Nuclear matter as a liquid phase of spontaneously broken semiclassical $SU(2)_L \times SU(2)_R$ chiral perturbation theory: Static chiral nucleon liquids

Bryan W. Lynn*
Department of Physics/CERCA/ISO, Case Western Reserve University, Cleveland, Ohio 44106, USA and Department of Physics/Astronomy, University College London, London WC1E 6BT, United Kingdom

Brian J. Coffey†
Department of Physics/CERCA/ISO, Case Western Reserve University, Cleveland, Ohio 44106, USA

Kellen E. McGee‡
Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

Glenn D. Starkman§
Department of Physics/CERCA/ISO and Department Astronomy, Case Western Reserve University, Cleveland, Ohio 44106, USA

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The standard model of particle physics (SM), augmented with neutrino mixing, is either the complete theory of interactions of known particles at energies naturally accessible on earth, or very nearly so, with a Lagrangian symmetric under the global $SU(2)_L \times SU(2)_R$ symmetry of two-massless-quark QCD, *spontaneously* broken to $SU(2)_{L+R}$. Using naive dimensional operator power counting that enables perturbation and truncation in inverse powers of $\Lambda_{\chi SB} \approx 1$ GeV, we show that, to $\mathcal{O}(\Lambda_{\chi SB})$ and $\mathcal{O}(\Lambda^0_{\chi SB})$, SU(2) chiral perturbation theory [SU(2)χPT] of protons, neutrons, and pions admits a liquid phase, with energy required to increase or decrease the nucleon density. We further show that in the semiclassical approximation—i.e., quantum nucleons and classical pions—"pionless SU(2)χPT" emerges in that chiral liquid: soft static infrared Nambu-Goldstone-boson pions decouple from "static chiral nucleon liquids" (StaticχNLs). This vastly simplifies the derivation of saturated nuclear matter (the infinite liquid phase) and of finite microscopic liquid drops (ground-state heavy nuclides). StaticχNLs are made entirely of nucleons. They have even parity, total spin zero, even proton number Z, and even neutron number N. The nucleons are arranged so local expectation values for spin and momentum vanish. We derive the StaticχNL effective Lagrangian from semiclassical SU(2)χPT symmetries to order $\Lambda_{\chi SB}$ and $\Lambda^0_{\chi SB}$, including all relativistic four-nucleon operators that survive Fierz rearrangement in the nonrelativistic limit and SU(2)χPT fermion exchange operators and isovector exchange operators which are important when $Z \neq N$. Mean-field StaticχNL nontopological solitons are true solutions of SU(2)χPT semiclassical symmetries; e.g., they obey all conserved vector current (CVC) and partially conserved axial current (PCAC) conservation laws. They have zero internal and external pressure. The nuclear liquid-drop model and Bethe–von Weizsäcker semiempirical mass formula emerge—with correct nuclear density and saturation and asymmetry energies—in an explicit Thomas-Fermi construction.

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I. INTRODUCTION

In the standard model (SM) of particle physics, quantum chromodynamics (QCD) describes the strong interactions among quarks and gluons. At low energies, quarks and gluons are confined inside hadrons, concealing their degrees of freedom in such a way that we must employ an effective field theory (EFT) of hadrons. In doing so, we acknowledge as a starting point a still-mysterious experimental fact: Nature first makes hadrons and then assembles nuclei from them [1–4].

Since nuclei are made of hadrons, the fundamental challenge of nuclear physics is to identify the correct EFT of hadrons and use it to characterize all nuclear physics observations. (See the recent review by Hammer *et al.* [5].) Ultimately, the correct choice of EFT will both match the observations and be derivable from the SM, i.e., QCD.

Chiral perturbation theory (χPT) [6–11] is a low-energy perturbative approach to identifying the operators in the EFT of hadrons that are allowed by the global symmetries of the SM. It builds on the observation that the up and down

*Deceased.
†bjc118@case.edu
‡mcgee@frib.msu.edu
§glenn.starkman@case.edu

quarks ($m_u \simeq 6$ MeV, $m_d \simeq 12$ MeV), as well as the three pions ($\pi^{\pm}, \pi^0$, $m_\pi \simeq 140$ MeV)—which are pseudo-Nambu-Goldstone bosons (pNGBs) of the chiral symmetry—are all nearly massless compared to the cutoff energy scale in low-energy hadronic physics $\Lambda_{\chi SB} \approx 1$ GeV.

With naive power counting [12], the effective Lagrangian of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ χPT incorporates explicit breaking. The resultant perturbation expansion in the inverse of the chiral-symmetry-breaking scale $\Lambda_{\chi SB}^{-1} \approx 1\ \mathrm{GeV}^{-1}$ renders SU(2)χPT's strong interaction predictions calculable in practice. Its low-energy dynamics of a proton-neutron nucleon doublet and three pions as a pNGB triplet are our best understanding, together with lattice QCD, of the experimentally observed low-energy dynamics of QCD strong interactions. The predictive power of χPT [6,8–15] derives from its ability to maintain a well-ordered low-energy perturbation expansion that can be truncated.

Lynn [16] first introduced the idea that SU(2)χPT could also admit a liquid phase and introduced the idea of an "$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral liquid" as a statistically significant number of baryons interacting via chiral operators with an almost constant saturated density which can can survive as localized liquid drops at zero external pressure. The Lagrangian included all analytic SU(2)χPT terms of $\mathcal{O}(\Lambda_{\chi SB})$ and $\mathcal{O}(\Lambda^0_{\chi SB})$. Lynn argued that, in the exact chiral limit, nucleons in the liquid phase interact with each other only via the contact terms in (23). Study of chiral liquids in [16] focused on those explicit chiral symmetry breaking terms whose origin lies entirely in the nonzero light quark masses.

The result is a semiclassical nuclear picture, where Thomas-Fermi nucleons with contact interactions move in a mean spherically symmetric "classical" pion field, which in turn generates a "no-core" radial potential for nucleons. Finite saturating heavy nuclei, with well-defined surfaces, emerge as microscopic droplets of chiral liquid. Saturating infinite nuclear matter emerges as very large drops of chiral liquid, while neutron stars (Q stars) emerge as oceans of chiral liquid: These droplets emerge as non-topological-soliton semiclassical solutions of explicitlybroken SU(2)χPT. Lynn [16] conjectured the possible emergence of shell structure in that no-core spherical potential based on the observation that the angular momentum of each nucleon is a good quantum number. Reference [16] did not derive semiclassical pionless SU(2)χPT. Here, we focus our study of chiral liquids in the chiral limit, and prove the emergence of semiclassical pionless SU(2)χPT solutions.

There is a long history of viewing nuclear matter as a nontopological soliton. In the mid 1970s Lee and co-workers [17–19], Chin and Walecka [20], and Serber [21] first identified certain fermion nontopological solitons with the ground state of heavy nuclei (as well as possible superheavy nuclei) in "normal" and "abnormal" phases, thus making a crucial connection to the older (but still persistently predictive) insight of nuclear liquids, such as Gamow's nuclear liquid-drop model (NLDM) and Bethe and von Weizsäcker's semiempirical mass formula (SEMF). Breaking all precedent, these workers proposed for the first time a theory of liquid nuclear structure composed entirely of nucleons and a static scalar field, with no pions.

Mathematically, such solutions emerge as a subspecies of nontopological solitons or Q balls [17,22–35], a certain subset of which are composed of fermions along with the usual scalars. A practical goal was to identify mean-field nucleon nontopological solitons with the ground state of ordinary even-even spin-zero spherically symmetric heavy nuclei, such as ${}^{40}_{20}$Ca, ${}^{90}_{40}$Zr, and ${}^{208}_{82}$Pb.

Nuclear nontopological solitons identified as nuclear liquids became popular with the work of Chin and Walecka [20], carried forward by Serot [36]. Walecka's nuclear quantum-hadrodynamics-1 (QHD-1) models [37–39] contain four dynamical particles: protons, neutrons, the Lorentz-scalar isoscalar σ, and the Lorentz-vector isoscalar ω_μ. Nucleons are treated as locally free particles in Thomas-Fermi approximation. Finite-width nuclear surfaces are generated by dynamical attractive σ-particle exchange, allowing them to exist at zero external pressure.

The empirical success of QHD-1 is based on balancing σ-boson-exchange attraction against ω_μ-boson-exchange repulsion. That that balance must be fine tuned remains a famous mystery of the structure of the QHD-1 ground state. In the absence of long-ranged electromagnetic forces, infinite symmetric $Z = N$ nuclear matter, as well as finite microscopic ground state $Z = N$ nuclides, appear as symmetric nuclear liquid drops.

These nuclear nontopological solitons are to be classified as liquids because

(i) they have no crystalline or other solid structure;
(ii) it costs energy to either increase or decrease the density of the constituent nucleons compared to an optimum value;
(iii) they survive at zero external pressure, e.g., in the absence of gravity, so they are not a "gas."

Despite their successes, such topological-soliton models suffer from the flaw that higher loop corrections do not necessarily decrease in size and importance, which can significantly renormalize the parameters at each order. This was first demonstrated by Furnstahl *et al.* [40] for the Walecka model in the two-loop case. (See also the discussion in [41].)

This paper cures those problems, and resurrects nuclear liquids as a good starting point toward understanding the properties of bound nuclear matter (with Z and N both even) by strict compliance with the requirements of SU(2)χPT effective field theory of protons, neutrons, and pions. The static chiral nucleon liquids (StaticχNLs) studied below are true solutions to semi-classical SU(2)χPT, and have all of the semi-classical symmetries of spontaneously broken SU(2)χPT found in Appendix A: they obey all conserved vector current (CVC) and partially conserved axial current (PCAC) Ward identities; they are dependent on just a few experimentally measurable chiral coefficients; and, by the symmetries of spontaneously broken SU(2)χPT, they restore (cf. Appendix A) theoretical predictive power over heavy nuclides.

II. THE EMERGENCE OF SEMICLASSICAL PIONLESS STATICχNLS

In this paper we focus on the chiral limit and postpone treatment of departures from the chiral limit to future work. The SU(2)χPT Lagrangian with all terms of order $\Lambda_{\chi SB}$ and $\Lambda^0_{\chi SB}$ in the chiral limit is

$$
\begin{aligned}
L^{\rm Sym}_{\chi PT} =& L^{\pi;Sym}_{\chi PT} + L^{N;Sym}_{\chi PT} + L^{4\text{-}N;Sym}_{\chi PT},\\
L^{\pi;Sym}_{\chi PT} =& \frac{f_\pi^2}{4}{\rm Tr}\partial_\mu\Sigma\partial^\mu\Sigma^\dagger,\\
L^{N;Sym}_{\chi PT} =& \overline{N}[i\gamma^\mu(\partial_\mu + V_\mu) - m^N\mathbb{1}]N\\
&- g_A\overline{N}\gamma^\mu\gamma^5 A_\mu N,\\
=& \overline{N}(i\gamma^\mu\partial_\mu - m^N\mathbb{1})N + i\vec{J}^\mu\cdot\vec{V}_\mu\\
&- g_A\vec{J}^{\mu,5}\cdot\vec{A}_\mu,\\
L^{4\text{-}N;Sym}_{\chi PT} =& C_{\mathscr{A}}\frac{1}{2f_\pi^2}(\overline{N}\gamma^{\mathscr{A}}N)(\overline{N}\gamma_{\mathscr{A}}N) + + +,
\end{aligned}
\tag{1}
$$

where the pion field is

$$
\Sigma \equiv \exp\left(2i\pi_a\frac{t_a}{f_\pi}\right),
\tag{2}
$$

and we defined the fermion bilinear and pionic currents:

$$
\begin{aligned}
\vec{J}^\mu &= \overline{N}\gamma^\mu\vec{t}N, \quad \vec{J}^{\mu,5} = \overline{N}\gamma^\mu\gamma^5\vec{t}N,\\
\vec{J}^\mu &= \overline{N}\gamma^\mu\vec{t}N, \quad \vec{J}^{\mu,5} = \overline{N}\gamma^\mu\gamma^5\vec{t}N,\\
V_\mu &= \vec{t}\cdot\vec{V}_\mu, \quad \vec{V}_\mu = 2i\,{\rm sinc}^2\left(\frac{\pi}{2f_\pi}\right)[\vec{\pi}\times\partial_\mu\vec{\pi}],\\
A_\mu &= \vec{t}\cdot\vec{A}_\mu,\\
\vec{A}_\mu &= -\frac{2}{\pi^2}\left[\vec{\pi}(\vec{\pi}\cdot\partial_\mu\vec{\pi}) + {\rm sinc}\left(\frac{\pi}{f_\pi}\right)[\vec{\pi}\times(\partial_\mu\vec{\pi}\times\vec{\pi})]\right],
\end{aligned}
\tag{3}
$$

with $\vec{t} \equiv \frac{1}{2}\vec{\tau}$, τ are the Pauli isospin matrices, $\pi = |\vec{\pi}| = \sqrt{\vec{\pi}^2}$, and ${\rm sinc}(x) \equiv \sin(x)/x$. The pion → dileptons decay constant is $F_\pi = 130.4 \pm 0.04 \pm 0.2$ MeV [42]. We use $f_\pi \equiv F_\pi/\sqrt{2} = 92.2$ MeV.

The parentheses in the four-nucleon Lagrangian indicate the order of SU(2) index contraction, while + + + indicates that one should include all possible combinations of such contractions. As usual, $\gamma^{\mathscr{A}} \equiv (1, \gamma^\mu, i\sigma^{\mu\nu}, i\gamma^\mu\gamma^5, \gamma^5)$, for $\mathscr{A} = 1, \ldots, 16$ (with $\sigma^{\mu\nu} \equiv \frac{1}{2}[\gamma^\mu, \gamma^\nu]$). These are commonly referred to as scalar (S), vector (V), tensor (T), axial-vector (A), and pseudoscalar (P) respectively. $C_{\mathscr{A}}$ are a set of chiral constants.

In the chiral limit, where $\vec{\pi}$'s are massless, the presence of quantum nucleon sources could allow the massless NGB to build up, with tree-level interactions only, a nonlinear quantum pion cloud. If we minimize the resultant action with respect to variations in the pion field, the equations of motion[1] capture the part of the quantum cloud that is to be characterized as a classical soft-pion field, thus giving us the pion ground state in the presence of the ground state "chiral nucleon liquid" (χNL) with fixed baryon number $A = Z + N$:

$$
\begin{aligned}
0 =& \left[\partial_\nu\frac{\partial}{\partial(\partial_\nu\pi^m)} - \frac{\partial}{\partial\pi^m}\right]L^{\pi;Sym}_{\chi PT}\\
&+ i\vec{J}^\mu\cdot\left[\partial_\nu\frac{\partial}{\partial(\partial_\nu\pi^m)} - \frac{\partial}{\partial\pi^m}\right]\vec{V}_\mu\\
&- g_A\vec{J}^{\mu,5}\cdot\left[\partial_\nu\frac{\partial}{\partial(\partial_\nu\pi^m)} - \frac{\partial}{\partial\pi^m}\right]\vec{A}_\mu\\
&- 2\partial_\mu\vec{J}^\mu\cdot{\rm sinc}^2\left(\frac{\pi}{2f_\pi}\right)(\vec{\pi}\times\hat{m})\\
&+ \frac{2}{\pi^2}g_A\partial_\mu\vec{J}^{\mu,5}.\\
&\times\left[\vec{\pi}(\vec{\pi}\cdot\hat{m}) + {\rm sinc}\left(\frac{\pi}{f_\pi}\right)[\vec{\pi}\times(\hat{m}\times\vec{\pi})]\right].
\end{aligned}
\tag{4}
$$

We divide the classical pion field into "IR" and "non-IR" parts. By definition, only IR pions survive the internal projection operators associated with taking expectation values of the classical NGB $\vec{\pi}$ s in the $|\chi{\rm NL}\rangle$ quantum state:

$$
\begin{aligned}
&\langle\chi{\rm NL}|F(\partial_\mu\vec{\pi}, \vec{\pi})|\chi{\rm NL}\rangle\\
&\quad= \langle\chi{\rm NL}|\text{IR-part}[F(\partial_\mu\vec{\pi}, \vec{\pi})]|\chi{\rm NL}\rangle\\
&\quad\equiv \{F(\partial_\mu\vec{\pi}, \vec{\pi})\}_{IR},
\end{aligned}
\tag{5}
$$

where F is an unspecified function. The IR part does not change the χNL. It could in principle play an important role in the excited states of the χNL: a $\vec{\pi}$ condensate, a giant resonance, a breathing mode, or a time-dependent flashing-pion mode. To ignore such classical IR $\vec{\pi}$ s would therefore be an incorrect definition of the excited states of χNL.

We call these "IR pions" by keeping in mind a simple picture, where the $\vec{\pi}$ wavelength is longer than the scale within the χNL over which the local mean values of nucleon spin and momentum vanish. Only such IR pions survive the internal projection operators associated with taking expectation values of the classical NGB $\vec{\pi}$'s in the $|\chi{\rm NL}\rangle$ quantum state.

We now take expectation values of the $\vec{\pi}$ equations of motion. In the presence of the quantum χNL source, the classical NGB $\vec{\pi}$ cloud obeys

$$
\begin{aligned}
0 =& \langle\chi{\rm NL}|\left[\partial_\nu\frac{\partial}{\partial(\partial_\nu\pi^m)} - \frac{\partial}{\partial\pi^m}\right]L^{\rm Sym}_{\chi PT}|\chi{\rm NL}\rangle\\
=& \left\{\left[\partial_\nu\frac{\partial}{\partial(\partial_\nu\pi^m)} - \frac{\partial}{\partial\pi^m}\right]L^{\pi;Sym}_{\chi PT}\right\}_{IR} + i\langle\chi{\rm NL}|\vec{J}^\mu|\chi{\rm NL}\rangle\cdot
\end{aligned}
$$

[1] This is a chiral-limit SU(2)χPT analog of QED where, in the presence of quantum lepton sources, a specific superposition of massless infrared photons builds up into a classical electromagnetic field. Important examples are the "exponentiation" of IR photons in $e^+e^- \to \mu^+\mu^-$ asymmetries, and $e^+e^- \to e^+e^-$ Bhabha scattering, at LEP1. Understanding the classical fields generated by initial-state and final-state soft-photon radiation [43,44] is crucial to disentangling high-precision electroweak loop effects, such as the experimentally confirmed precise standard model predictions for the top-quark [45] and Higgs' masses [45,46].

$$\times \left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]\vec{V}_\mu\right\}_{IR}$$
$$- g_A\langle\chi\mathrm{NL}|\vec{J}^{\mu,5}|\chi\mathrm{NL}\rangle \cdot \left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]\vec{A}_\mu\right\}_{IR}$$
$$- 2\langle\chi\mathrm{NL}|\partial_\mu\vec{J}^{\mu}|\chi\mathrm{NL}\rangle \cdot \left\{\mathrm{sinc}^2\left(\frac{\pi}{2f_\pi}\right)\vec{\pi}\times\hat{m}\right\}_{IR}$$
$$+ \frac{2}{\pi^2}g_A\langle\chi\mathrm{NL}|\partial_\mu\vec{J}^{\mu,5}|\chi\mathrm{NL}\rangle \cdot \left\{\vec{\pi}(\vec{\pi}\cdot\hat{m}) + \mathrm{sinc}\left(\frac{\pi}{f_\pi}\right)\vec{\pi}\right.$$
$$\left.\times(\hat{m}\times\vec{\pi})\right\}_{IR}. \tag{6}$$

We examine the following semiclassical nuclear current components,

$$J^\mu_\pm = J^\mu_1 \pm iJ^\mu_2 = \begin{Bmatrix}\overline{p}\gamma^\mu n\\ \overline{n}\gamma^\mu p\end{Bmatrix},$$
$$J^\mu_3 = \frac{1}{2}(\overline{p}\gamma^\mu p - \overline{n}\gamma^\mu n),$$
$$J^{5\mu}_\pm = J^{5\mu}_1 \pm iJ^{5\mu}_2 = \begin{Bmatrix}\overline{p}\gamma^\mu\gamma^5 n\\ \overline{n}\gamma^\mu\gamma^5 p\end{Bmatrix},$$
$$J^{5\mu}_3 = \frac{1}{2}(\overline{p}\gamma^\mu\gamma^5 p - \overline{n}\gamma^\mu\gamma^5 n), \tag{7}$$

and find that the ground-state expectation values of these currents and their divergences in (6) vanish,

$$\langle\chi\mathrm{NL}|J^\pm_\mu|\chi\mathrm{NL}\rangle = \langle\chi\mathrm{NL}|J^{\pm,5}_\mu|\chi\mathrm{NL}\rangle = 0,$$
$$\langle\chi\mathrm{NL}|\partial^\mu J^\pm_\mu|\chi\mathrm{NL}\rangle = \langle\chi\mathrm{NL}|\partial^\mu J^{\pm,5}_\mu|\chi\mathrm{NL}\rangle = 0, \tag{8}$$

because $J^\pm_\mu$ and $J^{\pm,5}_\mu$ change neutron and proton number. Since the liquid ground state is homogeneous, isotropic and spherically symmetric, spatial components of vector currents vanish, in particular

$$\langle\chi\mathrm{NL}|J^3_i|\chi\mathrm{NL}\rangle \simeq 0 \tag{9}$$

for Lorentz index $i=1,2,3$. Because there are separately equal numbers of left-handed and right-handed protons and neutrons in the nuclear ground state we have

$$\langle\chi\mathrm{NL}|J^{3,5}_\mu|\chi\mathrm{NL}\rangle \simeq 0 \tag{10}$$

for all μ. Note that (8)–(10) follow because the liquid ground state is assumed to have definite numbers of fully paired nucleons in a spherically symmetric, homogeneous, and isotropic arrangement. Current conservation enforces

$$\langle\chi\mathrm{NL}|\partial^\mu J^3_\mu|\chi\mathrm{NL}\rangle = \langle\chi\mathrm{NL}|\partial^\mu J^{3,5}_\mu|\chi\mathrm{NL}\rangle = 0, \tag{11}$$

which leaves only a single nonvanishing current expectation value:

$$\langle\chi\mathrm{NL}|J^3_0|\chi\mathrm{NL}\rangle \neq 0. \tag{12}$$

Equation (6), governing the classical pion cloud, is thus enormously simplified:

$$0 \simeq \left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]L^{\pi;Sym}_{\chi PT}\right\}_{IR}$$
$$+i\langle\chi\mathrm{NL}|J^{3;0}|\chi\mathrm{NL}\rangle\left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]V^3_0\right\}_{IR} \tag{13}$$

with

$$\left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]V^3_0\right\}_{IR} = \left\{2i\left[(\partial_0\vec{\pi})\times\hat{m} + \vec{\pi}\times\hat{m}\partial_0 - \hat{m}\times(\partial_0\vec{\pi}) - \vec{\pi}\times(\partial_0\vec{\pi})\frac{\partial}{\partial\pi^m}\right]^3 \mathrm{sinc}^2\left(\frac{\pi}{2f_\pi}\right)\right\}_{IR}. \tag{14}$$

A crucial observation is that (14) is linear in $\partial_0\vec{\pi}$; i.e., in the energy of the classical NGB IR $\vec{\pi}$ field. Expecting the nuclear ground state, and thus its classical IR $\vec{\pi}$ field, to be static, we enforce

$$\{\partial_o\vec{\pi}\}_{IR} = 0. \tag{15}$$

It now follows that

$$\left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]V^3_0\right\}_{IR} = 0, \tag{16}$$

independently of $\langle\chi\mathrm{NL}|J^{3;0}|\chi\mathrm{NL}\rangle$. The IR pion equation of motion

$$\left\{\left[\partial_\nu \frac{\partial}{\partial(\partial_\nu \pi^m)} - \frac{\partial}{\partial \pi^m}\right]L^{\pi;Sym}_{\chi PT}\right\}_{IR} = 0, \tag{17}$$

therefore has no nucleon source. The ground state nucleons are not a source of any static IR NGB $\vec{\pi}$ classical field. The nuclear ground state in the chiral liquid is thus a static chiral nucleon liquid (StaticχNL), with no $\vec{\pi}$ condensate[2] or time-dependent pion-flashing modes.

We want to quantize the nucleons in the background field of the StaticχNL, and so consider the expectation value of the nucleon equation of motion in the chiral nucleon liquid static ground state. For brevity, we denote expectations in this ground state using "⟨" and "⟩":

$$0 = \left\langle\overline{N}\frac{\partial}{\partial\overline{N}}L^{\mathrm{Sym}}_{\chi PT}\right\rangle$$
$$= \langle\overline{N}(i\gamma^\mu\partial_\mu - m^N\mathbb{1})N\rangle$$
$$+ i\langle\vec{J}^\mu\rangle\cdot\{\vec{V}_\mu\}_{IR} - g_A\langle\vec{J}^{\mu,5}\rangle\cdot\{\vec{A}_\mu\}_{IR}$$
$$+ \frac{1}{f_\pi^2}\langle C_{\mathscr{A}}(\overline{N}\gamma^{\mathscr{A}}N)(\overline{N}\gamma_{\mathscr{A}}N) + {+}{+}\rangle. \tag{18}$$

[2]After explicit chiral symmetry breaking, with nonzero u, d quark and resultant pion masses, and with partially conserved axial currents (PCAC), a static S-wave $\vec{\pi}$ condensate is a logical possibility [16].

Since most of the nucleon $SU(2)_L \times SU(2)_R$ currents vanish in the StaticχNL, and since $\{\partial_o \vec{\pi}\}_{IR} = 0$, we find

$$0 \simeq \langle \overline{N}(i\gamma^\mu \partial_\mu - m^N \mathbb{1})N\rangle \tag{19}$$

$$+ \frac{1}{f_\pi^2}\langle C_{\mathscr{A}}(\overline{N}\gamma^{\mathscr{A}}N)(\overline{N}\gamma_{\mathscr{A}}N) + ++\rangle. \tag{20}$$

Equations (17) and (19) show that, to order $\Lambda_{\chi SB}$ and $\Lambda^0_{\chi SB}$, StaticχNLs are composed entirely of nucleons. That is also the basic premise of many empirical models and we have shown that that empirical nuclear premise can be traced (to good approximation) directly to the global $SU(2)_L \times SU(2)_R$ symmetries of two-massless-quark QCD of the standard model.

The effective Lagrangian derived from $SU(2)_L \times SU(2)_R$ χPT governing StaticχNLs can now be written:

$$\begin{aligned}
&\langle L^{\rm Sym}_{\chi PT}\rangle \equiv L_{S\chi NL},\\
&L_{S\chi NL} = L^{\rm Free}_{S\chi NL} + L^{4\text{-}N}_{S\chi NL}\\
&L^{\rm Free}_{S\chi NL} = \langle \overline{N}(i\gamma^\mu \partial_\mu - m^N \mathbb{1})N\rangle\\
&L^{4\text{-}N}_{S\chi NL} = \left\langle \frac{1}{2f_\pi^2}C_{\mathscr{A}}(\overline{N}\gamma^{\mathscr{A}}N)(\overline{N}\gamma_{\mathscr{A}}N) + ++\right\rangle.
\end{aligned} \tag{21}$$

Semiclassical pionless $SU(2)\chi$PT thus emerges inside StaticχNLs. Within all-loop-orders renormalized analytic $SU(2)\chi$PT to $\mathcal{O}(\Lambda_{\chi SB})$ and $\mathcal{O}(\Lambda^0_{\chi SB})$, infrared NGB pions effectively decouple from StaticχNLs, vastly simplifying the derivation of the properties of saturated nuclear matter (the infinite liquid phase) and of finite microscopic liquid drops (the nuclides). StaticχNLs thus explain the (previously puzzling) power of pionless EFT to capture experimental ground state facts of certain specific nuclides, by tracing that empirical success directly to the global symmetries of two-massless-quark QCD.

It will be shown below that StaticχNLs satisfy all relevant $SU(2)_L \times SU(2)_R$ vector and axial-vector current-conservation equations in the liquid phase. StaticχNLs are therefore solutions of the semiclassical-liquid equations of motion, possessing the symmetries of spontaneously broken $SU(2)\chi$PT (cf. Appendix A 1).

III. SEMICLASSICAL PIONLESS STATICχNLS AS THE APPROXIMATE GROUND STATE OF CERTAIN NUCLEI

To further elucidate the properties of the StaticχNL, we must address the effects of the four-nucleon interactions. In this paper, we ignore fluctuations in all bilinear nucleon operators. For our purposes this is equivalent to ignoring any and all nuclear excited states.

A priori there are ten possible contact interactions representing isosinglet and isotriplet channels for each of five spatial current types: scalar, vector, tensor, pseudoscalar and axial vector. There are therefore ten chiral coefficients parametrizing four-nucleon contact terms: C_K^T with $K \in \{S, V, T, A, P\}$ and $T \in \{0, 1\}$.

The inclusion of exchange interactions induces the isospin ($T = 1$) operators to appear [16], and potentially greatly complicates the effective chiral Lagrangian. Fortunately, we are interested here in the liquid limit of this Lagrangian. Spinor-interchange contributions are properly obtained by Fierz rearranging before imposing the properties of the semiclassical liquid (see Appendix B). The appropriate StaticχNL Lagrangian is given by

$$L_{S\chi NL} = \bar{N}(i\gamma^\mu \overrightarrow{\partial}_\mu + \Theta)N + L^{4\text{-}N;BE}_{S\chi NL}, \tag{22}$$

where the contact interactions can be approximated by

$$\begin{aligned}
-L^{4\text{-}N;BE}_{S\chi NL} = &\frac{C^S_{200}}{2f_\pi^2}\langle\overline{N}N\rangle\langle\overline{N}N\rangle\\
&- \frac{\overline{C^S_{200}}}{4f_\pi^2}\{\langle\overline{N}N\rangle\langle\overline{N}N\rangle + 4\langle\overline{N}t_3N\rangle\langle\overline{N}t_3N\rangle\}\\
&+ \frac{C^V_{200}}{2f_\pi^2}\{\langle N^\dagger N\rangle\langle N^\dagger N\rangle\}\\
&- \frac{\overline{C^V_{200}}}{4f_\pi^2}\{\langle N^\dagger N\rangle\langle N^\dagger N\rangle + 4\langle N^\dagger t_3N\rangle\langle N^\dagger t_3N\rangle\}
\end{aligned} \tag{23}$$

with only *four* independent chiral coefficients:

$$\begin{aligned}
C^S_{200} &= C_S^{T=0}\\
-\overline{C^S_{200}} &= \tfrac{1}{4}\big[C_S^{T=0} + 5C_S^{T=1}\\
&\quad + 6\big(C_T^{T=0} + C_T^{T=1}\big) + \big(C_P^{T=0} + C_P^{T=1}\big)\big],\\
C^V_{200} &= C_V^{T=0},\\
-\overline{C^V_{200}} &= \tfrac{1}{2}\big[-C_V^{T=0} + C_A^{T=0} + C_V^{T=1} + C_A^{T=1}\big].
\end{aligned} \tag{24}$$

To simplify the notation and to retain the connection with previous work [20] we introduce

$$\mathcal{C}_V^2 \equiv \frac{1}{f_\pi^2}\left(C^V_{200} - \frac{1}{2}\overline{C^V_{200}}\right), \tag{25}$$

$$\mathcal{C}_S^2 \equiv -\frac{1}{f_\pi^2}\left(C^S_{200} - \frac{1}{2}\overline{C^S_{200}}\right). \tag{26}$$

For brevity, we also define

$$\overline{\mathcal{C}_V^2} \equiv \frac{1}{f_\pi^2}\overline{C^V_{200}}, \tag{27}$$

$$\overline{\mathcal{C}_S^2} \equiv \frac{1}{f_\pi^2}\overline{C^S_{200}}. \tag{28}$$

In (22) the operator Θ is given by

$$\Theta \equiv -m^N - \widehat{C^S_{200}} - \widehat{C^V_{200}}\gamma^0, \tag{29}$$

with

$$\begin{aligned}
\widehat{C^V_{200}} &\equiv \mathcal{C}_V^2\,\langle N^\dagger N\rangle - 2\,\overline{\mathcal{C}_V^2}\langle N^\dagger t_3N\rangle t_3,\\
\widehat{C^S_{200}} &\equiv -\mathcal{C}_S^2\,\langle\overline{N}N\rangle - 2\,\overline{\mathcal{C}_S^2}\langle\overline{N}t_3N\rangle t_3,\\
0 &= \big[t_3, \widehat{C^S_{200}}\big] = \big[t_3, \widehat{C^V_{200}}\gamma^0\big] = [t_3, \Theta].
\end{aligned} \tag{30}$$

We have ignored possible excited states that contribute to fluctuations in the nuclear density and which are beyond the scope of this paper.

The StaticχNL Lagrangian offers a significant improvement in the predictive power of the theory, while still providing sufficient free parameters to balance vector repulsive forces against scalar attractive forces when fitting (to order $\Lambda^0_{\chi SB}$) nontopological-soliton and Skyrme nuclear models to the experimentally observed structure of ground state nuclei. Further simplification results for a sufficiently large number of nucleons: simple Hartree analysis of (23) is equivalent to more accurate Hartree-Fock analysis of the same Lagrangian without spinor-interchange terms.

We now see that, inside the StaticχNL, a nucleon living in the self-consistent field of the other nucleons obeys the Dirac equation

$$0 = (i\gamma^\mu \partial_\mu + \Theta)N. \tag{31}$$

Baryon-number and the third component of isospin are both conserved; i.e., the associated currents $J^\mu_{\text{Baryon}} \equiv \overline{N}\gamma^\mu N$ and $J^\mu_3 \equiv \overline{N}\gamma^\mu t_3 N$ are both divergence free. The neutral axial-vector current $J^{5,\mu}_8 \equiv \frac{\sqrt{3}}{2}\overline{N}\gamma^\mu\gamma^5 N$, corresponding to the projection onto SU(2) of the NGB η particle, part of the unbroken $SU(3)_L \times SU(3)_R$ meson octet, is also divergence free,

$$\begin{aligned}\frac{2}{\sqrt{3}}\langle i\partial_\mu J^{5,\mu}_8\rangle &= \langle \overline{N}\{\Theta, \gamma^5\}N\rangle \\ &= 2\langle \overline{N}\big(-m^N - \widehat{C^S_{200}}\big)\gamma^5 N\rangle \\ &\simeq 0.\end{aligned} \tag{32}$$

This result can be understood as a statement that the η particle cannot survive in the parity-even interior of a StaticχNL, since it is a NGB pseudoscalar in the chiral limit. Similarly, the third component of the axial vector current is divergence free; i.e.,

$$\begin{aligned}\langle i\partial_\mu J^{5,\mu}_3\rangle &= \langle \overline{N}\{\Theta, \gamma^5\}t_3 N\rangle \\ &= 2\langle \overline{N}\big(-m^N - \widehat{C^S_{200}}\big)\gamma^5 t_3 N\rangle \\ &\simeq 0,\end{aligned} \tag{33}$$

because the SU(2)χPT π_3 particle is also a NGB pseudoscalar in the chiral limit, and cannot survive in the interior of a parity-even StaticχNL.

Even though explicit pion and eta fields vanish in StaticχNLs, their quantum numbers reappear in its PCAC properties from nucleon bilinears and four-nucleon terms in the divergences of axial vector currents. That these average to zero in StaticχNLs plays a crucial role in the conservation of axial-vector currents within the liquid.

It is now straightforward to see that, in the liquid approximation, a homogeneous SU(2)χPT nucleon liquid drop with no meson condensate satisfies all relevant CVC and PCAC equations. As shown in Appendix C, most of the space-time components of the three $SU(2)_{L+R}$ vector currents J^μ_a and three axial vector currents $J^{5\mu}_a$ vanish: only J^0_3 is nonzero in StaticχNLs.

The neutral $SU(3)_L \times SU(3)_R$ currents are conserved, $\langle\partial_\mu J^\mu_8\rangle = 0$ and $\langle\partial_\mu J^{5;\mu}_8\rangle = 0$, in the Static$\chi$NL mean field. In addition, the neutral $SU(3)_{L+R}$ vector current's spatial components $J^{\mu=1,2,3}_8$ and the axial-vector currents $J^{5;\mu}_8$ all vanish. Only J^0_8, proportional to the baryon number density, survives in the StaticχNL mean field.

Since StaticχNL chiral nuclear liquids satisfy all relevant χPT CVC and PCAC equations in the liquid phase, they are true solutions of the all-orders-renormalized tree-level semiclassical liquid equations of motion truncated at $\mathcal{O}(\Lambda^0_{\chi SB})$.

IV. NUCLEI AND NEUTRON STARS AS MEAN-FIELD STATICχNLS

A. Thomas-Fermi nontopological solitons, liquid drops, and the semiempirical mass formula

Mean-field StaticχNL nontopological solitons are solutions of χPT semiclassical symmetries, obeying all CVC and PCAC conservation laws. They have zero internal and external pressure. The nuclear liquid-drop model and Bethe–von Weizsäcker SEMF emerge—with correct nuclear density, and saturation and asymmetry energies—in an explicit Thomas-Fermi construction.

In Appendix D, we construct explicit liquid mean field StaticχNL solutions based on (22), constrained to order $4\pi f_\pi \approx \Lambda_{\chi SB} \simeq 1$ GeV and $\Lambda^0_{\chi SB}$ naive power counting, in an independent-nucleon model, using the Thomas-Fermi free-particle approximation.[3]

Constant-density nontopological solitons, i.e., liquid drops comprised entirely of nucleons, emerge as homogeneous and isotropic semiclassical static solutions with internal and external pressures both zero. Their surface is a step function. Ignoring electromagnetism, nuclear matter and finite nuclei then have identical microscopic structure, serving as a model of the ground state of both infinite nuclear matter and finite liquid drops. There is no need for an additional confining interaction to define the finite-drop surface. With even proton number Z, and even neutron number N, nucleons are arranged in pairs so that local expectation values for spin vanish, $\langle\vec{s}\rangle \simeq 0$. The microscopic structure is also spherically symmetric, so that local momenta have a vanishing expectation value, $\langle\vec{k}\rangle \simeq 0$. Consequently, total spin $\vec{S} = 0$ and total momentum $\vec{K} = 0$ in the center of mass.

The semiempirical mass formula [48,49] is

$$\begin{aligned}M(Z,N) &= Zm_p + Nm_n - E_B,\\ E_B &= E^{\text{Vol}}_B + E^{\text{Surf}}_B + E^{\text{Pair}}_B,\\ E^{\text{Vol}}_B/A &\equiv a_V - a_{\text{Asym}}X^2 - a_C\frac{Z(Z-1)}{A^{4/3}},\\ E^{\text{Surf}}_B/A &\equiv -\frac{a_S}{A^{1/3}},\\ E^{\text{Pair}}_B/A &\equiv a_{Pair}\frac{\delta_0(Z,N)}{A^{3/2}}.\end{aligned} \tag{34}$$

[3]An effective Lagrangian, built from $\mathcal{O}(\Lambda_{\chi SB})$ free nucleons and $\mathcal{O}(\Lambda^0_{\chi SB})$ point-coupling interaction-operators, was also identified by Gelmini and Ritzi [47]. However, it does not correspond to Chin-Walecka infinite symmetric $Z = N$ nuclear matter, and the authors constructed no $Z = N$ bound-state nontopological solitons with zero internal and external pressure, which could therefore survive in an external vacuum.

with $A = Z + N$; X is the neutron excess,

$$X \equiv \left(\frac{N-Z}{N+Z}\right), \tag{35}$$

and

$$\delta_0 \equiv \begin{cases} +1 & \text{for } Z \text{ even, } N \text{ even,} \\ -1 & \text{for } Z \text{ odd, } N \text{ odd,} \\ 0 & \text{for } A = Z + N \text{ odd.} \end{cases} \tag{36}$$

From [49] we use $a_V = 15.75$ MeV, $a_S = 17.8$ MeV, $a_C = 0.711$ MeV, $a_{\text{Asym}} = 23.7$ MeV, and $a_{Pair} = 11.18$ MeV.

We show in Appendix D that the SEMF is (almost) an $SU(2)_L \times SU(2)_R$ χPT nontopological-soliton prediction. We first display symmetric $Z = N$ ground state zero-pressure Hartree-Fock nontopological-soliton solutions, fit to inferred experimental values for symmetric-nuclear-matter density and volume binding energy, and find

$$\begin{aligned} C_V^2 &= 1.893 \frac{1}{f_\pi^2}, \\ C_S^2 &= 2.580 \frac{1}{f_\pi^2}. \end{aligned} \tag{37}$$

These values were obtained by fitting to a Fermi momentum ($k_{\text{Fermi}} = 1.42$/fm) and saturated volume energy (E_{binding}/nucleon = 15.75 MeV). We observe that for heavy nuclei $X^2 \ll 1$, and work to leading order in that small quantity. In Appendix D 2, we derive asymmetric $Z \neq N$ nuclear matter, for which fermion-exchange terms are crucial, fitting to $a_{\text{Asym}} = 23.7$ MeV. For $k_F = 1.42$/fm we find

$$\overline{C_V^2} = 0.61 \frac{1}{f_\pi^2}. \tag{38}$$

Additional results for $\overline{C_{200}^V}$ are given in Appendix D 3. Combining (37) and (38) using (25) and (27) gives:

$$C_{200}^V = 2.198. \tag{39}$$

In practice, there is very little sensitivity to our fourth independent chiral coefficient $\overline{C_{200}^S}$: this in agreement with Niksic [50] *et al.*, who argue that, although the total isovector strength has a relatively well-defined value, the distribution between the isovector Lorentz-scalar $\vec{\delta}$ exchange channel and the isovector Lorentz-vector $\vec{\rho}_\mu$ exchange channel is not determined by ground state data. We have assumed $(\frac{Z-N}{Z+N})^2 \ll 1$. In addition, we have

$$\langle N^\dagger N \rangle - \langle \bar{N} N \rangle \simeq \frac{3}{10} \frac{k_F^2}{m_{*8}^2} \langle N^\dagger N \rangle = (0.0762) \langle N^\dagger N \rangle \ll \langle N^\dagger N \rangle, \tag{40}$$

where $k_F = 279.7$ MeV and where $m_{*8} \equiv \frac{1}{2}(m_{*p} + m_{*n}) = 555$ MeV, which follows from the Thomas-Fermi solution in as found in Appendix D 1. It follows that only the combination $(\overline{C_{200}^V} + \overline{C_{200}^S})$ can strictly be fit to our $\mathcal{O}(\Lambda_{\chi SB}^0)$ StaticχNL accuracy. Therefore, for convenience and without loss of generality, we choose

$$\overline{C_{200}^S} = 0. \tag{41}$$

All coefficients in (37), (38), (39), and (41) then obey naive $\sim\mathcal{O}(1)$ dimensional power counting, and so are legitimate natural chiral coefficients. Note the fine tuning between $C_{200}^V = 2.198$ and $C_{200}^S = -2.580$ in (37) and (39) inherited from Serber's and Walecka's 1974 quadratic models [20], [51], and [21]. That fine tuning is alleviated in (37) by the our inclusion of $\vec{\rho}_\mu$ exchange, necessary to StaticχNLs. Equations (37), (38), (39), and (41) all satisfy naive dimensional power-counting $\mathcal{O}(1)$ naturalness, and so are legitimate chiral coefficients. The astute reader will notice that the difference (40) is of the same order as the next terms in the chiral expansion. Although we have calculated self-consistently in powers of $\Lambda_{\chi SB}$ in chiral perturbation theory, terms of order $\Lambda_{\chi SB}^{-1}$ must still play an important role in the nontopological-soliton solutions. Indeed, it is inconsistent to neglect them. We hope to return to this question in future work.

The SEMF is closely associated with Gamow's nuclear liquid-drop model (NLDM). Recall that, following Walecka's infinite symmetric nuclear matter (and neutron matter), we have imposed on the Thomas-Fermi mean field the condition that the pressure vanishes both internally and externally, not only at the surface of a finite "drop." Our nontopological-soliton nuclei therefore resemble ice cream balls scooped from an infinite vat [52], more than they do conventional liquid drops.

We clearly have no right to use the Thomas-Fermi approximation to calculate the surface and pairing energies, E_B^{Surf} and E_B^{Pair} of (34), at order $\Lambda_{\chi SB}$ and $\Lambda_{\chi SB}^0$ in the spontaneously broken theory. Unsurprisingly, the surface energy calculated entirely as a change in density gives incorrect a_S. However, there exist $\mathcal{O}(\Lambda_{\chi SB}^{-2})$ nuclear-surface $SU(2)\chi$PT terms that might replace the scalar σ particle in the Chin-Walecka model in describing the nuclear surface [20,38,39], namely

$$\mathcal{L}_{\chi PT}^{\text{Surf}} = -\frac{1}{2} \frac{C_{220}}{\Lambda_{\chi SB}^2} \partial_\nu (\bar{N}N) \partial^\nu (\bar{N}N), \tag{42}$$

with an $\mathcal{O}(1)$ constant C_{220}, obeying naturalness. $\mathcal{L}_{\chi PT}^{\text{Surf}}$ is invariant under nonlinear $SU(2)_L \times SU(2)_R$ transformations including pions, but is automatically pionless, even without the liquid approximation. It contains no dangerous $\partial_0 \sim m_N$ nucleon mass terms, so nonrelativistic reordering is unnecessary. Nucleon-exchange and spinor-interchange interactions must also be included.

Meanwhile, calculation of a_{Pair} involves understanding low-level excited states, such as Z-odd N-odd states which we have ignored in our study of the Lagrangian (23), which are beyond the scope of this paper, and will likely require explicit pions lying outside semiclassical pionless $SU(2)\chi$PT.

B. Neutron stars

Putting aside exotica (i.e., quark condensates, pion condensates, strange-kaon condensates, etc.), we conjecture that much of the structure of neutron stars may be traced directly to two-massless-quark QCD, and thus directly to the standard model. This will be explored further in a companion paper. Here we note only that the models of Harrison and Wheeler [53], Salpeter [54], and Baym, Pethic, and Sutherland [55] are all based on the Bethe–von Weizsäcker semiempirical mass formula [56]. They would therefore seem to follow from StaticχNLs; however, we do not yet know how well the observed chart of nuclides and these neutron-star models match the "ice-cream scoop" StaticχNL no-surface SEMF, augmented by Coulomb repulsion; i.e., (34) with $E_B^{\rm Pair}$ set to zero.

C. Shell structure from chiral symmetry breaking?

We conjecture here that nontopological StaticχNL solitons could, with inclusion of explicit axial symmetry breaking, be requantized to incorporate no-core nuclear shell structure and magic numbers, as imagined in [16]. Lynn first introduced the idea [16] that $SU(2)_L \times SU(2)_R$ χPT could admit a liquid phase. Like ours, his Lagrangian included only terms of $\mathcal{O}(\Lambda_{\chi SB})$ and $\mathcal{O}(\Lambda^0_{\chi SB})$. Though he did not anticipate StaticχNLs, he was careful to include only and all those terms that respect the $SU(2)_L \times SU(2)_R$ semiclassical symmetries—i.e., of quantum nucleons and classical pions—discussed in this paper. These included strong interaction terms that survive the chiral limit, as well as explicit axial breaking terms that do not.

The purpose of [16] was to generate a "no-core" classical static spherical central potential for $|\vec{\pi}|$, in which all of the quantum nucleons moved, and thus plausibly shell structure for certain heavy even-even ground state spin-zero spherical nuclei. It now seems advantageous to focus on doubly magic or spherically magic nuclides.

Such shell structure is plausible in semiclassical $SU(2)_L \times SU(2)_R$ χPT because the explicit symmetry-breaking terms have naive operator power counting $m=0$, $l=1$, $n=1$ in (A14). Ignoring $\pi^\pm - \pi^0$ mass splitting, these are

$$\begin{aligned}\langle L^{N;\chi SB}_{\chi PT}\rangle &\simeq \overline{m}(a_1+2a_3)\left[1-\cos\frac{|\vec{\pi}|}{f_\pi}\right]\langle\bar{N}N\rangle,\\ &\equiv \beta\sigma_{\pi N}\left[1-\cos\frac{|\vec{\pi}|}{f_\pi}\right]\langle\bar{N}N\rangle,\end{aligned}\tag{43}$$

with

$$\overline{m} \equiv \tfrac{1}{2}(m_u+m_d),\tag{44}$$

and with experimental parameters

$$\begin{aligned}(a_1,a_2,a_3) &= (0.28, -0.56, 1.3\pm 0.2),\\ (m_u,\ m_d,\ \sigma_{\pi N}) &= (6,\ 12,\ 60)\ \text{MeV},\\ \beta &= 0.864\pm 0.120\end{aligned}\tag{45}$$

measured in $SU(3)_L \times SU(3)_R$ χPT processes [13] and [57].

Since $\langle L^{N;\chi SB}_{\chi PT}\rangle > 0$, the explicit symmetry-breaking terms lower the effective nucleon mass inside a static $\pi = |\vec{\pi}|$ condensate.

We conjecture that semiclassical $SU(2)_L \times SU(2)_R$ χPT [i.e., including all $\mathcal{O}(\Lambda_{\chi SB})$ and $\mathcal{O}(\Lambda^0_{\chi SB})$ nonstrange analytic naive operator power-counting terms, both those from the chira limit and those from explicit $m_u, m_d \neq 0$ chiral symmetry breaking] applied to certain *finite* nuclei, nuclear and neutron matter, and neutron stars will give a reasonable match to their structure.

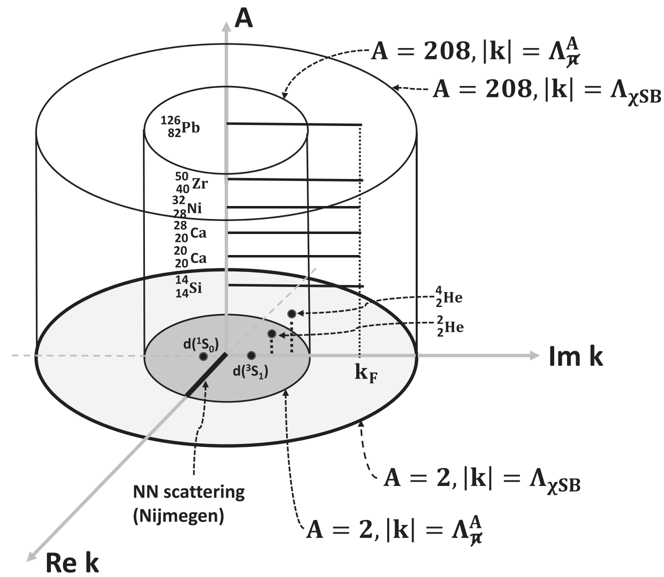

FIG. 1. Illustration (not to scale) of the domains of applicability of various analytic treatments of nuclear systems plotted in the three-dimensional space defined by complex momentum (Rek, Imk) and atomic/baryon number A. At the base sits the $A = 2$ complex k plane. Pionless effective field theory is valid inside the cylinder whose base is the disk with radius $\Lambda^A_{\not\pi} < m_\pi$. The even-even spin-zero nuclei to which the chiral nuclear liquid treatment of this paper are applicable are shown here: $^{28}_{14}$Si, $^{40}_{20}$Ca, $^{48}_{20}$Ca, $^{60}_{28}$Ni, $^{90}_{40}$Zr, and $^{208}_{82}$Pb. Their treatment incorporates k's along the Imk axis from 0 to $k_{\rm Fermi} \ll \Lambda_{\chi SB}$. See text for further details.

V. RELATION OF STATICχNLS TO PIONLESS EFT

Our pionless static chiral nuclear liquid solution bears superficial resemblance to results from pionless EFT [5]: both are "pionless." They are both pionless for different reasons, however. Pionless EFTS are pionless because the pions have been "integrated out" and so are valid for momenta less than the pion mass. StaticχNLs are pionless because the pionic source terms vanish in even-even, spin-zero spherical nuclei: here we work in the chiral limit of vanishing pion mass. The soliton solution has $k_F \simeq 280$ MeV. In fitting the parameters $C_{\mathcal{A}}$ in Eq. (2), we must fit to inferred infinite-nuclear-matter data. As pointed out by Hammer *et al.* [5], perturbation theory cannot be used to relate the coupling constants in the two theories. In future work one might hope to relate the coupling constants of StaticχNLs to those of pionless and of halo/cluster EFTs [5].

van Kolck and the pionless EFT community like to reveal relationships among their results by plotting them on the complex Re(k)-Im(k) momentum plane inside the circle $|k| \leqslant \Lambda^A_{\not\pi} < m_\pi$. In Fig. 1 we add an orthogonal $A = Z + N$ axis—forming a three-dimensional cylindrical Re(k)-Im(k)-A volume—and highlight some pionless EFT results. In the $A = 2$ plane, N-N elastic scattering is properly compared to Nijmegen data and lies along positive Re(k). The -2.2 MeV bound deuteron is at $k^{^3S_1}_{\rm Pole}$ on the positive Im(k) axis, while the shallow resonance is at $k^{^1S_0}_{\rm Pole}$ on the negative Im(k) axis. The $A = 4$ plane places the deeply bound (-28.296 Mev) α particle ($\sim {}^4_2\mathrm{He}_2$) at positive $\mathrm{Im}(k)$.

Halo/cluster EFT at $A \geqslant 5$ has no pions, and is mathematically similar to pionless EFT, becoming pionless EFT for light nuclei when the cores are nucleons. We plot only the classic example ${}^6_2\mathrm{He}_4$, where the energy required to remove the cluster (α particle), or either of the two halo nucleons, is much less than to break up the cluster. It lies on the $A = 6$ plane at positive $\mathrm{Im}(k)$.

In order to plot our Thomas-Fermi StaticχNL results from Appendix D and show their position relative to pionless EFT, we add an annulus to that pionless EFT cylinder, extending the radius of its $\mathrm{Re}(k)$- $\mathrm{Im}(k)$ base to the region $\Lambda^A_{\not\pi} < |k| \leqslant \Lambda^A_{\mathrm{Static}\chi NL} = \Lambda_{\chi SB} \approx 1$ GeV. Our bound-state StaticχNL "ice-cream-scoop" nuclei are then horizontal lines along $\mathrm{Im}(k)$, in the positive $\mathrm{Im}(k)$-A quarter-plane, with $0 \leqslant |k| \leqslant k_F \simeq 280$ MeV: they intersect the A axis at $A_{\mathrm{EvenEven}} = Z_{\mathrm{Even}} + N_{\mathrm{Even}} \geqslant 4$. For visual simplicity, we plot symmetric $Z = N$ StaticχNL nuclei only for ${}^{28}_{14}\mathrm{Si}$ and ${}^{40}_{20}\mathrm{Ca}$, at $A = 28, 40$. We show asymmetric StaticχNLs only for ${}^{48}_{20}\mathrm{Ca}$, ${}^{60}_{28}\mathrm{Ni}$, ${}^{90}_{40}\mathrm{Zr}$, and ${}^{208}_{82}\mathrm{Pb}$ with $X^2 \ll 1$. For further pedagogical simplicity, we have averaged $\frac{1}{2}(k^p_F + k^n_F) \approx k_F \simeq 280$ MeV.

Going forward, an important challenge is to find an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ χPT integration of the physics of StaticχNLs and that of pionless EFT and halo/cluster EFT.

In the Summary of the 1985 Paris Conference on Nuclear Physics with Electro-magnetic Probes, Ericson [58] showed just how many facets there are to the nuclear "truth": different physical domains require different descriptions, each of which is the truth for that domain. If the StaticχNL as derived from the symmetries of QCD describes heavy (spin-zero even-even) spherical nuclei, its truth may be difficult to relate directly to accurate descriptions of other physical domains.

VI. CONCLUSIONS

In this paper, we have explored heavy symmetric nuclei in a semiclassical approach starting with chiral EFT that respects the global symmetries of QCD. In this, we have been guided by two key observations: that nuclei are made of protons and neutrons, not quarks, and that the up and down quarks, which are the fermionic constituents of the protons and neutrons, are much lighter than the principal mass scales of QCD, such as the proton and neutron masses. Taken together, these strongly suggest that the full complexity of the standard model can largely be captured, for the purposes of nuclear physics, by an effective field theory (EFT): $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral perturbation theory [SU(2)χPT] of protons and neutrons.

Building on this long-standing insight, we have studied the chiral limit of spontaneously broken $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ [i.e., SU(2)χPT], including only operators of order $\Lambda_{\chi SB}$ and $\Lambda^0_{\chi SB}$. We find that SU(2)χPT of protons, neutrons, and three pseudo-Nambu-Goldstone boson pions admits a semiclassical liquid phase, a static chiral nucleon liquid (StaticχNL). StaticχNLs are made entirely of nucleons, with approximately zero antiproton and antineutron content. They are parity even and time independent. As we have studied them so far, not just the total nuclear spin $\vec{S} = 0$, but also the local expectation value for spin $\langle \vec{s} \rangle \simeq 0$. Similarly, the nucleon momenta vanish locally in the spherically symmetric StaticχNL rest frame. For these reasons, our study of StaticχNLs is applicable to bulk ground state spin-zero nuclear matter, and to the ground state of appropriate spin-zero parity-even nuclei with an even number Z of protons and an even number N of neutrons.

We classify these solutions of SU(2)χPT as "liquid" because energy is required both to pull the constituent nucleons further apart and to push them closer together. This is analogous with the balancing of the attractive Lorentz-scalar σ-exchange force and the repulsive Lorentz-vector ω_μ-exchange force in the Walecka model. The nucleon number density therefore takes a saturated value even in zero external pressure (e.g., in the absence of gravity), so the material is not a "gas." Meanwhile they are statistically homogeneous and isotropic, lacking the reduced symmetries of crystals or other solids.

We have shown that in this ground state liquid phase the expectation values of many of the allowed operators of the most general SU(2)χPT Lagrangian vanish or are small. Going forward, it is imperative to understand the effects of of excited nucleon states to the spectra of heavy nuclei.

We have also shown that this spontaneously broken ground state liquid phase does not support a classical pion field; infrared pions decouple from this solution. We expect that this emergence of "semiclassical pionless SU(2)χPT" is at the heart of the apparent theoretical independence of much successful nuclear structure physics from pion properties such as the pion mass.

We have constructed explicit StaticχNLs in the Thomas-Fermi approximation, demonstrating the existence of zero-pressure nontopological-soliton StaticχNL solutions with macroscopic (infinite nuclear matter) and microscopic (heavy nuclear ground states).

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APPENDIX A: $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ χPT OF A NUCLEON DOUBLET AND A PION TRIPLET IN THE SPONTANEOUSLY BROKEN (I.E., CHIRAL) LIMIT

The chiral symmetry of two light quark flavors in QCD, together with the symmetry breaking and Goldstone's theorem, makes it possible to obtain an approximate solution to QCD at low energies using a $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ EFT, where the degrees of freedom are hadrons [6–13,59]. In particular, the nonlinear SU(2)χPT effective Lagrangian has been shown to successfully model the interactions of pions with nucleons, where a perturbation expansion (e.g., in soft momentum $|\vec{k}|/\Lambda_{\chi SB} \ll 1$, baryon number density $\frac{\langle N^\dagger N\rangle}{f_\pi^2 \Lambda_{\chi SB}} \ll 1$, for chiral symmetry breaking scale $\Lambda_{\chi SB} \approx 1$ GeV) has demonstrated predictive power. Such *naive* power-counting in $\Lambda_{\chi SB}^{-1}$ includes all analytic quantum-loop effects into experimentally measurable coefficients of $SU(2)_L \times SU(2)_R$ current-algebraic operators obedient to the global symmetries of QCD, with light-quark masses generating additional explicit chiral-symmetry-breaking terms. Therefore, $SU(2)_L \times SU(2)_R$ χPT tree-level calculations with a naive power-counting effective Lagrangian are to be regarded as true predictions of QCD and the standard model of elementary particles.

1. Nonlinear transformation properties

We present the Lagrangian of $SU(2)_L \times SU(2)_R$ χPT of a nucleon doublet and a pNGB triplet. We employ the defining SU(2) strong isospin representation of unitary 2×2 Pauli matrices τ_a, with asymmetric structure constants $f_{abc} = \epsilon_{abc}$:

$$\begin{aligned} t_a &= \frac{\tau_a}{2}, \quad a = 1, 3, \\ \mathrm{Tr}(t_a t_b) &= \frac{\delta_{ab}}{2}, \\ [t_a, t_b] &= i f_{abc} t_c, \\ \{t_a, t_b\} &= \frac{\delta_{ab}}{2}. \end{aligned} \tag{A1}$$

The vector and axial-vector charges obey the algebra

$$\begin{aligned} \left[Q_a^{L+R}, Q_b^{L+R}\right] &= i f_{abc} Q_c^{L+R}, \\ \left[Q_a^{L-R}, Q_b^{L-R}\right] &= i f_{abc} Q_c^{L+R}, \\ \left[Q_a^{L+R}, Q_b^{L-R}\right] &= i f_{abc} Q_c^{L-R}. \end{aligned} \tag{A2}$$

We consider a triplet representation of NGBs,

$$\pi_a t_a = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & \frac{\pi^0}{\sqrt{2}} \end{bmatrix} \tag{A3}$$

and a doublet of nucleons,

$$N = \begin{bmatrix} p \\ n \end{bmatrix}. \tag{A4}$$

For pedagogical simplicity, representations of higher mass are neglected, even though the $SU(2)_L \times SU(2)_R$ baryon decuplet (especially Δ_{1232}) is known to have important nuclear structure [1] and scattering [60] effects.

Since SU(2)χPT matrix elements are independent of representation [8,9], we choose a representation [12,13,59] where the NGB triplet has only derivative couplings,

$$\Sigma \equiv \exp\left(2i\pi_a \frac{t_a}{f_\pi}\right). \tag{A5}$$

Under a unitary global $SU(2)_L \times SU(2)_R$ transformation, given by $L \equiv \exp(i l_a t_a)$ and $R \equiv \exp(i r_a t_a)$,

$$\Sigma \rightarrow \Sigma' = L\Sigma R^\dagger. \tag{A6}$$

It also proves useful to introduce the "square root" of Σ,

$$\xi \equiv \exp\left(i\pi_a \frac{t_a}{f_\pi}\right), \tag{A7}$$

which transforms as

$$\xi \rightarrow \xi' = \exp\left(i\pi'_a \frac{t_a}{f_\pi}\right). \tag{A8}$$

We observe that

$$\xi' = L\xi U^\dagger = U\xi R^\dagger, \tag{A9}$$

for a certain unitary local transformation matrix $U(L, R, \pi_a(t, x))$.

The vector and axial-vector NGB currents

$$\begin{aligned} V_\mu &\equiv \tfrac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\ A_\mu &\equiv \tfrac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \end{aligned} \tag{A10}$$

transform straightforwardly as

$$\begin{aligned} V_\mu &\rightarrow V' = U V_\mu U^\dagger + U \partial_\mu U^\dagger, \\ A_\mu &\rightarrow A' = U A_\mu U^\dagger. \end{aligned} \tag{A11}$$

Meanwhile the nucleons transform as

$$N \rightarrow N' = U N \tag{A12}$$

and

$$D_\mu N \equiv \partial_\mu N + V_\mu N \rightarrow U(D_\mu N). \tag{A13}$$

2. Naive $\Lambda_{\chi SB}$ operator power counting

The SU(2)χPT Lagrangian, including all analytic quantum-loop effects for soft momenta ($\ll 1$ GeV), can be written as [12,59]

$$L_{\chi PT} = - \sum_{\substack{l,m,n \\ l+m \geqslant 1}} C_{lmn} f_\pi^2 \Lambda_{\chi SB}^2 \left(\frac{\partial_\mu}{\Lambda_{\chi SB}}\right)^m \left(\frac{\overline{N}N}{f_\pi^2 \Lambda_{\chi SB}}\right)^l \left(\frac{m_{\text{quark}}}{\Lambda_{\chi SB}}\right)^n f_{lmn}\left(\frac{\pi_a}{f_\pi}\right), \tag{A14}$$

where f_{lmn} is an analytic function, and the dimensionless constants C_{lmn} are $\mathcal{O}(\Lambda_{\chi SB}^0)$ and, presumably, ≈ 1. As a power series in $\Lambda_{\chi SB}$ we take, self-consistently, $\Lambda_{\chi SB} \simeq 1$ GeV and, in higher orders, reorder the nonrelativistic perturbation expansion in ∂_0 to converge with large nucleon mass $m^N \approx \Lambda_{\chi SB}$ [2,61,62].

3. The chiral symmetric limit

For the purposes of this paper, we retain from (A14) only terms of order $\Lambda_{\chi SB}$ and $\Lambda^0_{\chi SB}$, i.e., $1 \leqslant m+l+n \leqslant 2$. We can further divide $L_{\chi PT}$ into a symmetric piece (i.e., with spontaneous breaking and massless Goldstones) and a symmetry-breaking piece (i.e., explicit breaking, arising from nonzero quark masses) generating three massive pNGB:

$$L_{\chi PT} = L^{\text{Sym}}_{\chi PT} + L^{\text{Sym-Breaking}}_{\chi PT}. \tag{A15}$$

In this paper, we are interested only in unbroken SU(2)χPT and so take $n=0$ in (A14):

$$L^{\text{Sym-Breaking}}_{\chi PT} = 0. \tag{A16}$$

We separate $L^{\text{Sym}}_{\chi PT}$ into pure meson terms, terms quadratic in baryons (i.e., nucleons), and four-baryon terms:

$$L^{\text{Sym}}_{\chi PT} = L^{\pi;Sym}_{\chi PT} + L^{N;Sym}_{\chi PT} + L^{4\text{-}N;Sym}_{\chi PT} \tag{A17}$$

with [as in (1)]

$$\begin{aligned}
L^{\pi;Sym}_{\chi PT} &= \frac{f_\pi^2}{4}\mathrm{Tr}\partial_\mu\Sigma\partial^\mu\Sigma^\dagger,\\
L^{N;Sym}_{\chi PT} &= \overline{N}(i\gamma^\mu D_\mu - m^N\mathbb{1})N\\
&\quad - g_A\overline{N}\gamma^\mu\gamma^5 A_\mu N,\\
L^{4\text{-}N;Sym}_{\chi PT} &\sim \frac{1}{f_\pi^2}(\overline{N}\gamma^{\mathscr{A}}N)(\overline{N}\gamma_{\mathscr{A}}N) + ++.
\end{aligned} \tag{A18}$$

As described below (1), the parentheses in the four-nucleon Lagrangian indicate the order of SU(2) index contraction, and $+++$ indicates that one should include all possible combinations of such contractions. As usual, $\gamma^{\mathscr{A}} \equiv (1, \gamma^\mu, i\sigma^{\mu\nu}, i\gamma^\mu\gamma^5, \gamma^5)$, for $\mathscr{A} = 1, \dots, 16$ (with $\sigma^{\mu\nu} \equiv \frac{1}{2}[\gamma^\mu, \gamma^\nu]$). These are commonly referred to as scalar (S), vector (V), tensor (T), axial-vector (A), and pseudoscalar (P) respectively.

In this paper, we will focus on the *semiclassical symmetries* of chiral (i.e., spontaneously broken) SU(2)χPT. Nucleons are treated as quantized fermions. Pions are classical fields; i.e., ξ, V_μ, A_μ, U, Σ, π_a, R, L defined in Sec. A 1 are not quantized: their nontrivial commutation properties are entirely due to strong isospin.

4. SU(2)$_L$ × SU(2)$_R$ invariant four-nucleon contact interactions

We focus on the four-fermion terms in (A18). We use the completeness relation for 2×2 matrices:

$$\begin{aligned}
\delta_{cf}\delta_{ed} &= 2\sum_{B=0}^{3} t^B_{cd}t^B_{ef},\\
[\vec{t},\, U(\vec{\pi}(x), r, l)] &\neq 0,
\end{aligned} \tag{A19}$$

with $t^B = (\frac{1}{2}I, \vec{t})$. (We use Greek letters for relativistic spinor indices, and Roman letters for isospin indices.) Both isoscalar and isovector four-nucleon contact interactions appear in the SU(2)$_L$ × SU(2)$_R$ invariant Lagrangian:

$$\begin{aligned}
L^{4\text{-}N;Sym}_{\chi PT} &= \frac{C^{T=0}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_a\gamma^{\mathscr{A}\alpha\beta}N^\beta_a\big)\big(\overline{N}^\lambda_b\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_b\big)\\
&\quad + \frac{C^{T=1}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_a\gamma^{\mathscr{A}\alpha\beta}N^\beta_b\big)\big(\overline{N}^\lambda_b\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_a\big)\\
&= \frac{C^{T=0}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_c U^\dagger_{ca}\gamma^{\mathscr{A}\alpha\beta}U_{ad}N^\beta_d\big)\big(\overline{N}^\lambda_e U^\dagger_{eb}\gamma^{\lambda\sigma}_{\mathscr{A}}U_{bf}N^\sigma_f\big)\\
&\quad + \frac{C^{T=1}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_c U^\dagger_{ca}\gamma^{\mathscr{A}\alpha\beta}U_{bd}N^\beta_d\big)\big(\overline{N}^\lambda_e U^\dagger_{eb}\gamma^{\lambda\sigma}_{\mathscr{A}}U_{af}N^\sigma_f\big)\\
&= \frac{C^{T=0}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_c\gamma^{\mathscr{A}\alpha\beta}N^\beta_c\big)\big(\overline{N}^\lambda_e\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_e\big)\\
&\quad + \frac{C^{T=1}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_c\gamma^{\mathscr{A}\alpha\beta}N^\beta_d\big)\big(\overline{N}^\lambda_e\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_f\big)\delta_{cf}\delta_{ed}\\
&= \frac{C^{T=0}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_c\gamma^{\mathscr{A}\alpha\beta}N^\beta_c\big)\big(\overline{N}^\lambda_e\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_e\big)\\
&\quad + 2\sum_{B=0}^{3}\frac{C^{T=1}_{\mathscr{A}}}{f_\pi^2}\big(\overline{N}^\alpha_c t^B_{cd}\gamma^{\mathscr{A}\alpha\beta}N^\beta_d\big)\big(\overline{N}^\lambda_e t^B_{ef}\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_f\big).
\end{aligned} \tag{A20}$$

APPENDIX B: FOUR-NUCLEON CONTACT INTERACTIONS IN STATICχNLs

1. Boson-exchange-inspired four-nucleon contact interactions

We wish to study the expectation value of $L^{4\text{-}N;Sym}_{\chi PT}$ in the ground state of the chiral nuclear liquid (which we continue to represent with $\langle\,\rangle$). Using (A20) we find

$$\begin{aligned}
-L^{BE}_{S\chi NL} &\equiv \big\langle -L^{4\text{-}N;Sym}_{\chi PT}\big\rangle\\
&= \sum_{\mathscr{A}}\frac{C^{T=0}_{\mathscr{A}}}{2f_\pi^2}\big\langle\overline{N}^\alpha_c\gamma^{\mathscr{A}\alpha\beta}N^\beta_c\overline{N}^\lambda_e\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_e\big\rangle\\
&\quad + \sum_{\mathscr{A}B}\frac{C^{T=1}_{\mathscr{A}}}{f_\pi^2}\big\langle\overline{N}^\alpha_c t^B_{cd}\gamma^{\mathscr{A}\alpha\beta}N^\beta_d\overline{N}^\lambda_e t^B_{ef}\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_f\big\rangle.
\end{aligned} \tag{B1}$$

In what follows we ignore any and all excited states and consider the effective Lagrangian:

$$\begin{aligned}
-L^{BE}_{S\chi NL} &= \frac{1}{2f_\pi^2}\sum_{\mathscr{A}}\\
&\quad\times\Big\{C^{T=0}_{\mathscr{A}}\big\langle\overline{N}^\alpha_c\gamma^{\mathscr{A}\alpha\beta}N^\beta_c\big\rangle\big\langle\overline{N}^\lambda_e\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_e\big\rangle\\
&\quad + 2\sum_B C^{T=1}_{\mathscr{A}}\big\langle\overline{N}^\alpha_c t^B_{cd}\gamma^{\mathscr{A}\alpha\beta}N^\beta_d\big\rangle\\
&\quad\times\big\langle\overline{N}^\lambda_e t^B_{ef}\gamma^{\lambda\sigma}_{\mathscr{A}}N^\sigma_f\big\rangle\Big\}.
\end{aligned} \tag{B2}$$

A useful identity is

$$\begin{aligned}
&\tfrac{1}{4}\langle \overline{N}_c^{\alpha}\,\gamma^{\mathscr{A}\alpha\beta} N_c^{\beta}\rangle\langle \overline{N}_e^{\lambda}\,\gamma_{\mathscr{A}}^{\lambda\sigma} N_e^{\sigma}\rangle \\
&\quad + \langle \overline{N}_c^{\alpha}\, t_{3;cd}\,\gamma^{\mathscr{A}\alpha\beta} N_d^{\beta}\rangle\langle \overline{N}_e^{\lambda}\, t_{3;ef}\,\gamma_{\mathscr{A}}^{\lambda\sigma} N_f^{\sigma}\rangle \\
&= \tfrac{1}{2}\langle \overline{p}_c^{\alpha}\,\gamma^{\mathscr{A}\alpha\beta} p_c^{\beta}\rangle\langle \overline{p}_e^{\lambda}\,\gamma_{\mathscr{A}}^{\lambda\sigma} p_e^{\sigma}\rangle \\
&\quad + \tfrac{1}{2}\langle \overline{n}_c^{\alpha}\,\gamma^{\mathscr{A}\alpha\beta} n_c^{\beta}\rangle\langle \overline{n}_e^{\lambda}\,\gamma_{\mathscr{A}}^{\lambda\sigma} n_e^{\sigma}\rangle. \qquad \text{(B3)}
\end{aligned}$$

2. Contact interactions that mimic hadronic boson exchange

Taking expectation values inside the StaticχNL as in (B2), we obtain

$$-L_{S\chi NL}^{BE} \simeq \frac{1}{2f_\pi^2}\left(L_S^{T=0} + L_V^{T=0} + L_S^{T=1} + L_V^{T=1}\right), \qquad \text{(B4)}$$

where

$$\begin{aligned}
L_S^{T=0} &= C_S^{T=0}\langle \overline{N}_c^{\alpha}\, N_c^{\alpha}\rangle\langle \overline{N}_e^{\lambda}\, N_e^{\lambda}\rangle, \\
L_V^{T=0} &= C_V^{T=0}\langle \overline{N}_c^{\alpha}\,\gamma^{0;\alpha\beta} N_c^{\beta}\rangle\langle \overline{N}_e^{\lambda}\,\gamma_0^{\lambda\sigma} N_e^{\sigma}\rangle, \\
L_V^{T=1} &= 2\, C_S^{T=1}\big\{\tfrac{1}{4}\langle \overline{N}_c^{\alpha}\, N_c^{\alpha}\rangle\langle \overline{N}_e^{\lambda}\, N_e^{\lambda}\rangle \\
&\quad + \langle \overline{N}_c^{\alpha}\, t_{3;cd}\, N_d^{\alpha}\rangle\langle \overline{N}_e^{\lambda}\, t_{3;ef}\, N_f^{\lambda}\rangle\big\}, \\
L_S^{T=1} &= 2\, C_V^{T=1}\big\{\tfrac{1}{4}\langle \overline{N}_c^{\alpha}\,\gamma^{0;\alpha\beta}\, N_c^{\beta}\rangle\langle \overline{N}_e^{\lambda}\,\gamma_0^{\lambda\sigma}\, N_e^{\sigma}\rangle \\
&\quad + \langle \overline{N}_c^{\alpha}\, t_{3;cd}\gamma^{0;\alpha\beta}\, N_d^{\beta}\rangle\langle \overline{N}_e^{\lambda}\, t_{3;ef}\,\gamma_0^{\lambda\sigma} N_f^{\sigma}\rangle\big\}. \qquad \text{(B5)}
\end{aligned}$$

The factorization in $L_{S\chi NL}^{BE}$, and its name, are inspired by a simple picture of forces carried by heavy hadronic-boson exchange, which is commonly envisioned in Walecka-like, nuclear-Skyrme and density-functional models; i.e., we have integrated out the auxiliary fields

(i) Lorentz-scalar isoscalar σ, with chiral coefficient $C_S^{T=0}$,
(ii) Lorentz-vector isoscalar ω_μ, with chiral coefficient $C_V^{T=0}$,
(iii) Lorentz-scalar isovector $\vec{\delta}$, with chiral coefficient $C_S^{T=1}$,
(iv) Lorentz-vector isovector $\vec{\rho}_\mu$, with chiral coefficient $C_V^{T=1}$.

To order $\Lambda_{\chi SB}^0$, the only four-nucleon contact terms allowed by local SU(2)χPT symmetry are exhibited in (B2) and (B 2). Note that isospin operators $\vec{t} = \frac{1}{2}\vec{\tau}$ have appeared. However, quantum-loop naive power counting requires inclusion of nucleon Lorentz-spinor-interchange interactions, in order to enforce antisymmetrization of fermion wave functions. These are the same order as direct interactions, i.e., $\mathcal{O}(\Lambda_{\chi SB}^0)$. The empirical nuclear models of Manakos and Mannel [63,64] were specifically built to include such spinor-interchange terms.

Explicit inclusion of spinor-interchange terms yields a great technical advantage for the liquid approximation: it allows us to treat StaticχNLs in Hartree-Fock approximation, i.e., including fermion wave function anti-symmetrization, rather than in less-accurate Hartree approximation.

3. Contact-interactions, including spinor-interchange terms enforcing effective antisymmetrization of fermion wave functions in the Hartree-Fock approximation

In this section, we write an effective StaticχNL Lagrangian for the four-nucleon contact interactions in terms of the ten independent chiral coefficients: C_K^T with $K \in \{S, V, T, A, P\}$ and $T \in \{0, 1\}$.

For pedagogical simplicity, we first focus on the "boson-exchange-inspired" terms, with power-counting contact-interactions of order ($\Lambda_{\chi SB}^0$). "Direct" terms depend only on $C_S^{T=0}, C_V^{T=0}, C_S^{T=1}$, and $C_V^{T=1}$, because isoscalar ($C_T^{T=0}$, $C_A^{T=0}$, and $C_P^{T=0}$) and isovector ($C_T^{T=1}$, $C_A^{T=1}$, $C_P^{T=1}$) vanish when evaluated in the liquid. "Spinor-interchange" terms depend on all ten coefficients after Fierz rearrangement. [Such terms do not appear in the SU(2)χPT analysis of the deuteron ground state, because it only has one proton and one neutron.] The combination of direct and spinor-interchange terms (which we refer to below as "Total") depends on all ten coefficients.

Because of the inclusion of spinor exchange terms, Hartree treatment of the StaticχNL Lagrangian is equivalent to Hartree-Fock treatment of the liquid. When building the semiclassical liquid quantum state, this enforces the anti-symmetrization of the fermion wave functions. A crucial observation is that the resultant liquid depends on only four independent chiral coefficients: $\mathcal{C}_S^2$, $\mathcal{C}_V^2$, $\overline{\mathcal{C}_S^2}$, and $\overline{\mathcal{C}_V^2}$. These provide sufficient free parameters to balance the scalar attractive force carried by $\mathcal{C}_S^2$ and $\overline{\mathcal{C}_S^2}$ against the vector repulsive force carried by $\mathcal{C}_V^2$ and $\overline{\mathcal{C}_V^2}$ when fitting to the experimentally observed structure of ground state nuclei [as reflected, e.g., in the different signs in definitions of $\mathcal{C}_V^2$ and $\mathcal{C}_S^2$ in (25) and (26)].

Motivated by the empirical success of nontopological soliton models we conjecture that excited-nucleon-inspired contact-interaction terms are small, and that the simple picture of scalar attraction balanced against vector repulsion persists when including them. Such analysis is beyond the scope of this paper.

a. *Lorentz vector (V) and axial-vector (A) forces*

Proceeding in a similar manner for the vector and axial vector terms we find

$$-L_{S\chi NL}^{V,A} \equiv -\langle L^{4\text{-}N;V,A}\rangle = \frac{1}{2f_\pi^2}\sum_{\mathscr{A}=V,A}\left\{C_{\mathscr{A}}^{T=0}\langle \overline{N}_c^{\alpha}\,\gamma^{\mathscr{A}\alpha\beta}\, N_c^{\beta}\rangle\langle \overline{N}_e^{\lambda}\,\gamma_{\mathscr{A}}^{\lambda\sigma}\, N_e^{\sigma}\rangle + 2\sum_B C_{\mathscr{A}}^{T=1}\langle \overline{N}_c^{\alpha}\, t_{cd}^B\,\gamma^{\mathscr{A}\alpha\beta}\, N_d^{\beta}\rangle\langle \overline{N}_e^{\lambda}\, t_{ef}^B\,\gamma_{\mathscr{A}}^{\lambda\sigma}\, N_f^{\sigma}\rangle\right\}, \qquad \text{(B6)}$$

which is

$$-L^{V,A}_{S\chi NL}=\frac{1}{2f_\pi^2}\sum_{\mathscr{A}=V,A}\{2C^{T=0}_{\mathscr{A}}\langle\overline{p}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,p^\beta_c\rangle\langle\overline{n}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,n^\sigma_e\rangle+[C^{T=0}_{\mathscr{A}}+C^{T=1}_{\mathscr{A}}][\langle\overline{p}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,p^\beta_c\rangle\langle\overline{p}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,p^\sigma_e\rangle+\langle\overline{n}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,n^\beta_c\rangle\langle\overline{n}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,n^\sigma_e\rangle]\}. \tag{B7}$$

Direct terms. The properties of StaticχNLs enable this expression to be written as

$$-L^{V,A}_{S\chi NL;D}=\frac{1}{2f_\pi^2}C^{T=0}_V\{2\langle p^\dagger p\rangle\langle n^\dagger n\rangle\} + \frac{1}{2f_\pi^2}[C^{T=0}_V+C^{T=1}_V]\{\langle p^\dagger p\rangle^2+\langle n^\dagger n\rangle^2\}, \tag{B8}$$

where $\langle p^\dagger p\rangle$ and $\langle n^\dagger n\rangle$ represent $\langle p^{\alpha\dagger}_c p^\alpha_c\rangle$ and $\langle n^{\lambda\dagger}_e n^\lambda_e\rangle$, respectively.

Spinor-interchange terms. After interchanging the appropriate spinors, normal ordering creation and annihilation operators, and Fierz re-arrangement, spinor-interchange contributions depend on $C^{T=0}_V$, $C^{T=0}_A$, $C^{T=1}_V$, and $C^{T=1}_A$:

$$-L^{V,A}_{S\chi NL;Ex}=\frac{1}{2f_\pi^2}\times[-(C^{T=0}_V+C^{T=1}_V)+(C^{T=0}_A+C^{T=1}_A)] \times\{\langle p^\dagger_L p_L\rangle^2+\langle p^\dagger_R p_R\rangle^2+\langle n^\dagger_L n_L\rangle^2+\langle n^\dagger_R n_R\rangle^2\}, \tag{B9}$$

where we have expanded the spinors p and n into left-handed and right-handed components via $p=p_L+p_R$ and $n=n_L+n_R$.

Total direct and spinor-interchange terms. Combining the direct and exchange terms yields:

$$\begin{aligned}-L^{V,A}_{S\chi NL;\mathrm{Total}}&=\frac{1}{f_\pi^2}C^{T=0}_V\{\langle p^\dagger p\rangle\langle n^\dagger n\rangle\}\\&+\frac{C^0_V+C^1_V}{f_\pi^2}\{\langle p^\dagger_L p_L\rangle\langle p^\dagger_R p_R\rangle+\langle n^\dagger_L n_L\rangle\langle n^\dagger_R n_R\rangle\}\\&+\frac{C^0_A+C^1_A}{2f_\pi^2}\sum_{h=L,R}\{\langle p^\dagger_h p_h\rangle^2+\langle n^\dagger_h n_h\rangle^2\}.\end{aligned} \tag{B10}$$

The reader should note the cancellation of the term

$$\frac{(C^{T=0}_V+C^{T=1}_V)}{2f_\pi^2}\sum_{h=L,R}\{\langle p^\dagger_h p_h\rangle^2+\langle n^\dagger_h n_h\rangle^2\}, \tag{B11}$$

showing that vector-boson exchange cannot carry forces between same-handed fermion protons, or between same-handed fermion neutrons.

Significant simplification follows because StaticχNLs are defined to have equal left-handed and right-handed densities; i.e.,

$$\begin{aligned}\langle p^\dagger_L p_L\rangle&=\langle p^\dagger_R p_R\rangle=\tfrac{1}{2}\langle p^\dagger p\rangle,\\ \langle n^\dagger_L n_L\rangle&=\langle n^\dagger_R n_R\rangle=\tfrac{1}{2}\langle n^\dagger n\rangle.\end{aligned} \tag{B12}$$

Using (25) the contribution of (B10) to the Lorentz-spinor-interchange Lagrangian can be written as

$$-L^{V,A}_{S\chi NL;\mathrm{Total}}=\tfrac{1}{2}\mathcal{C}^2_V\langle N^\dagger N\rangle^2-\overline{\mathcal{C}^2_V}\langle N^\dagger t_3 N\rangle^2 \tag{B13}$$

with

$$\begin{aligned}C^V_{200}&=C^{T=0}_V,\\ -\overline{C^V_{200}}&=\tfrac{1}{2}[-C^{T=0}_V+C^{T=0}_A+C^{T=1}_V+C^{T=1}_A].\end{aligned} \tag{B14}$$

The crucial observation is that (B13) and (B14) depend on just *two* independent chiral coefficients, $\mathcal{C}^2_S$ and $\overline{\mathcal{C}^2_V}$, (or equivalently C^V_{200} and $\overline{C^V_{200}}$), instead of four, while still providing sufficient free parameters to fit the vector repulsive force (i.e., within nontopological soliton, density functional, and Skyrme nuclear models), up to naive power-counting order ($\Lambda^0_{\chi SB}$), to the experimentally observed structure of ground state nuclei.

b. Lorentz scalar (S), tensor (T), and pseudoscalar (P) forces

Proceeding in a similar manner we define

$$L^{STP}_{S\chi NL}\equiv\langle L^{4\text{-}N;STP}_{\chi PT}\rangle \tag{B15}$$

with

$$-L^{STP}_{S\chi NL}=\frac{1}{2f_\pi^2}\sum_{\mathscr{A}=S,T,P}\times\Big\{C^{T=0}_{\mathscr{A}}\langle\overline{N}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,N^\beta_c\rangle\langle\overline{N}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,N^\sigma_e\rangle + 2\sum_B C^{T=1}_{\mathscr{A}}\langle\overline{N}^\alpha_c\,t^B_{cd}\gamma^{\mathscr{A}\alpha\beta}\,N^\beta_d\rangle\langle\overline{N}^\lambda_e\,t^B_{ef}\gamma^{\lambda\sigma}_{\mathscr{A}}\,N^\sigma_f\rangle\Big\} \tag{B16}$$

which is

$$\begin{aligned}-L^{STP}_{S\chi NL}&=\frac{1}{2f_\pi^2}\\&\times\sum_{\mathscr{A}=S,T,P}\{2\,C^{T=0}_{\mathscr{A}}\langle\overline{p}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,p^\beta_c\rangle\langle\overline{n}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,n^\sigma_e\rangle\\&+[C^{T=0}_{\mathscr{A}}+C^{T=1}_{\mathscr{A}}][\langle\overline{p}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,p^\beta_c\rangle\langle\overline{p}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,p^\sigma_e\rangle\\&+\langle\overline{n}^\alpha_c\,\gamma^{\mathscr{A}\alpha\beta}\,n^\beta_c\rangle\langle\overline{n}^\lambda_e\,\gamma^{\lambda\sigma}_{\mathscr{A}}\,n^\sigma_e\rangle]\}\end{aligned} \tag{B17}$$

Direct terms. The properties of StaticχNLs give

$$-L^{STP}_{S\chi NL;D}=\frac{1}{2f_\pi^2}\,C^{T=0}_S\langle\overline{N}N\rangle\langle\overline{N}N\rangle + \frac{1}{2f_\pi^2}\,C^{T=1}_S(\langle\overline{p}p\rangle\langle\overline{p}p\rangle+\langle\overline{n}n\rangle\langle\overline{n}n\rangle). \tag{B18}$$

Spinor-interchange terms. Spinor-interchange contributions depend on six chiral coefficients: isoscalars $C^{T=0}_S$, $C^{T=0}_T$, $C^{T=0}_P$ and isovectors $C^{T=1}_S$, $C^{T=1}_T$, $C^{T=1}_P$:

$$\begin{aligned}-L^{STP}_{\chi NL;Ex}&=\frac{1}{4f_\pi^2}[(C^{T=0}_S+C^{T=1}_S)\\&+6(C^{T=0}_T+C^{T=1}_T)+(C^{T=0}_P+C^{T=1}_P)]\\&\times\{\langle\overline{p}_L p_R\rangle^2+\langle\overline{p}_R p_L\rangle^2+\langle\overline{n}_L n_R\rangle^2+\langle\overline{n}_R n_L\rangle^2\}\end{aligned} \tag{B19}$$

Total direct and spinor-interchange terms. As above, since StaticχNLs have equal left-handed and right-handed scalar densities by definition, the total direct and spinor-interchange contribution is considerably simplified:

$$-L^{STP}_{S\chi NL;\text{Total}} = -\tfrac{1}{2}\mathcal{C}_S^2\langle\overline{N}N\rangle^2 - \overline{\mathcal{C}_S^2}\langle\overline{N}t_3N\rangle^2, \quad \text{(B20)}$$

where in (26) and (28) we have

$$\begin{aligned} C^S_{200} &= C_S^{T=0}, \\ -\overline{C^S_{200}} &= \tfrac{1}{2}\big[\tfrac{1}{2}C_S^{T=0} + \tfrac{5}{2}C_S^{T=1} \\ &\quad + 3\left(C_T^{T=0} + C_T^{T=1}\right) + \left(C_P^{T=0} + C_P^{T=1}\right)\big]. \end{aligned} \quad \text{(B21)}$$

Once again we find that (B20) and (B21) depend on just *two* independent chiral coefficients, C^S_{200} and $\overline{C^S_{200}}$, instead of six, while still providing sufficient free parameters to fit the scalar attractive force (i.e., within nontopological soliton, density functional, and Skyrme nuclear models), up to naive power-counting order $\Lambda^0_{\chi SB}$, to the experimentally observed structure of ground state nuclei.

APPENDIX C: NUCLEON BILINEARS AND SEMICLASSICAL NUCLEAR CURRENTS IN STATICχNLs

The structure of StaticχNLs suppresses various nucleon bilinears:

(i) Vectors' space components: because it is a three-vector, parity odd, and stationary,

$$\left\langle \overline{N}^\alpha_c\, \vec{\gamma}^{\alpha\beta} N^\beta_c \right\rangle \sim \langle\vec{k}\rangle \simeq 0. \quad \text{(C1)}$$

(ii) Tensors: because the local expectation value of nuclear spin $\langle\vec{s}\rangle = \frac{1}{2}\langle\vec{\sigma}\rangle \simeq 0$,

(1) σ^{0j}:

$$\begin{aligned} \left\langle \overline{N}^\alpha_c\, \sigma^{0j;\alpha\beta}\, N^\beta_c \right\rangle &= \left\langle \overline{N}_L\, \sigma^{0j} N_R \right\rangle + \left\langle \overline{N}_R\, \sigma^{0j} N_L \right\rangle \\ &= 2\left\langle \overline{N}_L \begin{bmatrix} 0 & \vec{s}_j \\ \vec{s}_j & 0 \end{bmatrix} N_R \right\rangle \\ &\quad + 2\left\langle \overline{N}_R \begin{bmatrix} 0 & \vec{s}_j \\ \vec{s}_j & 0 \end{bmatrix} N_L \right\rangle \\ &\simeq 0; \end{aligned} \quad \text{(C2)}$$

(2) σ^{ij}:

$$\begin{aligned} \left\langle \overline{N}^\alpha_c\, \sigma^{ij;\alpha\beta} N^\beta_c \right\rangle &= \langle \overline{N}_L\, \sigma^{ij} N_R\rangle + \langle \overline{N}_R\, \sigma^{ij} N_L\rangle \\ &= -2i\epsilon_{ijk}\langle \overline{N}_L\, \vec{s}_k N_R\rangle \\ &\quad - 2i\epsilon_{ijk}\langle \overline{N}_R\, \vec{s}_k N_L\rangle \simeq 0. \end{aligned} \quad \text{(C3)}$$

(iii) Axial vectors: because p_L, p_R are equally represented in StaticχNLs, as are n_L, n_R,

$$\begin{aligned} \left\langle \overline{N}^\alpha_c\, \gamma^{A;\alpha\beta} N^\beta_c \right\rangle &= \langle \overline{N}_L\, \gamma^\mu\gamma^5 N_L\rangle + \langle \overline{N}_R\, \gamma^\mu\gamma^5 N_R\rangle \\ &= -\langle \overline{N}_L\, \gamma^\mu N_L\rangle + \langle \overline{N}_R\, \gamma^\mu N_R\rangle \\ &\simeq 0. \end{aligned} \quad \text{(C4)}$$

(iv) Pseudoscalars: because StaticχNLs are of even parity,

$$\begin{aligned} \left\langle \overline{N}^\alpha_c\, \gamma^{P;\alpha\beta} N^\beta_c \right\rangle &= \langle \overline{N}_R\, \gamma^5 N_L\rangle + \langle \overline{N}_L\, \gamma^5 N_R\rangle \\ &= -\langle \overline{N}_R\, N_L\rangle + \langle \overline{N}_L\, N_R\rangle \\ &\simeq 0. \end{aligned} \quad \text{(C5)}$$

Therefore, various Lorentz and isospin contributions are suppressed in StaticχNLs. In summary, for isoscalars,

$$\begin{aligned} \left\langle \overline{N}^\alpha_c\, N^\alpha_c \right\rangle &\neq 0, \\ \left\langle \overline{N}^\alpha_c\, \gamma^{0;\alpha\beta} N^\beta_c \right\rangle &\neq 0, \\ \left\langle \overline{N}^\alpha_c\, \vec{\gamma}^{\alpha\beta} N^\beta_c \right\rangle &\simeq 0, \\ \left\langle \overline{N}^\alpha_c\, \gamma^{T;\alpha\beta} N^\beta_c \right\rangle &\simeq 0, \\ \left\langle \overline{N}^\alpha_c\, \gamma^{A;\alpha\beta} N^\beta_c \right\rangle &\simeq 0, \\ \left\langle \overline{N}^\alpha_c\, \gamma^{P;\alpha\beta} N^\beta_c \right\rangle &\simeq 0, \end{aligned} \quad \text{(C6)}$$

and for isovectors,

$$\begin{aligned} \left\langle \overline{N}^\alpha_c\, t^\pm_{cd}\gamma^{\mathscr{A}\alpha\beta} N^\beta_d \right\rangle &= 0, \\ \left\langle \overline{N}^\alpha_c\, t^3_{cd} N^\alpha_d \right\rangle &\neq 0, \\ \left\langle \overline{N}^\alpha_c\, t^3_{cd}\gamma^{0;\alpha\beta} N^\beta_d \right\rangle &\neq 0, \\ \left\langle \overline{N}^\alpha_c\, t^3_{cd}\vec{\gamma}^{\alpha\beta} N^\beta_d \right\rangle &\simeq 0, \\ \left\langle \overline{N}^\alpha_c\, t^3_{cd}\gamma^{T\alpha\beta} N^\beta_d \right\rangle &\simeq 0, \\ \left\langle \overline{N}^\alpha_c\, t^3_{cd}\gamma^{A\alpha\beta} N^\beta_d \right\rangle &\simeq 0, \\ \left\langle \overline{N}^\alpha_c\, t^3_{cd}\gamma^{P\alpha\beta} N^\beta_d \right\rangle &\simeq 0 \end{aligned} \quad \text{(C7)}$$

Now we form the semiclassical nuclear currents

$$\begin{aligned} J^\mu_k &= \overline{N}\,\gamma^\mu t_k N, \quad k = 1, 2, 3, \\ J^\mu_\pm &= J^\mu_1 \pm iJ^\mu_2 = \begin{Bmatrix} \overline{p}\gamma^\mu n \\ \overline{n}\gamma^\mu p \end{Bmatrix}, \\ J^\mu_3 &= \frac{1}{2}(\overline{p}\gamma^\mu p - \overline{n}\gamma^\mu n), \\ J^\mu_8 &= \frac{\sqrt{3}}{2}(\overline{p}\gamma^\mu p + \overline{n}\gamma^\mu n), \\ J^\mu_{QED} &= \frac{1}{\sqrt{3}}J^\mu_8 + J^\mu_3 = \overline{p}\gamma^\mu p \\ J^\mu_{\text{Baryon}} &= \frac{2}{\sqrt{3}}J^\mu_8 = \overline{p}\gamma^\mu p + \overline{n}\gamma^\mu n, \\ J^{5\mu}_k &= \overline{N}\gamma^\mu\gamma^5 t_k N, \quad k = 1, 2, 3, \\ J^{5\mu}_\pm &= J^{5\mu}_1 \pm iJ^{5\mu}_2 = \begin{Bmatrix} \overline{p}\gamma^\mu\gamma^5 n \\ \overline{n}\gamma^\mu\gamma^5 p \end{Bmatrix}, \\ J^{5\mu}_3 &= \frac{1}{2}(\overline{p}\gamma^\mu\gamma^5 p - \overline{n}\gamma^\mu\gamma^5 n), \\ J^{5\mu}_8 &= \frac{\sqrt{3}}{2}(\overline{p}\gamma^\mu\gamma^5 p + \overline{n}\gamma^\mu\gamma^5 n). \end{aligned} \quad \text{(C8)}$$

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ nuclear currents within StaticχNLs are obedient to its semiclassical symmetries. Thus we have

$$\langle J^{\mu}_{\pm}\rangle = \langle J^{\mu,5}_{\pm}\rangle = \langle \partial_\mu J^{\mu}_{\pm}\rangle = \langle \partial_\mu J^{\mu,5}_{\pm}\rangle = 0 \tag{C9}$$

and

$$\begin{aligned} \langle \partial_\mu J^{\mu}_3\rangle, \langle J^{\mu,5}_3\rangle, \langle J^{\mu,5}_8\rangle &\simeq 0,\\ \langle J^{\mu=1,2,3}_3\rangle, \langle J^{\mu=1,2,3}_8\rangle &\simeq 0,\\ \langle \partial_\mu J^{\mu}_8\rangle, \langle \partial_\mu J^{\mu}_{\mathrm{Baryon}}\rangle, \langle \partial_\mu J^{\mu}_{QED}\rangle &\simeq 0,\\ \langle J^{\mu=1,2,3}_{\mathrm{Baryon}}\rangle, \langle J^{\mu=1,2,3}_{QED}\rangle &\simeq 0, \end{aligned} \tag{C10}$$

$$\begin{aligned} \frac{1}{\sqrt{3}}\langle \partial_\mu J^{\mu,5}_8\rangle &\propto \langle (m^N + \widehat{C^S_{200}})\gamma^5\rangle \sim \eta \simeq 0,\\ \tfrac{1}{2}\langle \partial_\mu J^{\mu,5}_3\rangle &\propto \langle (m^N + \widehat{C^S_{200}})\gamma^5 t_3\rangle \sim \pi_3 \simeq 0. \end{aligned} \tag{C11}$$

The remaining nonzero contributions to the currents are

$$\begin{aligned} \langle J^0_{\mathrm{Baryon}}\rangle &\neq 0,\\ \langle J^0_3\rangle &\neq 0,\\ \langle J^0_8\rangle &\neq 0,\\ \langle J^0_{QED}\rangle &\neq 0. \end{aligned} \tag{C12}$$

APPENDIX D: THOMAS-FERMI NONTOPOLOGICAL SOLITONS AND THE SEMIEMPIRICAL MASS FORMULA

We are interested here in semiclassical solutions to (31), identifiable as quantum chiral nucleon liquids, that are, for reasons laid out in the main body of the paper, in the ground state, spin zero, spherically symmetric, and even-even (i.e., have an even number of protons and of neutrons). We employ relativistic mean-field point-coupling Hartree-Fock and Thomas-Fermi approximations, ignoring the antinucleon sea.

We seek solutions that are static, homogeneous, and isotropic. Given the absence of any surface terms at the order $\Lambda^0_{\chi SB}$ in chiral symmetry breaking to which we are working, we avoid the *ad hoc* imposition of such terms. We therefore impose the condition that the pressure vanishes everywhere, rather than just at the surface of a finite "liquid drop." Our finite StaticχNL nuclei therefore resemble "ice cream balls" scooped from an infinite vat [65], more than they do conventional liquid drops (which have surface tension).

The Thomas-Fermi approximation replaces the neutrons and protons with homogeneous and isotropic expectation values over free neutron and proton spinors, with (for $j = n$ and p) effective reduced mass m^j_*, three-momentum $\vec{k}^j$, energy $E^j = \sqrt{(\vec{k}^j)^2 + (m^j_*)^2}$, and zero spin. Most of these vanish because of the absence of any preferred direction for spin or momenta in StaticχNLs:

$$\begin{aligned} \bar{n}n &\to \langle \bar{n}n\rangle = \frac{m^n_*}{E_n},\\ \bar{n}(\gamma^0,\vec{\gamma})n &\to \langle \bar{n}(\gamma^0,\vec{\gamma})n\rangle = (1,\vec{0}),\\ \bar{n}(\sigma^{0j},\sigma^{ij})n &\to \langle \bar{n}(\sigma^{0j},\sigma^{ij})n\rangle = 0,\\ \bar{n}(\gamma^0,\vec{\gamma})\gamma^5 n &\to \langle \bar{n}(\gamma^0,\vec{\gamma})\gamma^5 n\rangle = 0,\\ \bar{n}\gamma^5 n &\to \langle \bar{n}\gamma^5 n\rangle = 0, \end{aligned} \tag{D1}$$

and similarly for the proton. To simplify our notation, we drop the $\langle\cdots\rangle$ in the remainder of this Appendix.

Within the liquid drop, the baryon number density

$$N^\dagger N = p^\dagger p + n^\dagger n \tag{D2}$$

and scalar density

$$\bar{N}N = \bar{p}p + \bar{n}n. \tag{D3}$$

The neutron contributions to these densities are

$$\begin{aligned} n^\dagger n &= 2\int_0^{k^n_F} \frac{d^3k}{(2\pi)^3} = \frac{(k^n_F)^3}{3\pi^2},\\ \bar{n}n &= 2\int_0^{k^n_F} \frac{d^3k}{(2\pi)^3}\frac{m^n_*}{\sqrt{k^2+(m^n_*)^2}},\\ &= \frac{m^n_*}{2\pi^2}\left[k^n_F\mu^n_* - \frac{1}{2}(m^n_*)^2\ln\left(\frac{\mu^n_*+k^n_F}{\mu^n_*-k^n_F}\right)\right], \end{aligned} \tag{D4}$$

with

$$\begin{aligned} m^n_* &\equiv m_n + C^2_S\bar{N}N - \tfrac{1}{2}\overline{C^S_{200}}(\bar{n}n - \bar{p}p),\\ \mu^n_* &\equiv \sqrt{(k^n_F)^2 + (m^n_*)^2}. \end{aligned} \tag{D5}$$

The equivalent proton contributions are obtained by straightforward substitution of $n \leftrightarrow p$.

It is convenient to define

$$\begin{aligned} \epsilon^{\int n} &\equiv 2\int_0^{k^n_F}\frac{d^3k}{(2\pi)^3}\sqrt{k^2+(m^n_*)^2},\\ &= \frac{3}{4}\mu^n_* n^\dagger n + \frac{1}{4}m^n_*\bar{n}n, \end{aligned} \tag{D6}$$

$$\begin{aligned} P^{\int n} &\equiv 2\int_0^{k^n_F}\frac{d^3k}{(2\pi)^3}\frac{k^2}{3\sqrt{k^2+(m^n_*)^2}},\\ &= \frac{1}{4}\mu^n_* n^\dagger n - \frac{1}{4}m^n_*\bar{n}n, \end{aligned} \tag{D7}$$

and equivalently for protons. These look conveniently like the neutron and proton energy density and pressure, and indeed

$$\begin{aligned} \epsilon^{\int n} - 3P^{\int n} &= m^n_*\bar{n}n,\\ \epsilon^{\int n} + P^{\int n} &= \mu^n_* n^\dagger n. \end{aligned} \tag{D8}$$

The actual nucleon energy density and pressure are properly constructed from the stress-energy tensor:

$$\left(T^N_{\chi PT}\right)^{\mu\nu} = \frac{\partial L^N_{\chi PT}}{\partial(\partial_\mu N)}\partial^\nu N - g^{\mu\nu}L^N_{\chi PT}, \tag{D9}$$

with

$$\begin{aligned} \epsilon^N &\equiv \left(T^N_{\chi PT}\right)^{00},\\ P^N &\equiv \tfrac{1}{3}\left(T^N_{\chi PT}\right)^{jj}. \end{aligned} \tag{D10}$$

The total nucleon energy and pressure are thus

$$\epsilon^N = \epsilon^{\int p} + \epsilon^{\int n} + \frac{1}{2}\left(\mathcal{C}_V^2(N^\dagger N)^2 - \frac{\overline{\mathcal{C}_V^2}}{2}(p^\dagger p - n^\dagger n)^2\right) + \frac{1}{2}\left(\mathcal{C}_S^2(\overline{N}N)^2 + \frac{\overline{\mathcal{C}_{200}^S}}{2}(\overline{p}p - \overline{n}n)^2\right),$$
$$P^N = P^{\int p} + P^{\int n} + \frac{1}{2}\left(\mathcal{C}_V^2(N^\dagger N)^2 - \frac{\overline{\mathcal{C}_V^2}}{2}(p^\dagger p - n^\dagger n)^2\right) - \frac{1}{2}\left(\mathcal{C}_S^2(\overline{N}N)^2 + \frac{\overline{\mathcal{C}_S^2}}{2}(\overline{p}p - \overline{n}n)^2\right). \quad \text{(D11)}$$

Using

$$\mu_B^n \equiv \mu_*^n + \mathcal{C}_V^2 N^\dagger N - \overline{\mathcal{C}_V^2}(n^\dagger n - p^\dagger p),$$
$$\mu_B^p \equiv \mu_*^p + \mathcal{C}_V^2 N^\dagger N + \overline{\mathcal{C}_V^2}(n^\dagger n - p^\dagger p), \quad \text{(D12)}$$

it follows that ϵ^N and P^N are related by the baryon number densities:

$$\epsilon^N + P^N = \mu_B^p\, p^\dagger p + \mu_B^n\, n^\dagger n. \quad \text{(D13)}$$

The objects of our calculations are therefore the six quantities $\mu_B^{n,p}$, $m_*^{n,p}$, and $k_F^{n,p}$. These are, respectively, the chemical potential, reduced mass, and Fermi-momentum for neutrons and protons.

1. $Z = N$ heavy nuclei in the chiral symmetric limit

To calculate binding energies, we work in the chiral symmetric limit, $m_p = m_n$, e.g., zero electromagnetic breaking, and $m_8 = \frac{1}{2}(m_p + m_n)$. We first study the case $Z = N$, so $m_*^n = m_*^p \equiv m_*$ for equal numbers of protons and neutrons. We search for a solution of the chiral-symmetric liquid equations that has $P^N = 0$. In this simple case, $\mu_B^p = \mu_B^n \equiv \mu_B$, $\mu_*^p = \mu_*^n \equiv \mu_*$, $m_p = m_n \equiv m_N$, and $k_{Fn} = k_{Fp} \equiv k_F$. Thus

$$k_F = \sqrt{\mu_*^2 - m_*^2}. \quad \text{(D14)}$$

We also have $n^\dagger n = p^\dagger p = \frac{1}{2}N^\dagger N$, and $\overline{n}n = \overline{p}p = \frac{1}{2}\overline{N}N$. We are therefore able to write the baryon density as

$$N^\dagger N = \frac{\mu_B - \mu_*}{\mathcal{C}_V^2}, \quad \text{(D15)}$$

and the scalar density as

$$\overline{N}N = \frac{m_N - m_*}{\mathcal{C}_S^2}, \quad \text{(D16)}$$

where, to make connection to Walecka's model of nuclear matter, we use $\mathcal{C}_V^2$ and $\mathcal{C}_S^2$ defined in (26) and (26), respectively.

The baryon number and scalar densities are simply twice the values in (D4); i.e.,

$$N^\dagger N = \frac{2k_F^3}{3\pi^2},$$
$$\overline{N}N = \frac{m_*}{\pi^2}\left(\mu_* k_F - m_*^2 \ln\left[\frac{\mu_* + k_F}{m_*}\right]\right). \quad \text{(D17)}$$

The fermion pressure is now

$$P^N = \tfrac{1}{4}\big[\mu_B N^\dagger N + \mathcal{C}_V^2(N^\dagger N)^2 - m_N\overline{N}N - \mathcal{C}_S^2(\overline{N}N)^2\big]. \quad \text{(D18)}$$

To these six equations (D14)–(D18) in the seven variables k_F, μ_*, $\delta_\mu \equiv \mu - \mu_*$, m_*, $\overline{N}N$, $N^\dagger N$, and P^N, we add the physical condition that the StaticχNL nontopological-soliton pressure vanishes internally, in order that it remain stable when immersed in the physical vacuum:

$$P^N = 0, \quad \text{(D19)}$$

eliminating P^N as a free variable. Equations (D14)–(D17) can be solved analytically to give k_F, μ_*, $N^\dagger N$, and $\overline{N}N$ as functions of m_* and δ_μ:

$$k_F = \left(\frac{3\pi^2}{2}\frac{\delta_\mu}{\mathcal{C}_V^2}\right)^{1/3},$$
$$\mu_* = \sqrt{k_F^2 + m_*^2}. \quad \text{(D20)}$$

Equation (D18), with $P^N = 0$, then becomes a quartic equation for m_* in terms of δ_μ:

$$0 = m_*^2 + \left(\frac{3\pi^2\delta_\mu}{2\mathcal{C}_V^2}\right)^{2/3} - \left[\frac{\mathcal{C}_V^2}{\mathcal{C}_S^2}\frac{(m_N - m_*)(2m_N - m_*)}{\delta_\mu} - 2\delta_\mu\right]^2, \quad \text{(D21)}$$

which has up to four roots, $m_*(\delta_\mu;\mathcal{C}_S^2,\mathcal{C}_V^2)$, for every value of δ_μ, $\mathcal{C}_S^2$, and $\mathcal{C}_V^2$.[4] To be an actual solution of the complete set of $Z = N$ chiral-symmetric pressure-less liquid equations, the root must also satisfy (D16) and the second of (D17); i.e.,

$$\Delta_{\bar{N}N} \equiv 1 - \frac{\mathcal{C}_S^2}{m_N - m_*}\overline{N}N = 0, \quad \text{(D22)}$$

where we use (D20) for $k_F(\delta_\mu;\mathcal{C}_V^2)$ and $\mu_*(\delta_\mu;\mathcal{C}_S^2,\mathcal{C}_V^2)$.

Now $m_N/f_\pi \approx 939/93 \approx 10.10$, but in principle $\mathcal{C}_S^2$ and $\mathcal{C}_V^2$ are free parameters. For given values of $\mathcal{C}_S^2$ and $\mathcal{C}_V^2$, we must search for a value of $\delta\mu$ such that (D22) holds. The existence of such a value of $\delta\mu$ is not assured for arbitrary values of $\mathcal{C}_S^2$ and $\mathcal{C}_V^2$.

Fitting to experimental values, Chin and Walecka found that their parameters $\mathcal{C}_V^2 = 222.65\ \text{GeV}^{-2}$ and $\mathcal{C}_S^2 = 303.45\ \text{GeV}^{-2}$. In Fig. 2, we show that there does indeed exist a pressureless chiral-symmetric nuclear liquid for $\mathcal{C}_S^2$ and $\mathcal{C}_V^2$

[4]But only one of these four roots might be an infinite StaticχNL, and then only if it were the $P^N \to 0$ limit of a *finite* Walecka nontopological soliton. Those solitons satisfy *Newtonian roll-around-ology* [16,22–26], where the mean field nucleons move within a dynamic σ field. $P^N_{Internal} \neq 0$ and $P^N_{External} = 0$ are then connected by the dynamic σ surface.

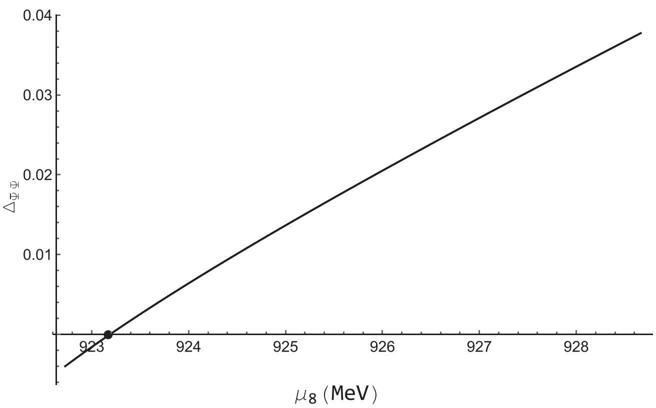

FIG. 2. $\Delta_{\bar{N}N}$ [cf. (D22)] as a function of baryon chemical potential μ_B for $\mathcal{C}_V^2 = 222.65\ \mathrm{GeV}^{-2}$ and $\mathcal{C}_S^2 = 303.45\ \mathrm{GeV}^{-2}$, the Chin and Walecka values [20] equivalent to ours. A solution of the complete set of $Z = N$ chiral-symmetric pressureless liquid equations must have $\Delta_{\bar{N}N} = 0$, and thus is found at $\mu_B \simeq 923.17$ MeV, where the curve intersects the μ_B axis. This value equals the Chin-Walecka value shown as a black dot.

equal to the Chin and Walecka [20] values. Furthermore, the inferred value of the baryon chemical potential is 923.17 MeV, and is consistent with Chen and Walecka's value. Figure 3 shows representative values of $\mathcal{C}_V^2$ and $\mathcal{C}_S^2$ for different values of k_F using the first approach. Remarkably, we can now understand Chin and Walecka's nuclear matter to *be* a pressureless chiral-symmetric nuclear liquid. We also perhaps thereby gain some insight into the relative insensitivity of nuclear properties to pion properties.

2. $Z \neq N$ heavy nuclei in the chiral-symmetric limit

Here we outline the analytic and numerical treatment of the case where $Z \neq N$ in the chiral limit. The approach may be summarized as follows:

(1) The starting point is the zeroth-order solution for the case $Z = N$ which determines the coupling constants $\mathcal{C}_S^2$ and $\mathcal{C}_V^2$ for a given Fermi level and binding energy as in the previous section.

(2) All proton and neutron specific quantities are expanded in a Taylor series.

(3) The general rule is that quantities vanishing in zeroth order have a first-order variation while those not vanishing in zeroth order have only a second-order variation; thus all terms up to second order must be retained.

(4) The vanishing of pressure to second order provides an additional equation which allows all variations to be expressed in terms of the first-order change in density only.

(5) Since there appears no way to infer separately the value of $\overline{\mathcal{C}_S^2}$ we follow Niksic and co-workers [50] and set this constant to zero. This leads to significant simplification. In particular, changes in the proton and neutron reduced masses are equal in first order.

(6) We then solve for $\overline{\mathcal{C}_V^2}$ by setting the asymmetry energy of the liquid model to the second-order variation in the Thomas-Fermi energy.

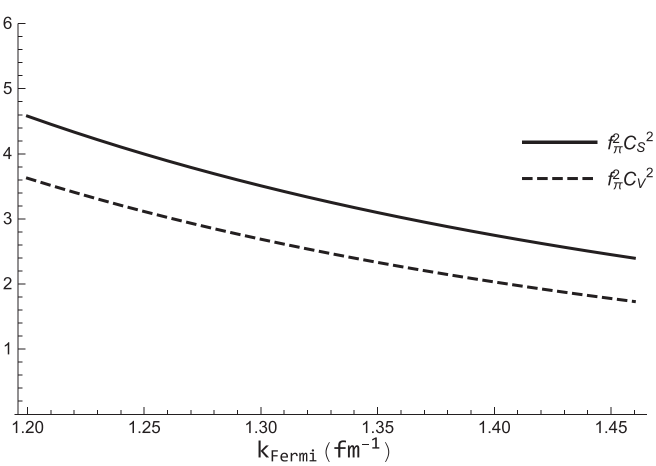

FIG. 3. Plot of $f_\pi^2 \mathcal{C}_S^2$ and $f_\pi^2 \mathcal{C}_V^2$ against the Fermi level in inverse F. The calibration used a bulk binding energy $E_{Vol} = 15.75$ MeV.

In this section we use the following notation for the number and scalar densities:

$$\rho_p \equiv p^\dagger p, \qquad \rho_n \equiv n^\dagger n, \qquad \rho_\pm = \rho_p \pm \rho_n,$$
$$\rho_{Sp} \equiv \overline{p}\, p, \qquad \rho_{Sn} \equiv \overline{n}\, n, \qquad \rho_{S\pm} = \rho_{Sp} \pm \rho_{Sn}. \tag{D23}$$

We define the changes in densities as follows:

$$d\rho_p - d\rho_n = \epsilon d\rho_-,$$
$$d\rho_p + d\rho_n = \epsilon^2 d\rho_+ \tag{D24}$$

where ϵ is merely a placeholder for the order of the variation. It then follows that

$$\rho_p = \frac{1}{2}\rho_+ + \frac{\epsilon}{2} d\rho_- + \frac{\epsilon^2}{2} d\rho_+,$$
$$\rho_n = \frac{1}{2}\rho_+ - \frac{\epsilon}{2} d\rho_- + \frac{\epsilon^2}{2} d\rho_+. \tag{D25}$$

Since the number density for each species is given by the first of (D4) we get the following expansions for the Fermi levels:

$$\delta k_{Fp} - \delta k_{Fn} = \epsilon \frac{2k_F}{3\rho_+}\delta\rho_-,$$
$$\delta k_{Fp} + \delta k_{Fn} = \epsilon^2 \left(\frac{2k_{Fp}}{3\rho_+}\delta\rho_+ - \frac{2k_F}{9\rho_+^2}\delta\rho_-^2 \right). \tag{D26}$$

It follows that

$$m_{*8} \equiv \frac{1}{2}(m_{*p} + m_{*n}) = m_N - \mathcal{C}_S^2\, \rho_{S+},$$
$$m_{*3} \equiv \frac{1}{2}(m_{*p} - m_{*n}) = \frac{\overline{\mathcal{C}_S^2}}{2}\, \rho_{S-}, \tag{D27}$$

where we used the second of (26). We also define

$$\mu_{*8} = \tfrac{1}{2}(\mu_{*p} + \mu_{*n}),$$
$$\mu_{*3} = \tfrac{1}{2}(\mu_{*p} - \mu_{*n}), \tag{D28}$$

with $\mu_{*n,p}$ as in (D5). We now enforce $\overline{\mathcal{C}_S^2}=0$: it follows immediately from the second of (D27) that $m_{*3}=\delta m_{*3}=0$ with considerable simplification. First, $\delta\mu_{*3}$ is a linear function of $\delta\rho_-$ only; i.e.,

$$\delta\mu_{*3}=\frac{\pi^2}{2\,k_F\,\mu_{*8}}\,\delta\rho_-. \tag{D29}$$

Second, $\delta\mu_{*8}$ is also simplified:

$$\delta\mu_{*8}=\frac{m_{*8}}{\mu_{*8}}\,\delta m_{*8}+\frac{\pi^2}{2\mu_{*8}k_{Fp}}\delta\rho_+ \\ -\frac{\pi^4}{8\,k_F^4\,\mu_{*8}^3}\left(m_{*8}^2+2k_F^2\right)\delta\rho_-^2. \tag{D30}$$

(As noted, $\delta\mu_{*3}$ is first order, while $\delta\mu_{*8}$ is second order.) The variation in the first of (D27) gives

$$\delta\rho_{S+}=-\frac{\delta m_{*8}}{\mathcal{C}_S^2}, \tag{D31}$$

where the variation in ρ_{S+} is obtained using

$$\delta\rho_{Sp,n}=3\left(\frac{\rho_{Sp,n}}{m_{*p,n}}-\frac{\rho_{p,n}}{\mu_{*p,n}}\right)\delta m_{*p,n} \\ +\quad\frac{m_{*p,n}}{\mu_{*p,n}}\frac{k_{Fp,n}^2}{\pi^2}\,\delta k_{Fp,n}. \tag{D32}$$

After some algebra and substituting the variations in μ_{*3} and μ_{*8} from (D29) and (D30), we find

$$3\left(\frac{\rho_{S+}}{m_{*8}}-\frac{\rho_+}{\mu_{*8}}+\frac{1}{\mathcal{C}_S^2}\right)\delta m_{*8}+\frac{m_{*8}}{\mu_{*8}}\delta\rho_+-\frac{m_{*8}\pi^2}{\mu_{*8}^3\,k_F}\frac{\delta\rho_-^2}{4}=0. \tag{D33}$$

We must also enforce the vanishing of the Fermi pressure. The first-order variation of the Fermi pressure vanishes identically. The second-order term is

$$\delta P_2^N=\frac{1}{4}(\rho_+\delta\mu_{*8}+\mu_{*8}\delta\rho_+\;+\;\delta\mu_{*3}\delta\rho_-)+C_V^2\rho_+\delta\rho_+ \\ -\frac{\overline{\mathcal{C}_V^2}}{4}\delta\rho_-^2+\frac{1}{4\mathcal{C}_S^2}(3m_N-2m_{*8})\delta m_{*8}. \tag{D34}$$

After using (D29) and (D30), the zero pressure equation becomes

$$\left(\frac{3m_N-2m_{*8}}{4\mathcal{C}_S^2}+\frac{\rho_+m_{*8}}{2\,\mu_{*8}}\right)\delta m_{*8} \\ +\left(\frac{\mu_{*8}}{4}+C_V^2\rho_++\frac{k_F^2}{12\,\mu_{*8}}\right)\delta\rho_+ \\ +\left(\frac{\pi^2\,(5m_{*8}^2-4k_F^2)}{48\,k_F\,\mu_{*8}^3}-\frac{\overline{\mathcal{C}_V^2}}{4}\right)\delta\rho_-^2 \\ =0. \tag{D35}$$

Equations (D33) and (D35) are solved to express δm_{*8} and $\delta\rho_+$ in terms of $\delta\rho_-^2$. To determine $\overline{\mathcal{C}_V^2}$ we need the second variation in the energy density $\mathscr{E}$. This quantity is discussed below.

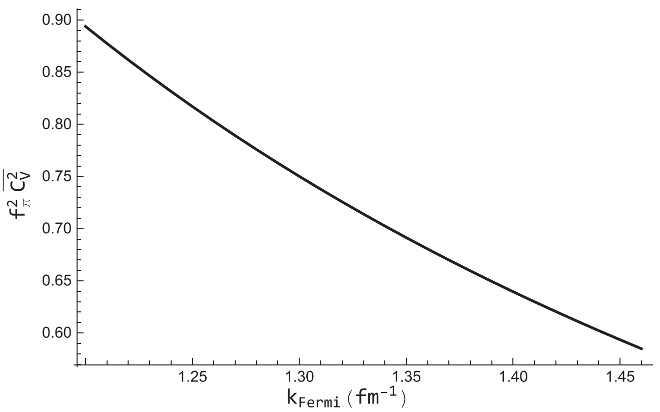

FIG. 4. Plot of $\overline{C_{200}^V}\equiv f_\pi^2\,\overline{\mathcal{C}}_V^2$ against the Fermi level in fm^{-1}. The behavior is roughly linear in the range considered and corresponds to a one-third power of the number density.

3. Calibration of $\overline{\mathcal{C}_V^2}$

We start with the vanishing of the pressure and the relationship:

$$\epsilon^N+P^N=\mu_p\rho_p+\mu_n\rho_n=\mu_8\,\rho_++\mu_3\,\rho_-, \tag{D36}$$

where

$$\mu_8=\mu_{*8}+C_V^2\rho_+, \\ \mu_3=\mu_{*3}-\tfrac{1}{2}\overline{\mathcal{C}_V^2}\rho_-. \tag{D37}$$

The zeroth-order energy density when $Z=N$ follows at once:

$$\epsilon_0^N=\mu_{*8}\,\rho_++C_V^2\,\rho_+^2. \tag{D38}$$

The first-order energy term vanishes. The second-order term is

$$\delta\epsilon_2^N=\rho_+\,\delta\mu_{*8}+\mu_{*8}\,\delta\rho_++\delta\mu_{*3}\,\delta\rho_- \\ +2C_V^2\,\rho_+\,\delta\rho_+-\tfrac{1}{2}\overline{\mathcal{C}_V^2}\,\delta\rho_-^2. \tag{D39}$$

Finally, we can express $\delta\rho_-$ in terms of the relative neutron excess as

$$\delta\rho_-=\frac{Z-N}{Z+N}\,\rho_+. \tag{D40}$$

The parameter $\overline{\mathcal{C}_V^2}$ can be calibrated in two ways. In the first, we merely ascribe all of the second-order energy to the asymmetry term in the liquid drop formula (34) for $\overline{\mathcal{C}_V^2}$:

$$\delta\epsilon_2^N=a_{\rm Asym}\left(\frac{Z-N}{Z+N}\right)^2\rho_+=a_{\rm Asym}\frac{\delta\rho_-^2}{\rho_+}, \tag{D41}$$

where $a_{\rm Asym}$ is fit to SEMF observation. In the second approach, we calibrate directly to the binding energies of isotopes, possibly using the liquid drop formula to correct for effects that we have ignored in this paper such as the Coulomb and surface terms. Both approaches give comparable results. Figure 4 shows the behavior of $\overline{\mathcal{C}_V^2}$ for different values of k_F.

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