Effect of nuclear deformation on the observation of a low-energy super-Gamow-Teller state

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A well-known structure with concentrated Gamow-Teller (GT) transitions is the Gamow-Teller resonance, which has been observed at higher-energy regions (usually >6 MeV) of nuclear excitation. It has been found that the GT strength can also concentrate in the lowest $J^\pi = 1^+$ GT state named the "low-energy super-GT (LeSGT) state" when the initial even-even nucleus has the structure of "LS-closed-shell core nucleus + 2 neutrons (or 2 protons)" and the final nucleus "LS-closed-shell core nucleus + 1 proton and 1 neutron." Such concentrations are realized with the core nuclei ^{4}He, ^{16}O, and ^{40}Ca, corresponding to the shell closures of s, p, and sd shells, respectively. It is natural to speculate that the LeSGT state may also be observed in the $A = 82$ systems if the $N = Z = 40$ shell gap is significant and ^{80}Zr represents a good core nucleus corresponding to the pf shell closure. Possible conditions that allow the formation of the LeSGT state in the ^{82}Zr → ^{82}Nb charge-exchange reaction (or ^{82}Mo → ^{82}Nb β decay) are discussed by evaluating the results of projected shell model calculations, which are based on deformed model space. Our calculations show that with increasing deformation, the LeSGT feature found in the spherical limit (zero deformation) evolves gradually into a broad distribution in the higher-energy region. This lets us conclude that no LeSGT state is expected in ^{82}Nb because the shape of the ^{80}Zr core nucleus is ellipsoidal.

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I. INTRODUCTION

Gamow-Teller (GT) transitions in nuclei are sensitive to shell structure, deformation, and effective residual interactions. At the spherical limit, the number of configurations that can contribute to GT transitions is rather limited. This is because of the simplicity of the GT operator, i.e., the spin-isospin ($\sigma\tau$) operator having no spatial components. For deformed nuclei, however, different density distributions modify the spatial part of wave functions significantly [1]. The effect of deformation on GT strengths was noticed already in the 1950s by Alaga [2,3] and by Mottelson and Nilsson [4], who attempted to introduce additional selection rules for GT transitions in deformed nuclei. Especially, the K quantum number associated with the projection of the total angular momentum on the symmetry axis of a deformed nucleus enters into the discussion. The K quantum number is introduced because spontaneous symmetry breakings lift the degeneracy in a j orbit in the spherical limit, causing a distribution of $2j + 1$ deformed K states. As a consequence, for allowed Gamow-Teller β decays, one adds the K selection rule into the usual selection rules about angular momentum and parity,

$$\Delta J = 0, \pm 1 \ (0 \nrightarrow 0); \quad \Delta\pi = \text{no}; \quad \Delta K = \Delta J. \tag{1}$$

Experimentally, the most direct information on the GT transition strength B(GT) is obtained from β-decay studies [5,6], but the accessible range of excitation energy (E_x) is limited by the relatively small Q values. On the other hand, charge-exchange (CE) reactions, such as (p, n) and (^{3}He, t) reactions, can also access GT transitions. In particular, those performed at 0° and intermediate energies ($E_{\text{in}} > 100$ MeV/nucleon) were found to be a good probe of GT transition strengths owing to the relatively simple proportionality between the zero-degree cross sections and the B(GT) values [6,7]. These CE reactions could overcome the Q-value limitation of β-decay studies, and resonance-like structures, i.e., the Gamow-Teller resonances (GTRs), have been observed in high-energy regions of around 9–18 MeV in various nuclei with mass number A larger than ≈50 [8]. It was found that GTRs consume about 50%–60% of the full GT sum-rule strength.

A possible concentration of low-energy "super-allowed" beta decays was suggested early by Sagawa, Hamamoto, and Ishihara for very neutron-rich nuclei with $Z \leqslant 8$ [9]. Remarkably, it was found [10] that the GT transition strength can also be concentrated in the lowest $J^\pi = 1^+$ GT state named the "low-energy super-GT (LeSGT) state" when the initial even-even nuclei have the structure of "LS-closed-shell core nucleus + 2 neutrons (or 2 protons)" and the final nucleus "LS-closed-shell core nucleus + 1 proton and 1 neutron." Here, the initial nuclei are either $T_z = +1$ or -1 mirror nuclei

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and the final nuclei are the $T_z = 0$ nuclei situated in between, where T_z is the z component of isospin T and defined by $(N - Z)/2$. Such concentrations have been experimentally observed in the $A = 6$ system (^{6}He $\to$ ^{6}Li β decay) [11], the $A = 18$ system (^{18}O $\to$ ^{18}F CE reaction [12,13], and ^{18}Ne $\to$ ^{18}F β decay [14]), and the $A = 42$ system (^{42}Ca $\to$ ^{42}Sc CE reaction [15], and ^{42}Ti $\to$ ^{42}Sc β decay [16]), where the core nuclei are ^{4}He, ^{16}O, and ^{40}Ca, representing the shell closures for the *s*, *p*, and *sd* shell, respectively.

It is natural to speculate that the next LeSGT state may be observed in the $A = 82$ system if ^{80}Zr is regarded as a good core nucleus representing the pf shell closure. However, the answer may not be obvious prior to investigation because, differently from all the LeSGT examples in the *s*, *p*, and *sd* shells, nuclei near ^{80}Zr are known to be well deformed [17]. Note that deformation is an important parameter that can significantly change the distribution of single particles. In the present article, possible conditions for observing the LeSGT state in the ^{82}Zr $\to$ ^{82}Nb charge-exchange (CE) reaction are discussed by examining the results of the theoretical calculations using the model that is optimized for the deformed $A \approx 80$ mass region.

II. SHAPE-DEPENDENT GT STRENGTH DISTRIBUTIONS

As one moves up along the $N = Z$ line, shape phase transition occurs around $N = Z = 36$ [18,19]. Thus, beyond the phase transition, conventional shell-model calculations based on spherical basis cannot be applied, for example, to the $A \approx 80$ mass region. In the region with large deformation, single-particle distributions near the Fermi surfaces are qualitatively different from those at the spherical limit [20]. To discuss physics in deformed nuclei, it is convenient to use the deformed Nilsson single-particle states [21]. One then has to answer the question on how to efficiently treat the many-body problems for large, deformed systems using the shell-model concept. Toward answering this question there have been various theoretical efforts to develop many-body techniques [22,23]. The solution lies in the numerical angular momentum projection, which was a subject started in the 1970s [24], and becomes popular today in the nuclear structure study. One application of this technique is the projected shell model (PSM) [25–27]. It has been shown that the PSM can be successfully applied to the deformed $A \approx 80$ nuclei [28–32].

It has been discussed [6] that GTRs are formed by the repulsive isovector (IV) type effective residual interaction that is active in the proton-particle–neutron-hole configurations, while LeSGT states are formed by the attractive isoscalar (IS) type effective residual interaction that is active in the proton-particle–neutron-particle configurations. Therefore, the existence of the low-energy and high-energy collective states, i.e., the LeSGT and the GTR states, is attributed to the two fermionic degrees of freedom in nuclear physics, i.e., protons and neutrons [10,33]. Let us take a conceptual discussion in Fig. 1, where we show that before and after the shape phase transition from spherical to deformed nuclei, there exist two distinct transitions: (1) transitions if both the parent and daughter nuclei are spherical (connected by green arrows), and (2) transitions if both nuclei are well

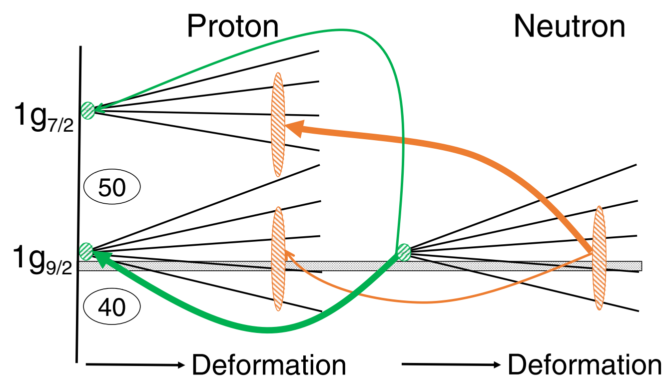

FIG. 1. Schematic illustration for the configurations that are involved in the β^--type ^{82}Zr $\to$ ^{82}Nb GT transitions: (1) Transition if the nuclei are spherical (indicated by green arrows), and (2) transition if the nuclei are well deformed (indicated by orange arrows). The shaded bar shows roughly the Fermi surfaces. Thick (thin) arrows represent stronger (weaker) GT strengths.

deformed (connected by orange arrows). It can be seen that in case 1, strong transitions occur between two spherically symmetric $1g_{9/2}$ states (indicated by a thick green arrow from $\nu 1g_{9/2}$ to $\pi 1g_{9/2}$) and weak ones (indicated by a thin green arrow from $\nu 1g_{9/2}$ to $\pi 1g_{7/2}$). The stronger GT transition connects the states close to the Fermi surfaces, corresponding to the lowest excitation where the LeSGT state is expected. In contrast, in case 2, transitions occur among extended K states with quite different excitation energies (see Fig. 1). Thus, qualitatively, one expects that after the shape phase transition [18], the entire GT strength concentrated in one transition in the spherical case becomes a broad distribution in the deformed case. This is the main origin of fragmentation of GT strength in the deformed single-particle picture.

III. RESULTS AND DISCUSSION

In the known example where the LeSGT state is observed in the ^{42}Ca $\to$ ^{42}Sc CE reaction (see Ref. [15] and the upper panel of Fig. 2), both ^{42}Ca and ^{42}Sc are spherical with no deformation. In the lower panel of Fig. 2, our PSM calculation reproduces correctly the observed LeSGT feature if we assume the deformation parameter $\varepsilon_2 = 0.01$. (Note that the PSM will break down with the absolute zero deformation as the QQ force in the model is related to deformation [25]. For this reason, we use $\varepsilon_2 = 0.01$ to simulate the spherical case.) For numerical calculation of these $B(\mathrm{GT}^-)$ values, we use the code developed in Refs. [34,35]. As usual, a quenching factor of 0.74 is applied to the coupling constant of the GT force. The GT transitions are calculated from the ground state of the parent nucleus to excited states up to 12 MeV in the daughter.

Based on this model, we can now study $B(\mathrm{GT}^-)$ distribution for ^{82}Zr $\to$ ^{82}Nb. With the set of realistic parameters that we usually use for this mass region [30] and $\varepsilon_2 = 0.29$, we have checked that the model can reproduce correctly all existing spectral data [36]. As stated in the Introduction, with the relatively good subshell closure at $N = Z = 40$, the conditions for forming a LeSGT state [10] in the

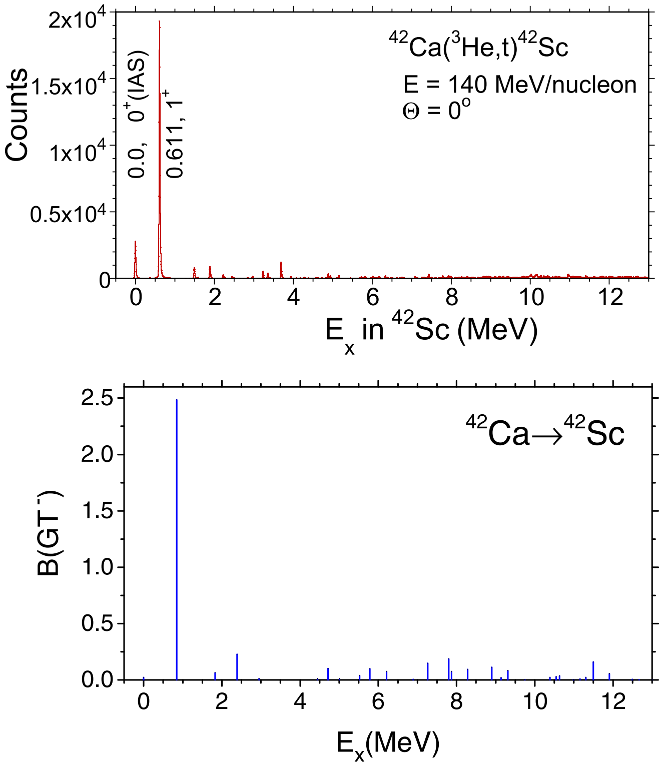

FIG. 2. Upper panel: Experimental ^{42}Ca(^{3}He, t)^{42}Sc spectrum taken at 0 degrees. Lower panel: The B(GT$^-$) distribution calculated for the ^{42}Ca $\to$ ^{42}Sc GT transition from the ground state of ^{42}Ca to excited states up to 12 MeV in ^{42}Sc by assuming deformation parameter $\varepsilon_2 = 0.01$.

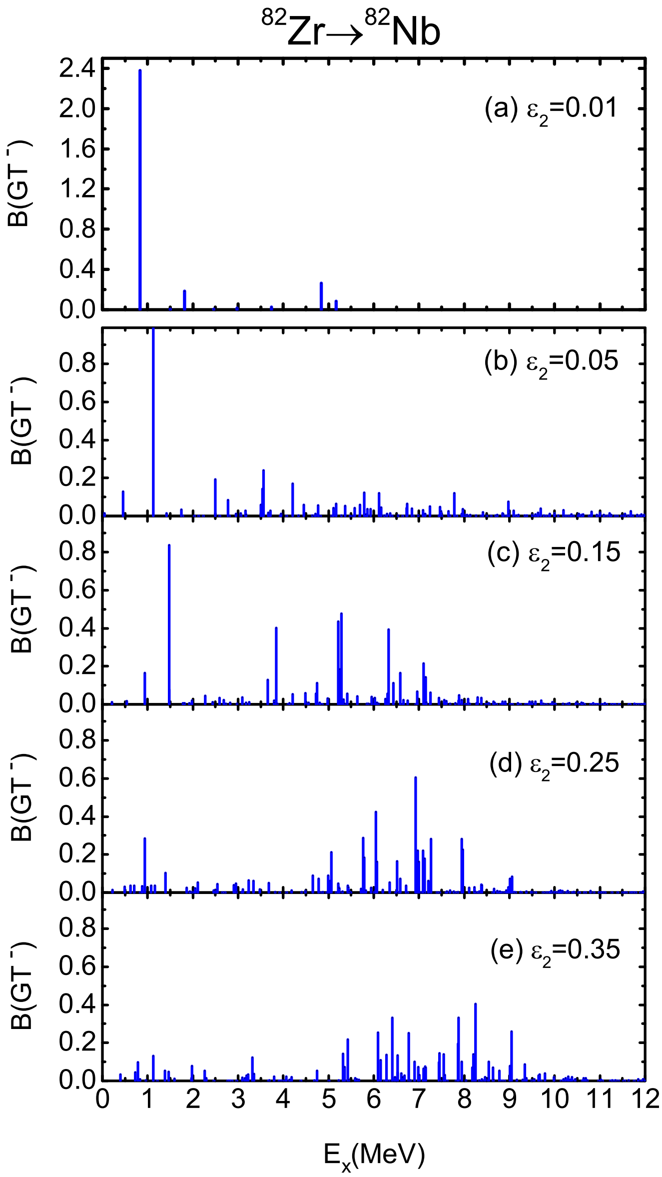

FIG. 3. Theoretical B(GT$^-$) distributions for the ^{82}Zr $\to$ ^{82}Nb transition from the ground state of ^{82}Zr to excited states up to 12 MeV in ^{82}Nb by assuming different deformations. Note the plot (a) has different vertical scales than (b)–(e).

^{82}Zr $\to$ ^{82}Nb $\leftarrow$ ^{82}Mo transitions are fulfilled. However, the calculated B(GT$^-$) distribution for ^{82}Zr $\to$ ^{82}Nb suggests that the distribution is not LeSGT-like, but GTR-like. In addition, we have checked, and confirmed that an identical GT spectrum is obtained for ^{82}Mo $\to$ ^{82}Nb β decay, under the assumption that the $A = 82$ systems hold the isospin symmetry exactly. Therefore, the following discussion and conclusions for ^{82}Zr $\to$ ^{82}Nb are valid also for ^{82}Mo $\to$ ^{82}Nb β decay.

To understand why the GT spectrum from the ^{82}Zr $\to$ ^{82}Nb reaction shows a GTR feature, not a LeSGT, we perform gedanken calculations in which the deformation parameters for ^{82}Zr and ^{82}Nb are allowed to vary from its realistic value of $\varepsilon_2 = 0.29$. The variation range includes those that usually happen in nuclei. The model itself (such as the model space and the forces in the Hamiltonian) remains unchanged.

In Fig. 3(a) with (near) zero deformation ($\varepsilon_2 = 0.01$), one indeed observes the feature of the LeSGT state where the strength concentrates in the lowest state of the ^{82}Nb GT spectrum. Detailed analysis indicates that this strong concentration of the GT strength at 0.83 MeV corresponds to the transition of the $\nu g_{9/2}$ to the $\pi g_{9/2}$ orbits with the dominant low-K component of $K = 1/2$, satisfying the selection rule $\Delta K = 0$ [see Eq. (1)]. With increasing deformation, Figs. 3(b)–3(e) indicate a gradual shift of GT peak to higher excitations in ^{82}Nb. In Fig. 3(b) with a small deformation $\varepsilon_2 = 0.05$, the strongest GT line is shifted to a higher energy of 1.13 MeV and the strengths developed in the 2–4 MeV region correspond to mixtures from the $K = 1/2$, 3/2, and 5/2 states of the $g_{9/2}$ orbit. By increasing the deformation to $\varepsilon_2 = 0.15$, the LeSGT strength is suppressed to about 1/3 of its value at zero deformation, and at the same time, GT peaks at higher excitations become stronger. In this region, the GT lines correspond to transitions to several K components of the $\pi g_{9/2}$ orbit that exhibit large splitting in energy due to large deformation. With $\varepsilon_2 = 0.25$, the GTR is developed centered at 6–7 MeV. The strongest GT strengths in GTR are found to be the transitions that start from the $\nu g_{9/2}$ orbit, across the $N = Z = 50$ major shell gap, and connect to several K states of the $\pi g_{7/2}$ orbit, as schematically illustrated in Fig. 1. The $g_{9/2}$ and $g_{7/2}$ orbits have both $l = 4$, and GT transitions between them correspond

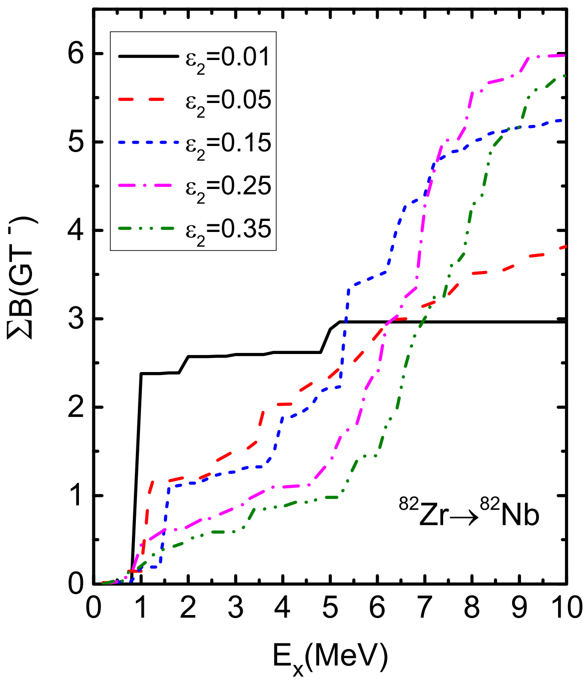

FIG. 4. Accumulated $B(\mathrm{GT}^-)$ values for the $^{82}\mathrm{Zr} \rightarrow {}^{82}\mathrm{Nb}$ transition from the ground state of ^{82}Zr to excited states in ^{82}Nb by calculations with different deformations.

to a spin-flip process, and are expected to be strong. Finally in Fig. 3(e) with $\varepsilon_2 = 0.35$, wider spread strengths can be seen where the stronger GT lines in the 5–9 MeV energy region correspond to highly mixed K states due to larger deformation.

Fujita *et al.* [10,15] have found that in the $^{42}\mathrm{Ca} \rightarrow {}^{42}\mathrm{Sc}$ reaction the GT strengths concentrate in the lowest excited 1^+ state at 0.6 MeV in ^{42}Sc. Moreover, in the same reaction for all other $f_{7/2}$-shell nuclei, which ends up with the odd-odd $N = Z$ ($T_z = 0$) nuclei (i.e., ^{46}V, ^{50}Mn, and ^{54}Co), the GT strength distributions are found to be qualitatively different. As mass number A increases, the low-energy strength becomes fragmented and the bumplike structure in the high-energy region begins to develop. Finally in ^{54}Co, the GT strength is mainly in the typical bumplike GTR structure. These results suggest drastic differences occurring along the $N = Z$ line when moving away from the $N = Z = 20$ shell closure. Large-scale shell model calculations based on the spherical basis were performed to study the $f_{7/2}$-shell nuclei and as a result, the SU(4)-symmetry breaking in GT transitions was discussed [37]. In Fig. 3, we see a similar development in the $^{82}\mathrm{Zr} \rightarrow {}^{82}\mathrm{Nb}$ reaction if the gedanken calculations with "artificial" values in deformation parameter are performed.

Figure 4 shows calculated accumulated $B(\mathrm{GT}^-)$ values for the $^{82}\mathrm{Zr} \rightarrow {}^{82}\mathrm{Nb}$ transition for different deformation parameters. One sees clearly that with zero deformation the $\sum B(\mathrm{GT}^-)$ jumps to 2.4 at very low energy less than 1 MeV, and afterward remains approximately constant. This is the character of the LeSGT suggested by Fujita *et al.* [10]. With increasing deformation, the general trend of the curves is that the low-energy GT concentration moves gradually to higher energies with rapid climbs later in the curves.

It should be mentioned that the deformation discussed here is not the only mechanism for spreading GT strengths. The complex configurations due to quasiparticle-phonon or phonon-phonon couplings can produce similar fragmentation. The extended quasiparticle random-phase approximation (QRPA) model accounting for both phonon-phonon couplings and tensor interaction effects has been applied for calculations of the GT strength functions and β decay [38,39].

IV. SUMMARY AND FUTURE PROSPECTS

To summarize, it was established by Fujita *et al.* [10,15] that the condition for the formation of the LeSGT state is that the initial even-even nucleus has the structure of "LS-closed-shell core nucleus + 2 neutrons (or 2 protons)" and the final nucleus "LS-closed-shell core nucleus + 1 proton and 1 neutron." The present work has investigated the possibility of whether one can observe, at least in principle, the LeSGT state in the nuclei near the subshell closure at $N = Z = 40$ where the above condition is met. The employed model for this study, the projected shell model, has a unique property that the model allows constructing deformed bases with different deformation parameters. We have shown that with the realistic model parameters, the deformed structure of the relevant nuclei, ^{82}Zr and ^{82}Nb, can be well described. The calculated GT strengths of the $^{82}\mathrm{Zr} \rightarrow {}^{82}\mathrm{Nb}$ transition, however, do not show anticipated LeSGT feature. Subsequent discussions have explored that occurrence of the LeSGT feature for a system is determined by, in addition to the conditions in Refs. [10,15], also the one that the involved nuclei must be (near) spherical.

The discussion in the present paper is based on the study within the PSM. However, the main physics is a competition between the $T = 1$ (repulsive) and $T = 0$ (attractive) components of the NN interaction. It was schematically captured already by the simplified QRPA approach [40]. The role of $T = 0$ pn dynamical pairing including a possibility of the $T = 0$ pairing condensate was studied in the deformed HFB+QRPA model in Ref. [41].

For experimental confirmation of present theoretical discussions, we expect that the $^{82}\mathrm{Mo} \rightarrow {}^{82}\mathrm{Nb}$ β decay can be measured at RIKEN. In the near future, this decay can also be measured at IMP, Lanzhou, and at FRIB, MSU.

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