k(\mathbf{q}')\big\}F(\mathbf{p}).\end{aligned} \tag{19}$$

Employing Eq. (18) in Eq. (16), we obtain the nonequilibrium correction to the HQ drag coefficient described in Eq. (5) as,

$$A_0^{(\text{aniso})} = A_0 + \delta A_0, \tag{20}$$

where A_0 is the isotropic part and δA_0 represents the anisotropic corrections to the drag coefficient in the QGP medium and can be obtained from Eq. (19). Furthermore, the term $A_1^{(\text{aniso})}$ is the additional component of the drag coefficient that arises due to the anisotropy of the medium. To decompose the HQ diffusion, one needs to construct the appropriate tensor basis for the symmetric matrix B_{ij} with the momentum vector p^i and anisotropy vector n^i. Following Ref. [47], we decompose the B_{ij} into four components as follows:

$$\begin{aligned}B_{ij} &= \left(\delta_{ij}-\frac{p_ip_j}{p^2}\right)B_0^{(\text{aniso})} + \frac{p_ip_j}{p^2}B_1^{(\text{aniso})}\\ &+\frac{\tilde{n}_i\tilde{n}_j}{\tilde{n}^2}B_2^{(\text{aniso})} + (p^i\tilde{n}^j + p^j\tilde{n}^i)B_3^{(\text{aniso})}.\end{aligned} \tag{21}$$

The components of the momentum diffusion can be obtained by taking the appropriate projections of the Eq. (21) and have the following forms:

$$B_0^{(\text{aniso})} = \left[\left(\delta_{ij}-\frac{p_ip_j}{p^2}\right)-\frac{\tilde{n}_i\tilde{n}_j}{\tilde{n}^2}\right]B_{ij} = \frac{1}{2}\left[\langle\langle p'^2\rangle\rangle - \frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2} - \frac{\langle\langle(\mathbf{p}'\cdot\tilde{\mathbf{n}})^2\rangle\rangle}{\tilde{n}^2}\right], \tag{22}$$

$$B_1^{(\text{aniso})} = \frac{p_ip_j}{p^2}B_{ij} = \frac{1}{2}\left[\frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2} - 2\langle\langle(\mathbf{p}'\cdot\mathbf{p})\rangle\rangle + p^2\langle\langle 1\rangle\rangle\right], \tag{23}$$

$$B_2^{(\text{aniso})} = \left[\frac{2\tilde{n}_i\tilde{n}_j}{\tilde{n}^2} - \left(\delta_{ij}-\frac{p_ip_j}{p^2}\right)\right]B_{ij} = \frac{1}{2}\left[-\langle\langle p'^2\rangle\rangle + \frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2} + \frac{2\langle\langle(\mathbf{p}'\cdot\tilde{\mathbf{n}})^2\rangle\rangle}{\tilde{n}^2}\right], \tag{24}$$

$$B_3^{(\text{aniso})} = \frac{1}{2p^2\tilde{n}^2}\big(p^i\tilde{n}^j + p^j\tilde{n}^i\big)B_{ij} = \frac{1}{2p^2\tilde{n}^2}\big[-p^2\langle\langle(\mathbf{p}'\cdot\tilde{\mathbf{n}})\rangle\rangle + \langle\langle(\mathbf{p}'\cdot\mathbf{p})(\mathbf{p}'\cdot\tilde{\mathbf{n}})\rangle\rangle\big]. \tag{25}$$

Note that Eqs. (22)–(25) will reduce back to the results of Ref. [60] in the case of $\langle\langle\tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle = 0$. Note that we have obtained $\langle\langle\tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle = 0$ for the isotropic case. However, for the anisotropic medium, by employing the general tensor decomposition, we observe that the term $\langle\langle\tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle_a$ is nonzero and modify the HQ transport coefficients. Now, we proceed with the estimation of $\langle\langle\tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle$ in center-of-mass frame. We have considered $\mathbf{n} = (\sin\theta_n, 0, \cos\theta_n)$, where angle θ_n

is the angle between anisotropy vector and $\hat{\mathbf{z}}$. It is important to note that the analysis is also valid for the choice $\mathbf{n} = (0, \sin\theta_n, \cos\theta_n)$. Light quark momentum can be decomposed as $\mathbf{q} = (q\sin\chi\cos\phi,\ q\sin\chi\sin\phi,\ q\cos\chi)$ and HQ momentum chosen as $\mathbf{p} = (0, 0, p)$ such that we have

$$\mathbf{p}\cdot\mathbf{q} = pq\cos\chi, \tag{26}$$

$$\mathbf{p}\cdot\mathbf{n} = p\cos\theta_n, \tag{27}$$

$$\mathbf{q}\cdot\mathbf{n} = q\sin\chi\cos\phi\sin\theta_n + q\cos\chi\cos\theta_n. \tag{28}$$

We have $\tilde{n}^i p'^i = p'^i(\delta^{ij} - \frac{p^i p^j}{p^2})n^j$. Hence, we have

$$\langle\langle\tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle = \langle\langle\mathbf{n}\cdot\mathbf{p}'\rangle\rangle - \langle\langle\mathbf{p}\cdot\mathbf{p}'\rangle\rangle\frac{\cos\theta_n}{p}. \tag{29}$$

First, we need to obtain $(\mathbf{n}\cdot\mathbf{p}')$ in terms of other variables of integration. The Lorentz transformation that relates laboratory frame and center-of-mass frame has the following form:

$$\mathbf{p}' = \gamma_{cm}(\hat{\mathbf{p}}'_{cm} + \mathbf{v}_{cm}\hat{E}'_{cm}), \tag{30}$$

where $\gamma_{cm} = \frac{E_p+E_q}{\sqrt{s}}$ and the velocity of the center-of-mass $\mathbf{v}_{cm} = \frac{\mathbf{p}+\mathbf{q}}{E_p+E_q}$. Energy conservation leads to $\hat{p}'^2_{cm} = \hat{p}^2_{cm}$. In the center-of-mass frame, $\hat{\mathbf{p}}'_{cm}$ can be decomposed as follows:

$$\hat{\mathbf{p}}'_{cm} = \hat{p}_{cm}(\cos\theta_{cm}\hat{\mathbf{x}}_{cm} + \sin\theta_{cm}\sin\phi_{cm}\hat{\mathbf{y}}_{cm} + \sin\theta_{cm}\cos\phi_{cm}\hat{\mathbf{z}}_{cm}), \tag{31}$$

where $\hat{p}_{cm} = \frac{s-m^2_{\rm HQ}}{2\sqrt{s}}$ is the HQ momentum and $\hat{E}_{cm} = (\hat{p}^2_{cm} + m^2_{\rm HQ})^{1/2}$ is the energy in the center-of-mass frame. The axes $\hat{\mathbf{x}}_{cm}$, $\hat{\mathbf{y}}_{cm}$, and $\hat{\mathbf{z}}_{cm}$ are defined in Ref. [66]. Employing the above definitions, we obtain

$$\begin{aligned}\tilde{\mathbf{n}}\cdot\mathbf{p}' =& \frac{\gamma_{cm}}{1+\gamma^2_{cm}v^2_{cm}}\Bigg\{\hat{p}_{cm}[\cos\theta_{cm}(\hat{\mathbf{x}}_{cm}\cdot\mathbf{n}) + \sin\theta_{cm}\sin\phi_{cm}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{n}) + \sin\theta_{cm}\cos\phi_{cm}(\hat{\mathbf{z}}_{cm}\cdot\mathbf{n})] \\ &+ \gamma_{cm}E'_p\frac{p\cos\theta_n + q\cos\chi\cos\theta_n + q\sin\chi\cos\phi\sin\theta_n}{E_p+E_q}\Bigg\} \\ &- \frac{\gamma_{cm}}{1+\gamma^2_{cm}v^2_{cm}}\frac{\cos\theta_n}{p}\Bigg\{\hat{p}_{cm}[\cos\theta_{cm}(\hat{\mathbf{x}}_{cm}\cdot\mathbf{p}) + \sin\theta_{cm}\sin\phi_{cm}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{p})] + \gamma_{cm}E'_p\frac{p^2+pq\cos\chi}{E_p+E_q}\Bigg\}.\end{aligned} \tag{32}$$

Note that we have obtained

$$\begin{aligned}\mathbf{p}\cdot\mathbf{p}' &= \frac{\gamma_{cm}}{1+\gamma^2_{cm}v^2_{cm}}\Bigg\{\hat{p}_{cm}[\cos\theta_{cm}(\hat{\mathbf{x}}_{cm}\cdot\mathbf{p}) + \sin\theta_{cm}\sin\phi_{cm}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{p})] + \gamma_{cm}E'_p\frac{p^2+pq\cos\chi}{E_p+E_q}\Bigg\} \\ &= E_pE'_p - \hat{E}^2_{cm} + \hat{p}^2_{cm}\cos\theta_{cm}.\end{aligned}$$

The respective projections of the anisotropy vector and HQ momentum with the center-of-mass axis are defined in Appendix B.

III. RESULTS AND DISCUSSIONS

A. Heavy quark transport coefficients in an anisotropic medium

We initiate the discussions with the momentum dependence of the components of the HQ drag force in an anisotropic QCD medium. HQ drag force has two components, namely, $A_0^{(\rm aniso)}$ and $A_1^{(\rm aniso)}$, in the anisotropic medium as described in Eq. (15). The anisotropic effects are entering through the nonequilibrium part of the momentum distribution function. For the quantitative estimation, we have considered $m_{\rm HQ} = 1.3$ GeV for charm quarks, $a_k = 0$, and one-loop running coupling constant from Ref. [25]. The impact of anisotropy on the momentum dependence of A_0 is depicted in Fig. 1 (top panel). The momentum and temperature of A_0 in the isotropic QCD medium have been well explored in Refs. [23,59]. The anisotropic part δA_0 considerably reduces the drag coefficient A_0, especially in the low-momentum regimes. It is seen that the anisotropic correction has a strong dependence on the direction of anisotropy (with respect to the direction of HQ motion, θ_n) and the strength of anisotropy in the medium. However, the dependence of the angle θ_n on the drag coefficient is observed to be opposite for low-momentum regimes in comparison with the high-momentum regimes. The additional drag coefficient $A_1^{(\rm aniso)}$ arises due to the anisotropy of the medium. The relative significance of $A_1^{(\rm aniso)}$ with that with $A_0^{(\rm aniso)}$ is plotted as a function of HQ momentum at $T = 360$ MeV and $T = 480$ MeV in Fig. 1 (middle panel and bottom panel). We have observed that the additional component is negligible in the high-momentum regimes. However, the additional drag coefficient may have an important role at the higher-temperature regimes. It is important to note that the same decomposition of HQ drag force holds true in a strongly anisotropic medium, and the additional component may have a more visible impact on HQ motion with an increase in the strength of anisotropy because $\langle\langle\tilde{\mathbf{n}}\cdot\mathbf{p}'\rangle\rangle \propto \xi$.

The anisotropic corrections to the HQ diffusion coefficients $B_0^{(\rm aniso)} = B_0 + \delta B_0$ and $B_1^{(\rm aniso)} = B_1 + \delta B_1$ are plotted in Fig. 2. The impact of medium anisotropy is more pronounced in the low HQ momentum regimes. Unlike in the case of $B_1^{(\rm aniso)}$, the coefficient $B_0^{(\rm aniso)}$ gets anisotropic

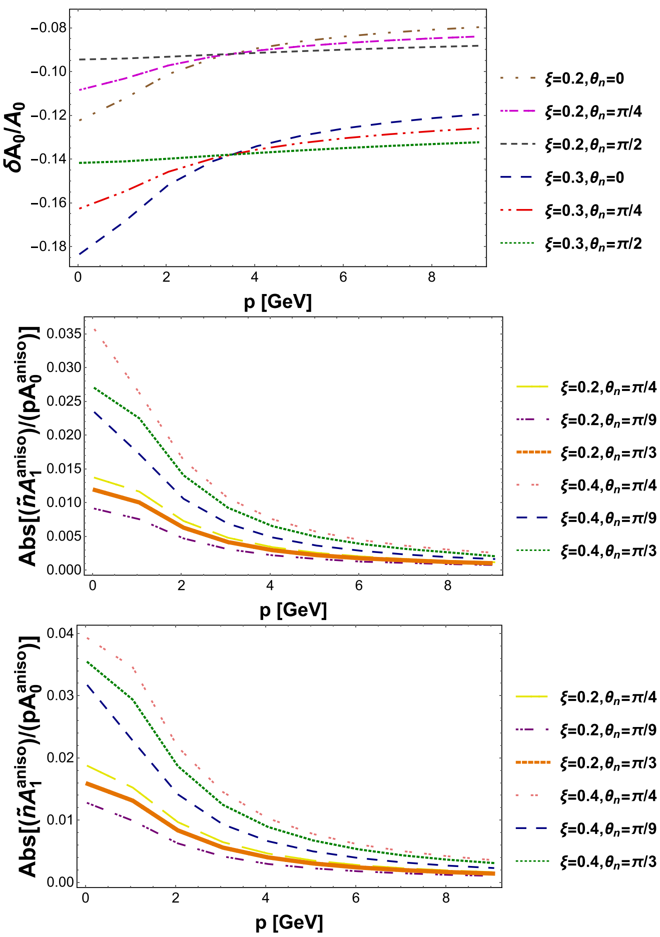

FIG. 1. Anisotropic correction to A_0 as a function of its initial momentum at $T = 360$ MeV (top panel). Relative significance of $A_1^{(\text{aniso})}$ in comparison with $A_0^{(\text{aniso})}$ at $T = 360$ MeV (middle panel) and $T = 480$ MeV (bottom panel).

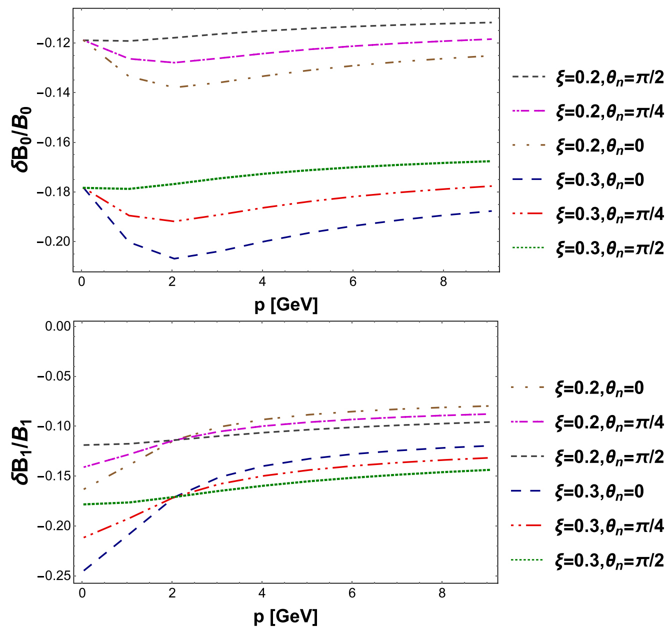

FIG. 2. Momentum dependence of anisotropic corrections to B_0 (top panel) and B_1 (bottom panel) at $T = 360$ MeV.

contribution from $\langle\langle(\mathbf{p}'\cdot\tilde{\mathbf{n}})^2\rangle\rangle$ along with nonequilibrium part of the thermal distribution function. In the limit $\xi \to 0$, the forms of $B_0^{(\text{aniso})}$ and $B_1^{(\text{aniso})}$ as described in Eqs. (22) and (23) will reduce back to the results of Ref. [66] (if we use same parameters as used in Ref. [66]). Both the momentum behavior of diffusion coefficients are seen to have a strong dependence on the angle between the anisotropic vector and HQ velocity in the QCD medium. The momentum anisotropy in the medium further give rise to additional components of HQ diffusion coefficients, namely, $B_2^{(\text{aniso})}$ and $B_3^{(\text{aniso})}$. Note that these additional coefficients vanish in the isotropic limit as $\xi \to 0$. The relative significance of HQ diffusion coefficients in an anisotropic QCD medium is depicted in Fig. 3. The additional diffusion coefficients seem to be more prominent in the low-momentum regimes in comparison with high-momentum regimes. In the static limit $p \to 0$, these coefficients are non-negligible, especially for the case of a strongly anisotropic medium. However, the coefficient $B_1^{(\text{aniso})}$ is dominant over $B_0^{(\text{aniso})}$ at high momenta. Similar to the case of isotropic medium, in the static limit, we obtain $B_0^{(\text{aniso})} = B_1^{(\text{aniso})}$. The direction of anisotropy in the medium has a visible impact on the low-momentum behavior of $B_2^{(\text{aniso})}$ and $B_3^{(\text{aniso})}$. Whereas the angle θ_n dependence is negligible for the momentum behavior of $B_1^{(\text{aniso})}$.

B. Heavy quark energy loss in an anisotropic medium

HQs may lose its energy while traveling through the anisotropic QCD medium due to the collisional processes with the in-medium particles. The differential collisional energy loss can be quantified in terms of the HQ drag coefficient due to the elastic collisions in the medium as [59]

$$\left(-\frac{dE}{dx}\right)_{\text{aniso}} = A_0^{(\text{aniso})}(p^2, T)p. \qquad (33)$$

It is important to note that the current focus is on the energy loss in the direction of initial HQ momentum. Hence, the contribution from $A_1^{(\text{aniso})}$ will vanish as $\mathbf{p}\cdot\tilde{\mathbf{n}} = 0$. However, the energy loss will have an anisotropic contribution through the δA_0. We have plotted the ratio of collisional energy loss of the charm quark in an anisotropic medium $(-\frac{dE}{dx})_{\text{aniso}}$ to that in the isotropic QCD medium $(-\frac{dE}{dx})_0$ for the RHIC and LHC energies in Fig. 4. The quark energy loss of HQ in the QCD medium seems to have a dependence on its initial momentum and temperature of the background medium. The energy loss of HQ gets suppressed in the anisotropic QCD medium with an increase in the strength of the anisotropy factor. However, the direction of anisotropy in the medium seems to have a weaker dependence on the HQ energy loss for the RHIC energy. These observations on charm quark energy loss hold true for the LHC energy, too.

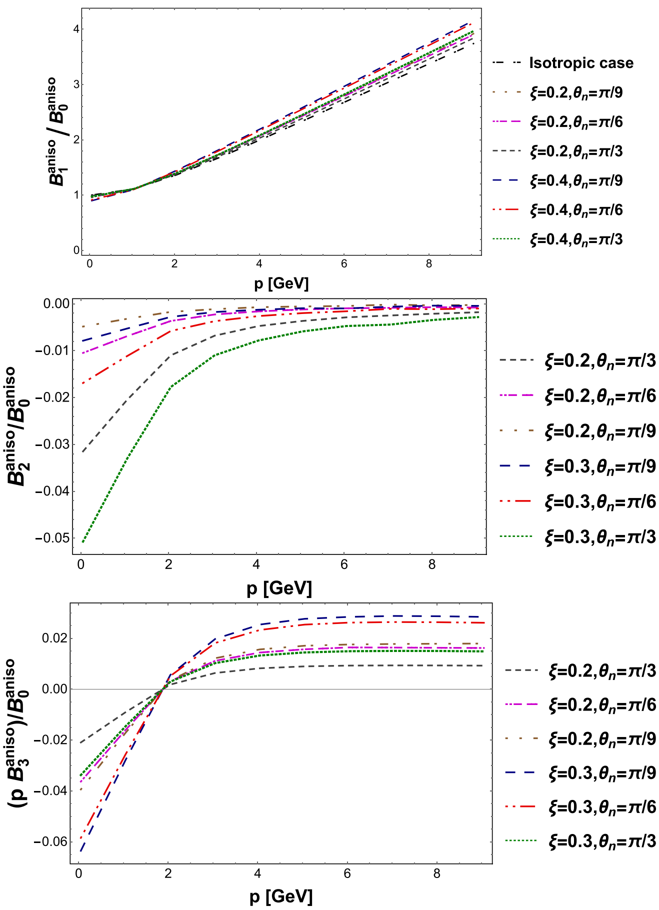

FIG. 3. Relative significance of HQ diffusion coefficients in an anisotropic medium: $B_1^{(\rm aniso)}/B_0^{(\rm aniso)}$ (top panel), $B_2^{(\rm aniso)}/B_0^{(\rm aniso)}$ (middle panel), $(pB_3^{(\rm aniso)})/B_0^{(\rm aniso)}$ (bottom panel) at $T = 360$ MeV.

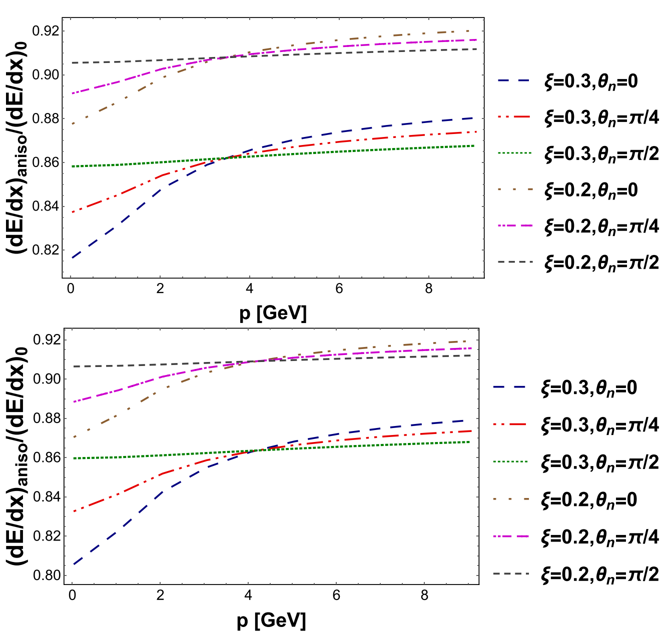

FIG. 4. Impact of anisotropy on the momentum behavior of collisional energy loss of charm quark for the RHIC energy at $T =$ 360 MeV (top panel) and for the LHC energy at $T = 480$ MeV.

IV. CONCLUSION AND OUTLOOK

We have studied the HQ transport coefficients and energy loss in an anisotropic QCD medium within the Fokker-Planck approach. The anisotropic aspect of the medium has been incorporated in the analysis through the nonequilibrium part of the quarks and antiquarks and gluonic momentum distribution. We have employed a proper decomposition to HQ drag force with two components in the anisotropic QCD medium. Similarly, we have constructed the second rank HQ diffusion tensor with four diffusion coefficients in the anisotropic medium. We have realized that the anisotropic effects have a strong dependence on the orientation of HQ motion with the direction of anisotropy in the medium. The relative significance of these anisotropic transport coefficients has been studied as a function of HQ initial momentum. It is seen that the additional components of drag and diffusion coefficients that arise due to the momentum anisotropy of the medium are subdominant in comparison with the isotropic components for weakly anisotropic medium. Moreover, these anisotropic contributions are essential for the theoretical consistency for studying the HQ transport in an anisotropic QCD medium. Furthermore, we have analyzed the anisotropic contribution to HQ collisional energy loss. It is observed that the HQ energy loss depends on the relative orientation of anisotropy with the HQ motion, especially in the low-momentum regimes.

In the anisotropy medium, the overall magnitude of the transport coefficients at the chosen momentum regimes decreases, which can reduce the nuclear suppression factor and elliptic flow. However, the anisotropy in the heavy quark transport will be able to generate an additional contribution to the HQ elliptic flow, which can enhance it at low momentum. In the current analysis, we have observed that the effect of anisotropy on HQ drag coefficient (and diffusion coefficients) is more significant in the lower-momentum regimes. This indicates that the heavy quark average momentum transfer will depend on the direction and strength of anisotropy in the medium. We expect a positive contribution to the elliptic flow of HQs as the momentum transfer will be different along and the direction transverse to the anisotropy vector. The same prediction holds true for the case of HQs in the magnetized medium [54]. However, the quantitative difference needs to be explored with detailed numerical simulations.

An enhanced directed flow of $D^0/\bar{D}^0$ mesons in comparison with that of charged kaons have been recently reported at the LHC and RHIC [56,57]. HQ v_1 has been computed recently in the presence of electromagnetic fields and tilted bulk distributions. To compute the HQ directed flow along with the bulk, one needs to consider the effect of anisotropy on HQ transport coefficients, which has been ignored in the previous calculations. The impact of anisotropy of the medium, more specifically, the anisotropic drag force and momentum diffusion due to the magnetic field in the medium, will help in better understanding the recent data. The current work can be the first step towards the understanding of the recent LHC and RHIC data on the flow coefficient of D mesons. Furthermore, the heavy quark scattering process (matrix element) in a general magnetized medium and the inclusion of $(1+3)$D QGP medium expansion need to be considered systematically in the analysis to explore the phenomenological aspects of heavy quarks at RHIC and the LHC.

We intend to study the phenomenological aspect of the anisotropic HQ transport coefficients in follow-up work. It is an interesting task to study HQ dynamics in a strongly anisotropic medium. Perhaps the additional drag and diffusion coefficients due to the anisotropy in the medium may have a significant role in a strongly anisotropic medium. The radiative process by HQs (inelastic process) in the anisotropic magnetized QCD medium is another interesting direction to explore.

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APPENDIX A: $\langle\langle F(p')\rangle\rangle_a$ IN AN ANISOTROPIC MEDIUM

The most general form for the average of a function $F(\mathbf{p})$ in the COM frame for an anisotropic medium (nonequilibrium medium) can be described as follows:

$$\begin{aligned}\langle\langle F(\mathbf{p})\rangle\rangle =& \frac{1}{(1024\pi^5)E_p\gamma_{\rm HQ}}\int_0^\infty dqq\left(\frac{s-m_{\rm HQ}^2}{s}\right)\\ &\times\int_0^\pi d\chi\sin\chi\int_0^{2\pi}d\phi\int_0^\pi d\theta_{cm}\sin\theta_{cm}\\ &\times\sum|\mathcal{M}_{2\to 2}|^2\int_0^{2\pi}d\phi_{cm}f_k^{(\rm aniso)}(E_q)\\ &\times\left[1+a_kf_k^{(\rm aniso)}(E_{q'})\right]F(\mathbf{p}),\end{aligned} \tag{A1}$$

where $f_k^{(\rm aniso)}(E_{q'})$ is the anisotropic momentum distribution function. As the current focus is on a weakly anisotropic medium $\xi \ll 1$, the distribution function takes the form

$$f_k^{(\rm aniso)}(\mathbf{q}) = f_k^0 + \delta f_k, \tag{A2}$$

in which δf_k is the anisotropic contribution to the isotropic distribution function f_k^0. From Eq. (14), we have $\delta f_k \propto \xi(\mathbf{q}\cdot\mathbf{n})^2$, which depends on the scattering angles and θ_n. Note that, in the isotropic limit, we have $f_k^{(\rm aniso)}(\mathbf{q}) \to f_k^0$, and the $d\phi$ integration can be carried out directly because $\sum|\mathcal{M}_{2\to 2}|^2$ is independent of the angle ϕ. Hence, Eq. (A1) reduces back to the expression in Eq. (9) for the isotropic case. By substituting Eq. (A2) into Eq. (A1) and arranging terms in order of ξ we obtain

$$\begin{aligned}\langle\langle F(\mathbf{p})\rangle\rangle =& \frac{1}{(1024\pi^5)E_p\gamma_{\rm HQ}}\int_0^\infty dqq\left(\frac{s-m_{\rm HQ}^2}{s}\right)\\ &\times\int_0^\pi d\chi\sin\chi\int_0^{2\pi}d\phi\int_0^\pi d\theta_{cm}\sin\theta_{cm}\\ &\times\sum|\mathcal{M}_{2\to 2}|^2\int_0^{2\pi}d\phi_{cm}\{f_k^0(E_q)\\ &\times\left[1+a_kf_k^0(E_{q'})\right]+\delta f_k(E_q)\left[1+a_kf_k^0(E_{q'})\right]\\ &+f_k^0(E_q)\delta f_k(E_{q'})+O(\xi^2)\}F(\mathbf{p}).\end{aligned} \tag{A3}$$

As we are considering the case of weakly anisotropic case, we can neglect the term with $(\delta f_k)^2$ because it is of higher order in ξ. Equation (A3) can be further decomposed into isotropic and anisotropic parts as

$$\langle\langle F(p')\rangle\rangle = \langle\langle F(p')\rangle\rangle_0 + \langle\langle F(p')\rangle\rangle_{\rm a},$$

in which $\langle\langle F(p')\rangle\rangle_0$ and $\langle\langle F(p')\rangle\rangle_{\rm a}$ take the forms as described in Eqs. (9) and (19), respectively.

Substituting Eq. (18) into Eq. (5) and employing Eqs. (9) and (19), we obtain Eq. (20). Following Ref. [47], a general decomposition of diffusion matrix and drag force need to be done to describe the dynamics of HQ in an anisotropic medium as described in Eqs. (15) and in (21).

APPENDIX B: PROJECTIONS OF ANISOTROPY VECTOR AND HEAVY QUARK MOMENTUM WITH THE CENTER-OF-MASS AXIS

By employing the following definitions in the center-of-mass frame:

$$(\mathbf{v}_{cm}\cdot\hat{\mathbf{p}}_{cm}) = \gamma_{cm}\left[\frac{p^2+pq\cos\chi}{E_p+E_q} - v_{cm}^2E_p\right], \tag{B1}$$

$$N^2 = v_{cm}^2 - \frac{(\mathbf{v}_{cm}\cdot\hat{\mathbf{p}}_{cm})^2}{\hat{p}_{cm}^2}, \tag{B2}$$

$$v_{cm}^2 = \frac{p^2+q^2+2pq\cos\chi}{(E_p+E_q)^2}, \tag{B3}$$

we have

$$(\hat{\mathbf{x}}_{cm}\cdot\mathbf{n}) = \frac{\gamma_{cm}}{\hat{p}_{cm}}\left[p\cos\theta_n - E_p\frac{(p\cos\theta_n + q\cos\chi\cos\theta_n + q\sin\chi\cos\phi\sin\theta_n)}{E_p+E_q}\right], \tag{B4}$$

$$\begin{aligned}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{n}) =& N^{-1}\Bigg[\frac{(p\cos\theta_n + q\cos\chi\cos\theta_n + q\sin\chi\cos\phi\sin\theta_n)}{E_p+E_q}\\ &-(\mathbf{v}_{cm}\cdot\hat{\mathbf{p}}_{cm})\frac{\gamma_{cm}}{\hat{p}_{cm}^2}\left(p\cos\theta_n - E_p\frac{(p\cos\theta_n + q\cos\chi\cos\theta_n + q\sin\chi\cos\phi\sin\theta_n)}{E_p+E_q}\right)\Bigg],\end{aligned} \tag{B5}$$

$$(\hat{\mathbf{z}}_{cm}\cdot\mathbf{n}) = \gamma_{cm}N^{-1}\frac{1}{\hat{p}_{cm}(E_p+E_q)}pq\sin\chi\sin\phi\sin\theta_n, \tag{B6}$$

$$(\hat{\mathbf{x}}_{cm} \cdot \mathbf{p}) = \frac{\gamma_{cm}}{\hat{p}_{cm}}\left[p^2 - E_p\frac{(p^2 + pq\cos\chi)}{E_p + E_q}\right], \tag{B7}$$

$$(\hat{\mathbf{y}}_{cm} \cdot \mathbf{p}) = N^{-1}\left[\frac{(p^2 + pq\cos\chi)}{E_p + E_q} - (\mathbf{v}_{cm} \cdot \hat{\mathbf{p}}_{cm})\frac{\gamma_{cm}}{\hat{p}_{cm}^2}\left(p^2 - E_p\frac{(p^2 + pq\cos\chi)}{E_p + E_q}\right)\right]. \tag{B8}$$

[1] J. Adams *et al.* (STAR Collaboration), Nucl. Phys. A **757**, 102 (2005).
[2] B. Back *et al.* (PHOBOS Collaboration), Nucl. Phys. A **757**, 28 (2005).
[3] I. Arsene *et al.* (BRAHMS Collaboration), Nucl. Phys. A **757**, 1 (2005).
[4] K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. A **757**, 184 (2005).
[5] K. Aamodt *et al.* (ALICE Collaboration), Phys. Rev. Lett. **105**, 252301 (2010).
[6] A. Jaiswal *et al.*, Int. J. Mod. Phys. E **30**, 2130001 (2021).
[7] A. Beraudo *et al.*, Nucl. Phys. A **979**, 21 (2018).
[8] X. Dong and V. Greco, Prog. Part. Nucl. Phys. **104**, 97 (2019).
[9] F. Prino and R. Rapp, J. Phys. G **43**, 093002 (2016).
[10] G. Aarts *et al.*, Eur. Phys. J. A **53**, 93 (2017).
[11] A. Andronic *et al.*, Eur. Phys. J. C **76**, 107 (2016).
[12] S. K. Das, Jan-e. Alam, and P. Mohanty, Phys. Rev. C **82**, 014908 (2010).
[13] S. K. Das, F. Scardina, S. Plumari, and V. Greco, Phys. Rev. C **90**, 044901 (2014).
[14] G. D. Moore and D. Teaney, Phys. Rev. C **71**, 064904 (2005).
[15] P. B. Gossiaux, V. Guiho, and J. Aichelin, J. Phys. G **32**, S359 (2006).
[16] N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, Phys. Rev. D **102**, 074503 (2020).
[17] R. Rapp and H. van Hees, arXiv:0903.1096.
[18] H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C **73**, 034913 (2006).
[19] P. B. Gossiaux and J. Aichelin, Phys. Rev. C **78**, 014904 (2008).
[20] S. K. Das, Jan-e. Alam, and P. Mohanty, Phys. Rev. C **80**, 054916 (2009).
[21] M. He, R. J. Fries, and R. Rapp, Phys. Rev. Lett. **110**, 112301 (2013).
[22] S. Cao, G.-Y. Qin, and S. A. Bass, Phys. Rev. C **88**, 044907 (2013).
[23] S. Cao, G. Coci, S. K. Das, W. Ke, S. Y. F. Liu, S. Plumari, T. Song, Y. Xu, J. Aichelin, S. Bass, E. Bratkovskaya, X. Dong, P. B. Gossiaux, V. Greco, M. He, M. Nahrgang, R. Rapp, F. Scardina, and X. N. Wang, Phys. Rev. C **99**, 054907 (2019).
[24] Y. Xu, S. A. Bass, P. Moreau, T. Song, M. Nahrgang, E. Bratkovskaya, P. Gossiaux, J. Aichelin, S. Cao, V. Greco, G. Coci, and K.Werner, Phys. Rev. C **99**, 014902 (2019).
[25] S. K. Das, F. Scardina, S. Plumari, and V. Greco, Phys. Lett. B **747**, 260 (2015).
[26] F. Scardina, S. K. Das, V. Minissale, S. Plumari, and V. Greco, Phys. Rev. C **96**, 044905 (2017).
[27] S. Cao, T. Luo, G.-Y. Qin, and X.-N. Wang, Phys. Rev. C **94**, 014909 (2016).
[28] T. Song, H. Berrehrah, D. Cabrera, J. M. Torres-Rincon, L. Tolos, W. Cassing, and E. Bratkovskaya, Phys. Rev. C **92**, 014910 (2015).
[29] A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **98**, 172301 (2007).
[30] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **96**, 032301 (2006).
[31] W. M. Alberico, A. Beraudo, A. De Pace, A. Molinari, M. Monteno, M. Nardi, F. Prino, and M. Sitta, Eur. Phys. J. C **73**, 2481 (2013).
[32] H. van Hees, M. Mannarelli, V. Greco, and R. Rapp, Phys. Rev. Lett. **100**, 192301 (2008).
[33] S. Li, C. Wang, R. Wan, and J. Liao, Phys. Rev. C **99**, 054909 (2019).
[34] Y. Akamatsu, T. Hatsuda, and T. Hirano, Phys. Rev. C **79**, 054907 (2009).
[35] J. Uphoff, O. Fochler, Z. Xu, and C. Greiner, Phys. Rev. C **84**, 024908 (2011).
[36] H.-j. Xu, X. Dong, L.-j. Ruan, Q. Wang, Z.-b. Xu, and Y.-f. Zhang, Phys. Rev. C **89**, 024905 (2014).
[37] D. Zigic, I. Salom, J. Auvinen, M. Djordjevic, and M. Djordjevic, Phys. Lett. B **791**, 236 (2019).
[38] S. Mrowczynski, Phys. Lett. B **314**, 118 (1993).
[39] J. Randrup and S. Mrowczynski, Phys. Rev. C **68**, 034909 (2003).
[40] V. Chandra, Phys. Rev. D **86**, 114008 (2012).
[41] V. Chandra, Phys. Rev. D **84**, 094025 (2011).
[42] M. Asakawa, S. A. Bass, and B. Muller, Phys. Rev. Lett. **96**, 252301 (2006).
[43] M. Asakawa, S. A. Bass, and B. Muller, Prog. Theor. Phys. **116**, 725 (2007).
[44] C. Shen, J.-F. Paquet, U. Heinz, and C. Gale, Phys. Rev. C **91**, 014908 (2015).
[45] B. S. Kasmaei and M. Strickland, Phys. Rev. D **102**, 014037 (2020).
[46] B. S. Kasmaei and M. Strickland, Phys. Rev. D **99**, 034015 (2019).
[47] P. Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003).
[48] B. Schenke, M. Strickland, C. Greiner, and M. H. Thoma, Phys. Rev. D **73**, 125004 (2006).
[49] A. Kumar, M. Y. Jamal, V. Chandra, and J. R. Bhatt, Phys. Rev. D **97**, 034007 (2018).
[50] S. Hauksson, S. Jeon, and C. Gale, Phys. Rev. C **105**, 014914 (2022).
[51] M. Alqahtani, M. Nopoush, and M. Strickland, Prog. Part. Nucl. Phys. **101**, 204 (2018).
[52] M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, Phys. Rev. C **96**, 044910 (2017).
[53] M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, Phys. Rev. Lett. **119**, 042301 (2017).
[54] K. Fukushima, K. Hattori, H.-U. Yee, and Y. Yin, Phys. Rev. D **93**, 074028 (2016).
[55] B. Singh, M. Kurian, S. Mazumder, H. Mishra, V. Chandra, and S. K. Das, arXiv:2004.11092.

[56] S. Acharya *et al.* (ALICE Collaboration), Phys. Rev. Lett. **125**, 022301 (2020).

[57] J. Adam *et al.* (STAR Collaboration), Phys. Rev. Lett. **123**, 162301 (2019).

[58] M. Kurian, M. Singh, V. Chandra, S. Jeon, and C. Gale, Phys. Rev. C **102**, 044907 (2020).

[59] M. G. Golam Mustafa, D. Pal, and D. K. Kumar Srivastava, Phys. Rev. C **57**, 889 (1998); **57**, 3499(E) (1998).

[60] P. K. Srivastava and B. K. Patra, Eur. Phys. J. A **53**, 116 (2017).

[61] T. Song, P. Moreau, J. Aichelin, and E. Bratkovskaya, Phys. Rev. C **101**, 044901 (2020).

[62] J. Prakash, M. Kurian, S. K. Das, and V. Chandra, Phys. Rev. D **103**, 094009 (2021).

[63] A. Shaikh, M. Kurian, S. K. Das, V. Chandra, S. Dash, and B. K. Nandi, Phys. Rev. D **104**, 034017 (2021).

[64] P. Romatschke and M. Strickland, Phys. Rev. D **71**, 125008 (2005).

[65] S. K. Das, V. Chandra, and Jan-e. Alam, J. Phys. G **41**, 015102 (2013).

[66] B. Svetitsky, Phys. Rev. D **37**, 2484 (1988).

[67] B. L. Combridge, Nucl. Phys. B **151**, 429 (1979).

[68] P. K. Srivastava, L. Thakur, and B. K. Patra, Phys. Rev. C **91**, 044903 (2015).