New Coulomb barrier scaling law with reference to the synthesis of superheavy elements

P. W. Wen,[1] C. J. Lin,[1,2,*] H. M. Jia,[1] L. Yang,[1] F. Yang,[1] D. H. Huang,[1,2] T. P. Luo,[1]
C. Chang,[1] M. H. Zhang,[1] and N. R. Ma[1]
[1]*China Institute of Atomic Energy, 102413 Beijing, China*
[2]*College of Physics and Technology and Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, 541004 Guilin, China*

(Received 29 November 2021; revised 9 February 2022; accepted 25 February 2022; published 9 March 2022)

The synthesis of superheavy elements based on fusion reactions and multinucleon transfer reactions is sensitive to the reaction energies, where the Coulomb barrier plays a crucial role as it must be overcome for the projectile and target to contact each other. The Coulomb barrier cannot be measured directly, and the synthesis of superheavy elements is sensitive to it. In this study, we systematically extract the barrier information from the experimental fusion excitation functions by the empirical cross section formula, which is based on a single-Gaussian distribution of fusion barrier heights. A total of 403 sets of experimental data are fitted, among which 243 sets have good results with χ^2/pt less than one. The extracted fusion barrier is a dynamical barrier, which includes the overall effect of coupled channels. Different from the prediction of the WKJ formula, the new scaling law proposed in this work is almost identical to the CW76 Coulomb barrier at the $z = 170$–300 region and reproduces well the centroid barrier extracted from quasielastic scattering. The comparison to the other 14 bare potential models and exploration of the very large z region is also discussed. It is also remarkable to find that the predictions of the DP2015 potential, including of the contributions of the macroscopic and shell correction terms, are highly consistent with the extracted results in this work and the CW76 potential. The numerical results demonstrated that the dynamical fusion barrier and the centroid barrier extracted from the quasielastic reaction (the reaction threshold) could be the same physical quantity for superheavy reactions. This study could provide important references for the synthesis of superheavy nuclei based on fusion reactions and very heavy multinucleon transfer reactions.

DOI: 10.1103/PhysRevC.105.034606

I. INTRODUCTION

Study of heavy-ion fusion reactions is important for extending the periodic tables [1], understanding the stellar evolution [2], and exploring the quantum many body dynamics [3–5]. Heavy-ion fusion cross sections are extremely sensitive to the height, position, and shape of the Coulomb barrier. However, the Coulomb barrier is not a direct experimentally measured quantity. The extraction of the Coulomb barrier from experiments is important for both experimental and theoretical studies. From an experimental point of view, it is crucial to know the barrier height for the setup of near-barrier incident energy, and the estimation of barrier-sensitive quantities. The Coulomb barrier extracted from the experiment will also serve as a benchmark for different kinds of theoretical potential models and nuclear reaction approaches.

There are many types of research that exist to study the Coulomb barrier systematically. According to the systematical study on the fusion and elastic scattering experimental data of ^{19}F + ^{208}Pb and neighboring reaction systems, it was shown that dynamical polarization effects around the barrier and static deformation effects below the barrier play an important role in enhancing the fusion cross sections [6]. In Refs. [7,8], a large number of above-barrier fusion excitation functions were fitted using the Woods-Saxon type nuclear potential embedded in the simplified coupled-channels model. It is found that the fitted empirical diffuseness parameters are abnormally larger than those extracted from elastic scattering data, and tend to increase strongly with the reaction charge product Z_PZ_T. It is suggested that the Woods-Saxon form potential may be inappropriate and a new dynamical calculation including deep-inelastic scattering is probably needed.

The fusion barrier is known to have a certain kind of distribution concerning incident energy E, due to the coupling of the collective movements or neutron transfers of two colliding nuclei [9–11]. The distribution of the fusion barrier can be extracted by taking the second derivative of the product of experimental cross section and energy, i.e., $d^2(E\sigma_{\rm fus})/dE^2$. However, high precision and small energy steps of the measured cross sections are required. The statistical error of the fusion barrier distribution is usually large due to its energy dependence. The shape of the extracted barrier distribution has a certain degree of uncertainty due to the energy step used in the point-difference approximation of the double differentiation. By considering the barrier distribution as multiple Gaussian distributions, systematic extractions of the fusion barrier were studied in Ref. [12].

*Corresponding author: cjlin@ciae.ac.cn

Another way to extract the Coulomb barrier is through the prediction of the theoretical interaction potential. By comparison with the recently proposed energy scaling approach for experimental fusion reactions with 14 different versions of proximity potentials, it is found that proximity potentials 77 and 80 agree with experimental values better and the corresponding Coulomb barrier is systematically studied [13,14]. By comparing the experimental fusion cross sections and the predictions of the empirical barrier distribution approach with 14 potential models, it is found that proximity 77 potential is more preferable [15,16].

One could also adopt the sophisticated coupled-channels approaches to fit the experimental data [17–19]. However, this method depends deeply on the theoretical fusion models. It is unclear how many coupled channels should be included to fit the results. Many coupling mechanisms, such as the impact of positive Q-value neutron transfer [10,11] and the decoherence effect [20], are still in debate now. When different reaction channels are included, the fitted fusion barrier could be significantly varied.

In Ref. [21], an empirical fusion cross section formula based on a single-Gaussian distribution of the barrier height is proposed by Siwek-Wilczyńska and Wilczyński (SW), which is simple and proved to have wide applicability [22,23]. Based on the fitting of 45 fusion reactions, a scaling law (WKJ) of the experimentally extracted fusion barrier concerning the Coulomb parameter $z = Z_P Z_T/(A_P^{1/3} + A_T^{1/3})$ is proposed [22], which is named as the WKJ formula in this work. It is aimed at providing reference for the fusion of superheavy elements with $z = 170$–300.

It is first reported in Ref. [24] that the derivation of the barrier distribution function from the quasielastic scattering cross section at backward angles is more feasible than fusion reactions, especially for the synthesis of the superheavy elements. Later, it was experimentally confirmed for the first time that the barrier distributions extracted from fusion, quasielastic, and spin distributions are consistent in Ref. [25]. And there are extensive efforts on the measurement of quasielastic scattering reactions [26–32]. However, there is still controversy on the relationship between the barrier extracted from fusion reactions, quasielastic reactions, and elastic scattering reactions. In Refs. [33,34], the centroid barrier of the quasielastic backscattering is extracted, and it is found that the experimental values are about 3–10 MeV lower than several potential models and also the empirical WKJ formula, with the CW76 potential as an exception. After analyzing these results, Zagrebaev proposed that the experimental centroid barrier extracted from the quasielastic reaction is not the fusion barrier but the reaction threshold distribution [35], with the latter one being smaller than the centroid barrier.

However, there are limitations in the fitting of the WKJ formula in Ref. [22]. Firstly, the experimental data is limited with 45 fusion reactions with $50 < z < 170$. Secondly, the higher orders of z are adopted, which has no physical foundation. Thirdly, the empirical fusion barrier is explored by considering the trend of the Bass proximity potential at the $z = 170$–300 region, where lowering of the Bass potential by 3 MeV is considered.

Considering the above problems of the WKJ formula, in this work, we reanalyze the experimental fusion reactions for a large scale of fusion reactions. It is known that the NRV website has collected the most abundant experimental fusion cross sections [36]. In this study, these data are systematically analyzed for the first time.

The present paper is organized as follows. In Sec. II, the theoretical framework and fitting procedure of experimental fusion cross sections are briefly described. Section III presents the fitting results of Coulomb barriers and systematical comparisons with different kinds of experimental data and various potential models. Finally, the summary of the article is given in Sec. IV.

II. THEORETICAL FRAMEWORK

A. Fusion models

How to define and extract a relatively model-independent fusion barrier is a critical problem. In this work, we define the experimental fusion barrier as the position where V_B satisfies the condition

$$\left.\frac{d(E\sigma_{\mathrm{fus}})}{dE}\right|_{E=V_B} = \frac{1}{2}\left[\frac{d(E\sigma_{\mathrm{fus}})}{dE}\right]_{\max}, \tag{1}$$

where E is the center of mass incident energy, and σ_{fus} is the fusion cross sections. It is worth noticing that this definition involves only the experimental incident energies and fusion cross sections. The definition is model independent under ideal circumstances when the experimental data points are measured well, and distributed uniformly and densely. However, for most cases, the experimental data is not so ideal. One could extract the fusion barrier by fitting the fusion excitation function with various theoretical methods.

We could adopt the SW formula [21], Wong formula [37], or the exactly solved single-channel tunneling method (SC) [17,18] to fit the experimental data. Among these three methods, the SC method is deeply dependent on specific potential models. For the usually used Woods-Saxon type potentials, it is reported that many experimental data cannot be described well simultaneously for the above barrier and sub-barrier fusion cross sections [7,8]. In contrast, the SW formula and the Wong formula are more flexible due to their potential model-independent character. And the SW formula is demonstrated to be able to fit many deep sub-barrier fusion cross sections [23].

The SW formula is given as

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi\nu}{2E}R_B^2\left[X(1+\mathrm{erf}X) + \frac{1}{\sqrt{\pi}}\exp(-X^2)\right], \tag{2}$$

where $X = (E - V_B)/\nu$. V_B is the height of the barrier, R_B is the barrier position. ν is the normalized Gaussian variance, which could be constructed by considering various couplings, including the the quantum effect of sub-barrier tunneling, as well as for static quadrupole deformations and collective surface vibrations of the colliding nuclei. It is obtained by initially considering the classical formula for fusion cross section as

$$\sigma_{\rm fus}(E) = \begin{cases} \pi R_{\rm B}^2(1 - V_{\rm B}/E) & E \geqslant V_{\rm B}, \\ 0 & E \leqslant V_{\rm B}. \end{cases} \tag{3}$$

And then

$$\sigma_{\rm fus}(E)E/\pi R_{\rm B}^2 = \begin{cases} (E - V_{\rm B}) & E \geqslant V_{\rm B}, \\ 0 & E \leqslant V_{\rm B}. \end{cases} \tag{4}$$

Differentiating Eq. (4) with respect to E results in a step function, and the second differentiation produces a δ function at $E = V_{\rm B}$. Replacing the δ function by a Gaussian distribution, and integrating reversely, one could obtain Eq. (2). Since the peak position of the symmetric Gaussian distribution is at $E = V_{\rm B}$, the $V_{\rm B}$ defined in Eq. (2) coincides with the definition in Eq. (1). The ν, $R_{\rm B}$, and $V_{\rm B}$ could be fitted directly.

The situation is similar to the Wong formula. According to the Wong formula [37], fusion cross sections can be predicted as

$$\sigma_{\rm fus}(E) = \frac{\hbar\omega}{2E}R_{\rm B}^2 \ \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\omega}(E - V_{\rm B})\right)\right], \tag{5}$$

where $\hbar\omega$ is the s-wave barrier curvature. The first order derivative of the fusion cross section according to the Wong formula is

$$\frac{d(E\sigma_{\rm fus})}{dE} = \frac{\pi R_{\rm B}^2}{1 + e^x} = \pi R_{\rm B}^2 P_0(E), \tag{6}$$

where $x = -\frac{2\pi}{\hbar\omega}(E - V_{\rm B})$, and $P_0(E) = 1/(1 + e^x)$, which is the s-wave Hill-Wheeler penetration probability. It can be seen that the maximum of Eq. (6) is when $P_0(E)$ tends to one and

$$\lim_{x\to-\infty} \frac{d(E\sigma_{\rm fus})}{dE} = \pi R_{\rm B}^2. \tag{7}$$

When $E = V_{\rm B}$,

$$\left.\frac{d(E\sigma_{\rm fus})}{dE}\right|_{E=V_{\rm B}} = \frac{\pi R_{\rm B}^2}{2} = \frac{1}{2}\lim_{x\to-\infty} \frac{d(E\sigma_{\rm fus})}{dE}, \tag{8}$$

which is also coincides with the definition of experimental fusion barrier in Eq. (1). $\hbar\omega$, $R_{\rm B}$, and $V_{\rm B}$ could be fitted directly.

It should be noted that these empirical formulas have certain limitations in the description of fusion cross sections due to the assumptions in the derivation. However, according to Eq. (1), any theory that satisfies the definition will produce a similar barrier height considering experimental errors. The defect of the empirical formulas could be implemented by adjusting its parameters to fit the experimental data to a certain extent.

B. Collection of experimental data

The fusion cross sections in the NRV website include not only capture cross sections, but also fusion-fission and evaporation residue data. The experimental data are both obtained from the table and read from the graph of the corresponding publications. The procedure to select the experimental fusion data in this study is as follows. There are 1621 sets of experimental fusion cross sections in the NRV fusion database up to now (from 1959 to 2016) in total. Firstly, sub-barrier fusion cross sections and above barrier fusion cross sections are both important to determine the fitting parameters. The fittings can be remarkably influenced by the concentration of the data points. 972 data sets with less than two energy points in the sub-barrier energy region or less than two points in the above barrier region are not included. The barrier used to clarify sub-barrier and above barrier is determined by the WKJ formula. Secondly, the error bars of 91 experimental data sets are not collected in the NRV database, which are also neglected in the following fitting process. Besides, experimental data sets consist of different types of experimental data including fission, fusion, and evaporation residue, or repeated published, which are not included. Finally, there are 403 data sets left for the fitting process.

In the present context, we use the popular Minuit minimization program [38] to determine the fitting parameters by searching the global minimum in the hypersurface of the χ^2 function. The χ^2 per energy point is expressed as

$$\chi^2/pt = \frac{1}{n}\sum_i \left(\frac{\sigma_i^{\rm The} - \sigma_i^{\rm Exp}}{\delta\sigma_i^{\rm err}}\right)^2, \tag{9}$$

where i, n are the ith and total number of experimental energy points for each reaction system. $\sigma_i^{\rm The}$, $\sigma_i^{\rm Exp}$, $\delta\sigma_i^{\rm err}$ represent the calculated fusion cross sections, corresponding experimental data, and experimental error, respectively.

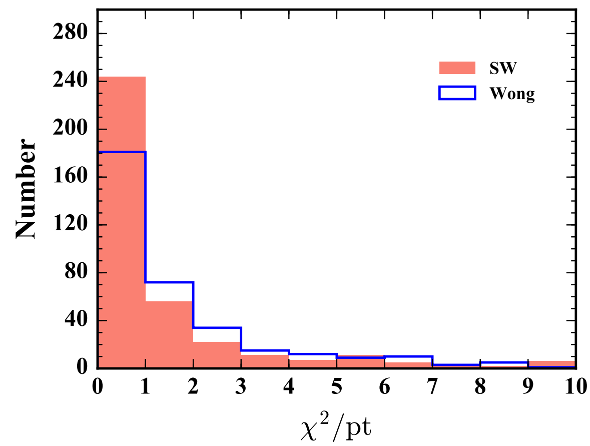

FIG. 1. Number distribution of experimental data sets with respect to corresponding fitted χ^2/pt value based on the SW formula (red filled bar) and the Wong formula (blue empty bar).

III. RESULTS AND DISCUSSIONS

The number distribution of experimental data sets with respect to the fitted χ^2/pt value by the SW formula and Wong formula is shown in Fig. 1. It is found that the SW formula (red filled bar) and the Wong formula (blue empty bar) are both capable to fit a large number of experimental data. Finally, 243 (182) sets out of the 403 experimental fusion data sets can be fitted with χ^2/pt smaller than 1.0 by the SW (Wong) formula, which means the average difference between theoretical data and experimental data are located within the range of the experimental errors. For data sets with

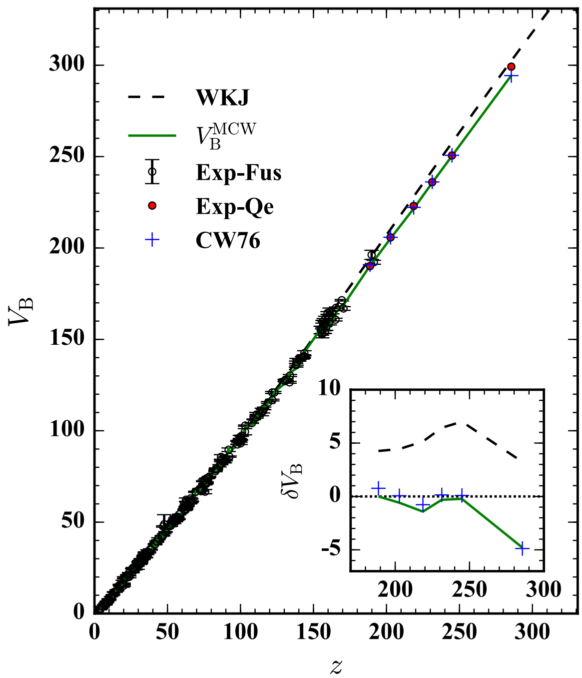

FIG. 2. The barrier height V_B with respect to Coulomb parameter $z = Z_1Z_2/(A_P^{1/3} + A_T^{1/3})$. The open circles (Exp-Fus) represent the results extracted from experimental data based on the SW formula. The solid circles (Exp-Qe) denote the experimental barrier extracted from quasielastic scattering [33,34], and the predictions of CW76 for the same reactions are shown as the plus symbols. Prediction of the WKJ formula [22] is shown as the dashed line. The fitting to the circles of this work V_B^{MCW} is shown as the solid line based on Eq. (15). The insert is an enlargement and transformation of the barrier at large z region, where δV_B is the difference between the barrier height and the Exp-Qe barrier.

χ^2/pt larger than 1.0, it may be caused by the experimental errors, or that the shapes of the experimental data are complex compared to the simple empirical formulas. We could also see that the SW formula behaves much better for the barrier fitting than the Wong formula. Therefore, the 243 sets of data with fitted χ^2/pt smaller than 1.0 by the SW formula are used in the following discussions. The SW formula is a simple expression for the cross-section description. It is not correct at energies very far from the value of the barrier height, where the deep-inelastic reactions or other reaction channels could suppress the fusion cross sections. However, when such effects appear, the obtained χ^2/pt are very large based on this simple formula. The way we choose the data with χ^2/pt smaller than one imposed a strict criterion to remove these effects such as the deep inelastic scattering.

Figure 2 displays the changes of barrier height V_B with respect to Coulomb parameter z. The experimental barrier height V_B represented by the open circles (Exp-Fus) is extracted by the SW formula. The fitting error bars are given by the Minuit program [38]. From the figure, it can be seen clearly that V_B (Exp-Fus) is approximately linear proportional to z. As mentioned in the Introduction, the SW formula is used to fit 45 fusion reactions, and the cases with the lowing of the Bass barriers. The scaling law WKJ based on the experimentally extracted fusion barrier with respect to z is proposed accordingly [22], which is

$$V_B^{WKJ} = 0.85247z + 0.001361z^2 - 0.00000223z^3. \quad (10)$$

The prediction of the WKJ formula is also presented in Fig. 2 as the dashed line. One could see that the current extracted experimental fusion barrier Exp-Fus overlaps with the prediction of WKJ at $z < 170$.

Due to the second and third order of z being included, the barrier height predicted by the WKJ formula is not linear with respect to z, but decreases quickly when $z > 600$. It is reduced to zero when z is increased to around 1000. For very heavy multinucleon transfer reaction (MNT) reactions, the Coulomb parameter z could be much larger than 300. For instance, z is 682.9 for $^{238}U + ^{238}U$. The prediction of CW76 potential is 726.3 MeV, while the prediction by the WKJ formula is 506.6 which is obviously too small. Due to this drawback of the WKJ formula, we adopt a similar barrier formula as the CW76 potential. The CW76 potential is obtained by analyzing the heavy-ion elastic scattering data, and was proposed by Christensen and Winther in 1976 [39]. The nuclear part of CW76 potential is expressed as

$$V_N(r) = -S_0\bar{R}_{PT} \exp\left(-\frac{r - R_P - R_T}{a}\right), \quad (11)$$

where the parameters $S_0 = 50$ MeV fm^{-1}, $a = 0.63$ fm, and

$$R_i = \left(1.233A_i^{1/3} - 0.978A_i^{-1/3}\right) \text{ fm},$$
$$\bar{R}_{PT} = R_PR_T/(R_P + R_T). \quad (12)$$

According to Ref. [39], the barrier position of this potential is approximately given by

$$R_B = \left[1.07\left(A_P^{1/3} + A_T^{1/3}\right) + 2.72\right] \text{ fm}, \quad (13)$$

which is linear to the system size $(A_P^{1/3} + A_T^{1/3})$. The height of the barrier could be derived as

$$V_B = \frac{Z_PZ_Te^2}{R_B}\left(1 - \frac{a}{R_B}\right). \quad (14)$$

Due to the smallness of a/R_B compared with 1, V_B is approximately proportional to the Coulomb parameter z. The two coefficients in R_B are newly fitted to the experimental data Exp-Fus as

$$V_B = \frac{Z_PZ_Te^2}{0.9782\left(A_P^{1/3} + A_T^{1/3}\right) + 4.2833}. \quad (15)$$

The above formula is plotted as the solid line in Fig. 2. The predicted barrier heights are named V_B^{MCW}, as the modified version of those predicted by the CW76 potential model. It should be noted that the barrier predicted by the above formula is not a single variable of z. It is drawn as the solid line in this figure simply to guide the eyes. In addition, the centroid potential barrier extracted based on experimental quasiprojectile scattering (Exp-Qe) is shown as solid circles in the figure. The prediction of the CW76 model is marked with plus signs.

In Fig. 2, an insert picture is plotted to gain a better sense of the δV_B, which is the difference between the calculated barrier height and the Exp-Qe barrier, including the V_B^{MCW}, CW76,

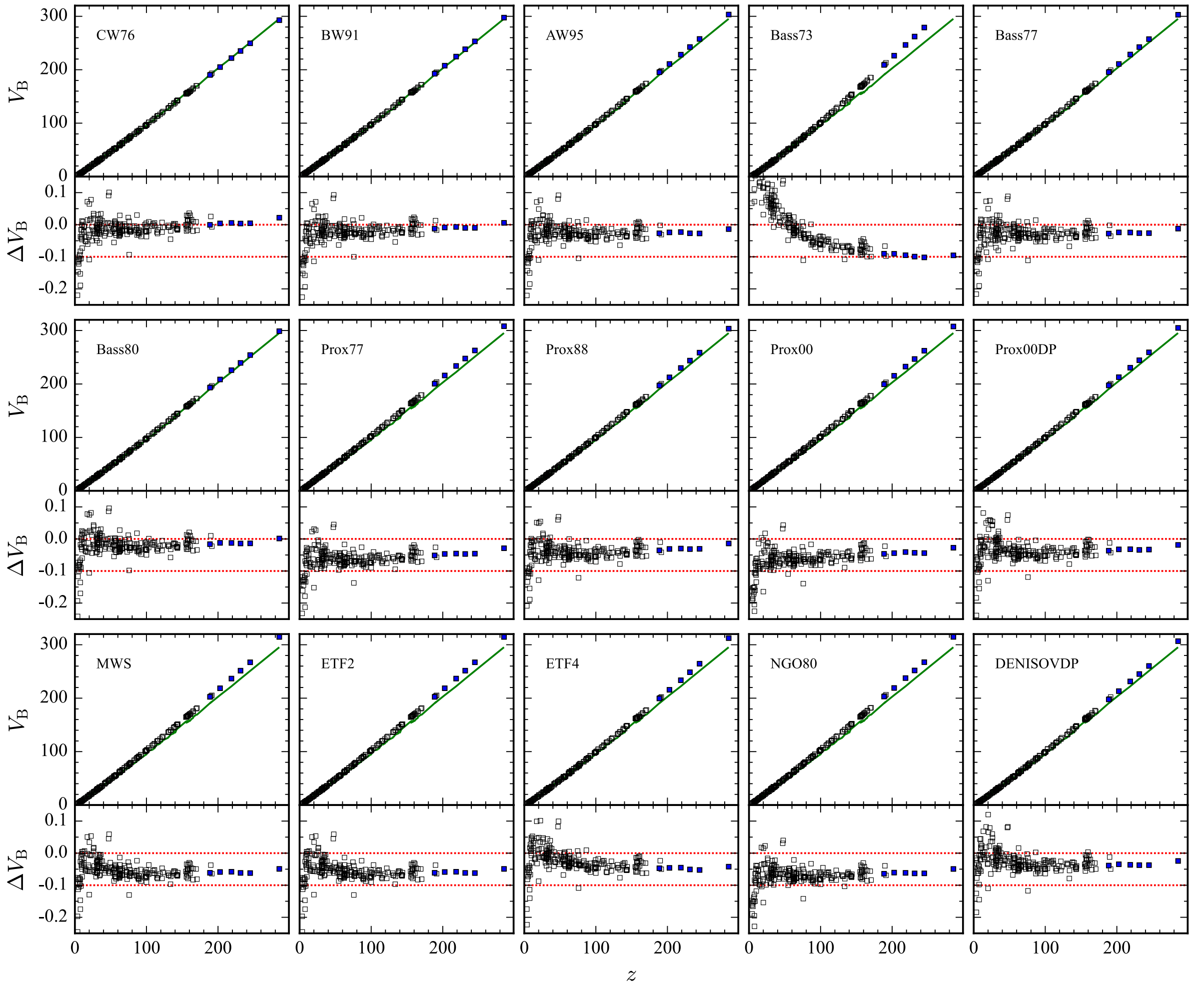

FIG. 3. The predicted barrier height V_B and barrier deviation $\Delta V_B = (V_B - V_B^{\mathrm{The}})/V_B$ with respect to Coulomb parameter for 15 theoretical potential models. The reactions corresponding to the Exp-Fus and Exp-Qe in Fig. 2 are denoted as open and solid squares, respectively. The barrier heights V_B^{MCW} predicted by Eq. (15) are shown as the solid lines. ΔV_B is also plotted in the lower part of each subpanel for reference.

and WKJ formulas. The experimental quasielastic reactions (experimental centroid barrier height) include ^{48}Ti + ^{208}Pb (190.1 MeV), ^{54}Cr + ^{208}Pb (205.8 MeV), ^{56}Fe + ^{208}Pb (223.0 MeV), ^{64}Ni + ^{208}Pb (236.0 MeV), ^{70}Zr + ^{208}Pb (250.6 MeV) from Ref. [33], and ^{86}Kr + ^{208}Pb (299.2 MeV) from Ref. [34]. The last point for the quasielastic reaction ^{86}Kr + ^{208}Pb shows a larger deviation. The distinguish of this point compared to the others might be due to fluctuations. The Exp-Qe barrier heights are about 4–7 MeV lower than the predictions of the WKJ formula. the results of V_B^{MCW} are remarkably consistent with the results Exp-Qe except the last point. These results demonstrate that the barriers drawn from the fusion reaction share the same rule with respect to z as those drawn from the quasielastic reaction for these reactions. The numerical results in this study suggest that the dynamical fusion barrier and the centroid barrier extracted from quasielastic reaction (the reaction threshold) are the same physical quantity for the above-mentioned heavy reactions.

From the current scale of Fig. 2, the difference between different lines seems not so large, with several MeV deviations from each other. However, the fusion cross section around the barrier is so sensitive that if the experimental incident energy deviates by several MeV from the potential barrier, the fusion cross section may change by an order of magnitude. This is critical for the synthesis of superheavy nuclei.

In addition, we can also see from the figure that the results of V_B^{MCW} are highly consistent with the interaction potential of CW76, especially when $170 < z < 300$. The extracted fusion barrier is a dynamical barrier, which includes the overall effect of coupled channels. Many factors, such as the collective vibration, rotation, positive Q-value neutron transfer, deep inelastic scattering, and quasifission, would affect the barrier. It is said in Ref. [7] that the experimental fusion barriers are always lower than the theoretical bare potentials, which is the result of coupling to high energy collective states. Other studies indicate similar results and show a preference for certain theoretical potential [14,22,40,41]. To see the difference between the result of V_B^{MCW} and other bare potentials, we compare this result with the results of 15 potential models in Fig. 3. The 15 potentials are CW76 [39], BW91 [42], AW95 [43], BASS74 [44], BASS77 [45], BASS80 [46], PROX77 [15], PROX88 [47], PROX00 [48], PROX00DP [49], MWS

[50], ETF2 [51], ETF4 [52], NGO80 [53], and DENISOVDP [54]. Most theoretical formulas of these potentials can be found in Ref. [40]. The barrier heights V_B^{MCW} and the barrier deviations ΔV_B are also plotted in the lower part of each subpanel in Fig. 3.

From Fig. 3, one could see that the barrier prediction by the CW76 potential agrees best with the barrier V_B^{MCW} at the larger z. This demonstrates that the fitting of Exp-Fus in Fig. 2 is not an accident and is not solely limited to the form of Eq. (15). The other 14 interaction potentials excluding the CW76 potential also show a linear relationship with z. However, predictions of most potential models are higher than the V_B^{MCW} at large z region. The barrier deviation $\Delta V_B = (V_B - V_B^{The})/V_B$ is also presented in the lower panels of Fig. 3. We could see that ΔV_B moves down from the zero line (Exp-Fus barrier) nearly in parallel. According to these results, for $z > 100$, the ΔV_B is always larger than -0.1 with only a few points as the exception. It indicates that most of the bare barrier heights do not exceed 10% of the centroid fusion barrier.

Over the past decade, researchers have proposed new nuclear interaction potentials. For example, Dutt *et al.* and Gharaei *et al.* fitted the existing potentials, such as Bass80, AW95 potentials [55,56], and they obtained new simple potential barrier laws. Since it is the theoretical potential models that are studied in these works, it could be expected that they were consistent with the original results shown in Fig. 3. In 2015, Denisov proposed another sophisticated relaxed-density potential (DP2015) including the contributions of the macroscopic and shell-correction terms [57]. The parameters of the potential are found using both data for empirical nucleus-nucleus barriers and the values of macroscopic nucleus-nucleus potentials around the barrier for various systems. It should be noted that the empirical barrier extracted from quasielastic scattering data (the first five solid squares in Fig. 2) by Mitsuoka *et al.* in Ref. [33] were used in the fitting of the potential parameters. And it is demonstrated that the DP2015 could predict well the empirical potential barrier for very heavy reactions, and one could see Table 1 in Ref. [58] for more details.

In Fig. 4(a), we show the predicted barrier height V_B by the DP2015 potential and V_B^{MCW}. It could be found that it is not easy to distinguish them from each other, and the prediction of DP2015 and other predictions are highly consistent. The barrier deviation ΔV_B between each other as labeled in each subpanel with respect to Coulomb parameter z are plotted in In Fig. 4(b)–4(d). Based on the results of Fig. 4(b) and 4(c), the deviations of different models are all within 10% of the experimental data. The prediction of DP2015 is overall slightly smaller than the zero line in Fig. 4(c), and its distribution is similar to that of CW76 potential in Fig. 3. When we plotted the barrier deviations of DP2015 and CW76 in Fig. 4(d), it is remarkable to find that they are very close to each other when $z > 20$ with the deviation smaller than 1%. These comparisons increase the credibility and the model-independence feature obtained in this work.

During the last two decades, MNT has been a hot research topic due to its potential to synthesize new superheavy neutron-rich nuclei [59]. For very heavy MNT reactions, the

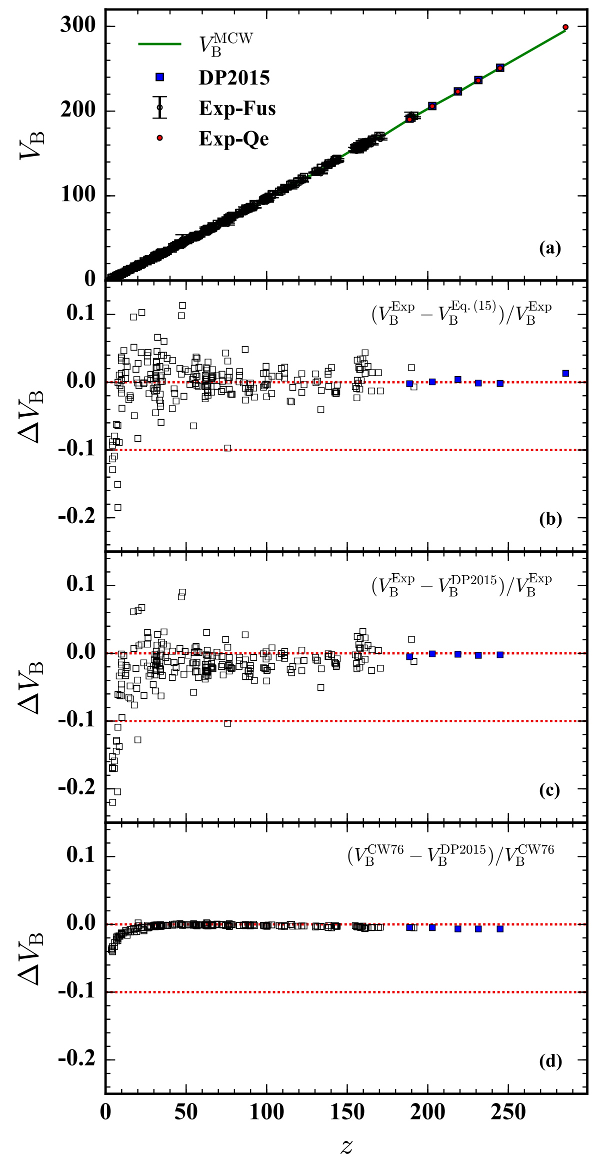

FIG. 4. The barrier height V_B and barrier deviations between ΔV_B with respect to Coulomb parameter z. The experimental data Exp-Fus and Exp-Qe are the same as those in Fig. 2. The barrier heights V_B^{MCW} predicted by Eq. (15) are shown as the solid lines. The barrier heights predicted by the DP2015 potential and its deviations concerning other results as labeled in each subpanel are shown as open (solid) squares for reactions having the fusion (quasielastic) experimental data.

Coulomb parameter z could be much larger than 300. For MNT reactions such as $^{136}Xe + ^{208}Pb$ and $^{238}U + ^{238}U$, there is no nominal Coulomb barrier for most potential models due to the strong Coulomb repulsion. However, models such as the CW76, BASS73, BASS77, BASS80 could still provide a maximum of the potentials. The predicted barriers, but not the potential at all distances, are widely used to provide benchmark energy for the experimental and theoretical MNT investigations for the setup of the incident energy [60,61]. It is expected that near-barrier MNT collisions are most favorable for the synthesis of superheavy neutron-rich elements

TABLE I. The Coulomb barrier height for typical heavy reactions predicted by WKJ, V_B^{MCW}, and V_B^{CW76} in MeV. The Coulomb parameter z is also listed for reference.

Reaction	z	WKJ	V_B^{MCW}	V_B^{CW76}
$^{136}Xe+^{198}Pt$	383.92	401.69	403.94	398.65
$^{136}Xe+^{208}Pb$	400.09	416.10	422.01	416.25
$^{136}Xe+^{209}Bi$	404.62	420.02	426.89	421.18
$^{136}Xe+^{249}Cm$	462.84	465.01	492.67	484.81
$^{204}Hg+^{198}Pt$	532.64	503.20	570.76	560.86
$^{208}Pb+^{238}U$	622.33	520.14	673.02	659.50
$^{238}U+^{238}U$	682.89	506.67	742.84	726.30
$^{238}U+^{248}Cm$	707.70	494.53	771.21	753.72
$^{238}U+^{249}Cf$	721.95	485.68	786.88	769.52

considering the balance of transfer and fission probabilities [62–64]. In the following, we made a comparison of the barrier heights between different models. The Coulomb barriers for typical MNT reactions predicted by the WKJ formula, V_B^{MCW}, and V_B^{CW76} are listed in Table I. It can be seen that the barrier height predicted by the WKJ formula reaches the maximum value at $z = 622.33$ for reaction $^{208}Pb + ^{238}U$. And the prediction by V_B^{MCW} and the CW76 formula produce similar results. The scaling law proposed in this work could provide a reference for these heavy MNT reactions.

IV. CONCLUSIONS

In summary, the NRV experimental fusion cross section data are systematically analyzed for the first time. 403 high-quality data sets out of the 1621 data sets are analyzed by using the Wong formula and the SW formula. It is demonstrated that the SW formula is more preferable to extract the fusion barrier from extensive experimental fusion data sets than the Wong formula. Finally, 243 experimental data sets fitted with χ^2/pt smaller than 1 are used to extract the fusion barrier. Similar to previous results in Refs. [21,22], it is confirmed that the phenomenological V_B is in a good linear relationship concerning z, which suggests that the extracted barrier is steady.

It is found that the results V_B^{MCW} are remarkably consistent with the predictions of the CW76 potential, and also the Exp-Qe results for some heavy reactions which could lead to the synthesis of superheavy nuclei. And they are several MeV lower than the predictions of the WKJ formula at $z > 170$.

Compared with the predictions of 15 potential models, the calculations show that the CW76 potential agrees best with the newly proposed scaling law than other potential models. The other static potentials are mostly higher than the CW76 potential and ΔV_B is within 10%. The sophisticated DP2015 potential considering contributions from density distribution and shell corrections is also studied. The potential parameters are obtained by fitting from experimental fusion and quasielastic data, as well as theoretical predictions. It is remarkable to find that the predictions of the barrier heights are highly consistent with the extracted results in this work and the CW76 potential.

The fusion cross sections are very sensitive to the barrier height especially for the fusion of superheavy nuclei. This study suggests the barrier information extracted from the quasielastic scattering could be the same quantity as that extracted from the fusion reaction for very heavy reactions. Due to the great advantage of measuring quasielastic reaction rather than fusion reaction, the former one could provide a significant reference for the synthesis of superheavy nuclei based on fusion reaction. One could also use the new scaling law of V_B^{MCW} as the experimental benchmark or theoretical reference for superheavy MNT collisions which has no potential barrier.

ACKNOWLEDGMENTS

This work is supported by the National Key R&D Program of China (Contract No. 2018YFA0404404), the National Natural Science Foundation of China (Grants No. 11635015, No. 11805280, No. U1732145, No. 11705285, No. U1867212, and 11961131012), the Continuous Basic Scientific Research Project (No. WDJC-2019-13), the Young Talent Development Foundation (Grant No. YC212212000101), and the Leading Innovation Project (Grants No. LC192209000701 and No. LC202309000201).

[1] J. H. Hamilton, S. Hofmann, and Y. T. Oganessian, Annu. Rev. Nucl. Part. Sci. **63**, 383 (2013).
[2] A. Pierre, Rep. Prog. Phys. **75**, 116901 (2012).
[3] M. Dasgupta, D. J. Hinde, N. Rowley, and A. M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).
[4] C. J. Lin, *Heavy-Ion Nuclear Reactions* (Harbin Engineering University Press, Harbin, 2015).
[5] G. Montagnoli and A. M. Stefanini, Eur. Phys. J. A **53**, 169 (2017).
[6] C. J. Lin, J. C. Xu, H. Q. Zhang, Z. H. Liu, F. Yang, and L. X. Lu, Phys. Rev. C **63**, 064606 (2001).
[7] J. O. Newton, R. D. Butt, M. Dasgupta, D. J. Hinde, I. I. Gontchar, C. R. Morton, and K. Hagino, Phys. Rev. C **70**, 024605 (2004).
[8] J. O. Newton, R. D. Butt, M. Dasgupta, D. J. Hinde, I. Gontchar, C. R. Morton, and K. Hagino, Phys. Lett. B **586**, 219 (2004).
[9] N. Rowley, G. R. Satchler, and P. H. Stelson, Phys. Lett. B **254**, 25 (1991).
[10] H. M. Jia, C. J. Lin, F. Yang, X. X. Xu, H. Q. Zhang, Z. H. Liu, Z. D. Wu, L. Yang, N. R. Ma, P. F. Bao, and L. J. Sun, Phys. Rev. C **89**, 064605 (2014).
[11] H. M. Jia, C. J. Lin, L. Yang, X. X. Xu, N. R. Ma, L. J. Sun, F. Yang, Z. D. Wu, H. Q. Zhang, Z. H. Liu, and D. X. Wang, Phys. Lett. B **755**, 43 (2016).
[12] N. Wang, Z. Li, and W. Scheid, J. Phys. G: Nucl. Part. Phys. **34**, 1935 (2007).
[13] W. W. Qu, G. L. Zhang, H. Q. Zhang, and R. Wolski, Phys. Rev. C **90**, 064603 (2014).

[14] G. L. Zhang, W. W. Qu, M. F. Guo, H. Q. Zhang, R. Wolski, and J. Q. Qian, Eur. Phys. J. A **52**, 39 (2016).
[15] J. Blocki, J. Randrup, W. J. Świątecki, and C. F. Tsang, Ann. Phys. (NY) **105**, 427 (1977).
[16] R. Gharaei, V. Zanganeh, and N. Wang, Nucl. Phys. A **979**, 237 (2018).
[17] K. Hagino, N. Rowley, and A. T. Kruppa, Comput. Phys. Commun. **123**, 143 (1999).
[18] P. W. Wen, O. Chuluunbaatar, A. A. Gusev, R. G. Nazmitdinov, A. K. Nasirov, S. I. Vinitsky, C. J. Lin, and H. M. Jia, Phys. Rev. C **101**, 014618 (2020).
[19] P. W. Wen, C. J. Lin, R. G. Nazmitdinov, S. I. Vinitsky, O. Chuluunbaatar, A. A. Gusev, A. K. Nasirov, H. M. Jia, and A. Góźdź, Phys. Rev. C **103**, 054601 (2021).
[20] A. Diaz-Torres, Phys. Rev. C **82**, 054617 (2010).
[21] K. Siwek-Wilczyńska and J. Wilczyński, Phys. Rev. C **69**, 024611 (2004).
[22] W. J. Świątecki, K. Siwek-Wilczyńska, and J. Wilczyński, Phys. Rev. C **71**, 014602 (2005).
[23] C. L. Jiang, K. E. Rehm, B. B. Back, A. M. Stefanini, and G. Montagnoli, Eur. Phys. J. A **54**, 218 (2018).
[24] H. Timmers, J. R. Leigh, M. Dasgupta, D. J. Hinde, R. C. Lemmon, J. C. Mein, C. R. Morton, J. O. Newton, and N. Rowley, Nucl. Phys. A **584**, 190 (1995).
[25] H. Q. Zhang, F. Yang, C. J. Lin, Z. H. Liu, and Y. M. Hu, Phys. Rev. C **57**, R1047 (1998).
[26] E. Piasecki, M. Kowalczyk, K. Piasecki, Ł. Świderski, J. Srebrny, M. Witecki, F. Carstoiu, W. Czarnacki, K. Rusek, J. Iwanicki, J. Jastrzębski, M. Kisieliński, A. Kordyasz, A. Stolarz, J. Tys, T. Krogulski, and N. Rowley, Phys. Rev. C **65**, 054611 (2002).
[27] F. Yang, C. J. Lin, X. K. Wu, H. Q. Zhang, C. L. Zhang, P. Zhou, and Z. H. Liu, Phys. Rev. C **77**, 014601 (2008).
[28] C. J. Lin, H. M. Jia, H. Q. Zhang, F. Yang, X. X. Xu, Z. H. Liu, and S. T. Zhang, EPJ Web Conf. **17**, 05005 (2011).
[29] E. Piasecki, Ł. Świderski, N. Keeley, M. Kisieliński, M. Kowalczyk, S. Khlebnikov, T. Krogulski, K. Piasecki, G. Tiourin, M. Sillanpää, W. H. Trzaska, and A. Trzcińska, Phys. Rev. C **85**, 054608 (2012).
[30] H. M. Jia, C. J. Lin, F. Yang, X. X. Xu, H. Q. Zhang, Z. H. Liu, Z. D. Wu, L. Yang, N. R. Ma, P. F. Bao, and L. J. Sun, Phys. Rev. C **90**, 031601(R) (2014).
[31] E. Piasecki, M. Kowalczyk, S. Yusa, A. Trzcińska, and K. Hagino, Phys. Rev. C **100**, 014616 (2019).
[32] L. Yang, C. J. Lin, H. Yamaguchi, J. Lei, P. W. Wen, M. Mazzocco, N. R. Ma, L. J. Sun, D. X. Wang, G. X. Zhang, K. Abe, S. M. Cha, K. Y. Chae, A. Diaz-Torres, J. L. Ferreira, S. Hayakawa, H. M. Jia, D. Kahl, A. Kim, M. S. Kwag *et al.*, Phys. Lett. B **813**, 136045 (2021).
[33] S. Mitsuoka, H. Ikezoe, K. Nishio, K. Tsuruta, S. C. Jeong, and Y. Watanabe, Phys. Rev. Lett. **99**, 182701 (2007).
[34] S. S. Ntshangase, N. Rowley, R. A. Bark, S. V. Förtsch, J. J. Lawrie, E. A. Lawrie, R. Lindsay, M. Lipoglavsek, S. M. Maliage, L. J. Mudau, S. M. Mullins, O. M. Ndwandwe, R. Neveling, G. Sletten, F. D. Smit, and C. Theron, Phys. Lett. B **651**, 27 (2007).
[35] V. I. Zagrebaev, Phys. Rev. C **78**, 047602 (2008).
[36] http://nrv.jinr.ru/nrv/ (nrv).
[37] C. Y. Wong, Phys. Rev. Lett. **31**, 766 (1973).
[38] F. James and M. Roos, Comput. Phys. Commun. **10**, 343 (1975).
[39] P. R. Christensen and A. Winther, Phys. Lett. B **65**, 19 (1976).
[40] I. Dutt and R. K. Puri, Phys. Rev. C **81**, 064609 (2010).
[41] I. Dutt and R. K. Puri, Phys. Rev. C **81**, 044615 (2010).
[42] R. Broglia and A. Winther, *Heavy Ion Reactions: The Elementary Processes* (Addison-Wesley, Reading, MA, 1991).
[43] A. Winther, Nucl. Phys. A **594**, 203 (1995).
[44] R. Bass, Nucl. Phys. A **231**, 45 (1974).
[45] R. Bass, Phys. Rev. Lett. **39**, 265 (1977).
[46] R. Bass, *Nuclear Reactions with Heavy Ions* (Springer, Berlin/Heidelberg, 2010).
[47] W. Reisdorf, J. Phys. G: Nucl. Part. Phys. **20**, 1297 (1994).
[48] W. D. Myers and W. J. Świątecki, Phys. Rev. C **62**, 044610 (2000).
[49] G. Royer and R. Rousseau, Eur. Phys. J. A **42**, 541 (2009).
[50] J. L. Tian, N. Wang, and Z. X. Li, Chin. Phys. Lett. **24**, 905 (2007).
[51] M. Liu, N. Wang, Z. Li, X. Wu, and E. Zhao, Nucl. Phys. A **768**, 80 (2006).
[52] A. Dobrowolski, K. Pomorski, and J. Bartel, Nucl. Phys. A **729**, 713 (2003).
[53] H. Ngô and C. Ngô, Nucl. Phys. A **348**, 140 (1980).
[54] V. Y. Denisov, Phys. Lett. B **526**, 315 (2002).
[55] I. Dutt, Phys. At. Nucl. **74**, 1010 (2011).
[56] R. Gharaei and J. Sheibani, Eur. Phys. J. A **52**, 129 (2016).
[57] V. Y. Denisov, Phys. Rev. C **91**, 024603 (2015).
[58] V. Y. Denisov and I. Y. Sedykh, Chin. Phys. C **45**, 044106 (2021).
[59] P. W. Wen, A. K. Nasirov, C. J. Lin, and H. M. Jia, J. Phys. G: Nucl. Part. Phys. **47**, 075106 (2020).
[60] V. I. Zagrebaev and W. Greiner, Phys. Rev. C **83**, 044618 (2011).
[61] J. V. Kratz, M. Schädel, and H. W. Gäggeler, Phys. Rev. C **88**, 054615 (2013).
[62] R. Yanez and W. Loveland, Phys. Rev. C **91**, 044608 (2015).
[63] P. W. Wen, C. J. Lin, C. Li, L. Zhu, F. Zhang, F. S. Zhang, H. M. Jia, F. Yang, N. R. Ma, L. J. Sun, D. X. Wang, F. P. Zhong, H. H. Sun, L. Yang, and X. X. Xu, Phys. Rev. C **99**, 034606 (2019).
[64] F. S. Zhang, C. Li, L. Zhu, and P. Wen, Front. Phys. **13**, 132113 (2018).