Strange quark matter at finite temperature under magnetic fields with a quasiparticle model

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We investigate the properties of equation of state, the quark fraction, the isospin chemical potential, and the entropy per baryon of strange quark matter at finite temperature under constant magnetic field within the quasiparticle model. We find that both the effects of temperature and magnetic field can significantly influence the thermodynamical properties of quark matter. Our result also indicates that the maximum mass of protoquark stars (PQSs) increases with the heating process along the star evolution, and the core temperature of the maximum mass of PQSs depends on the different snapshots by considering the isentropic stages along the star evolution line.

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I. INTRODUCTION

The properties of the neutron stars can provide a natural and ideal way of exploring the thermodynamical properties of strongly interacting matter at low temperature (less than tens of MeV) and finite chemical potential. Since strange quark matter (SQM), which is totally made up of absolutely stable deconfined u, d, and s quarks and leptons (e and μ), has been considered as the true ground state of quantum chromodynamics (QCD) [1–7], SQM ought to exist in neutron stars (NSs) and generally becomes a hot topic in compact star physics. Furthermore, if a compact star is made entirely of SQM, one can then obtain these stars as quark stars (QSs), and the possible existence of QSs still has important research value in modern nuclear physics and astrophysics [1,2,8–17].

For compact stars, one of the most intriguing features is the maximum mass of compact stars from the observation results. In recent research on compact stars, the heavy pulsar PSR J0348 + 0432 with the star mass of $2.01 \pm 0.04 M_\odot$ [18] has been discovered, while a more massive compact star PSR J 2215 + 5135 whose star mass reaches $2.27^{+0.17}_{-0.15} M_\odot$ has been detected by fitting the radial velocity lines and the three-band light curves in the irradiated compact stars model in 2018 [19]. Last year, the newly discovered compact binary merger GW190814 [20] was reported by the LIGO/Virgo Collaborations whose secondary component m_2 owns the mass of $2.50 M_\odot$–$2.67 M_\odot$ at 90% credible level. Moreover, the star mass of PSR J0740 + 6620 has been updated as $2.08 \pm 0.07 M_\odot$ [21] in this year. Such supermassive compact star results indeed set very strict constraints on the equation of state (EOS) of SQM, whereas there still exist many models which are able to produce massive quark star cases considering strong isospin interaction inside the star matter, color-flavor locked (CFL) QSs, rotating QSs, hybrid star with a quark core, etc. [22–39]. For density-dependent quark mass model, like the confined-density-dependent mass (CDDM) model and confined-isospin-density-dependent mass (CIDDM) model [40], one cannot describe the unknown partner in the GW190814 as QSs due to the EOS not being stiff enough, while the EOS of SQM can still support 2.6 solar mass QSs within quasiparticle model in [33] by increasing the coupling constant g.

Newly born neutron stars are believed to be the remnants of the explosion of a supernova, which contain hot nuclear matter rich in leptons. These stars are usually called as protoneutron stars (PNSs) and the formation of PNSs is well studied in the work [41], while people still know little about the transition from PNS to protoquark stars (PQSs) owing to the complexity of the burning process from hadron matter to SQM during the type II supernova explosion. From [15,42–49], the results imply that PQSs could be found from NSs merging, and the properties of PQSs mainly depend on the thermodynamical properties of the SQM which comprises the QSs at finite temperature. Moreover, compact stars may be endowed with magnetic fields, and the strength of the magnetic field at the surface of magnetars is estimated about $B = 10^{14}$–10^{15} G in [50–52]. Considering the magnetic field inside the stars, the pressure of the star matter might be anisotropic because of the spatial rotational [$\mathcal{O}(3)$] symmetry being broken [53–57], and people usually introduce the density-dependent magnetic fields [58–62] to provide the strength distribution of the magnetic fields of the stars from the surface to the core. Since the spherically symmetric Tolman-Oppenheimer-Volkov (TOV) equations are not applicable under strong magnetic fields,

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combining the Einstein, Maxwell, and equilibrium equations with the EOS under magnetic fields is the ideal way to calculate the structure of magnetars, which is quite complicated [63]. In [64,65], the so-called universal magnetic field profile is proposed as a solution in the context of NSs, which is more suitable for calculating the properties of NSs in much stronger magnetic fields cases. From the research of [66,67], the authors show that the magnetic field might not increase exponentially inside the magnetars, which implies that the magnetic field in the core of the magnetars might be only several times the magnetic field of the surface. Then the magnetic field inside the magnetars in this case would be relatively not strong and the pressure anisotropy inside the star matter might be very small. For strong magnetic fields cases of magnetars, the modification of the TOV equations is still an open nontrivial question which goes beyond the scope of the present work, and we can use the LORENE code [68] to calculate the mass-radius relation of magnetars in future works.

The paper is organized as follows. In Sec. II, we introduce the models and methods for the quark matter at finite temperature under magnetic fields within the quasiparticle model. The properties of EOS, the quark fraction, the isospin asymmetry for SQM, and the maximum mass of PQSs at finite temperature within the quasiparticle model are studied in Sec. III. Finally, a conclusion is given in Sec. IV.

II. MODELS AND METHODS

A. The quasiparticle model at finite temperature under magnetic fields

Many works decades ago considered all the interactions in the medium into the equivalent quark mass [17,40,69–101]. From Ref. [97], using the one-loop self-energy diagrams in the hard dense loop approximation, the equivalent mass of the quasiparticle model can be derived as [97,102,103]

$$m_q = \frac{m_{q0}}{2} + \sqrt{\frac{m_{q0}^2}{4} + \frac{g^2\mu_q^2}{6\pi^2}}, \tag{1}$$

where the current mass of quarks is m_{q0} ($m_{u0} = 5.5$ MeV, $m_{d0} = 5.5$ MeV, and $m_{s0} = 95$ MeV), μ_q is the quark chemical potential, and g stands for the strongly interacting coupling constant adjusted freely in this work.

The total thermodynamic potential density for SQM within the quasiparticle model can be written as

$$\Omega = \sum_i [\Omega_i + B_i(\mu_i)] + B_m, \tag{2}$$

where Ω_i is the thermodynamic potential density for quarks and leptons, $B_i(\mu_i)$ is derived from the chemical dependence of the mass term, and B_m is the negative vacuum pressure term for confinement [103,104]. The contribution to the thermodynamic potential Ω_i from each particle at finite temperature under magnetic fields can be written as

$$\Omega_i = -\sum_\nu \alpha_\nu \frac{g_i T|q_i|B}{2\pi^2}\int_0^\infty \{\ln[1 + e^{-(E_{p,i}-\mu_i)/T}] + \ln[1 + e^{-(E_{p,i}+\mu_i)/T}]\}\mathrm{d}p_z, \tag{3}$$

where $\alpha_\nu = 2 - \delta_{\nu,0}$, and μ_i is the chemical potential. Following the previous works [57–59,105,106], we consider the direction of the magnetic field as the z axis. The degeneracy factor g_i is considered as 3 for quarks and 1 for leptons, and the energy spectrum for quarks and leptons with electric charge q_i from [107] can be written as

$$E_{p,i} = \sqrt{p_z^2 + 2\nu|q_i|B + m_i^2}, \tag{4}$$

where p_z is the momentum in the z direction, and $\nu = n + \frac{1}{2} - \frac{q_i}{|q_i|}\frac{s}{2}$ represents the Landau levels with $n = 0, 1, 2, 3, \ldots$ being the principal quantum number and $s = \pm 1$ for spin up and down. Additionally, the term $B_i(\mu_i)$ is determined by using the integration formula as

$$B_i(\mu_i) = -\int \frac{\partial \Omega_i}{\partial \mu_i}\frac{\partial m_i}{\partial \mu_i}\mathrm{d}\mu_i. \tag{5}$$

B. Properties of strange quark matter with finite temperature and strong magnetic fields

Strange quark matter is composed of u, d, and s quarks and leptons (e and μ) with electric charge neutrality in β equilibrium. The weak β-equilibrium condition can be written as

$$\begin{aligned} &\mu_d = \mu_s = \mu_u + \mu_e - \mu_{\nu_e}, \\ &\mu_\mu = \mu_e, \text{ and } \mu_{\nu_\mu} = \mu_{\nu_e}. \end{aligned} \tag{6}$$

The electric charge neutrality condition can be expressed as

$$\frac{2}{3}n_u = \frac{1}{3}n_d + \frac{1}{3}n_s + n_e. \tag{7}$$

The number density for quarks and leptons can be obtained as

$$n_i = \sum_\nu \frac{g_i(|q_i|B)T}{2\pi^2}\alpha_\nu \int_0^\infty \left[\frac{1}{1 + e^{(E_{p,i}-\mu_i)/T}} - \frac{1}{1 + e^{(E_{p,i}+\mu_i)/T}}\right]\mathrm{d}p_z. \tag{8}$$

The free-energy density $\mathcal{F}$ can be written as

$$\mathcal{F} = \sum_i (\Omega_i + \mu_i n_i) + \frac{B^2}{2}, \tag{9}$$

where the term $B^2/2$ comes from the magnetic field contribution, and we use gaussian natural units in this work. Owing to the $\mathcal{O}(3)$ rotational symmetry is broken for SQM under magnetic fields, the pressure becomes anisotropic which is defined as the longitudinal pressure $P_\parallel$ parallel to the magnetic field and the transverse pressure $P_\perp$ perpendicular to the magnetic field. Then the analytic forms of $P_\parallel$ and $P_\perp$ for SQM can be obtained as [54]

$$P_\parallel = \sum_i \mu_i n_i - \mathcal{F}, \tag{10}$$

$$P_\perp = \sum_i \mu_i n_i - \mathcal{F} + B^2 - MB, \tag{11}$$

where M is the system magnetization and is given by

$$M = -\partial\Omega/\partial B = \sum_{i=u,d,s,l} M_i \tag{12}$$

with

$$M_i = -\frac{g_i|q_i|}{2\pi^2}\sum_{\nu=0}^{\nu_{max}^i}(2-\delta_{\nu 0})\int_0^{k_{F,\nu}^i}\left\{\frac{\nu|q_i|B}{E_{p,i}} + E_{p,i} - \mu_i^*\right\}dk_z. \tag{13}$$

Since the amplitude of the magnetization of the system is much smaller than the magnetic field B, one can neglect it in the EOS of SQM under magnetic fields.

The energy density $\mathcal{E}_i$ for quarks and leptons can be written as

$$\mathcal{E}_i = -\sum_\nu \frac{g_i(|q_i|B)}{2\pi^2}\alpha_\nu\int_0^\infty\left[\frac{E_{p,i}}{1+e^{(E_{p,i}-\mu_i)/T}} + \frac{E_{p,i}}{1+e^{(E_{p,i}+\mu_i)/T}}\right]\mathrm{d}p_z - T\frac{\partial\Omega_i}{\partial m_i}\frac{\partial m_i}{\partial T}, . \tag{14}$$

Furthermore, the total entropy density can be obtained from

$$S = \sum_i S_i = \sum_i -\frac{\partial\Omega_i}{\partial T}. \tag{15}$$

Then one can find from the results above that the free energy density, energy density, and entropy density satisfy $\mathcal{F} = \mathcal{E} - TS$.

III. RESULTS AND DISCUSSIONS

A. SQM at finite temperature under constant magnetic fields

In this work, we calculate the thermodynamical properties of SQM and PQS under magnetic fields within thequasi-particle model with two sets of parameters, i.e., $g-2$ ($g = 2$, $B_m^{1/4} = 141$ MeV) and $g-5$ ($g = 5$, $B_m^{1/4} = 120$ MeV). In Ref. [33], the result has shown that the absolutely stable condition for SQM can be guaranteed for both parameter sets at zero temperature, which indicates that the minimum energy per baryon of SQM with the two parameter sets is less than 930 MeV (the minimum value of energy per baryon of the observed nuclei $M(^{56}\mathrm{Fe})/56$). This absolutely stable condition for SQM has been proposed in Ref. [14], which can put strong constraints on the parameter chosen region for most phenomenological quark models.

In Fig. 1, we calculate the energy per baryon and the free energy per baryon of SQM as functions of baryon density with $g-2$ and $g-5$ under zero magnetic fields and $B = 2\times10^{18}$ G at different temperatures within quasiparticle model. One can see in Fig. 1 that the minimum energy per baryon increases with the increment of the temperature, while the minimum free energy per baryon decreases with temperature. Furthermore, both the energy per baryon and the free energy per baryon increase once the constant magnetic field B increases from zero to $B = 2\times10^{18}$ G, which indicates that the magnetic field can stiffen the energy and free energy in the EOS of SQM.

In Fig. 2, we calculate the anisotropic pressures of SQM as functions of baryon density within the quasiparticle model at different temperatures when $B = 0$ and $B = 2\times10^{18}$ G with $g-2$ and $g-5$. One can find that the baryon number density of the minimum free energy per baryon of all the cases in Fig. 1 is exactly the baryon density of the zero pressure point with $B = 0$, while the density of the minimum free energy per baryon is identical to the density of zero longitudinal pressure point with $B = 2\times10^{18}$ G, which matches the thermodynamical self-consistency. It can also be found from Figs. 1 and 2 that the baryon density of the minimum free energy per baryon (which also means the zero longitudinal pressure point under magnetic fields) decreases with the increment of the temperature, and we can also obtain that the zero longitudinal pressure point decreases with the increment of the coupling constant g when the magnetic field and temperature

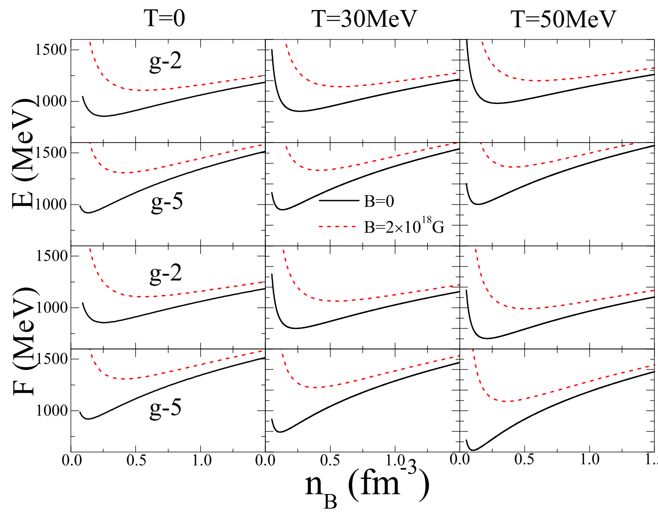

FIG. 1. Energy per baryon and free energy per baryon of SQM as functions of baryon density with $g-2$ and $g-5$ under zero magnetic field and $B = 2\times10^{18}$ G at different temperature within quasiparticle model.

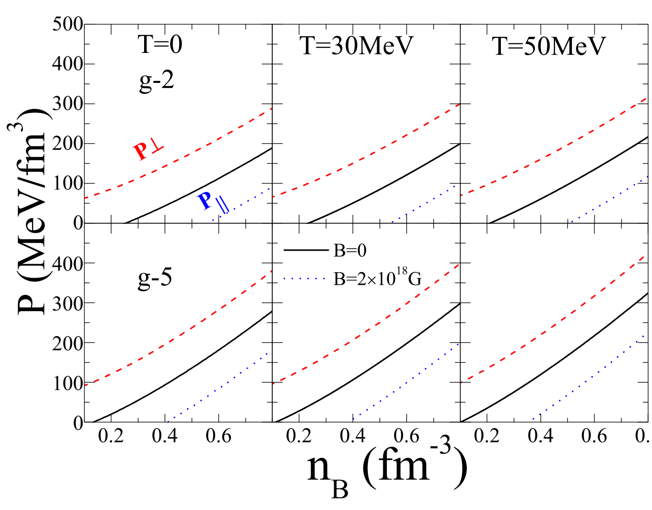

FIG. 2. Longitudinal pressure and transverse pressure for SQM as functions of baryon density within the quasiparticle model under zero magnetic field and $B = 2\times10^{18}$ G with $g-2$ and $g-5$.

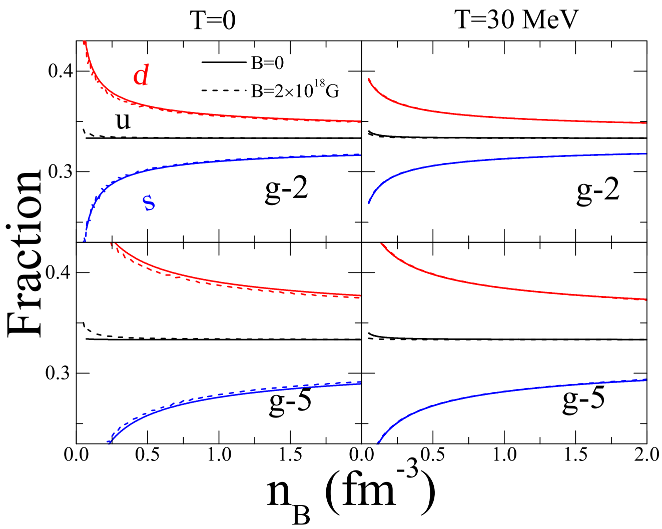

FIG. 3. Quark fraction of SQM as functions of baryon density with different sets of parameters at zero temperature and $T = 30$ MeV in the quasiparticle model when $B = 0$ and $B = 2 \times 10^{18}$ G.

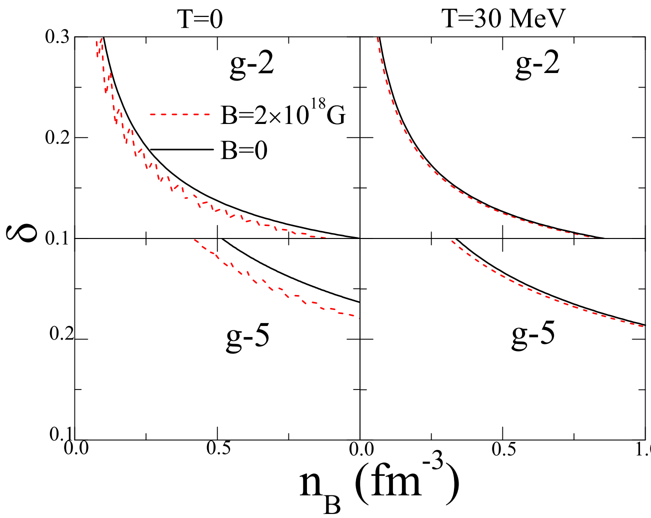

FIG. 4. Isospin asymmetry of SQM as functions of baryon density with different sets of parameters at zero temperature and $T = 30$ MeV in quasiparticle model when $B = 0$ and $B = 2 \times 10^{18}$ G.

are fixed. Furthermore, one can see that the transverse and/or longitudinal pressure of SQM increases and/or decreases with the magnetic field due to the additional term $B^2/2$ introduced by magnetic fields. Moreover, one can also see from Figs. 1 and 2 that the free energy per baryon, energy per baryon, and pressure of SQM all increase with the constant g, which indicates that the EOS of SQM within the quasiparticle model is stiffened by increasing the constant g.

In Fig. 3, we calculate the quark fraction of SQM as functions of baryon density with different sets of parameters at zero temperature and $T = 30$ MeV in the quasiparticle model when $B = 0$ and $B = 2 \times 10^{18}$ G. One can find that the d quark fraction is much larger than the fractions of u and s quarks at low baryon density, while the difference among these three quark fractions decreases at large baryon density for all parameter set cases. One can also see that the difference among u, d, and s quark fractions becomes smaller when increasing the magnetic field, and the quark fraction begins oscillating when we set $B = 2 \times 10^{18}$ G at zero temperature with $g-2$ and $g-5$. Furthermore, we can also find that the difference between the corresponding quark fraction with $B = 0$ and $B = 2 \times 10^{18}$ G becomes smaller at $T = 30$ MeV, and the oscillation of the quark fraction with $B = 2 \times 10^{18}$ almost "disappears" when temperature increases, which implies that the temperature can decrease the effects of the magnetic fields on the quark fractions.

As shown in Fig. 4, we calculate the isospin asymmetry of SQM as functions of baryon density with different sets of parameters at zero temperature and $T = 30$ MeV in the quasiparticle model when $B = 0$ and $B = 2 \times 10^{18}$ G (isospin asymmetry is defined as $\delta = \frac{3(n_d - n_u)}{n_d + n_u}$ in Refs. [108,109]). One can find in Fig. 4 that δ for all cases decreases with the increment of the baryon density, which is consistent with the result in Fig. 3 that the difference of the fractions of u and d quarks decreases with the baryon density, and a similar conclusion can also be found in the calculations of the color-flavor-locked (CFL) phase based on weak-coupling QCD at high baryon density, where u, d, and s quarks participate in a color condensate on an approximately equal footing with the density being identical [110–114]. One can also find the oscillation of δ as a function of n_B also appears when B reaches $B = 2 \times 10^{18}$ G at zero temperature, and this oscillation caused by a strong magnetic field effect can still be reduced when temperature increases. It can be also seen intuitively that δ decreases with B and T, respectively, at a certain baryon density, while the difference of the isospin asymmetry caused by the magnetic fields can also be reduced with an increment of the temperature. Furthermore, one can observe that δ increases with the coupling constant g, which implies that the coupling constant g is closely related to the isospin properties of SQM.

In Fig. 5, we calculate the isospin chemical potential $\mu_I = \mu_d - \mu_u$ as functions of the magnetic field in different temperature cases with $g-2$ and $g-5$. In this figure, we fix the

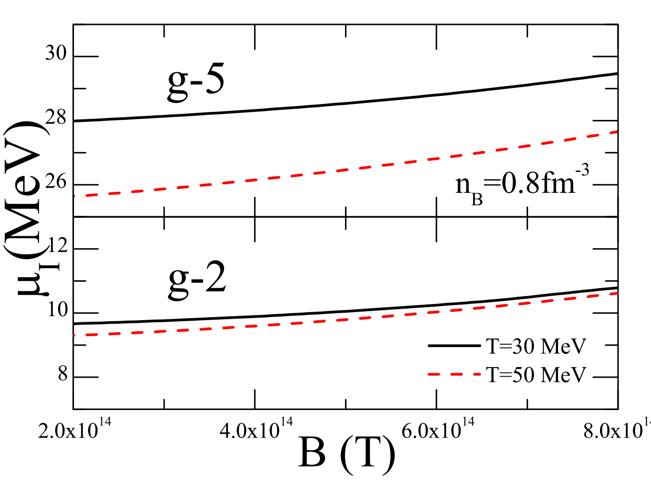

FIG. 5. μ_I as functions of the magnetic field in different temperature cases with $g-2$ and $g-5$.

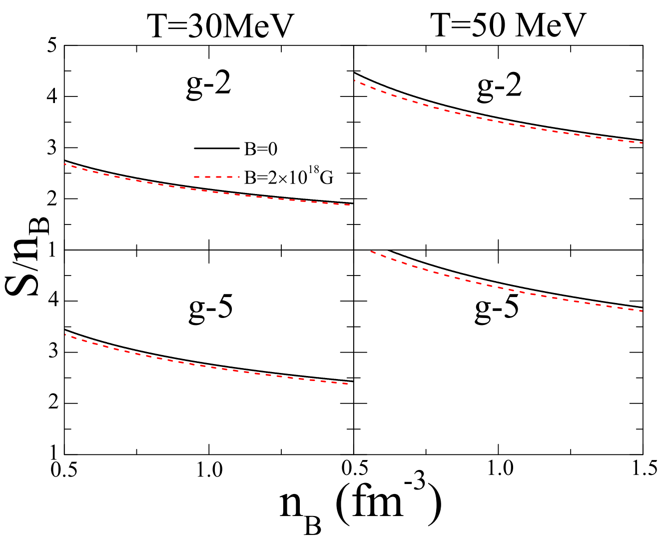

FIG. 6. Entropy per baryon of SQM as functions of the baryon number density in different temperature and magnetic field cases with $g-2$ and $g-5$.

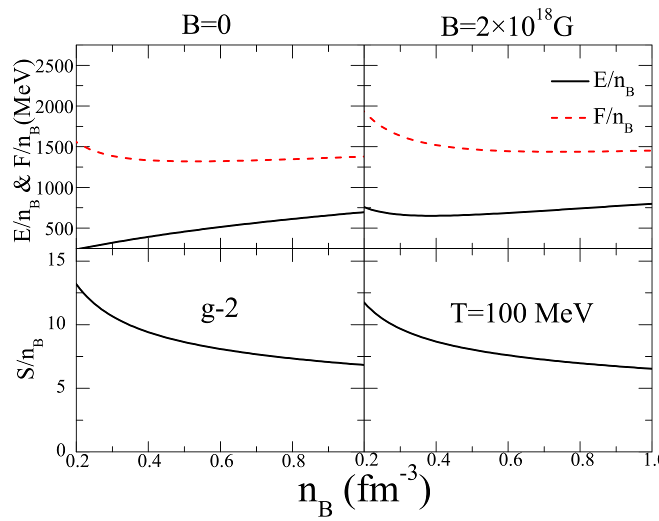

FIG. 7. Upper panel: Energy per baryon and free energy per baryon with $g-2$ as functions of the baryon density with $B=0$ and $B=2\times10^{18}$G at $T=100$ MeV. Lower panel: Entropy per baryon as a function baryon density within the corresponding cases.

baryon density as $n_B=0.8\,\mathrm{fm}^{-3}$, which is large and could be found in the compact star matter in the inner part of the stars. One can find that the isospin chemical potential μ_I increases with the magnetic field for all cases, which implies that isospin density $n_I=1/2(n_d-n_u)$ might decrease with magnetic field, and this phenomenon can also be found from Figs. 3 and 4 where the difference between u and d quark fractions and the isospin asymmetry both decrease with magnetic fields. One can also find in Fig. 5 that μ_I decreases with temperature at a certain magnetic field.

As shown in Fig. 6, we calculate the entropy per baryon of SQM as functions of the baryon density in different temperature and magnetic fields cases with $g-2$ and $g-5$. One can find in Fig. 6 that the entropy per baryon decreases with baryon density in all cases, and the entropy per baryon increases with increments of the temperature when B and g are fixed, which indicates that the degree of disorder of SQM becomes larger at high temperature. Furthermore, it can also be seen in Fig. 6 that the entropy per baryon decreases with increments of the magnetic field, while the entropy per baryon increases with increments of the coupling constant g at fixed temperatures. These results imply that the complexity of SQM increases with temperature and the coupling constant, while the complexity of SQM decreases with the magnetic field.

In Fig. 7, we calculate the energy per baryon and free energy per baryon of SQM within quasiparticle model with g-2 as functions of the baryon density with $B=0$ and $B=2\times10^{18}$ G at $T=100$ MeV. One can find in Fig. 7 that the energyand/or free energy per baryon at $T=100$ MeV for both cases is much larger and/or smaller than that in Fig. 1 for $T\leqslant 50$ MeV cases, and both the energy per baryon and the free energy per baryon are enhanced by the magnetic fields with $B=2\times10^{18}$ G, which can also be found in Fig. 1. We can also find the entropy per baryon is increased much at $T=100$ MeV while being decreased by magnetic fields, and then the difference between the energy per baryon and the free energy per baryon increased with both the entropy per baryon and the temperature. Such large temperature cases (100 MeV, for instance) are very relevant in the features of the QCD phase diagram.

In order to investigate the opposite roles which are played by the temperature and magnetic fields, we should check the number of the Landau level in finite temperature and magnetic fields cases. In this work, we neglect the Landau levels which contribute to the density with a fraction less than 10^{-5} in order to simplify the calculations which should consider all the Landau levels at finite temperature. Using this cutoff, the error of the thermodynamical quantities considering the limited Landau levels (e.g., energy density, free energy density, entropy, etc.) is far below 0.1%, which has almost no influence on the results in this work. From Table I, we calculate the Landau levels of SQM within the quasiparticle model for different temperatures and magnetic fields at $3\rho_0$ with $g-2$, and the results indicate that the number of the Landau levels decreases with the magnetic fields and increases with the temperature, which indicates the temperature and the magnetic field also play opposite roles in the number of Landau levels in this work.

B. Protoquark stars

We introduce three different snapshots by considering the isentropic stages along the star evolution line to investigate the

TABLE I. Landau levels of SQM within quasiparticle model for different temperatures and magnetic fields.

	$T=50$ MeV	$T=100$ MeV
$B=2\times10^{18}$ G	45	179
$B=1\times10^{19}$G	5	31

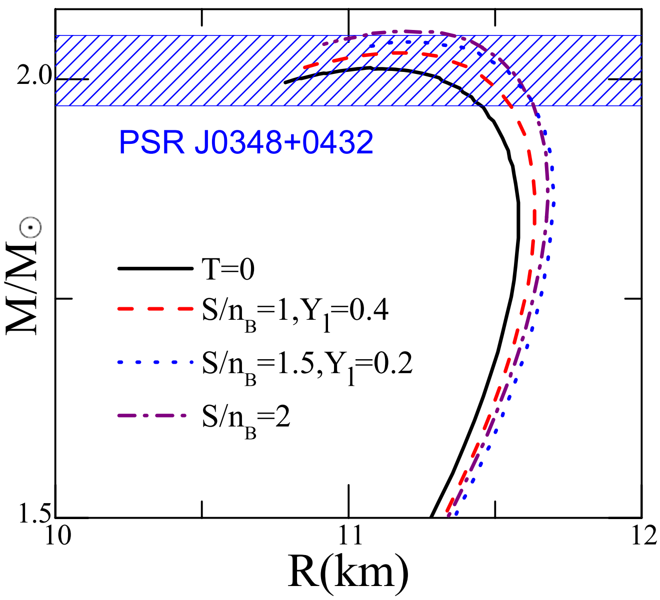

FIG. 8. Mass-radius relations of the stages along the star evolution line of PQS with $g-2$.

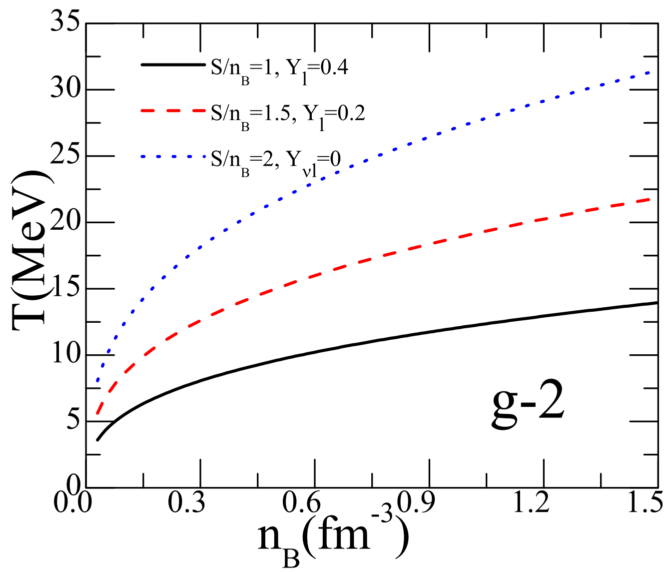

FIG. 9. The core temperature for the star matter as a function of the central baryon density with $g-2$.

properties of QSs at finite temperature, i.e., protoquark stars (PQSs),

$$\text{(I)}\ \ S/n_B = 1\ , Y_l = 0.4, \tag{16}$$

$$\text{(II)}\ \ S/n_B = 2\ , Y_{\nu_l} = 0, \tag{17}$$

$$\text{(III)}\ \ S/n_B = 0\ , Y_{\nu_l} = 0, \tag{18}$$

which is well discussed in previous studies [41,115–119] to describe the evolution of protocompact stars. For the first isentropic stage of PQS evolution at the beginning of the birth after the supernova explosion, the entropy per baryon is set as one and the fraction of leptons (including trapped neutrinos) is set as 0.4 ($Y_l = Y_e + Y_\mu + Y_{\nu_e} + Y_{\nu_\mu} = 0.4$). In the second stage, diffusing neutrinos boil the star matter and flee the star, while the corresponding entropy per baryon increases to 2. After the heating stage, the stars begin cooling down and finally transform to cold quark stars.

In Fig. 8, we calculate the mass-radius relations of the stages along the star evolution line of PQSs with $g-2$, where we consider the stages as ($T=0$), ($S/n_B = 1, Y_l = 0.4$), ($S/n_B = 1.5, Y_l = 0.2$), and ($S/n_B = 2$). One can find in Fig. 8 that the maximum star mass of PQSs with $g-2$ at $T=0$ and $B=0$ is $2.01\,M_\odot$, which can describe the PSR J0348 + 0432 with the mass of $2.01 \pm 0.04 M_\odot$ [18] as QSs. Furthermore, the maximum mass of the PQS increases from $2.03\,M_\odot$ at ($S/n_B = 1,\ Y_l = 0.4$) to $2.05\,M_\odot$ at ($S/n_B = 2,\ Y_l = 0$), which implies that the maximum star mass of PQSs within the quasiparticle model can be increased by the heating process in the star evolution under zero magnetic field.

In Fig. 9, we calculate the core temperature for the star matter as a function of the central baryon density with $g-2$ at ($S/n_B = 1, Y_l = 0.4$), ($S/n_B = 1.5, Y_l = 0.2$), and ($S/n_B = 2$). One can find the core temperature increases with central baryon density for all cases, and the core temperature at a certain baryon density increases from ($S/n_B = 1, Y_l = 0.4$) to ($S/n_B = 2$). Moreover, one can also find the core temperature of the maximum star mass increases from $T_c = 12$ MeV at ($S/n_B = 1, Y_l = 0.4$) to $T_c = 27$ MeV at ($S/n_B = 2$), and the central baryon density of the maximum star mass decreases from $n_B = 0.99\,\text{fm}^{-3}$ MeV at ($S/n_B = 1, Y_l = 0.4$) to $n_B = 0.98\,\text{fm}^{-3}$ MeV at ($S/n_B = 2$), which implies that the core temperature and the central baryon density of the maximum mass of PQSs might also depend on the heating process stages along the star evolution line. We should mention that for the properties of protoquark stars under strong magnetic field cases one need to consider new methods (like LORENE ways) instead of the TOV equations, which we expect to use to calculate the mass-radius relation of magnetars in future works.

IV. CONCLUSION AND DISCUSSION

In this work, we have investigated the properties of EOS, the quark fraction, the isospin chemical potential, the isospin asymmetry, and the entropy per baryon of SQM under magnetic field within quasiparticle model. We have found that both the effects of temperature and magnetic field can significantly influence the energy and/or free energy per baryon and the longitudinal and/or transverse pressure of SQM, and the difference among u, d, and s quark fractions becomes smaller when the strong magnetic field is considered, while the oscillation caused by a strong magnetic field can be decreased by temperature.

Furthermore, we also investigate the isospin properties of the star matter at finite temperature under magnetic fields. The results indicate that the isospin asymmetry decreases with both temperature and magnetic field while increasing with the coupling constant g, and the isospin chemical potential increases with magnetic field while decreasing with temperature. The entropy baryon of SQM is also calculated in this work, and the entropy per baryon decreases with magnetic field while increasing with temperature.

Moreover, we have investigated the maximum mass of PQSs at different isentropic stages along the star evolution line. The results indicate that we can describe PSR J0348 + 0432 as QSs within the quasiparticle model, and the maximum mass, the corresponding star radius, the core temperature, and the central density of the stars are all strongly influenced by the effects of the temperature.

Therefore our present results have shown that the effects of the temperature and the magnetic fields are very important for the thermodynamical properties and isospin properties of the strange quark matter. The maximum star mass, central baryon density, and the core temperature of protoquark stars within the quasiparticle model are influenced by the heating process in star evolution.

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