Coexistence and mixing in ^{98,100}Mo

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I have applied a simple two-state mixing model to matrix elements for $0 \leftrightarrow 2$ transitions in ^{98,100}Mo. Because the situation regarding the second and third 2^+ states is unclear, I have performed three separate fits for each nucleus. Results indicate that ^{100}Mo is somewhat more collective than ^{98}Mo, and that basis-state band g is significantly more deformed than band e.

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I. INTRODUCTION

The patterns of $E2$ strengths in ^{98,100}Mo are quite complicated. They are usually interpreted in terms of mixing of coexisting structures, but what are the natures of the underlying basis states: spherical deformed, vibrational-rotational, two different deformed shapes, or something else? For nuclei near $Z = 40$, $N = 60$, Heyde and Wood [1] discuss coexistence as it relates to α transfer and $E0$ strengths of 0^+ states. Wood *et al.* [2] examine shape coexistence in even-even nuclei from ^{16}O to ^{238}U. They discuss deformed bands in the neutron-rich zirconium isotopes and in ^{98}Sr. Otsuka *et al.* [3] discuss exotic nuclei in terms of the shell model. They stress the importance of the tensor force in the evolution of nuclei from ^{90}Zr to ^{100}Sn. Nomura *et al.* [4] performed calculations for several even neutron-rich Ru, Mo, Zr, and Sr nuclei using the interacting boson model (IBM) and energy density functional theory. The $2_1 \rightarrow 0_1$ strengths were reproduced reasonably well, but the model appears to be ill suited for properties of the 0_2^+ states in Mo nuclei. The calculated 0_2 energies are about 2 MeV higher than the experimental ones, and the calculated $0_2 \rightarrow 2_1$ strengths are only about 1–10% of the experimental strengths. Reference [4] does not provide calculated values for other strengths involved in the present analysis.

An additional complication in the present case is the fact that in both ^{98,100}Mo nuclei, the second and third 2^+ states are reasonably close together with little clarity concerning which is to be associated with the second 0^+ state and the nature of the other. I have used a simple two-state mixing model to examine these questions.

II. DATA AND ANALYSIS

Zielinska *et al.* [5] investigated Coulomb excitation of ^{98}Mo using beams of ^{20}Ne, ^{84}Kr, and ^{136}Xe. They extracted $E2$ transition matrix elements and compared them with the predictions of an extended version of the IBM1 model. Their parameters of their model were obtained by fitting excitation energies and three $E2$ matrix elements. They found that "the overall agreement between the measured and the calculated matrix elements shows visible discrepancies, which cannot be explained within the framework of the model used." Their matrix elements for low-lying $0 \leftrightarrow 2$ transitions are listed in Table I. An earlier experiment [6] reported $M_1 = 0.55(2)$ eb, but the smaller value of $0.36^{+0.02}_{0.05}$ is preferred [7]. A value of $B(E2) = 280(40)$ W.u. [8] quoted for this transition is erroneous and should be disregarded [7].

Thomas *et al.* [8] used a $\gamma\gamma$ angular-correlation experiment to investigate the low-energy states of ^{98}Mo. They concluded that their results "reveal evidence for shape coexistence and mixing in ^{98}Mo, arising from a proton intruder configuration." They attributed the intruder configuration to proton particle-hole excitations across the $Z = 40$ subshell closure. They ascribed the coexisting structures as spherical vibrational and γ-soft equilibrium shapes.

In a later paper, Thomas *et al.* [9] concluded that "The experimental data show a complex spectrum due to configuration mixing, which is confirmed by interacting boson model calculations based on a Skyrme energy density functional." They stated that "The description of ^{98}Mo in the framework of shape coexistence is supported by two-proton separation energies of even-even nuclei in the $A = 100$ region."

Coulomb excitation of ^{100}Mo was performed by Wrzosek-Lipska *et al.* [10] using a 76-MeV ^{32}S beam. Their $E2$ transition matrix elements are also listed in Table I. They concluded that "The overall deformation of the 0_1^+ and 0_2^+ states in ^{100}Mo is of similar magnitude, in both cases larger compared to what was found for the neighboring isotopes ^{96}Mo and ^{98}Mo." They compared their $E2$ strengths with model calculations using the general quadrupole collective Bohr Hamiltonian model. They found that their results indicated a triaxial shape for the ground state (g.s.) and a prolate shape in the excited 0^+ state.

Rusev *et al.* [11] discussed 0^+ mixing in ^{98,100}Mo in terms of 1^+ decays to the first two 0^+ states. I return to their results later.

For the mixing analysis, I write

$$\Psi(0_1) = a\,\Phi(0_g) + b\,\Phi(0_e),\ \Psi(0_2) = b\,\Phi(0_g) - a\,\Phi(0_e),$$

$$\Psi(2_1) = A\,\Phi(2_g) + B\,\Phi(2_e),\ \Psi(2') = B\,\Phi(2_g) - A\,\Phi(2_e),$$

TABLE I. $E2$ matrix elements for $0 \leftrightarrow 2$ transitions in ^{98,100}Mo.

Label	Initial	Final	IM (eb) ^{98}Mo[a]	IM (eb) ^{100}Mo[b]
M_0	0_1	2_1	$0.526^{+0.008}_{-0.006}$	0.68(1)
M_1	0_2	2_1	$0.36^{+0.02}_{-0.05}$[c]	$0.513^{+0.009}_{-0.004}$
M_2	0_1	2_3	−0.021(1)	−0.016(3)
M_3	0_2	2_3	0.311(6)	$0.506^{+0.008}_{-0.006}$
$M_{2\text{alt}}$	0_1	2_2	$0.123^{+0.006}_{-0.003}$	$-0.103^{+0.002}_{-0.001}$
$M_{3\text{alt}}$	0_2	2_2	0.251(9)	$0.32^{+0.03}_{-0.02}$
$M_{2\text{sum}}$	0_1	$2_2 + 2_3$	$0.125^{+0.006}_{-0.003}$	$-0.104^{+0.004}_{-0.003}$
$M_{3\text{sum}}$	0_2	$2_2 + 2_3$	0.400(11)	$0.599^{+0.031}_{-00.21}$

[a]Reference [5].
[b]Reference [10].
[c]An earlier experiment reported a matrix element of 0.55(2) eb [6], but the value of $0.36^{+0.02}_{-0.05}$ eb [5] is preferred [7]. A reported B($E2$) of 280(40) W.u. [8] is erroneous and should be disregarded [7].

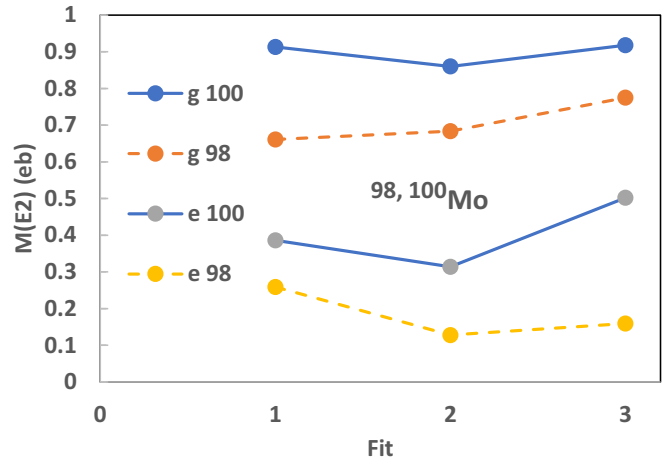

FIG. 1. $E2$ transition matrix elements between basis states that result from present fits.

where $2'$ is either the second or the third 2^+ state, and 0_g, 0_e, 2_g, and 2_e are basis states.

I define $M_g = \langle 0_g \parallel M(E2) \parallel 2_g \rangle$, $M_e = \langle 0_e \parallel M(E2) \parallel 2_e \rangle$, and I assume basis states g and e are not connected by the $E2$ operator.

Given the uncertainty concerning the second and third 2^+ states, I have performed three separate fits: One using the third 2^+, one using the second one, and one in which I have summed the two. Results of these fits are listed in Table II. Note that the 0^+ mixing is almost the same for all three fits in ^{98}Mo. But, of course, the 2^+ mixing changes.

Sambataro and Molnár [12] used an extension of the neutron-proton interacting boson model to explore coexistence and configuration mixing in Mo nuclei. Their 0^+ intruder basis state was a proton two-particle two-hole state. From their Fig. 5, it would appear that the mixing is about 60% normal and 40% intruder in the g.s. of ^{98}Mo but approximately the opposite in ^{100}Mo.

Rusev *et al.* [11] measured branching ratios of 1^+ decays to the first two 0^+ states in ^{98,100}Mo. Their analysis depends on a precise description of the normal and intruder 0^+ basis states and on the hypothesis that the 1^+ states are unable to mix. Within that model, they obtain mixing intensities of 0.78(3) and 0.22(3) in ^{98}Mo and 0.69(6) and 0.31(6) in ^{100}Mo. The assumptions of their model remain to be tested.

Thomas *et al.* [8] state that for ^{98}Mo, "In the resulting 0_1^+ and 0_2^+ states, normal and intruder configurations are almost equally mixed with fractions of 55.3% and 46.9%, respectively [12]." (Their Ref. [12] is my Ref. [11].) But I do not find those numbers in Rusev *et al.* They are, however, quite close to my 0^+ averages of 57.5% and 42.5% for the three fits in ^{98}Mo.

The basis-state matrix elements that emerge from the various fits are plotted in Fig. 1. This figure confirms the conclusion of Ref. [10] that ^{100}Mo is more collective than ^{98}Mo and that band g is significantly more collective than band e.

The fact that $a > b$ for ^{98}Mo means that the basis state 0_g is lower in energy than 0_e in that nucleus. This situation is less clear in ^{100}Mo. In all cases (both nuclei, three fits in each), 2_g is significantly lower than 2_e, further demonstrating that band g is more collective than band e.

The mixing amplitudes and energy separations can be combined to determine the potential matrix element responsible for the mixing. These are listed in Table III. If one expects V for 0^+ and 2^+ to be similar, then the ^{100}Mo results indicate that it is the third 2^+ that is to be associated with the second 0^+ in that nucleus.

TABLE II. Results of the fits.

Quantity	^{98}Mo Fit 1[a]	^{98}Mo Fit 2[b]	^{98}Mo Fit 3[c]	^{100}Mo Fit 1[a]	^{100}Mo Fit 2[b]	^{100}Mo Fit 3[c]
b	0.657	0.625	0.678	0.736	0.645	0.791
B	0.285	0.370	0.581	0.396	0.149	0.447
Mg (eb)	0.661	0.684	0.775	0.913	0.860	0.918
Me (eb)	0.259	0.128	0.159	0.386	0.314	0.502

[a]Used 0_1, 0_2, 2_1, and 2_3.
[b]Used 0_1, 0_2, 2_1, and 2_2.
[c]Used 0_1, 0_2, 2_1, and the sum of 2_2 and 2_3.

TABLE III. Potential matrix elements (keV) obtained from various fits.

Potential	^{98}Mo Fit 1	^{98}Mo Fit 2	^{98}Mo Fit 3	^{100}Mo Fit 1	^{100}Mo Fit 2	^{100}Mo Fit 3
V0	364	359	366	346	342	336
$V2_3$	265			337		
$V2_2$		222			78	

III. SUMMARY

I have applied a simple two-state mixing model to matrix elements for $0 \leftrightarrow 2$ transitions in ^{98,100}Mo. Because the situation regarding the second and third 2^+ states is unclear, I have performed three separate fits for each nucleus, one using 2_2, one using 2_3, and one for their sum. The 0^+ states are close to maximally mixed, whereas the lowest 2^+ states contain a significant majority of the g basis state. The 0^+ potential matrix elements that cause the mixing are nearly equal in ^{98}Mo and ^{100}Mo. In ^{100}Mo, Fit 1 has the same V for $J = 2$. Results indicate that ^{100}Mo is somewhat more collective than ^{98}Mo, and that basis-state band g is significantly more deformed than band e.

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