Probing collective enhancement in nuclear level density with evaporation α particle spectra

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Collective enhancement in nuclear level density (CELD) and its fadeout has been studied experimentally using neutron and high energy γ -ray spectra earlier. Attempts to probe CELD with α -particle spectra were made for compound nucleus (CN) ¹⁷⁸Hf in the excitation energy 54–124 MeV, but no signature of CELD was found. The possible reason behind this nonobservance has been discussed in a few subsequent reports. In our previous study, evidence of CELD was found using neutron spectra and enhancement faded away near 25 MeV of excitation energy. This implies that the effect of CELD on α particles, if any, would be found at excitation lower than 25 MeV. With the aim to observe CELD and its fadeout with α -particle spectra, two reactions ${}^{12}C + {}^{116}Sn$, ${}^{159}Tb$, forming CN in different mass region ($A \approx 128$ and $A \approx 171$) were studied. Evaporation α -particle spectra were measured for these reactions in singles as well as in coincidence with neutrons. Experimental data were compared with statistical model calculations and inverse level density parameter (k) were obtained from α -particle spectra. As a function of CN excitation energy, the k value showed peak like structure for the reaction ${}^{12}C + {}^{159}Tb$ which indicates fadeout of CELD. No such evidence was found for the reaction ${}^{12}C + {}^{116}Sn$. Collective enhancement factor for daughter nuclei populated in ${}^{12}C + {}^{159}Tb$ reaction was extracted. Critical energy of the fadeout was found to be similar to that of mass $A \approx 188$ region.

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I. INTRODUCTION

Experimental evidence of fadeout of collective enhancement of nuclear level density is limited. Recently, a few studies reported evidence of collective enhancement fadeout from evaporated neutron and γ -ray spectra [1–4]. The level density $\rho(U, J)$ of a spherical nucleus with excitation energy U and angular momentum J was given by Bethe *et al.* [5-7]to be

$$\rho(U,J) = \frac{(2J+1)}{\sqrt{8\pi\sigma^3}} e^{\{-\frac{J(J+1)}{2\sigma^2}\}} \rho(U).$$
(1)

Here σ is the spin cutoff factor and $\rho(U)$ is the total state density independent of spin. For a deformed axially symmetric nucleus, with total intrinsic state density $\rho_{int}(U)$, the level density for angular momentum J can be written as [6-8]

$$\rho(U,J) = \sum_{K=-J}^{J} \frac{1}{\sqrt{8\pi}\sigma_{\perp}} e^{-\frac{K^2}{2\sigma_{\perp}^2}} \rho_{\text{int}}(U - E_{\text{rot}}(K,J)).$$
(2)

Here, σ_{\perp} is spin cut-off factor perpendicular to the symmetry axis and $E_{rot}(K, J)$ is rotational energy of the nucleus.

These two equations indicate that level density is larger by a factor of $\approx \sigma^2$ in case of a deformed axially symmetric nucleus. This is known as collective enhancement of nuclear level density which then fades out with increase in energy as the nucleus becomes spherical. The enhancement factor (σ^2) and fadeout energy both depend on mass and ground-state deformations. For a deformed nucleus in mass 150 region, $\sigma \approx 10$, i.e., enhancement by a factor of ≈ 100 is expected [6,8–11]. However, in the case of state density, the enhancement is $\sqrt{\frac{2}{\pi}\sigma_{\perp}}$ instead of σ^2 [7]. Efforts were made to observe the collective enhancement experimentally. First experimental evidence of CELD was reported by Junghans et al. [12] from fission fragment yield distributions. However, in another study, reported by Komarov *et al.* [13], α -particle spectra for the reaction ${}^{18}O + {}^{160}Gd$ were studied over excitation energy range 54-124 MeV to investigate fadeout of CELD but no such effect was observed. The reason for the nonobservance was discussed later by Grimes [7]. It was argued that, in low-spin states, the nuclear state density dominates the evaporation spectra instead of nuclear level density and the amplitude of enhancement is small, making it difficult to observe.

Later, experimental evidence of fadeout of CELD was reported by Banerjee et al. [1] from evaporated neutron spectra.

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In this study, the inverse level density parameter k was extracted by fitting evaporated neutron spectra for ${}^{4}\text{He} + {}^{169}\text{Tm}$, ${}^{181}\text{Ta}$, and ${}^{197}\text{Au}$. For reactions ${}^{4}\text{He} + {}^{169}\text{Tm}$, ${}^{181}\text{Ta}$, high-k value was found over a certain range of compound nucleus (CN) excitation energy, which was attributed to the fadeout of collective enhancement. In a similar study, Pandit *et al.* [2] observed evidence of CELD using high-energy γ rays and enhancement factor was from γ spectra.

In our previous study [3], we measured evaporated neutron spectra in coincidence with α particles for the reactions $^{11}B + ^{181}Ta$, ^{197}Au , and k was extracted. A peak-like structure in k as a function of excitation energy was observed for ${}^{11}\text{B} + {}^{181}\text{Ta}$, which was then explained after incorporating CELD in the statistical model calculation. When daughter nuclei are formed in the fadeout energy range where slope of level density is modified, the change in slope is reflected in the evaporation spectra. The enhancement factor was chosen to be of Fermi form and enhancement of the order of 10 was observed, which was close to $\sqrt{\frac{2}{\pi}}\sigma_{\perp}$. The critical energy $U_{\rm crit}$ and width $d_{\rm crit}$ of the Fermi function were 16.0 and 3.0, respectively. The resultant collective enhancement faded out near 25 MeV of excitation energy of the daughter nuclei. As the excitation energy of the daughter nuclei, populated in ${}^{18}O + {}^{160}Gd$ reaction after α evaporation was 25 MeV or higher, no enhancement could be observed [13]. To explore the CELD and its fadeout using α particles, we have measured evaporated α -spectra for reactions ${}^{12}C + {}^{116}Sn$ and ${}^{12}C + {}^{159}Tb$ at various low excitation energies. Neutron spectra in coincidence with α were also measured at one beam energy for each of the reactions as an additional probe.

II. EXPERIMENT

The experiment was carried out at the BARC-TIFR Pelletron Linac Facility using ${}^{12}C$ beam of energy 48–64 MeV. Isotopically enriched self-supporting targets of ¹¹⁶Sn and ¹⁵⁹Tb were used with thicknesses 1.4 and 1.8 mg/cm², respectively. For detection of charged particles, two $E - \Delta E$ telescope detectors, made of silicon strip detectors, were placed at angles $\pm 155^{\circ}$ with respect to the beam direction. Thicknesses of ΔE and E detectors were 50 and 1500 μ m, respectively. α -particle spectra were measured at five beam energies in the range of 48-64 MeV. For the estimation of the cross section, a current integrator was used to integrate the beam current after the target. Uncertainty due to target thickness and the solid angle of the detector were major contributors in the error analysis which was $\approx 25\%$ of the measured cross section. In a separate experiment, neutrons were detected in coincidence with α particles for at 57 MeV (for ${}^{12}C + {}^{116}Sn$) and 63 MeV (for ${}^{12} + {}^{159}Tb$) beam energies. Neutrons were detected using 15 liquid scintillator detectors placed at a distance of 72.5 cm from the target. Pulsed beam was used for this measurement and energy of the neutrons were measured using time of flight technique. The radio frequency signal of the beam pulsing system was used as the reference for the time of flight signal. γ rays were discriminated from the neutrons using pulse shape discrimination method. To estimate the contamination in target as well as

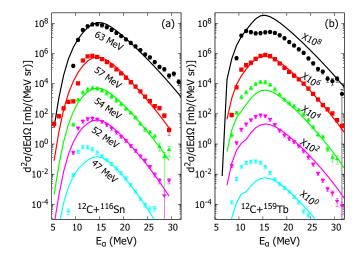


FIG. 1. α -particle spectra for the reaction (a) ${}^{12}C + {}^{116}Sn$ and (b) ${}^{12}C + {}^{159}Tb$ at different beam energies. Symbols and lines represent experimental and best fitted spectra, respectively.

beam hitting on target frame, data were collected with carbon target as well as with blank target frame. No high-energy α particles were detected. For neutron spectra, background subtraction was carried out to reject background events.

III. RESULTS AND DISCUSSION

For reactions ${}^{12}C + {}^{116}Sn$ and ${}^{12}C + {}^{159}Tb$, α particles were obtained at five (47,52,54,57,63 MeV) laboratory energies (E_{lab}) . Experimental α -particle spectra were compared with statistical model calculations using the code CASCADE [14]. The fusion cross sections, used as input in the CASCADE calculation, were calculated using the coupled channel code CCFULL [15]. Level density parameter (a) was varied in CAS-CADE to obtain the best agreement between experimental and calculated spectra. The effect of the shell correction on level density was taken care of in the calculation by considering energy-dependent level density parameter, prescribed by Ignatyuk [16], as $a(U) = \tilde{a}[1 - \frac{\Delta S}{U}(1 - e^{-\gamma U})]$. Here \tilde{a} is asymptotic value of a, ΔS is the shell correction energy, and γ is damping parameter. The value of γ was taken to be 0.055 and the value of \tilde{a} was taken as A/k, where A is the mass number of the nucleus and k is inverse level density parameter. In the calculation, the value of k was varied to obtain best fit by minimizing χ^2/f where, χ^2 is chi square and f is number of degrees of freedom. In order to avoid contamination in the α -particle spectra, if any, α particles with energy higher than 14.5 MeV for ${}^{12}C + {}^{116}Sn$ and 15.5 MeV for ${}^{12}C + {}^{159}Tb$ were considered for χ^2 estimation. All the experimental α -particle spectra for ${}^{12}C + {}^{116}Sn$ were in good agreement with the statistical model calculation as can be seen in Fig. 1 (a) $(\chi^2/f \approx 0.95-1.7)$. Similar plots for ${}^{12}C + {}^{159}Tb$ are shown in Fig. 1(b). For ${}^{12}C + {}^{159}Tb$, high values of k were needed in order to fit the data at low beam energies. At $E_{\text{lab}} = 63$ MeV, for all the values of k, CASCADE overpredicted the cross section. Hence, the value of k at this energy was not extracted from the spectra and Fig. 1(b) shows the calculation using the k value same as that of $E_{lab} = 57$ MeV.

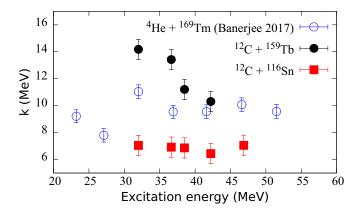
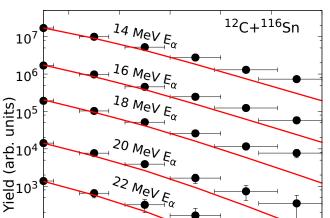


FIG. 2. k corresponding to best fits shown in Fig. 1 for reactions ${}^{12}C + {}^{116}Sn$, ${}^{159}Tb$. k parameters for reaction ${}^{4}He + {}^{169}Tm$, taken from Banerjee et al. [1] are shown by open circles.

The values of k for both the reactions corresponding to best fits are shown as a function of CN excitation energy E_x in Fig. 2. The same plot shows k values for another reaction ${}^{4}\text{He} + {}^{169}\text{Tm}$, taken from Banerjee *et al.* [1], which forms CN in the same mass region as of ${}^{12}C + {}^{159}Tb$. In the case of ${}^{4}\text{He} + {}^{169}\text{Tm}$, k was extracted by fitting evaporated neutron spectra. It can be seen from the plot that the value of k for $^{12}C + ^{116}Sn$, as expected, remains nearly unchanged over the studied energy range. For ${}^{4}\text{He} + {}^{169}\text{Tm}$, k is nearly constant (\approx 9.5) for $E_x > 35$ MeV but shows sudden variation near $E_x = 30$ MeV. On the other hand, k value for ${}^{12}C + {}^{159}Tb$ is as high as 14.2 near $E_x \approx 30.0$ MeV and then gradually decreases with energy and matches with the ${}^{4}\text{He} + {}^{169}\text{Tm}$ data at high energy. It is to be noticed here that the daughter nuclei, populated in reaction ${}^{12}C + {}^{159}Tb$ after α evaporation (165-167Tm), have very similar mass, atomic number, and deformation as of nuclei populated from ${}^{4}\text{He} + {}^{169}\text{Tm}$ after neutron evaporation (${}^{161,162}\text{Ho}$). Hence, a similar value of k is expected for these two reactions but the experimental α spectra for ¹²C + ¹⁵⁹Tb could not be explained by k = 9.5over the entire beam energy range studied. The reason behind this high k value needs to be investigated. Previously, high values of k over certain energy regions were reported for few other reactions [1,3] and increased k values were attributed to the fadeout of CELD. It was argued that when deformed daughter nuclei have excitation energy in the energy range where collectivity causes enhancement in the nuclear level density, the value of k, obtained by fitting evaporated particle spectra, is higher if calculations do not incorporate CELD. In the case of the ${}^{12}C + {}^{159}Tb$ reaction, populated daughter nuclei are highly deformed, which indicates higher k value in this reaction might be due to CELD and its fadeout.

For further study, neutrons in coincidence with α particles were studied for ${}^{12}C + {}^{116}Sn$ and ${}^{12}C + {}^{159}Tb$ at CN excitation energies (E_x) 44.8 and 45.2 MeV, respectively. Neutron spectra were obtained in coincidence with different α -particle energies (E_{α}) and compared with statistical model prediction. Calculated spectra were normalized to the experimental spectra at 2.5 MeV of neutron energy. A modified version of CASCADE [17] was used for this purpose to calculate neutron



 10^{7}

10²

 10^{1}

10⁰

3

4

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FIG. 3. Experimental neutron spectra (symbols) for reaction $^{12}C + ^{116}Sn$ in coincidence with α -particle of energy E_{α} . CASCADE calculations (with k = 7.0) are shown by lines.

5 E_n (MeV)

6

7

8

spectra in coincidence with α particle of specific energy, say E_{α} . For the ¹²C + ¹¹⁶Sn reaction, calculated spectra with k =7.0 (taken from Fig. 2) agree with the experimental spectra within the experimental uncertainty, as can be seen in Fig. 3. However, at high energy regions, the calculation slightly underpredicts the spectra. For ${}^{12}C + {}^{159}Tb$ too, a similar trend was observed with k = 9.5. However, due to large uncertainty associated with neutron spectra as well as uncertainty in kvalue, it was difficult to draw conclusions from the neutron spectra.

To investigate the role of CELD in ${}^{12}C + {}^{159}Tb$ reaction, α -particle spectra were calculated with k(=9.5) with and without including CELD. This value was taken from the highenergy part of the ${}^{4}\text{He} + {}^{169}\text{Tm}$ reaction (Fig. 2). Calculated α -particle spectra without CELD are shown by black dashed lines in Fig. 4. With this condition, neutron spectra too were calculated which are shown in Fig. 5 by black dashed lines. It can be seen that experimental α spectra can not be reproduced with k = 9.5 without CELD. Significant deviation is noticed at low E_{lab} where populated daughter nuclei have low excitation energy. In a next step, CELD was incorporated in the calculation. Collective enhancement factor $K_{coll}(U)$ modifies the level density $\rho_L(U)$ as, $\rho_L(U) = \rho_L^{int}(U)K_{coll}(U)$, where $\rho_L^{\text{int}}(U)$ is the intrinsic level density. Calculation with Fermi shaped K_{coll} , which was used in the previous work [3], could reproduce α spectra only at high E_{lab} but low E_{lab} spectra could not be reproduced. It was found that α spectra at higher E_{lab} are more sensitive to the decay part of CELD, i.e., the region U > 15 MeV in this case, whereas at low E_x , the spectra were more sensitive to $K_{coll}(U)$ at low energy, i.e., U < 15 MeV. Calculations were then performed with constant

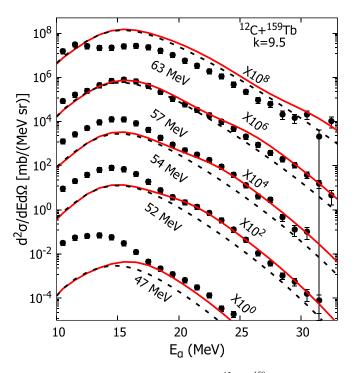


FIG. 4. α -particle spectra for reaction ${}^{12}\text{C} + {}^{159}\text{Tb}$ at different beam energies (mentioned with each plot). Symbols and lines represent experimental and CASCADE (k = 9.5) predictions, respectively. Black dashed lines show calculation without collective enhancement and red solid lines show calculations with CELD.

enhancement followed by a Gaussian decay with $K_{coll}(U)$ as

$$K_{\text{coll}} = 1 + A_{en} \exp\left[(U - U_{\text{crit}})^2 / 2d_{\text{crit}}^2\right] \text{ for } U > U_{\text{crit}}$$
$$= 1 + A_{en} \text{ for } U < U_{\text{crit}}.$$
(3)

Here A_{en} is amplitude of K_{coll} , U_{crit} is critical energy value for CELD fadeout, and d_{crit} is width of the decay. The enhancement factor $K_{coll}(U)$ with $A_{en} = 3.0$, $U_{crit} = 15.5$, and $d_{crit} = 2.35$ provided the best agreement between experimental and calculated α as well as neutron spectra $K_{coll}(U)$, used in the present calculation, is shown as a function of U in the Fig. 6. For all the E_{lab} values, experimental α -particle spectra were well reproduced, as shown by red lines in Figs. 4 and 5. α -particle spectra near $E_{\alpha} \approx 20$ –25 MeV were better reproduced after inclusion of the $K_{coll}(U)$. From neutron spectra, it can be seen that calculated spectra after CELD inclusion show better agreement with experimental values. However, in the present study the neutron spectra are less sensitive than α -particle spectra due to larger uncertainties.

In spite of the improvement in fitting after including CELD, it is noticed that fits are not good for ${}^{12}\text{C} + {}^{159}\text{Tb}$ at $E_{\alpha} < 15.0$ MeV for low E_{lab} . The possibility of contamination is negligible as freshly prepared self-supporting target of ${}^{159}\text{Tb}$ was used. It was also observed that with decrease in energy, the peak position of the α -particle spectra shifted toward the lower energy side. Similar observation was reported for reaction ${}^{12}\text{C} + {}^{93}\text{Nb}$ by Mirgule *et al.* [18] and was attributed to possible emission of α particle from a deformed complex. This needs further investigation by studying more reactions

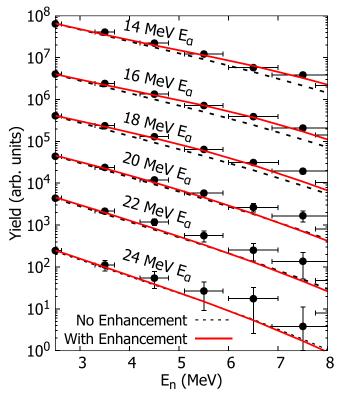


FIG. 5. Neutron spectra for reaction ${}^{12}\text{C} + {}^{159}\text{Tb}$ in coincidence with α -particle of energy E_{α} . Experimental and CASCADE predicted (with k = 9.5) spectra are shown by symbols and lines. Black dashed lines represent calculations without CELD and red solid lines represent calculations with CELD.

simultaneously over wider energy range. Another source of disagreement comes from the fact that the enhancement factor is approximated to have some regular functional form and this introduces some disagreement between the experimental and calculated spectra. Hence, in the present analysis, low-energy parts of the α spectra are not included in fitting or χ^2 estimation.

In the present study, the extracted value of $U_{\rm crit}$ is comparable to that obtained for nuclei in the $A \approx 188$ region in the previous study [3] but the enhancement factor is small.

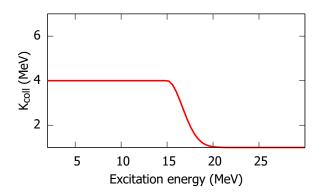


FIG. 6. $K_{\text{coll}}(U)$ as used in CASCADE calculations is shown as a function of energy U.

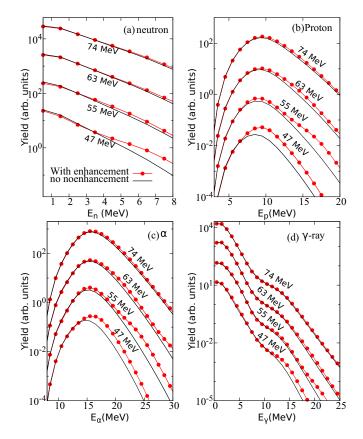


FIG. 7. CASCADE predicted evaporation spectra of n, p, α , and γ for $^{12}C + ^{159}Tb$ at different laboratory energies. Lines indicate calculations with no collective enhancement and line + symbol indicate calculations with collective enhancement.

The daughter nuclei populated in reaction ${}^{12}\text{C} + {}^{159}\text{Tb}$ are in the range $\approx 166-167$ and deformation is in the range 0.327-0.344 [19]. Same U_{crit} explains data over the mass region A = 167-188. In the case of the reaction ${}^{12}\text{C} + {}^{116}\text{Sn}$, the daughter nuclei (${}^{123-124}\text{Xe}$) have deformation 0.223-0.259 and excitation energy of daughter nuclei was as low as 8 MeV (for $E_n = 4.5 \text{ MeV}$ and $E_{\alpha} = 24$). No visible effect of CELD on α spectra was observed, even at this energy. This indicates absence of collective enhancement in ${}^{123,124}\text{Xe}$ nuclei within the studied energy range.

To compare the effect of fadeout of CELD on various evaporated particles and γ rays, we have performed CASCADE calculations for $^{12}C + ^{159}Tb$ at four E_{lab} values (47,55,63,74 MeV). Fusion barrier for the reaction is 52 MeV. Neutron (*n*), proton (*p*), α -particle, and γ -ray spectra were calculated at these four energies with and without CELD. Figures 7(a)–7(d) show these calculated spectra. It is noticed that depending on U_{crit} and d_{crit} , a small range of E_{lab} and particle energy will be sensitive to the fadeout, e.g., the effect is visible on protons of 15–20 MeV energy at $E_{lab} = 55$ MeV.

The CASCADE calculation suggest that measurement of evaporation spectra in this E_{lab} region with small energy steps will be helpful for observing the effect of fadeout of CELD. For the presently used $K_{\text{coll}}(U)$, neutron spectra are sensitive to CELD at low energies where fusion cross sections are small. However, for α -gated neutrons, higher E_{lab} data are sensitive to the fadeout. Effect of CELD on α spectra is easy to detect compared to neutrons due to wider energy range and better detection efficiency of the evaporated charged particles. This indicates that α -particle spectra might be an useful tool for exploring CELD.

IV. CONCLUSIONS

Evaporated α -particle spectra were measured for reactions ${}^{12}C + {}^{116}Sn$, ${}^{159}Tb$ at different laboratory energies in the energy range of 47-63 MeV. Further, neutron spectra in coincidence with evaporated α particles were also detected for both the reactions at $E_{\text{lab}} = 56 \text{ MeV}$ (for ${}^{12}\text{C} + {}^{116}\text{Sn}$) and $E_{\text{lab}} = 61 \text{ MeV}$ (for ${}^{12}\text{C} + {}^{159}\text{Tb}$). Statistical model calculations were performed to fit the experimental α spectra by varying inverse level density parameter k. For the reaction $^{12}C + ^{116}Sn$, value of $k \approx 7.0$) remained nearly constant over the studied beam energy region. For ${}^{12}C + {}^{159}Tb$, k value was high at low energy and then it decreased with increase in the excitation energy. This behavior of k was similar to the earlier reported studies where k values were extracted from evaporated neutron spectra [1,3]. Statistical model predictions with k = 9.5 improved after incorporating CELD using a factor $K_{coll}(U)$. This is in agreement with the earlier reports that noninclusion of CELD in statistical model calculation gives high k value over certain energy range. This indicates that charged particle spectra might also be affected by CELD and it might be useful in the study of CELD and its fadeout. However, the beam energy range as well as energy steps need to be chosen carefully for such experiments. The energy range and step size could be a possible reason behind the nonobservance of fadeout of CELD in ${}^{18}O + {}^{160}Gd$ [13]. For the reaction $^{12}C + ^{116}Sn$, no significant enhancement was noticed despite highly deformed ground state of daughter nuclei and the reason needs to be investigated further. More such measurements are needed to explore the potential of charged particle spectra in the study of CELD.

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