# Investigation of pseudospin and spin symmetries in relativistic mean field theory combined with a similarity renormalization group approach

Bo Huang, Jian-You Guo,* and Shou-Wan Chen
*School of Physics and Optoelectronics Engineering, Anhui University, Hefei 230601, People's Republic of China*

(Received 23 March 2022; accepted 6 May 2022; published 19 May 2022)

Similarity renormalization group (SRG) is combined with the relativistic mean-field (RMF) theory, the Schrödinger-like Hamiltonian describing the motion of nucleon is obtained. The Hamiltonian with the scalar and vector potential from the self-consistent RMF calculations is used to explore the pseudospin and spin symmetries and their evolution with mass number for Sn isotopes. The contribution of every term to the pseudospin (spin) energy splitting is extracted. The spin energy splitting comes almost entirely from spin-orbit coupling, whereas the pseudospin energy splitting is also dominated by nonrelativistic and dynamical terms. The magnitude of the splitting is related to the radial, angular, and isospin quantum numbers of the doublet. The isospin dependence of pseudospin (spin) symmetry comes from the nonrelativistic term (the spin-orbit coupling). The pseudospin symmetry of neutron is superior to that of proton from the weakening of nonrelativistic and dynamical breaking. These results are helpful to understand the origin and breaking mechanism of pseudospin and spin symmetries.

DOI: 10.1103/PhysRevC.105.054313

## I. INTRODUCTION

Pseudospin and spin symmetries play a critical role at many physical phenomena [1,2]. The introduction of spin-symmetry (SS) breaking successfully explains the existence of magic numbers in atomic nuclei. The accompanying consequence is the emergence of two quasidegenerate single-particle states with quantum numbers of $(n-1, l+2, j = l+3/2)$ and $(n, l, j = l+1/2)$ in heavy nuclei. In analogy to the SS, the quasidegeneracy states are defined as the pseudospin doublet with quantum numbers $(\tilde{n} = n-1, \tilde{l} = l+1, j = \tilde{l} \pm 1/2)$ [3,4]. The introduction of pseudospin symmetry (PSS) can explain many nuclear structure phenomena, such as deformed bands [5], superdeformed bands [6], magnetic moments [7–11], and identical bands [12–15]. Because of these successes, physicists have attempted to understand the origin of this symmetry. In Ref. [16], a helicity unitary transformation is introduced to map a normal state $(l, s)$ to a pseudostate $(\tilde{l}, \tilde{s})$. A special proportional relationship between the coefficients of spin-orbit and orbit-orbit terms was found to be responsible for the emergence of PSS [17]. From the Dirac equation given by the relativistic mean field (RMF) theory, PSS was confirmed to be a relativistic symmetry and be exact when the sum of scalar potential $S$ and vector potential $V$ equals to zero [18]. The condition is relaxed to $d\Sigma/dr = 0$ in Refs. [19,20], and the spin symmetry in the antinucleon spectrum was found by charge conjugation transformation [20]. Further exploration of spin symmetry in antinucleon spectrum was presented in Refs. [21,22]. In Ref. [23], the Dirac Hamiltonian was shown to possess SU(2) with SU(2) algebra established in the PSS and SS limit. Furthermore, U(3) symmetry [24] and chiral symmetry [25,26] were found in the Dirac Hamiltonian with special potentials in the PSS and SS limit. The supersymmetry of the Dirac Hamiltonian was investigated in Refs. [27–29]. Pseudospin symmetry in nucleon-nucleus scattering was researched in Refs. [30,31]. Also the phase shifts in nucleon-nucleon scattering are analyzed to show that pseudospin symmetry is better conserved in $T = 1$ phase shifts than in $T = 0$ phase sifts which demonstrates on a fundamental level why pseudspin symmetry conservation improves for heavy nuclei [32,33]. Moreover, it has also been found that the PSS and SS exist in resonant states [34,35].

In Refs. [22,36], it was pointed out that the pseudospin symmetry arises from the mutual cancellation of the energy splittings contributed by different components in the Schrödinger-like Hamiltonian and PSS possesses a dynamical character. The conclusion was also discussed in Refs. [37,38]. Unfortunately, it was inevitable to encounter the singularity in calculating the contribution of every component to the pseudospin splitting and the coupling between the operator and its eigenenergy in solving the Schrödinger-like equation for the lower component of the Dirac spinor. To avoid these defects, we have applied the similarity renormalization group (SRG) to transform the Dirac Hamiltonian into a diagonal form [39,40]. Because all the defects in the usual decoupling, such as the singularity and coupling, disappear, it is very appropriate to use this Hamiltonian analyzing pseudospin and spin symmetries. The quality of pseudospin approximation is shown to be related to the competition between the spin-orbit coupling and dynamical effect [41–43]. To improve the accuracy of the usual SRG, a reconstituted SRG method was proposed using the resummation technique in Ref. [44]. The results were compared with those obtained by Foldy-Wouthuysen transformation and SRG method [45]. In Ref. [46], we explored

*jianyou@ahu.edu.cn

 

the contributions from the various mesons and photon fields to the PSS. The relationship between the pseudospin energy splitting and these mesons and photon fields was checked in the framework of RMF theory. The origin of PSS and its breaking mechanism were explored by combining supersymmetry quantum mechanics, perturbation theory, and the SRG method in Ref. [47]. In Ref. [48], the roles of Coulomb and $\rho$ potentials were investigated at the pseudospin energy splitting for Sn isotopes. The influences of the different mesons and photon fields on the PSS and SS were investigated with the SRG method [49]. Sun *et al.* have applied the Green's function method to solve the Dirac equation with a Woods-Saxon potential and investigated the SS and PSS in single-particle resonant states [50]. Alberto *et al.* [36] have studied isospin asymmetry in the PSS and indicated that the PSS is better realized for neutrons than for protons in the bound states. More researches on the PSS and SS can be seen in the reviews [51,52]. Recently, the PSS and SS of single-particle resonant states for Pb isotopes, the influence of different meson fields on the PSS in single-neutron resonant states, and the dependence of PSS on shape of potential in resonant states have been explored [53–55]. Considering the RMF theory is very successful in describing many phenomena of nuclei, we combine SRG into RMF to explore the origin and breaking mechanism of PSS and SS, and their dependencies on isospin. In Sec. II, we state the theoretical framework. The numerical details and results are presented in Sec. III. A summary is given in Sec. IV.

## II. FORMALISM

For the convenience of readers, we sketch the theoretical formalism. The basic ansatz of the RMF model is a Lagrangian density, which nucleons are described as the Dirac particles with the interactions via the exchange of mesons ($\sigma$, $\omega$, and $\rho$) and photon,

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}(i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\tau}\vec{\rho}_\mu \\ & - e\frac{1-\tau_3}{2}\gamma^\mu A_\mu)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 \\ & - \frac{g_2}{3}\sigma^3 - \frac{g_3}{4}\sigma^4 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},\end{aligned} \tag{1}$$

where $M$ is the nucleon mass, and $m_\sigma$, $g_\sigma$, $m_\omega$, $g_\omega$, $m_\rho$, and $g_\rho$ are masses and coupling constants of the respective mesons. The field tensors for the vector mesons and photons are defined as

$$\begin{aligned}\Omega_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu, \\ \vec{R}_{\mu\nu} &= \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g_\rho(\vec{\rho}_\mu \times \vec{\rho}_\nu), \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu.\end{aligned} \tag{2}$$

By the classical variational principle, the Dirac equation for nucleon is obtained as

$$\{\vec{\alpha}\cdot\vec{p} + V(\vec{r}) + \beta[M + S(\vec{r})]\}\psi(\vec{r}) = \varepsilon\psi(\vec{r}), \tag{3}$$

where $V(\vec{r})$ and $S(\vec{r})$ are the vector and scalar potentials, respectively [56,57]. For the spherical nuclei, the Dirac spinor reads

$$\psi(\boldsymbol{r}) = \begin{pmatrix} f(r)Y_{jm}^{l}(\vartheta,\varphi,s) \\ ig(r)Y_{jm}^{\tilde{l}}(\vartheta,\varphi,s)\end{pmatrix}\chi_{t_\alpha}(t), \quad j = l + \frac{1}{2} = \tilde{l} - \frac{1}{2}, \tag{4}$$

where $Y_{jm}^{l}(\vartheta,\varphi,s)$ is the spin spherical harmonics. Then, the Dirac equation is simplified to the form

$$\begin{pmatrix} V+S+M & -\frac{d}{dr} - \frac{1}{r} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{1}{r} + \frac{\kappa}{r} & V-S-M \end{pmatrix}\begin{pmatrix} f(r) \\ g(r)\end{pmatrix} = \varepsilon\begin{pmatrix} f(r) \\ g(r)\end{pmatrix}, \tag{5}$$

with the Hamiltonian,

$$H = \beta M + \vec{\alpha}\cdot\vec{p} + (\beta S + V), \tag{6}$$

where $S$ and $V$ represent the scalar potential and vector potential, respectively. In order to transform $H$ into a diagonal form, we utilize Wegner's formulation of the SRG [58]. The initial Hamiltonian $H$ is transformed by the unitary operator $U(l)$ according to

$$H(l) = U(l)HU^\dagger(l), \quad H(0) = H, \tag{7}$$

where $l$ is a flow parameter. By differentiating Eq. (7), the flow equation is obtained as

$$\frac{d}{dl}H(l) = [\eta(l), H(l)], \tag{8}$$

with the generator,

$$\eta(l) = \frac{dU(l)}{dl}U^\dagger(l) = -\eta^\dagger(l). \tag{9}$$

The choice of generator $\eta(l)$ should achieve the goal of the off-diagonal matrix elements decay. A good choice based on Wegner's theory is defined by $\eta(l) = [H_d(l), H(l)]$, where $H_d(l)$ is the diagonal part of $H(l)$ [58]. For the Dirac Hamiltonian (7), it is appropriate to choose $\eta(l) = [\beta M, H(l)]$ [39]. Using $\eta(l)$ chosen, $H(l)$ can evolve into a diagonal form when $l \to \infty$. We use the technique in Ref. [39] and obtain the diagonalized Dirac operator as

$$H_D = \begin{pmatrix} H_P + M & 0 \\ 0 & -H_P^C - M\end{pmatrix}, \tag{10}$$

where

$$H_P = H_0 + H_d + H_c + H_k + H_w. \tag{11}$$

The expressions of these five Hermitian components are

$$\begin{aligned} H_0 &= \Sigma(r) + \frac{p^2}{2M}, \\ H_d &= -\frac{\left(Sp^2 - S'\frac{d}{dr}\right)}{2M^2} + \frac{S\left(Sp^2 - 2S'\frac{d}{dr}\right)}{2M^3}, \\ H_c &= (2S - M)\frac{\kappa}{r}\frac{\Delta'}{4M^3}, \\ H_k &= -\frac{p^4}{8M^3}, \\ H_w &= \frac{\Sigma''}{8M^2} - \frac{\Sigma'^2 - 2\Sigma'\Delta' + 4S\Sigma''}{16M^3}. \end{aligned}$$

$H_0$ corresponds to the operator describing a Dirac particle in the nonrelativistic limit. The spin-orbit interaction and the dynamical effect are reflected in $H_c$ and $H_d$, respectively. $H_k$ represents the relativistic modification of kinetic energy. $H_w$ can denote the Darwin term. $H_P$ is an operator describing Dirac particle with $p^2 = -\frac{d^2}{dr^2} + \frac{\kappa(\kappa+1)}{r^2}$, $H_P^C$ is the charge-conjugation of $H_P$ with $p^2 = -\frac{d^2}{dr^2} + \frac{\kappa(\kappa-1)}{r^2}$ [21,22]. The primes and double primes in $H_P$, respectively, denote first-order and second-order derivatives with respect to $r$. Obviously, the singularity disappears in every component of Eq. (11), and all the terms in $H_P$ are Hermitian. In addition, there is no coupling between the energy $\epsilon$ and the operator $H_P$. Thus, the energy spectra of $H_P$ can be calculated conveniently. The energy spectra of $H_P$ are calculated by expansion with a set of harmonic-oscillator functions. The contribution of the operator $O_i$ to the energy-level $E_k$ is calculated by the formula $\langle k|O_i|k\rangle = \int \psi_k^* O_i \psi_k d^3\vec{r}$, where $k$ marks the single-particle state considered.

## III. NUMERICAL DETAILS AND RESULTS

Based on the previous formulism, we perform the RMF-SRG calculations for Sn isotopes. With the available single-particle levels, we explore the PSS and SS. The obtained energies for neutron and proton are plotted in Figs. 1 and 2, respectively. For the neutron single-particle states, the energies increase with increasing mass number for those states far from the continuum threshold, whereas that is opposite for those states near the continuum threshold. There appears a small kink in several bound levels, and several large energy gaps between some levels, such as $1g_{7/2}$ and $1g_{9/2}$. Of particular note is the level inversion in the states $2f_{5/2}$ and $3p_{3/2}$, which has an important consequence for the PSS and SS. Unlike the neutron case, except for a few small kinks, the proton single-particle energies decrease with increasing mass number for all the available single-particle states. Furthermore, the speed of energy decreasing with increasing mass number is faster for the proton than that for neutron, which indicates that the proton single-particle energy is more sensitive to isospin. Moveover, the energy-level order of the proton is different from that of the neutron near the continuum threshold, which reflects that the PSS and SS are isospin dependent.

To understand the physical mechanism of SS and its correlation with isospin, we extract the energy splitting between the spin doublets for Sn isotopes. The energy splitting contributed by every term in $H_p$ and their variations with mass number are plotted in Figs. 3–5. For all the neutron spin doublets, the energy splitting and its variation with mass number is nearly identical to that contributed by the spin-orbit term. Although the energy splitting contributed by the nonrelativistic and dynamical terms are strong, they are nearly the same in magnitude, but opposite in sign, which results that their contributions to the total energy splitting almost cancel out each other. Comparably, the energy splitting contributed by the other term is negligible.

Although the spin-orbit coupling plays a key role in the SS breaking, the role of nonrelativistic and dynamical terms cannot be ignored because of their important contributions to single-particle energy levels. For the spin doublets $(1p_{1/2}, 1p_{3/2})$, $(1d_{3/2}, 1d_{5/2})$, $(1f_{5/2}, 1f_{7/2})$, and $(1g_{7/2}, 1g_{9/2})$, the dynamical term destroys spin symmetry, whereas the nonrelativistic term improves SS. For the doublet $(2p_{1/2}, 2p_{3/2})$, the spin energy splitting contributed by the nonrelativistic term changes from a negative value to a positive value whereas that by the dynamical term changes from a positive value to a negative value with increasing mass number. For the doublets far from the threshold, the spin energy splitting caused by the nonrelativistic term (the dynamical term) are changed to positive (negative) for the $2d_{3/2}$ and $2d_{5/2}$ partners. Namely, the improvement or breaking from the nonrelativistic term (the dynamical term) to the SS depends on the quantum number of the doublets and isospin.

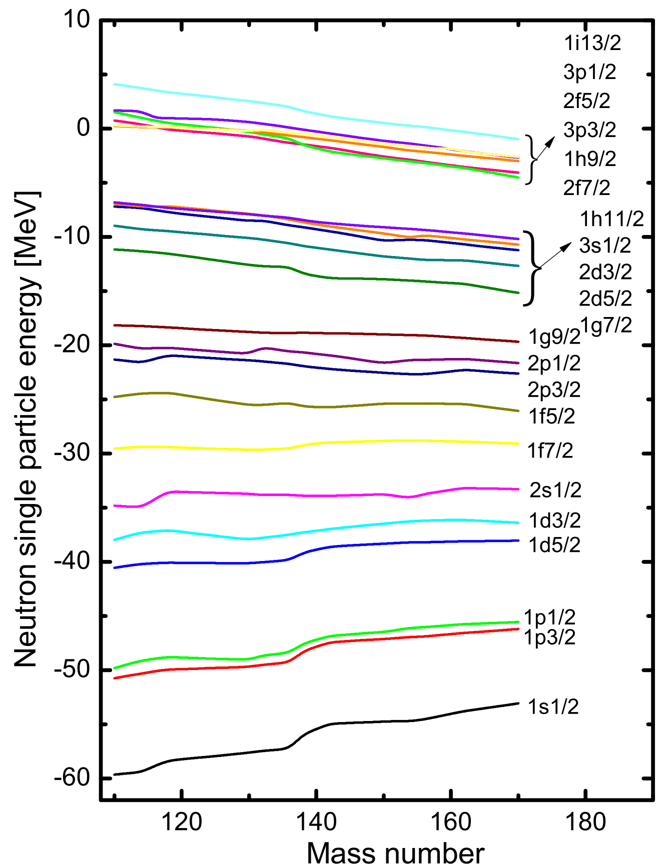


FIG. 1. Neutron single-particle energies and their evolution to mass number for Sn isotopes. The energies are obtained in the RMF-SRG calculations with the NL3 interaction.

The magnitude of energy splitting is correlated with the quantum numbers of the doublets. For the spin doublets with the same radial quantum number, the energy splitting caused by the spin-orbit term increases with increasing orbital angular momentum. For example, the energy splitting between the $1d_{3/2}$ and $1d_{5/2}$ partners is two times more than that of the doublet $(1p_{1/2}, 1p_{3/2})$. The similar case also appears in the other spin partners. For the spin doublets with the same orbital angular momentum, the energy splitting caused by

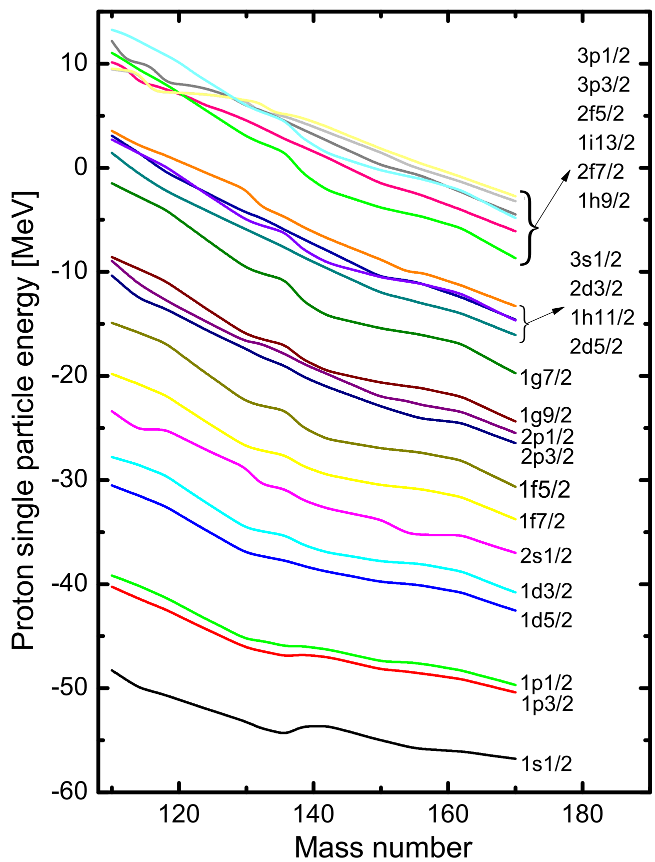


FIG. 2. The same as Fig. 1 but for proton single-particle levels.

the spin-orbit term increases with increasing radial quantum number. For instance, the energy splitting between the $2p_{1/2}$ and the $2p_{3/2}$ partners is two times more than that of the doublet $(1p_{1/2}, 1p_{3/2})$. Moreover, we have also observed that the energy of the single-particle state with the quantum numbers $(n, l, j = l - 1/2)$ is larger than that with the quantum numbers $(n, l, j = l + 1/2)$, which means that the single-particle state with higher total angular momentum is more stable than its spin partner.

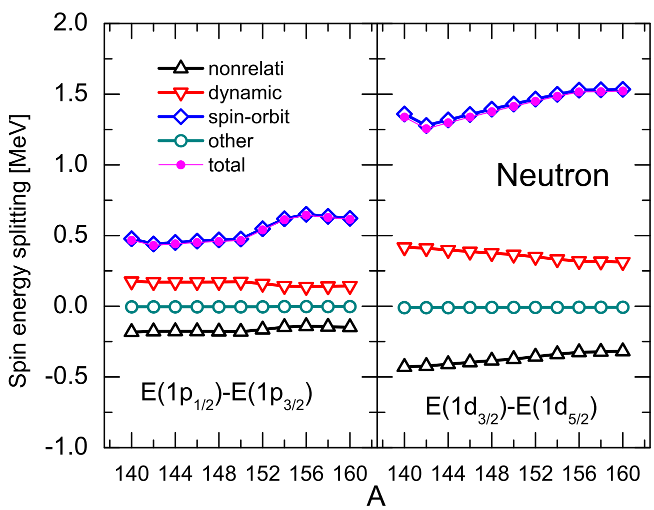


FIG. 3. Contribution of every term in $H_P$ to the spin energy splitting for the neutron spin doublets $(1p_{1/2}, 1p_{3/2})$ and $(1d_{3/2}, 1d_{5/2})$, and their evolutions to mass number for the Sn isotopes where "nonrelati," "dynamic," spin-orbit, and other denotes the nonrelativistic term, the dynamical term, the spin-orbit term, and the other term, respectively. The "total" marks a sum of the spin energy splitting.

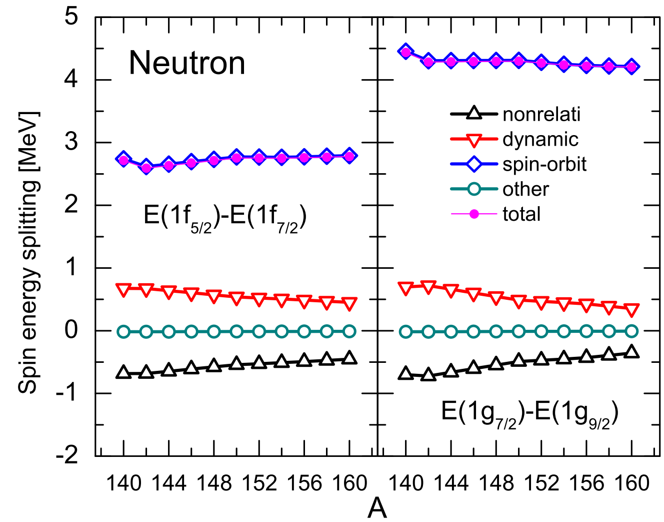


FIG. 4. The same as Fig. 3 but for the neutron spin doublets $(1f_{5/2}, 1f_{7/2})$ and $(1g_{7/2}, 1g_{9/2})$.

With the increasing of the mass number, the spin energy splitting increases for the two deeply bound doublets $(1p_{1/2}, 1p_{3/2})$ and $(1d_{3/2}, 1d_{5/2})$. For the energy levels closer to the continuum threshold, the energy splitting varies from increasing to almost constant to decreasing with increasing mass number. Especially for the energy levels very close to the continuum threshold as the $(2p_{1/2}, 2p_{3/2})$ and $(2d_{3/2}, 2d_{5/2})$ there appears a kink around $A = 150$. After $A = 150$, the energy splitting becomes increasing and then decreasing with

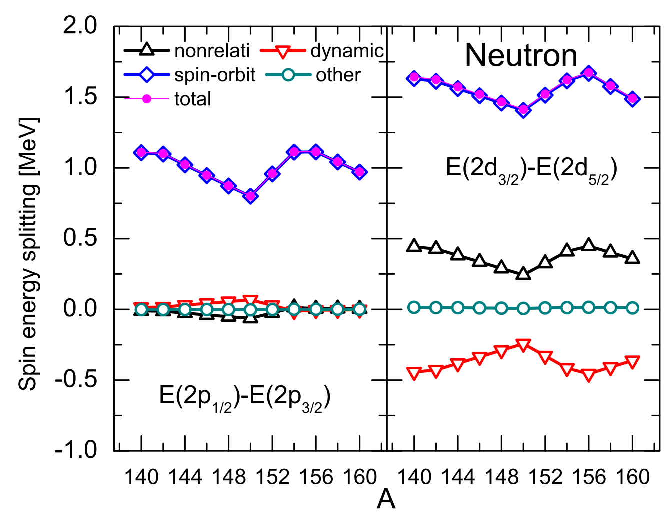


FIG. 5. The same as Fig. 3 but for the neutron spin doublets $(2p_{1/2}, 2p_{3/2})$ and $(2d_{3/2}, 2d_{5/2})$.

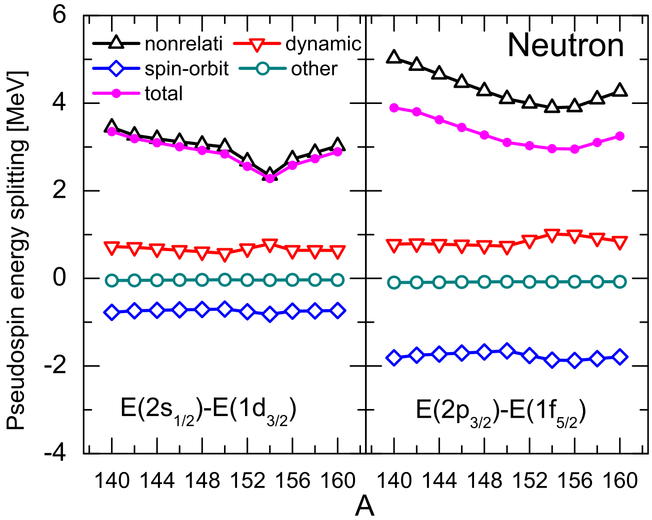


FIG. 6. The energy splitting of neutron pseudospin doublets $(2s_{1/2}, 1d_{3/2})$ and $(2p_{3/2}, 1f_{5/2})$ as a function of mass number for Sn isotopes where nonrelati, dynamic, spin-orbit, and other denote the nonrelativistic term, the dynamical term, the spin-orbit term, and the other term, respectively. The total marks a sum of the pseudospin energy splitting.

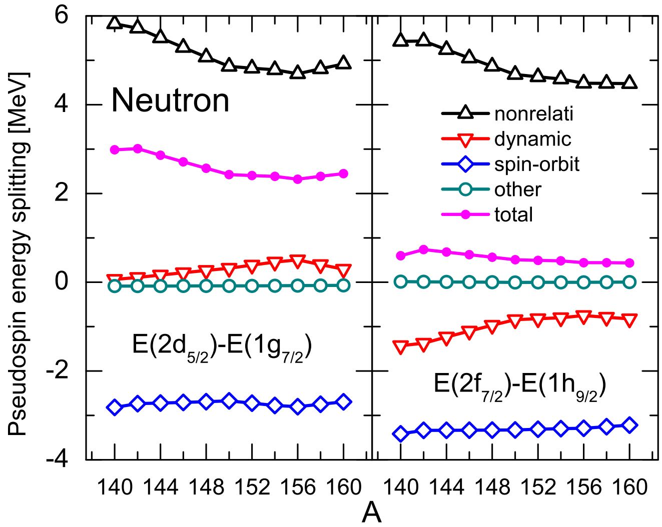


FIG. 7. The same as Fig. 6 but for the energy splitting for neutron pseudospin doublets $(2d_{5/2}, 1g_{7/2})$ and $(2f_{7/2}, 1h_{9/2})$.

increasing mass number. Namely, the isospin dependence of SS is related to the quantum number of the doublet.

These indicate that the breaking of SS mainly comes from the spin-orbit interaction, and the quality of SS is correlated with the quantum numbers of the doublets as well as isospin. The roles of nonrelativistic and dynamical terms does also play an important role at SS. Their influences on the SS is related to the quantum number of the doublets as well as isospin.

Next, we explore the physical mechanism of PSS and its dependence on isospin. The contribution of every term in $H_p$ to the pseudospin energy splitting and its evolution to mass number are displayed in Figs. 6–8 for all the available neutron pseudospin doublets. The variation of pseudospin energy splitting with mass number is quite consistent with that contributed by the nonrelativistic term, but the magnitude of splitting is significantly different. In addition to the contribution of the nonrelativistic term, the dynamical term and spin-orbit coupling also contribute significantly to the pseudospin energy splitting. Similar to the spin energy splitting, the contribution from the other term to pseudospin energy splitting is insignificant.

The nonrelativistic term always destroys the PSS. The magnitude of the destruction is very large, i.e., there is no PSS in the nonrelativistic limit. The spin-orbit term always improves the PSS. However, the dynamical term plays a role at destroying PSS for the doublets $(2s1/2, 1d_{3/2})$, $(2p_{3/2}, 1f_{5/2})$, and $(2d_{5/2}, 1g_{7/2})$, and improving PSS for the doublets $(2f_{7/2}, 1h_{9/2})$, $(3s_{1/2}, 2d_{3/2})$, and $(3p_{3/2}, 2f_{5/2})$. Whether improvement or breaking from the dynamical term depends on the particular pseudospin doublet.

For the pseudospin doublets with the same radial quantum number, the energy splitting decreases with increasing orbital angular momentum. For example, the energy splitting between the doublet $(2d_{5/2}, 1g_{7/2})$ is more than three times that of the doublet $(2f_{7/2}, 1h_{9/2})$. For the pseudospin doublets with the same orbital angular momentum, the energy splitting decreases with increasing radial quantum number. For instance, the energy splitting between the doublet $(2s_{1/2}, 1d_{3/2})$ is more than three times that of the doublet $(3s_{1/2}, 2d_{3/2})$. For the doublet closer to the continuum, there appears inversion in the pseudospin energy splitting. The energy splitting evolves from $E_{n,l,j=l+1/2} > E_{n-1,l+2,j=l+3/2}$ to $E_{n,l,j=l+1/2} < E_{n-1,l+2,j=l+3/2}$ with increasing orbital angular momentum as seen for the doublets $(3s_{1/2}, 2d_{3/2})$ and $(3p_{3/2}, 2f_{5/2})$ and indicated in Refs. [38,59–61]. This inversion is mainly due to the evolution of the dynamical term from improvement to destruction for the PSS.

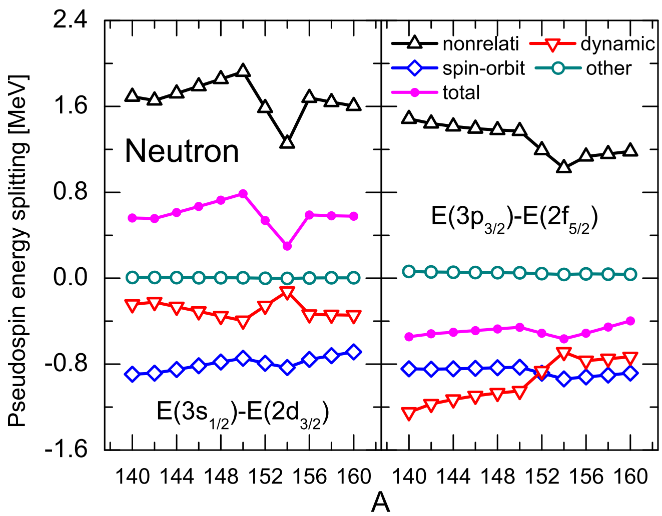


FIG. 8. The same as Fig. 6 but for the energy splitting for neutron pseudospin doublets $(3s_{1/2}, 2d_{3/2})$ and $(3p_{3/2}, 2f_{5/2})$.

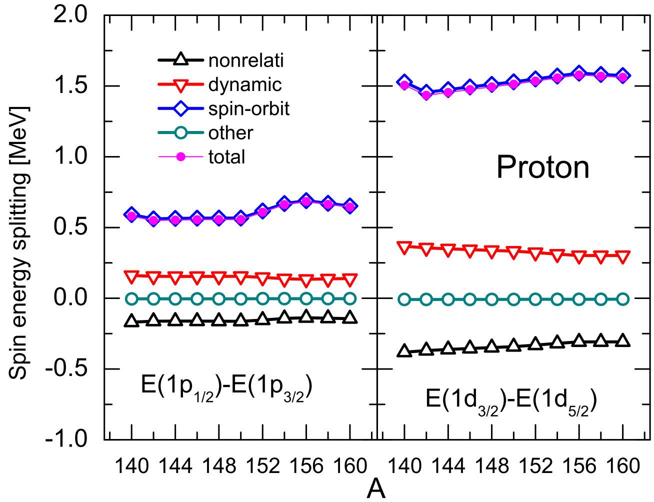


FIG. 9. The same as Fig. 3 but for the energy splittings for proton spin partners ($1p_{1/2}$, $1p_{3/2}$) and ($1d_{3/2}$, $1d_{5/2}$).

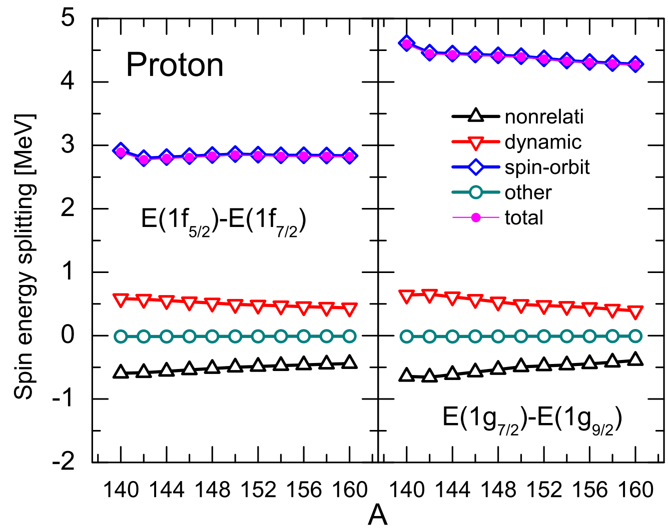


FIG. 10. The same as Fig. 3 but for the energy splittings for proton spin partners ($1f_{5/2}$, $1f_{7/2}$) and ($1g_{7/2}$, $1g_{9/2}$).

With the increasing of mass number, the PSS becomes better for all the doublets with several exceptions, which agree the result in Refs. [53,62,63]. Since the pseudospin energy splittings contributed by the spin-orbit term and the dynamical term are insensitive to mass number, the isospin dependence of PSS mainly originates from the nonrelativistic term. It is worth noting that there appears kink on the curve of pseudospin energy splitting as a function of mass number after $A = 150$ for the doublets ($2s_{1/2}$, $1d_{3/2}$), ($3s_{1/2}$, $2d_{3/2}$), and ($3p_{3/2}$, $2f_{5/2}$). The kink is mainly due to the contribution of the nonrelativistic term. The dynamical term has a considerable influence on the kink.

These indicate that the PSS originates from the relativistic effect. The nonrelativistic, spin-orbit coupling, and dynamical terms play important role at PSS. The quality of PSS is correlated with the quantum numbers of the doublets as well as isospin.

To understand better the isospin dependence of relativistic symmetry, we explore the PSS and SS for the proton single-particle states. In Figs. 9–11, we show the spin energy splitting contributed by every term in $H_p$ for all the available proton spin doublets. The variation of energy splitting with mass number is similar to that of neutron. The spin energy splitting and its evolution to mass number are almost entirely dominated by the spin-orbit term. The contributions of nonrelativistic and dynamic terms to spin energy splitting cancel each other out.

For the spin doublets with the same radial quantum number, the doublets with lower angular momentum have better SS. With the increasing of mass number, the spin energy splitting increases for the doublets ($1p_{1/2}$, $1p_{3/2}$) and ($1d_{3/2}$, $1d_{5/2}$), whereas decreases for the doublets ($1f_{5/2}$, $1f_{7/2}$) and ($1g_{7/2}$, $1g_{9/2}$). For the doublets ($2p_{1/2}$, $2p_{3/2}$) and ($2d_{3/2}$, $2d_{5/2}$), there appears a kink around $A = 150$ in the curve of spin energy splitting as a function of mass number. Before $A = 150$, the energy splitting decreases with increasing mass number. After $A = 150$, the energy splitting becomes increasing and then decreasing with increasing mass number. Namely, the isospin dependence of SS breaking is related to the quantum number of the doublet.

Compared with neutron, the energy splitting between the proton spin doublets is slightly larger than that of neutron spin doublets due to slightly stronger spin-orbit coupling.

In the following, we explore the PSS for the proton single particle states. The variation of energy splitting with mass number is shown in Figs. 12–14. Similar to the case of the neutron, the nonrelativistic term always destroys the PSS, and the spin-orbit term always improves the PSS for all the pseudospin doublets considered. Unlike the neutron case, the dynamical term always destroys the PSS for all the pseudospin doublets. For the doublets ($2f_{7/2}$, $1h_{9/2}$) and ($3s_{1/2}$, $2d_{3/2}$), the

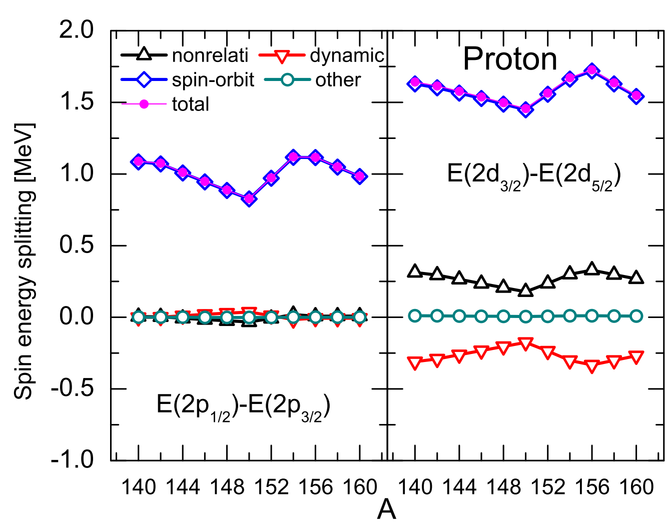


FIG. 11. The same as Fig. 3 but for the energy splittings for proton spin partners ($2p_{1/2}$, $2p_{3/2}$) and ($2d_{3/2}$, $2d_{5/2}$).

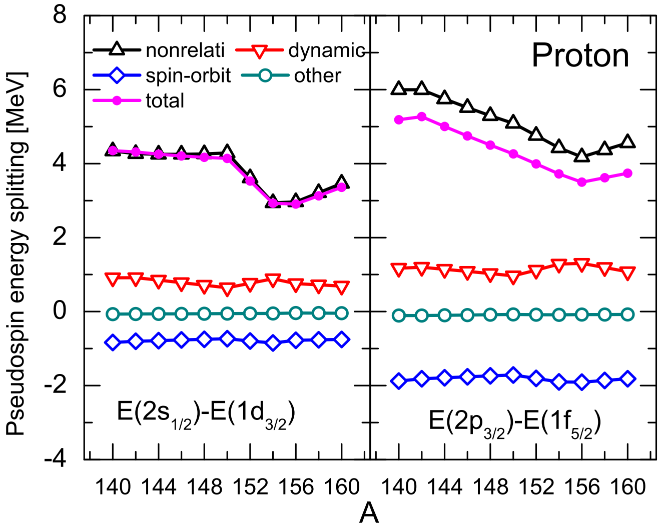


FIG. 12. The same as Fig. 6 but for the energy splitting for proton pseudospin partners ($2s1/2$, $1d_{3/2}$) and ($2p_{3/2}$, $1f_{5/2}$) .

dynamical term plays a role at improving the PSS for neutron, whereas destroying the PSS for the proton.

The variation of the total energy splitting with the mass number is quite consistent with that of the energy splitting contributed by the nonrelativistic term. The magnitude of energy splitting depends on the dynamical term and the spin-orbit coupling. Except for the small anomaly, the energy splitting decreases with increasing mass number, which means that the PSS is better in neutron-richer nuclei.

Compared with neutron, the PSS for proton is relatively worse since the energy splittings by the nonrelativistic and dynamical terms are stronger. For example, the energy splitting between the doublet ($2d_{5/2}$, $1g_{7/2}$) is 1 to 2 MeV larger for proton than that for neutron, which is consistent with this finding in Ref. [64].

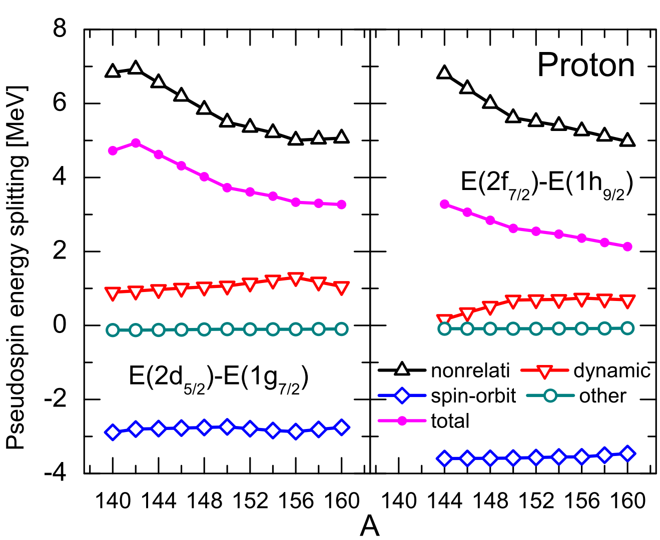


FIG. 13. The same as Fig. 6 but for the energy splitting for proton pseudospin partners ($2d_{5/2}$, $1g_{7/2}$) and ($2f_{7/2}$, $1h_{9/2}$).

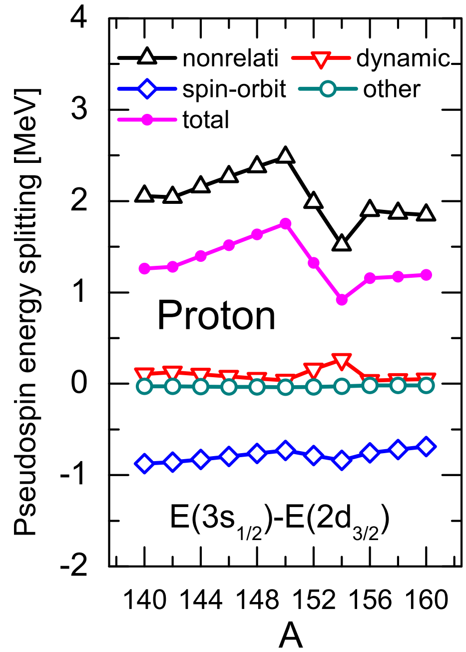


FIG. 14. The same as Fig. 6 but for the energy splitting for proton pseudospin partner ($3s_{1/2}$, $2d_{3/2}$).

## IV. SUMMARY

The SRG is combined with the RMF theory, and the Schrödinger-like Hamiltonian describing the motion of nucleon is obtained. This Hamiltonian consists of five terms, they are the nonrelativistic term, the spin-orbit coupling, the dynamical term, the relativistic modification of kinetic energy, and the Darwin term. All these terms are Hermitian. Combining the scalar and vector potentials from the self-consistent RMF calculations into this Hamiltonian, the contribution of every term to pseudospin (spin) energy splitting is extracted, and the origin and breaking mechanism of PSS and SS are explored. Sn isotopes are chosen as illustrated examples, the variations of pseudospin and spin energy splitting with mass number are checked, and the dependence of this symmetries on isospin is investigated.

For all the neutron spin doublets, the energy splitting and its variation with mass number are nearly identical to those contributed by the spin-orbit term. The breaking of SS mainly comes from the spin-orbit interaction. The energy splitting contributed by the nonrelativistic and dynamical terms are nearly the same in magnitude but opposite in sign, and their contributions to the total energy splitting almost cancel out each other. The magnitude of energy splitting increases with increasing angular momentum (radial quantum number) for the doublets with the same radial quantum number (angular momentum). The change in SS breaking with mass number is different for different doublets, and this isospin dependence is dominated by spin-orbit interactions.

For all the neutron pseudospin doublets, the variation of energy splitting with the mass number is quite consistent with that contributed by nonrelativistic term, but the magnitude of energy splitting is severely different. The energy splitting contributed by nonrelativistic term is extraordinarily large, i.e., there is no PSS in nonrelativistic limit. The spin-orbit coupling greatly reduces pseudospin energy splitting. The dynamical term evolves from breaking to improving PSS, and even reversing pseudospin splitting for the doublets closer to the continuum threshold. The PSS becomes better with increasing mass number for all the doublets with several exceptions. The isospin dependence of PSS is determined by the nonrelativistic term.

Similar to the case of the neutron, the energy splitting between the proton spin doublet and its variation with mass number are almost same to those contributed by the spin-orbit term. The breaking of SS mostly comes from the spin-orbit interaction. Comparably, the energy splitting between the proton doublet is slightly larger than that of the neutron. The magnitude of SS breaking is isospin dependent and related to the quantum numbers of the doublet.

Similarly, for the proton pseudospin doublets, the variation of energy splitting with the mass number is quite consistent with that contributed by the nonrelativistic term, but the magnitude of energy splitting is significantly changed by the dynamical term and the spin-orbit coupling. The dynamical term always destroys the PSS, which is not exactly the same as that of neutron. Compared with neutron, the PSS for proton is relatively worse since the energy splitting by the nonrelativistic and dynamical terms is stronger.

These indicate the nonrelativistic, dynamical, and spin-orbit terms play important roles at the PSS and SS. The quality of PSS and SS is related to the quantum numbers of the doublets. Isospin has also important influences on the symmetries. In the nonrelativistic limit, spin symmetry emerges, whereas no vestige of pseudospin symmetry remains in this limit, which reaffirms that pseudospin symmetry is a relativistic symmetry. The PSS of neutron is superior to that of proton from the weakening of nonrelativistic and dynamical breaking.

## ACKNOWLEDGMENTS

This work was partly supported by the National Natural Science Foundation of China under Grants No. 11935001 and No. 11575002; the Key Research Foundation of Education Ministry of Anhui Province under Grant No. KJ2018A0028, and the Doctoral Scientific Research Startup Fund of Anhui University (Grant No. J01001319-J10113190082).

[1] S. H. Shen, H. Z. Liang, P. W. Zhao, S. Q. Zhang, and J. Meng, Phys. Rev. C **88**, 024311 (2013).
[2] P. R. Page, T. Goldman, and J. N. Ginocchio, Phys. Rev. Lett. **86**, 204 (2001).
[3] K. T. Hecht and A. Adler, Nucl. Phys. A **137**, 129 (1969).
[4] A. Arima, M. Harvey, and K. Shimizu, Phys. Lett. B **30**, 517 (1969).
[5] A. Bohr, I. Hamamoto, and B. R. Mottelson, Phys. Scr. **26**, 267 (1982).
[6] J. Dudek, W. Nazarewicz, Z. Szymanski, and G. A. Leander, Phys. Rev. Lett. **59**, 1405 (1987).
[7] D. Troltenier, W. Nazarewicz, Z. Szymanski, and J. P. Draayer, Nucl. Phys. A **567**, 591 (1994).
[8] J. N. Ginocchio, Phys. Rev. C **59**, 2487 (1999).
[9] P. von Neumann-Cosel and J. N. Ginocchio, Phys. Rev. C **62**, 014308 (2000).
[10] A. E. Stuchbery, J. Phys. G: Nucl. Part. Phys. **25**, 611 (1999).
[11] A. E. Stuchbery, Nucl. Phys. A **700**, 83 (2002).
[12] W. Nazarewicz, P. J. Twin, P. Fallon, and J. D. Garrett, Phys. Rev. Lett. **64**, 1654 (1990).
[13] F. S. Stephens, M. A. Deleplanque, J. E. Draper, R. M. Diamond, A. O. Macchiavelli, C. W. Beausang, W. Korten, W. H. Kelly, F. Azaiez, J. A. Becker, E. A. Henry, S. W. Yates, M. J. Brinkman, A. Kuhnert, and J. A. Cizewski, Phys. Rev. Lett. **65**, 301 (1990).
[14] F. S. Stephens, M. A. Deleplanque, and A. O. Macchiavelli, R. M. Diamond, P. Fallon, I. Y. Lee, and C. Schuck, Phys. Rev. C **57**, R1565 (1998).
[15] J. Y. Zeng, J. Meng, C. S. Wu, E. G. Zhao, Z. Xing, and X. Q. Chen, Phys. Rev. C **44**, R1745 (1991).
[16] A. L. Blokhin, C. Bahri, and J. P. Draayer, Phys. Rev. Lett. **74**, 4149 (1995).
[17] C. Bahri, J. P. Draayer, and S. A. Moszkowski, Phys. Rev. Lett. **68**, 2133 (1992).
[18] J. N. Ginocchio, Phys. Rev. Lett. **78**, 436 (1997).
[19] J. Meng, K. Sugawara-Tanabe, S. Yamaji, P. Ring, and A. Arima, Phys. Rev. C **58**, R628 (1998).
[20] J. N. Ginocchio, Phys. Rep. **315**, 231 (1999).
[21] S. G. Zhou, J. Meng, and P. Ring, Phys. Rev. Lett. **91**, 262501 (2003).
[22] R. Lisboa, M. Malheiro, P. Alberto, M. Fiolhais, and A. S. de Castro, Phys. Rev. C **81**, 064324 (2010).
[23] J. N. Ginocchio, Phys. Rev. C **66**, 064312 (2002).
[24] J. N. Ginocchio, Phys. Rev. Lett. **95**, 252501 (2005).
[25] A. S. de Castro, P. Alberto, R. Lisboa, and M. Malheiro, Phys. Rev. C **73**, 054309 (2006).
[26] L. B. Castro, Phys. Rev. C **86**, 052201(R) (2012).
[27] A. Leviatan, Phys. Rev. Lett. **92**, 202501 (2004).
[28] A. Leviatan, Phys. Rev. Lett. **103**, 042502 (2009).
[29] A. D. Alhaidari, Phys. Lett. B **699**, 309 (2011).
[30] J. N. Ginocchio, Phys. Rev. Lett. **82**, 4599 (1999).
[31] H. Leeb and S. A. Sofianos, Phys. Rev. C **69**, 054608 (2004).
[32] J. N. Ginocchio, Phys. Rev. C **65**, 054002 (2002).
[33] J. N. Ginocchio, Int. J. Mod. Phys. E **14**, 105 (2005).
[34] J. Y. Guo, R. D. Wang, and X. Z. Fang, Phys. Rev. C **72**, 054319 (2005).
[35] B. N. Lu, E. G. Zhao, and S. G. Zhou, Phys. Rev. Lett. **109**, 072501 (2012).
[36] P. Alberto, M. Fiolhais, M. Malheiro, A. Delfino, and M. Chiapparini, Phys. Rev. Lett. **86**, 5015 (2001).
[37] S. Marcos, M. López-Quelle, R. Niembro, and L. N. Savushkin, Eur. Phys. J. A **37**, 251 (2008).
[38] S. Marcos, M. López-Quelle, R. Niembro, L. N. Savushkin, and P. Bernardos, Phys. Lett. B **513**, 30 (2001).
[39] J. Y. Guo, Phys. Rev. C **85**, 021302(R) (2012).
[40] J. Y. Guo, S. W. Chen, Z. M. Niu, D. P. Li, and Q. Liu, Phys. Rev. Lett. **112**, 062502 (2014).
[41] S. W. Chen and J. Y. Guo, Phys. Rev. C **85**, 054312 (2012).

[42] D. P. Li, S. W. Chen, and J. Y. Guo, Phys. Rev. C **87**, 044311 (2013).
[43] D. P. Li, S. W. Chen, Z. M. Niu, Q. Liu, and J. Y. Guo, Phys. Rev. C **91**, 024311 (2015).
[44] Y. X. Guo and H. Z. Liang, Phys. Rev. C **99**, 054324 (2019).
[45] Y. X. Guo and H. Z. Liang, Chin. Phys. C **43**, 114105 (2019).
[46] J. Y. Guo and X. Z. Fang, Eur. Phys. J. A **45**, 179 (2010).
[47] H. Liang, S. Shen, P. W. Zhao, and J. Meng, Phys. Rev. C **87**, 014334 (2013).
[48] Q. Xu, Eur. Phys. J. A **51**, 81 (2015).
[49] Q. Xu and L. F. Luo, Eur. Phys. J. A **56**, 129 (2020).
[50] T. T. Sun, W. L. Lu, L. Qian, and Y. X. Li, Phys. Rev. C **99**, 034310 (2019).
[51] J. N. Ginocchio, Phys. Rep. **414**, 165 (2005).
[52] H. Liang, J. Meng, and S. G. Zhou, Phys. Rep. **570**, 1 (2015).
[53] X. X. Shi, Q. Liu, J. Y. Guo *et al.*, Phys. Lett. B **801**, 135174 (2020).
[54] H. M. Dai, M. Shi, S. W. Chen, and Q. Liu, Chin. Phys. C **45**, 124102 (2021).
[55] Q. Liu, Y. Zhang, and J. Y. Guo, Phys. Lett. B **824**, 136829 (2022).
[56] D. Vretenar, A. V. Afanasjev, G. A. Lalazissis, and P. Ring, Phys. Rep. **409**, 101 (2005).
[57] J. Meng, H. Toki, S. G. Zhou, S. Q. Zhang, W. H. Long, and L. S. Geng, Prog. Part. Nucl. Phys. **57**, 470 (2006).
[58] F. Wegner, Ann. Physik **506**, 77 (1994).
[59] S. Marcos, L. N. Savushkin, M. Lopez-Quelle, and P. Ring, Phys. Rev. C **62**, 054309 (2000).
[60] G. A. Lalazissis, Y. K. Gambhir, J. P. Maharana, C. S. Warke, and P. Ring, Phys. Rev. C **58**, R45 (1998).
[61] J. Meng, K. Sugawara-Tanabe, S. Yamaji, and A. Arima, Phys. Rev. C **59**, 154 (1999).
[62] J. Y. Guo and X. Z. Fang, Phys. Rev. C **74**, 024320 (2006).
[63] J. Meng and I. Tanihata, Nucl. Phys. A **650**, 176 (1999).
[64] R. Lisboa, M. Malheiro, and P. Alberto, Phys. Rev. C **67**, 054305 (2003).