Letter

"Piston" mechanism in a time-dependent two-level model

A. O. Macchiavelli,[1] O. Civitarese,[2] S. M. Lenzi,[3] R. J. Liotta,[4] and D. R. Bès[5]

[1]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[2]*Departamento de Física, Universidad de La Plata, 49 y 115 Casilla de Correo 67 (1900) La Plata, Argentina*
[3]*Dipartimento di Fisica e Astronomia, Università degli Studi di Padova and INFN, Sezione di Padova, I-35131 Padova, Italy*
[4]*KTH (Royal Institute of Technology), Alba Nova University Center, S-10691 Stockholm, Sweden*
[5]*Centro Atómico Constituyentes, Av. Gral. Paz 1499, B1650 Villa Maipú, Prov. de Buenos Aires, Argentina*

(Received 4 August 2021; revised 3 November 2021; accepted 24 November 2021; published 16 December 2021)

We analyze a recent study of the one-proton knockout reaction on ^{19}N in the framework of a time-dependent two-level model. Focusing on the specific case of ^{18}C, we quantify the validity of the so-called "piston" mechanism in terms of the timescales involved in the process, namely, the reaction time, the period of oscillation between the mixed states, and the lifetime of the excited unbound state.

DOI: 10.1103/PhysRevC.104.L061304

Introduction. Pairing correlations play a crucial role in defining the properties of atomic nuclei. The evolution of these correlations in exotic nuclei is a subject which has received much attention in recent years as new accelerator facilities are providing unique radioactive beams for study. Of particular interest is the role of neutron-neutron (nn) pairing in neutron-rich isotopes.

In an elegant experiment carried out at the GSI Facility using the Fragment separator and the R^{3}B-LAND Collaboration setup [1], the emission of neutron pairs from the neutron-rich $N = 12$ isotones ^{18}C and ^{20}O was studied following the high-energy nucleon knockout reactions ^{19}N$(-1p)^{18}$C and ^{21}O$(-1n)^{20}$O to populate unbound states of the two isotones up to 15 MeV above their two-neutron emission thresholds. Their analysis of triple fragment-n-n correlations display significantly different features between ^{18}C and ^{20}O. The authors argue that in the case of ^{18}C, the knockout of a deeply bound proton from ^{19}N suddenly promotes neutron pairs of the final system into the continuum without affecting the pairing properties of the ground states and associate the process to a "piston" mechanism.

In this Letter we study more quantitatively the piston mechanism and discuss its validity in terms of the time dependence of a two-level model [2].

Formalism. Although we focus our discussion on the case of ^{18}C, the approach can be trivially extended to other cases. In a $(-1p)$ reaction the population of the excited states proceeds through the knockout of a deeply bound $p_{3/2}$ proton in ^{19}N. Let us start by considering the basic ingredients that may play a role in the low-lying 2^+ states[1] in the carbon isotopes [3–5]. Following the picture depicted in Fig. 1, we assume that the elementary excitations, which are relevant for the description of the structure of ^{16}C, are two-neutron particle and two-proton hole states built upon the double magic core ^{16}O [3]. We define $|2^+_p\rangle$ as the two-proton hole state in ^{14}C at 7 MeV excitation energy and $|2^+_n\rangle$, as the two-neutron particle state in ^{18}O at 1.98-MeV excitation energy.

The mixing between them in ^{16}C can then be expressed as

$$\begin{aligned} |2^+_1\rangle &= \alpha|2^+_n\rangle + \beta|2^+_p\rangle, \\ |2^+_2\rangle &= -\beta|2^+_n\rangle + \alpha|2^+_p\rangle. \end{aligned} \tag{1}$$

In the weak-coupling limit, the low-lying quadrupole excitation in ^{16}C will be dominated by the two-neutron component in Eq. (1) (that is, $\alpha \gg \beta$ for the $|2^+_1\rangle$ state).

Concerning the structure of the heavier carbons, the dependence of the effective single-particle energies (ESPE) of the $N = 2$ major shell with neutron number [6], shown in Fig. 2, exhibits a large gap ($Z = 16$) between the $2s_{1/2}$ $1d_{5/2}$ levels and the $1d_{3/2}$, in relation to a typical pairing gap Δ indicated by the yellow shaded area. In Refs. [4,5], a seniority inspired scheme was proposed that captures the main physical ingredients and correlates well with the available experimental data.

In this simple phenomenological model the 0^+ and 2^+_1 states of $^{18}_6$C$_{8+4}$ with four neutrons in the $(d_{5/2} + s_{1/2})$ combined shell are as follows:

$$\begin{aligned} |0^+;{}^{18}\mathrm{C}\rangle &= |\nu(sd)^4;J=0\rangle \otimes |\pi(p_{3/2})^4;J=0\rangle, \\ |2^+_1;{}^{18}\mathrm{C}\rangle &= \alpha|\nu(sd)^4;J=2\rangle \otimes |\pi(p_{3/2})^4;J=0\rangle \\ &\quad + \beta|\nu(sd)^n;J=0\rangle \otimes |\pi(p_{3/2})^3(p_{1/2})^1;J=2\rangle, \\ |2^+_2;{}^{18}\mathrm{C}\rangle &= -\beta|\nu(sd)^4;J=2\rangle \otimes |\pi(p_{3/2})^4;J=0\rangle \\ &\quad + \alpha|\nu(sd)^4;J=0\rangle \otimes |\pi(p_{3/2})^3(p_{1/2})^1;J=2\rangle. \end{aligned} \tag{2}$$

[1]Similar arguments can be applied to the 1^+ state that could be populated in the proton-knockout reaction.

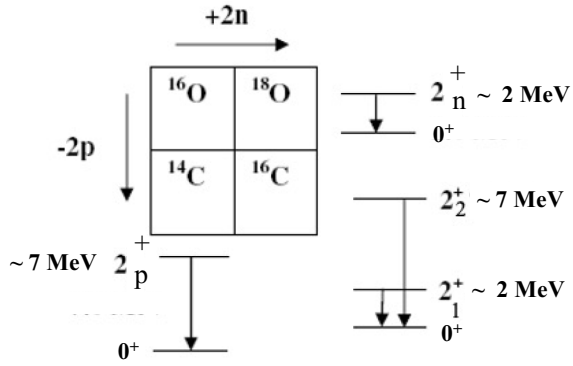

FIG. 1. Elementary 2^+ excitations relevant for the structure of ^{16}C. Adapted from Ref. [3].

Note that in a $(-1p)$ reaction the 0^+ state in ^{18}C is populated via the knockout of the $p_{1/2}$ proton from the ground state of the nitrogen target.

Time dependence. Given the fast nature of the reaction as well as the fast decay of the resonance state into $2n$, we need to consider three timescales, namely, the reaction time t_r, the lifetime of the excited unbound state τ, and the period of oscillation between the mixed states T.

In order to estimate the time evolution of the states prepared by the reaction, which are not eigenstates of the Hamiltonian in ^{18}C, we will resort to the time-dependent expression of these states [2]. We consider the Hamiltonian $H = H_0 + V$ where states $|2^+_p\rangle$ and $|2^+_n\rangle$ are eigenstates of H_0 with eigenvalues E_p and E_n and V is the interaction matrix element between them. The solution of the eigenvalue problem yields the stationary states $|2^+_1\rangle$ and $|2^+_2\rangle$ as given in Eqs. (2) with energies E_- and E_+, corresponding to 2^+ states in ^{18}C as discussed in Refs. [4,5]. We associate $|2^+_1\rangle$ with the first excited 2^+_1 state and $|2^+_2\rangle$ (mixed symmetry state) with the resonance observed in Ref. [1].

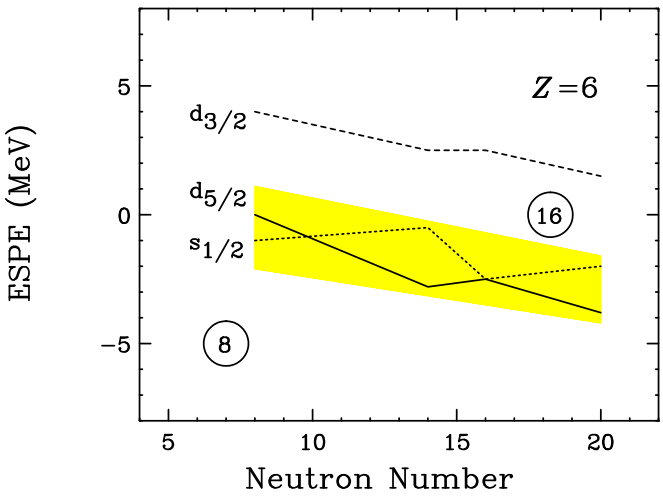

FIG. 2. ESPEs for the carbon isotopes obtained from the Warburton-Brown WBT interaction [6]. The yellow shaded area indicates the size of the pairing gap (Δ). Adapted from Ref. [4].

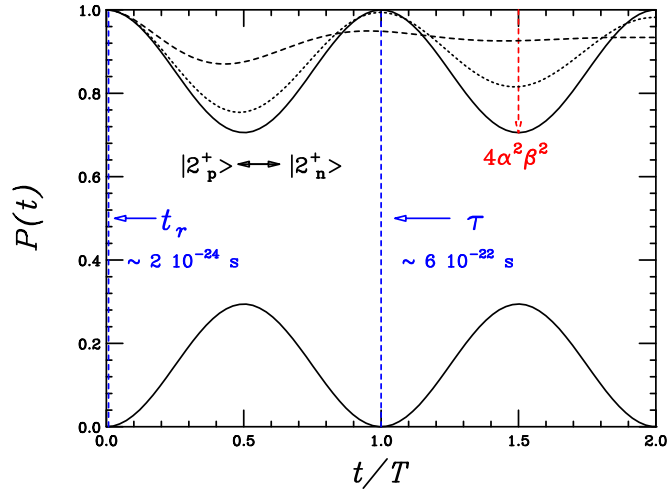

FIG. 3. Time oscillations between states $|2^+_p\rangle$ and $|2^+_n\rangle$ with time in units of T. The vertical (blue) dashed lines indicate the estimates of the reaction time t_r and the lifetime of the resonance state τ. The amplitude of the oscillations is also shown in red. The dotted and dashed lines include the effects of the decay for lifetimes τ and $\frac{1}{2}\tau$, respectively.

Suppose that at $t = 0$ we prepare the system in state $|2^+_p\rangle$, which is not an eigenstate of H, then the time evolution is given by the wave function,

$$|\psi(t)\rangle = \beta|2^+_1\rangle e^{-iE_-t/\hbar} + \alpha|2^+_2\rangle e^{-iE_+t/\hbar}, \tag{3}$$

from which the time dependence of the probabilities of finding $|\psi(t)\rangle$ in states $|2^+_n\rangle$, $|2^+_p\rangle$, $P_{2^+_n}(t)$, and $P_{2^+_p}(t)$, can be obtained

$$P_{2^+_n}(t) = 1 - P_{2^+_p}(t) = 4|\alpha|^2|\beta|^2 \sin^2(\omega t/\hbar), \tag{4}$$

with $\omega = (E_+ - E_-)/2$.

Results. The proton knockout reaction prepares the system in state $|2^+_p\rangle$ which undergoes oscillations with probabilities following Eq. (4). We show in Fig. 3 the time evolution of these probabilities with t given in units of the characteristic oscillation period $T = \pi\hbar/\omega = 2\pi\hbar/(E_+ - E_-)$. The amplitude of the oscillations $4\alpha^2\beta^2$ reaches its maximum of 1 when $\alpha = \beta = 1/\sqrt{2}$.

Based on the analysis of Refs. [4,5], we have $\beta \approx 0.27$ and $\omega = (E_+ - E_-)/2 \approx 3.4$ MeV, giving the oscillation period $T \approx 6\,10^{-22}$ s. For reference, we also indicate in Fig. 3 the reaction time, which at a bombarding energy of 400 MeV/A is on the order of 2×10^{-24} s, and the lifetime of the resonance state τ, which is on the order of 6×10^{-22} s [1]. Given these estimates, it appears that, although the reaction is very fast, the lifetime is long enough to average the probability of finding the state vector $|\psi(t)\rangle$ in $|2^+_p\rangle$ to

$$\langle P_{2^+_p}(t)\rangle = 1 - 2|\alpha|^2|\beta|^2 \approx 0.85. \tag{5}$$

We can then conclude that it is only for the cases where $\tau \approx t_r \ll T$ or in the unlikely scenario of negligibly mixing $\beta \approx 0$ that the concept of the piston mechanism might be applicable, and the decay properties of the populated resonance truly reflect the neutron correlations in the ground state of the final nucleus, in this case ^{18}C. This could be confirmed

experimentally by a measurement of the spectroscopic factors directly probing the amplitudes α and β. These arguments are, of course, applicable to other similar systems.

Finally, if we consider the fact that $|2_2^+\rangle$ is a decaying state with energy $E_+ - i\Gamma_+/2$, the results get modified [7] as shown with the dotted line in Fig. 3 where we have used $\Gamma_+ = \hbar/\tau$. We also show the results for a lifetime $\frac{1}{2}\tau$ (dashed line) to illustrate the effects of a faster decay. In the limit $\tau \to 0$, the probability approaches the value of $1 - |\alpha|^2|\beta|^2$.

Conclusions. We have studied the interplay between two-proton hole and neutron particle states, excited via a high-energy proton knockout reaction on ^{19}N leading to unbound states in ^{18}C [1]. The piston mechanism concept offers a unique motivation to use $(p, 2p)$ reactions to systematically study the evolution of nn correlations in exotic neutron-rich nuclei. It seems clear to us that such a program will feature prominently in current and future rare-isotopes facilities worldwide. With this in mind, we used a two-level mixing model and its time dependence to assess the validity of the piston mechanism and suggested some indicators for its applicability. In particular, differences in the timescales of the reaction, the lifetime and the oscillation period of the decaying state should be considered in relating the measured *n-n* correlations to the pairing properties of the ground state in the final nucleus. Although an extension of the two-level system to include more mixing states will be more realistic, our scenario captures the main physical ingredients of the problem.

Acknowledgements. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Contract No. DE-AC02-05CH11231 (LBNL) and the PIP 616 of the CONICET (Argentina). We would like to thank Dr. F. M. Marques and Dr. O. Sorlin for enlightening discussions on their experiment and the piston mechanism and Dr. R. M. Clark, Dr. V. Koch, and Dr. I.-Y. Lee for their comments on the paper. We also thank the referee for valuable comments and suggestions on our presentation of the time-dependent two-level model.

[1] A. Revel, F. M. Marques, O. Sorlin, T. Aumann, C. Caesar, M. Holl *et al.* (R^{3}B-LAND Collaboration), Phys. Rev. Lett. **120**, 152504 (2018).

[2] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Second ed. (Pergamon, Oxford, 1965).

[3] M. Wiedeking, P. Fallon, A. O. Macchiavelli, J. Gibelin, M. S. Basunia, R. M. Clark, M. Cromaz, M.-A. Deleplanque, S. Gros, H. B. Jeppesen, P. T. Lake, I.-Y. Lee, L. G. Moretto, J. Pavan, L. Phair, E. Rodriguez-Vietiez, L. A. Bernstein, D. L. Bleuel, J. T. Burke, S. R. Lesher, B. F. Lyles, and N. D. Scielzo, Phys. Rev. Lett. **100**, 152501 (2008).

[4] A. O. Macchiavelli, M. Petri, P. Fallon, S. Paschalis, R. M. Clark, M. Cromaz, and I. Y. Lee, Phys. Rev. C **90**, 067305 (2014).

[5] I. Syndikusa, M. Petri, A. O. Macchiavelli, S. Paschalis, C. A. Bertulani, T. Aumann *et al.*, Phys. Lett. B **809**, 135748 (2020).

[6] M. Stanoiu, D. Sohler, O. Sorlin, F. Azaiez, Z. Dombrádi, B. A. Brown, M. Belleguic, C. Borcea, C. Bourgeois, Z. Dlouhy, Z. Elekes, Z. Fülöp, S. Grévy, D. Guillemaud-Mueller, F. Ibrahim, A. Kerek, A. Krasznahorkay, M. Lewitowicz, S. M. Lukyanov, S. Mandal, J. Mrázek, F. Negoita, Y. E. Penionzhkevich, Z. Podolyák, P. Roussel-Chomaz, M. G. Saint-Laurent, H. Savajols, G. Sletten, J. Timár, C. Timis, and A. Yamamoto, Phys. Rev. C **78**, 034315 (2008).

[7] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, New Millennium Edition* (Basic Books, New York, 1965), Vol. III.