# Comment on "Quasielastic lepton scattering and back-to-back nucleons in the short-time approximation"

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The article of Pastore *et al.*, whereas proposing an interesting and potentially useful approach for the generalization of quantum Monte Carlo techniques to the treatment of the nuclear electromagnetic response, features an incorrect and misleading discussion of $y$ scaling. The response to interactions with transversely polarized virtual photons receives sizable contributions from nonscaling processes in which the momentum transfer is shared between two nucleons. It follows that, contrary to what is stated by the the authors, $y$ scaling in the transverse channel is accidental.

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The work of Pastore *et al.* [1] can be seen as a first step towards the implementation of the factorization scheme, which naturally emerges from the formalism of the impulse approximation [2], in the computational framework of quantum Monte Carlo (QMC). In view of the difficulties associated with the identification of specific final states in the nuclear responses obtained from QMC calculations, this is an interesting, and potentially useful, development.

The short-time approximation developed by the authors involves a number of strong simplifying assumptions—such as neglect of the energy dependence in the propagator of the spectator system, analyzed in Ref. [3]—the validity of which will only be fully appraised in years to come when the proposed approach will be extended to a broader kinematical range and nuclear targets other than the three- and four-nucleon systems. The discussion of scaling in Sec. IV, on the other hand, comprises incorrect and misleading statements requiring a prompt clarification.

The occurrence of $y$ scaling in electron-nucleus scattering—that is, the observation that the target response, which, in general, depends on both momentum and energy transfer $\mathbf{q}$ and $\omega$ in the limit of large $q = |\mathbf{q}|$ can be reduced to a function of the single variable $y = y(q, \omega)$ [4,5]—reflects the onset of the kinematical regime in which the dominant reaction mechanism is elastic scattering off individual nucleons [6].

The definition of the scaling variable $y$ follows from the assumption that the momentum transfer is absorbed by *only one* nucleon. In the absence of final-state interactions (FSIs) between the struck nucleon and the spectators, conservation of energy in the laboratory frame entails the relation [7],

$$\omega + M_A = \sqrt{m^2 + (y+q)^2} + \sqrt{(M_A - m + E_{\rm thr})^2 + y^2}, \tag{1}$$

where $M_A$ and $m$ are the target and nucleon mass, respectively, whereas $E_{\rm thr}$ denotes the nucleon emission threshold. Equation (1) shows that the scaling variable has a straightforward physical interpretation, being trivially related to the projection of the momentum of the struck nucleon along the direction of the momentum transfer $k_\parallel = \mathbf{k} \cdot \mathbf{q}/q$.

The scaling function of a nucleus of mass number $A$ and charge $Z$, defined as [7]

$$F(y) = \lim_{q\to\infty} F(q, y) \tag{2}$$

is obtained from

$$F(q, y) = \frac{d\sigma_{eA}}{Z\, d\sigma_{ep} + (A - Z) d\sigma_{en}} \left(\frac{d\omega}{dk_\parallel}\right), \tag{3}$$

where $d\sigma_{eA}$ is the measured nuclear cross section, whereas $d\sigma_{ep}$ and $d\sigma_{en}$ are the elastic electron-proton and electron-neutron cross sections, stripped of the energy-conserving $\delta$ function.[1] Large deviations from the scaling behavior, observed at $y > 0$, arise from processes other than elastic scattering, whereas smaller scaling violations at $y < 0$ are ascribed to FSI [7].

The above definitions imply that the scaling function is an intrinsic property of the target, providing information on the nucleon momentum distribution $n(\mathbf{k})$. In a deuteron, the relation between scaling function and momentum distribution takes the simple form [8]

$$n(k) = -\frac{1}{2\pi}\frac{1}{y}\frac{dF(y)}{dy}\bigg|_{|y|=k}, \tag{4}$$

with $k = |\mathbf{k}|$.

[1]Here, the elementary electron-nucleon cross sections, which explicitly depend on the nucleon momentum $\mathbf{k}$ and removal energy $E$ are evaluated at $|\mathbf{k}| = |y|$ and $E = E_{\rm thr}$.

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In the 1980s, Finn *et al.* [9] performed the first scaling analysis of the carbon responses to interactions with longitudinally and transversely polarized virtual photons, measured at Saclay [10]. The results of this work revealed a significant excess of strength in the transverse channel, which, however, did not appear to spoil the scaling behavior at $y < 0$. As a consequence, the analysis led to the determination of two distinct $q$-independent functions, $F_L(y)$ and $F_T(y)$, even though the interpretation of $F_T(y)$ as a scaling function cannot be reconciled with the presence of contributions arising from nonscaling processes, driven by two-nucleon currents. More recently, similar results have been obtained from the analysis of the longitudinal and transverse responses of light nuclei [11].

Pastore *et al.* [1], without introducing $y$ scaling and the interpretation of the scaling variable, explain the mechanism leading to $q$ independence of the response functions obtained from the Green's function Monte Carlo technique. However, their conclusion that $y$ scaling is preserved even in the presence of a mechanism other that single-nucleon knock out does not take into account the fact that $q$ independence and $y$ scaling are distinct properties and do not necessarily imply one another [12].

Accidental $y$ scaling—that is, scaling in the presence of nonscaling mechanisms, such as FSI, giving rise to sizable $q$-independent contributions to the nuclear response—is long known to occur in a variety of processes, ranging from electron-nucleus scattering [13] to neutron scattering from liquid helium [14]. Obviously, when scaling is accidental, the interpretation of both the scaling variable and the scaling function discussed above is no longer applicable.

Processes involving two-nucleon currents *do not scale* in the variable $y$ because the momentum transfer is shared between two nucleons, and conservation of energy cannot be written as in Eq. (1). This argument also applies to contributions arising from interference between one- and two-nucleon currents. It follows that $y$ scaling in the transverse channel is, in fact, accidental. As correctly noted by the authors of Ref. [9], a meaningful scaling function, providing information on initial-state dynamics, can only be obtained from the analysis of the longitudinal response, which is largely unaffected by two-nucleon currents.

On the constructive side, it should be noted that, after removal of the excess transverse strength arising from processes involving two-nucleon currents, the longitudinal and transverse responses obtained by Pastore *et al.* [1] may be employed to perform a fully consistent study of the universality of the scaling function. The results of such a study would be valuable for the ongoing efforts to exploit $y$ scaling as a tool for the analysis of the signals detected by accelerator-based searches of neutrino oscillations [15].


[1] S. Pastore, J. Carlson, S. Gandolfi, R. Schiavilla, and R. B. Wiringa, Phys. Rev. C **101**, 044612 (2020).

[2] N. Rocco, A. Lovato, and O. Benhar, Phys. Rev. Lett. **116**, 192501 (2016).

[3] O. Benhar, A. Fabrocini, and S. Fantoni, Phys. Rev. Lett. **87**, 052501 (2001).

[4] I. Sick, D. Day, and J. S. McCarthy, Phys. Rev. Lett. **45**, 871 (1980).

[5] D. B. Day, J. S. McCarthy, Z. E. Meziani, R. Minehart, R. Sealock, S. T. Thornton, J. Jourdan, I. Sick, B. W. Filippone, R. D. McKeown, R. G. Milner, D. H. Potterveld, and Z. Szalata, Phys. Rev. Lett. **59**, 427 (1987).

[6] G. B. West, Phys. Rep. **18**, 263 (1975).

[7] C. Ciofi degli Atti, E. Pace, and G. Salmè, Phys. Rev. C **43**, 1155 (1991).

[8] C. Ciofi degli Atti, E. Pace, and G. Salmè, Phys. Rev. C **36**, 1208 (1987).

[9] J. M. Finn, R. W. Lourie, and B. H. Cottman, Phys. Rev. C **29**, 2230 (1984).

[10] P. Barreau *et al.*, Nucl. Phys. **A402**, 515 (1983).

[11] J. Carlson, J. Jourdan, R. Schiavilla, and I. Sick, Phys. Rev. C **65**, 024002 (2002).

[12] T. W. Donnelly and I. Sick, Phys. Rev. C **60**, 065502 (1999).

[13] O. Benhar, Phys. Rev. Lett. **83**, 3130 (1999).

[14] J. J. Weinstein and J. W. Negele, Phys. Rev. Lett. **49**, 1016 (1982).

[15] G. D. Megias, J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and I. R. Simo, Phys. Rev. D **94**, 093004 (2016).