Light-front transverse momentum distributions for $\mathcal{J} = 1/2$ hadronic systems in valence approximation

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The semi-inclusive correlator for a $\mathcal{J}=1/2$ bound system, composed by A spin-1/2 fermions, is linearly expressed in terms of the light-front Poincaré covariant spin-dependent spectral function, in valence approximation. The light-front spin-dependent spectral function is fully determined by six scalar functions that allow for a complete description of the six T-even transverse momentum distributions, suitable for a detailed investigation of the dynamics inside the bound system. The application of the developed formalism to a case with a sophisticated dynamical content, like ^{3}He, reaches two goals: (i) to illustrate a prototype of an investigation path for gathering a rich wealth of information on the dynamics and also finding valuable constraints to be exploited from the phenomenological standpoint and (ii) to support for the three-nucleon system a dedicated experimental effort for obtaining a detailed three-dimensional picture in momentum space. In particular, the orbital angular momentum decomposition of the bound state can be studied through the assessment of relations among the transverse momentum distributions, as well as the relevance of the relativistic effect generated by the implementation of macroscopic locality. A fresh evaluation of the longitudinal and transverse polarizations of the neutron and proton is also provided, confirming essentially the values used in the standard procedure for extracting the neutron structure functions from both deep-inelastic scattering and semi-inclusive reactions, in the same kinematical regime.

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I. INTRODUCTION

A fully relativistic treatment is needed to describe hadronic bound systems, when high energy processes are studied and/or a high degree of accuracy is required. Among the phenomenological efforts to implement a Poincaré-covariant description of a bound system, we recall the proposal in Ref. [1], where the light-front Hamiltonian dynamics (LFHD) [2–7] was adopted in order to obtain the light-front (LF) spin-dependent spectral function, whose diagonal elements yield the distribution probability to find a constituent with given spin, LF momentum, and (off-shell) energy, inside the bound system. The spectral function is primarily used to study the nucleon momentum distributions in nuclei, but the formalism can be notably extended to a hadronic bound system and, as it will be illustrated in detail, to eventually obtain the six T-even transverse momentum distributions (TMDs) [8]. Through the latter quantities, one can achieve a detailed description of the system, much richer than the one given by the usual distribution in terms of the constituent momentum $|\mathbf{p}|$ in the laboratory frame. As a matter of fact, one can address the correlations between spin and momentum, substantially deepening our understanding of the inner dynamics. For the nucleon, TMDs (see, e.g., Refs. [9–19]) are the object of impressive theoretical and experimental efforts, both in semi-inclusive deep-inelastic scattering (SIDIS) and in Drell-Yan processes (see, e.g., Refs. [20–28] and Refs. [29–31], respectively). In particular light-cone models and phenomenological approaches for the TMDs have been used, e.g., (i) to study the three-dimensional nucleon structure [32–38], (ii) to address the nucleon-spin puzzle [39], and hence (iii) to disentangle the contributions of different angular momentum components to the spin of the nucleon [40,41]. Let us notice that the nucleon, namely, a spin-1/2 system, is composed by three quarks, in valence approximation. Therefore, in this approximation, the approach we have elaborated in LFHD can be applied both to the nucleon, as a system of three quarks, and to ^{3}He or ^{3}H, as systems of three nucleons.

The application of our formalism to the three-nucleon system has a twofold benefit. On one side, it allows one to illustrate a realistic example, with specific features of TMDs and constraints among them that can be traced back to the inner dynamics, e.g., the impact of the orbital momentum content generated by the interaction. On the other side, it establishes a first theoretical basis for supporting future experimental efforts aiming to investigate the TMDs of ^{3}He and eventually construct a three-dimensional (3D) tomography of the nucleus in momentum space. In a nucleus with total momentum P in the laboratory frame, TMDs describe the nucleon distribution as a function of $x = p^+/P^+$ and of the transverse momentum $p_\perp$, for any possible orientation of the spin of the nucleus and of the spin of the nucleon.[1] Hence, it is quite natural to look for a possible interplay with the spin-dependent spectral function. In Ref. [1] the LF spectral function, $\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$, is defined starting from the LF wave function for a three-nucleon system with polarization vector $\mathbf{S}$, spin 1/2, and third component $\mathcal{M}$, using nonsymmetric intrinsic variables. The energy ϵ is the energy of a fully interacting two-particle (23) subsystem and the variable $\tilde{\boldsymbol{\kappa}} = (\kappa^+, \boldsymbol{\kappa}_\perp)$ is the LF momentum of particle 1 in the intrinsic reference frame of the cluster [1,(23)]. From the spectator energy ϵ one can reconstruct the component κ^-, leading to the off-shell energy of the constituent. The spectral function is defined through the overlaps between the LF wave function of the system and tensor products of a plane wave of momentum $\tilde{\boldsymbol{\kappa}}$ and the intrinsic state of the two-particle spectator subsystem. The mentioned tensor product allows one to take care of macroscopic locality, i.e., cluster separability [3], and to introduce a new effect of binding in the spectral function [1]. With the help of the Bakamanjian-Thomas (BT) construction of the Poincaré generators [42], the LF wave function can be obtained from the usual nonrelativistic wave function of the system with a realistic interaction between the nucleons. Then the LF spectral function allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré-covariant framework and to satisfy at the same time both normalization of the three-body system bound state (i.e., the baryon number sum rule) and momentum sum rule. Interestingly, in Ref. [1] the definition of the LF spin-dependent spectral function was plainly generalized to a generic system of A spin-1/2 fermions. As a first test of our approach, the calculation of the European Muon Collaboration (EMC) effect for ^{3}He is under way [43]. Preliminary results which consider only the contribution of the two-body bound-state channel show encouraging improvements [44–47] with respect to a convolution approach with a momentum distribution [48].

In this work the *most general expressions* for the spin-dependent spectral functions and for the spin-dependent momentum distribution in terms of six scalar functions, $\mathcal{B}_i$ and b_i, respectively ($i = 0, \ldots, 5$), valid for any system of spin-1/2 fermions are presented. In valence approximation, we demonstrate that a linear relation between the LF spectral function and the semi-inclusive fermion correlator occurs (for preliminary results see Refs. [49–53]). In turn, for a spin-1/2 system it is straightforward to relate the six T-even twist-two TMDs to the LF spectral function and eventually to the system wave function. The results for the ^{3}He TMDs corresponding to a realistic nuclear interaction are also presented. In particular, the LF longitudinal and transverse effective polarizations, quantities relevant for the extraction of the neutron information from data collected with polarized nuclear targets, are evaluated for the proton and the neutron in ^{3}He and compared with the corresponding nonrelativistic results currently used by experimental collaborations. Moreover, we assessed the approximated relations between the TMDs, investigated in Refs. [13,35], with the aim to offer a guide for the extraction of TMDs from experimental data.

It is important to stress that the validity of these relations could be tested using ^{3}He as a playground, since to perform a similar test for the proton target is much more challenging, at the present stage, due to quark confinement and hadron fragmentation. Indeed the measurement of ^{3}He TMDs appears feasible at high luminosity facilities, such as Jefferson Lab and the future electron-ion collider (EIC) [54], through $^3\overrightarrow{\mathrm{He}}(\overrightarrow{e}, e'p)X$ experiments with proper polarization setups of beams and targets [55].

It should be pointed out that the relations investigated in Refs. [13,35] could be applied and experimentally tested for the nucleon, once one considers the nucleon formed by three constituents, i.e., when the valence regime of the dressed quarks is acting.

The paper is organized as follows. In Sec. II, the most general expressions of the LF spin-dependent spectral function and of the LF spin-dependent momentum distribution are presented for any bound system, composed by A fermions of spin 1/2 in terms of six suitable scalar functions. Explicit expressions for the six scalar functions defining the momentum distribution are given in Appendix C for a three-nucleon system of spin 1/2. In Sec. III, the linear relation in valence approximation between the semi-inclusive correlator and the LF spectral function is derived. In Sec. IV, the relation between the LF spectral function and the T-even twist-two TMDs is illustrated. In Sec. V, the numerical results for the ^{3}He nucleus are presented, ranging from the TMDs to the new calculations of LF longitudinal and transverse effective polarizations for the proton and the neutron in ^{3}He. Also the approximate relations between the T-even twist-two TMDs are discussed. In Sec. VI, our conclusions are drawn.

II. THE LF SPIN-DEPENDENT SPECTRAL FUNCTION AND THE LF SPIN-DEPENDENT MOMENTUM DISTRIBUTION

Following Ref. [1], within the LFHD a Poincaré-covariant definition of the spin-dependent spectral function for an A-particle bound system polarized along $\mathbf{S}$ can be simply obtained by replacing the nonrelativistic overlaps $\langle \vec{p}_1, \sigma\tau; \psi_{f_{(A-1)}} | \psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle$, which define the nonrelativistic spectral function, with their LF counterparts $_{LF}\langle \tau_S, T_S; \alpha, \epsilon; J_z J; \tau, \sigma, \tilde{\boldsymbol{\kappa}} | \psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle$.

Hence, the LF spin-dependent spectral function reads

$$\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z\alpha} \sum_{T_S\tau_S} {}_{LF}\langle \tau_S, T_S; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle \times \langle S, TT_z; \psi_{\mathcal{J}\mathcal{M}} | \tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; T_S, \tau_S\rangle_{LF}, \quad (1)$$

[1] For the definitions of the kinematical variables of the LFHD used in this paper, the reader can refer to Ref. [1]. Let us only recall here that the light-front components of a four-vector v are $(v^-, \tilde{\mathbf{v}})$, where $\tilde{\mathbf{v}} = (v^+, \mathbf{v}_\perp)$ with $v^\pm = v^0 \pm \hat{\mathbf{n}} \cdot \mathbf{v}$ and $\mathbf{v}_\perp = \mathbf{v} - \hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \mathbf{v})$. The vector $\hat{\mathbf{n}}$ is a generic unit vector. Then the scalar product of two four vectors a and b is $a \cdot b = \frac{a^-b^+ + a^+b^-}{2} - \mathbf{a}_\perp \cdot \mathbf{b}_\perp$. In this paper we choose $\hat{\mathbf{n}} \equiv \hat{z}$.

where $|\psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle$ is the A-fermion ground state, with total angular momentum $\mathcal{J}$ (third component $\mathcal{M}$), isospin TT_z, and rest-frame polarization $S \equiv \{0, \mathbf{S}\}$. The state $|\tilde{\boldsymbol{\kappa}}, \sigma, \tau; T_S, \tau_S; \alpha, \epsilon; JJ_z\rangle_{LF}$ is the tensor product of (i) a fully interacting intrinsic state of the $(A-1)$-particle spectator system, with intrinsic energy ϵ (negative for bound states and positive for the continuum spectrum ones), spin J (third component J_z), isospin T_S (third component τ_S), and α the set of quantum numbers needed to completely specify the state, and (ii) a plane wave for the acting particle with LF momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the cluster $[1, (A-1)]$. The total LF momentum of the cluster is $\tilde{\mathbf{P}}_{\rm intr}[1, (A-1)] \equiv \{\mathcal{M}_0, \mathbf{0}_\perp\}$ with $\mathcal{M}_0$ its free mass. In Eq. (1), $\rho(\epsilon)$ is the energy density of the $(A-1)$-particle states. Notice that for $(A-1) = 2$, one has $\rho(\epsilon) = 1$ for the bound states, e.g., for the deuteron, and $\rho(\epsilon) = m\sqrt{m\epsilon}/2$ for the continuum, with m the constituent mass.

Denoting with $\mathbf{p}_i$ $(i = 1, \ldots, A)$ and $\mathbf{P}$ the momenta of the particles and the whole system in the laboratory frame, respectively, one gets the following expression for the intrinsic momentum $\tilde{\boldsymbol{\kappa}}$:

$$\kappa^+ = \xi_1 \mathcal{M}_0[1, (A-1)], \quad \boldsymbol{\kappa}_\perp = \mathbf{p}_{1\perp} - \xi_1 \mathbf{P}_\perp, \tag{2}$$

where $\xi_1 = p_1^+/P^+$ and $\mathcal{M}_0[1, (A-1)]$, the previously mentioned free mass of the $[1, (A-1)]$ cluster [1], is given by

$$\mathcal{M}_0^2[1, (A-1)] = \frac{m^2 + \boldsymbol{\kappa}_\perp^2}{\xi_1} + \frac{M_S^2 + \boldsymbol{\kappa}_\perp^2}{(1-\xi_1)}, \tag{3}$$

with M_S the mass of the interacting $(A-1)$ system. Let us assume that the system is *at rest in the laboratory*. Then it follows that $\boldsymbol{\kappa}_\perp = \mathbf{p}_{1\perp}$ (recall that the LF momentum $\tilde{\boldsymbol{\kappa}}$ is relative to the cluster frame). For completeness let us introduce the LF momentum $\tilde{\mathbf{k}} \equiv (k^+, \mathbf{k}_\perp)$ of the acting particle in the intrinsic A-particle system, with transverse component $\mathbf{k}_\perp = \mathbf{p}_{1\perp} - \xi_1 \mathbf{P}_\perp = \mathbf{p}_{1\perp}$, plus component $k^+ = \xi_1 M_0 \neq p_1^+$ (M_0 is the free mass of A particles). In what follows the subscript 1 will be dropped out and the notations $\xi_1 = x$ and $\mathbf{p}_{1\perp} = \mathbf{p}_\perp$ will be adopted.

It is worth reminding that the states $|\tilde{\boldsymbol{\kappa}}, \sigma, \tau; T_S, \tau_S; \alpha, \epsilon; JJ_z\rangle_{LF}$, to be used for the definition of the spectral function in the LF overlaps ${}_{LF}\langle\tau_S, T_S; \alpha, \epsilon; J_z J; \tau\sigma, \tilde{\boldsymbol{\kappa}}|\psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle$, fulfill the macroscopic locality. The property of macroscopic locality means that the unitary representation of the Poincaré group for a system composed of two separated subsystems can be expressed as the tensor product of the unitary representations of the two subsystems. Hence, subsystem observables, associated with different space-time regions, must commute for large enough space-time separation (see Refs. [1,3]). This is the mathematical formulation of the physical insight that when a system is separated in disjoint subsystems, these subsystems must behave as independent subsystems. Obviously the notion of disjoint subsystems does not apply to systems of quarks, where asymptotic states do not exist due to the confinement.

Furthermore, the use of the momentum $\tilde{\boldsymbol{\kappa}}$, instead of the momentum $\mathbf{p}$ of the fermion in the laboratory frame, introduces a new effect of binding in the spectral function.

The LF overlaps where the ground state has a generic polarization vector $\mathbf{S}$, i.e., $|\psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle$, can be obtained from the overlaps where the ground state is polarized along the z axis, i.e., $|\mathcal{J}m; \epsilon^A, \Pi; TT_z\rangle_z$, by using the Wigner rotation matrices, $D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha, \beta, \gamma)$, viz.,

$$|\psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle = \sum\nolimits_m |\mathcal{J}m; \epsilon^A, \Pi; TT_z\rangle_z D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha, \beta, \gamma), \tag{4}$$

where α, β, and γ are the Euler angles describing the proper rotation from the z axis to the polarization vector $\mathbf{S}$ and ϵ^A and Π are the energy and the parity of the state, respectively. Let us recall that the rotations involved act on the bound system as a whole, and therefore they are interaction free. Through Eq. (4) one can relate the spin-dependent spectral function with a given polarization to the one with polarization $\mathbf{S} = \hat{\mathbf{z}}$ (see Appendix A).

As explained in Ref. [1], in the three-nucleon case the overlaps ${}_{LF}\langle\tau_S, T_S; \alpha, \epsilon; J_z J; \tau\sigma, \tilde{\boldsymbol{\kappa}}|\psi_{\mathcal{J}m}; S, TT_z\rangle_z$ can be evaluated in terms of canonical (or instant-form) two- and three-body wave functions, replaced by the nonrelativistic ones, after applying the Melosh rotation matrices [56,57] (needed for obtaining the LF spin states from the canonical ones). This follows, once the Bakamjian-Thomas construction of the Poincaré generators [42] is adopted. Then, it turns out that the two- and three-body nonrelativistic wave functions have all the needed properties with respect to rotations and translations of the corresponding canonical wave functions.

In conclusion the LF spin-dependent spectral function is a 2×2 matrix, $\widehat{\mathcal{P}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$, which depends on the direction of the polarization vector $\mathbf{S}$, and it is usually normalized for each isospin channel $\tau = p(n)$, i.e.,

$$\sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}}{2E(\kappa)(2\pi)^3} Tr\big[\widehat{\mathcal{P}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)\big] = 1, \tag{5}$$

with $E(\kappa) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2}$.

A general expression of $\widehat{\mathcal{P}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$ can be obtained in terms of the vectors at our disposal in the *rest frame of the system*, i.e., (i) the unit vector $\hat{\mathbf{n}}$, which defines the LF components of a four-vector, (ii) the polarization vector $\mathbf{S}$, and (iii) the transverse (with respect to the $\hat{\mathbf{n}}$ axis) momentum component $\mathbf{k}_\perp = \mathbf{p}_\perp = \boldsymbol{\kappa}_\perp$. Let us recall that we adopt $\hat{\mathbf{n}} \equiv \hat{z}$. Then the LF spin-dependent spectral function reads

$$\mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \tfrac{1}{2}\big[\mathcal{B}^\tau_{0,\mathcal{M}} + \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S})\big]_{\sigma'\sigma}, \tag{6}$$

where the function $\mathcal{B}^\tau_{0,\mathcal{M}}$ is the trace of $\widehat{\mathcal{P}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$ and yields the unpolarized spectral function, while

$$\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S}) = Tr\big[\widehat{\mathcal{P}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)\boldsymbol{\sigma}\big]. \tag{7}$$

The quantity $\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S})$ is a pseudovector and depends on the direction of the polarization vector $\mathbf{S}$. Therefore, it can be written as a linear combination of the independent pseudovectors at our disposal, viz., $\mathbf{S}$, $\hat{\mathbf{k}}_\perp(\mathbf{S} \cdot \hat{\mathbf{k}}_\perp)$, $\hat{\mathbf{k}}_\perp(\mathbf{S} \cdot \hat{z})$, $\hat{z}\,(\mathbf{S} \cdot \hat{\mathbf{k}}_\perp)$, and $\hat{z}\,(\mathbf{S} \cdot \hat{z})$. Furthermore, $\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}$ depends on x,

$$x = \frac{\kappa^+}{\mathcal{M}_0[1, (A-1)]}, \tag{8}$$

where $\mathcal{M}_0[1,(A-1)]$ (cf. Eq. (3)) is given in terms of $\tilde{\boldsymbol{\kappa}}$ by [1]

$$\mathcal{M}_0[1,(A-1)] = E(\kappa) + E_S = \frac{(\kappa^+)^2 + (m^2 + |\mathbf{k}_\perp|^2)}{2\kappa^+} + \left\{ \left[\frac{(\kappa^+)^2 + (m^2 + |\mathbf{k}_\perp|^2)}{2\kappa^+} \right]^2 + M_S^2 - m^2 \right\}^{1/2}, \tag{9}$$

where $E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2}$, with $M_S^2 = 4m^2 + 4m\epsilon$ for $(A-1) = 2$, and $\mathbf{k}_\perp = \boldsymbol{\kappa}_\perp = \mathbf{p}_\perp$.

Hence $\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}$ can be expressed as a sum of the five available independent pseudovectors multiplied by five scalar quantities, $\mathcal{B}^\tau_{i,\mathcal{M}}$ $(i = 1, \ldots, 5)$, viz.,

$$\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \epsilon, \mathbf{S}) = \mathbf{S}\mathcal{B}^\tau_{1,\mathcal{M}} + \hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^\tau_{2,\mathcal{M}} + \hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{z})\mathcal{B}^\tau_{3,\mathcal{M}} + \hat{z}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^\tau_{4,\mathcal{M}} + \hat{z}(\mathbf{S}\cdot\hat{z})\mathcal{B}^\tau_{5,\mathcal{M}}, \tag{10}$$

where the dependence upon $\hat{n} \equiv \hat{z}$ is understood for making light the notation. It should be pointed out that to fully address the issue of TMDs one has to distinguish between the transverse degrees of freedom (DOF) and the one associated to $\hat{n}$. Therefore, one cannot anymore use the parametrization with only three scalar functions considered in Ref. [58].

The six scalar quantities $\mathcal{B}^\tau_{i,\mathcal{M}}$ in general can depend on the possible scalars at our disposal, i.e., x, $|\mathbf{k}_\perp|$, ϵ, $(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2$, $(\mathbf{S}\cdot\hat{z})^2$, and $(\hat{\mathbf{k}}_\perp \times \hat{z})\cdot\mathbf{S}$. However, as shown in Appendix A, for a system of total angular momentum $\mathcal{J} = 1/2$, as ^{3}He or ^{3}H, the quantities $\mathcal{B}^\tau_{i,\mathcal{M}}$ can depend only on x, $|\mathbf{k}_\perp|$, and ϵ.

Through the trace of the spectral function $\widehat{\boldsymbol{\mathcal{P}}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$ one can define the LF *spin-independent* nucleon momentum distribution, averaged on the spin directions, as follows (see Ref. [1]):

$$\begin{aligned} n^\tau(x, \mathbf{k}_\perp) &= \sum\!\!\!\!\!\!\int d\epsilon \frac{1}{2\kappa^+(2\pi)^3} \frac{\partial\kappa^+}{\partial x} \mathrm{Tr}\mathcal{P}^\tau(\tilde{\boldsymbol{\kappa}}, \epsilon, S) \\ &= \sum\!\!\!\!\!\!\int d\epsilon \frac{1}{2(2\pi)^3} \frac{E_S}{(1-x)\kappa^+} \rho(\epsilon) \sum_\sigma \sum_{JJ_z\alpha} \sum_{T_S\tau_S} {}_{LF}\langle \tau_S, T_S; \alpha, \epsilon; J_z J; \tau\sigma, \tilde{\boldsymbol{\kappa}} | \psi_{\mathcal{J}\mathcal{M}}; S, TT_z\rangle \\ &\quad \times \langle TT_z, S; \psi_{\mathcal{J}\mathcal{M}} | \tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; T_S, \tau_S\rangle_{LF}. \end{aligned} \tag{11}$$

The completeness relation of the nonsymmetric basis for three-interacting-particle systems (see Eq. (51) of Ref. [1]) immediately leads to the normalization of the nucleon momentum distribution, i.e., the baryon number sum rule,

$$\int dx \int d\mathbf{k}_\perp\, n^\tau(x, \mathbf{k}_\perp) \;=\; 1, \tag{12}$$

and to the momentum sum rule,

$$\int x dx \int d\mathbf{k}_\perp n^\tau(x, \mathbf{k}_\perp) = \frac{1}{3}. \tag{13}$$

From the LF spin-dependent spectral function, after performing the integration shown in Eq. (11), one can obtain the LF *spin-dependent* momentum distribution, a 2×2 matrix defined by (see also Appendix C)

$$\left[\boldsymbol{\mathcal{N}}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})\right]_{\sigma'\sigma} = \sum\!\!\!\!\!\!\int d\epsilon \frac{1}{2(2\pi)^3} \frac{1}{1-x} \frac{E_S}{\kappa^+} \mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \frac{\pi}{4m} \int \frac{dp^+ dp^-}{(2\pi)^4} \delta(p^+ - \xi P^+) P^+ \frac{E_S}{\kappa^+} \mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S). \tag{14}$$

As shown in Appendix C, one can obtain the LF spin-dependent momentum distribution from the three-body wave function, using Eq. (4) and the expression for the LF spin-dependent spectral function given by Eq. (72) of Ref. [1].

As it occurs for the spectral function, the momentum distribution can be expressed through the three independent vectors available in the rest frame of the system, i.e., $\mathbf{k}_\perp$, $\mathbf{S}$, and $\hat{n} \equiv \hat{z}$, and six scalar functions $b^\tau_{i,\mathcal{M}}$ $(i = 0, 1, \ldots, 5)$, viz.,

$$\boldsymbol{\mathcal{N}}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) = \tfrac{1}{2}\{b_{0,\mathcal{M}} + \boldsymbol{\sigma}\cdot\boldsymbol{f}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})\}, \tag{15}$$

where $\boldsymbol{f}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})$ is a pseudovector (recall that the dependence upon $\hat{n} \equiv \hat{z}$ has been dropped out for simplicity) that can be decomposed as follows:

$$\boldsymbol{f}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) = \mathbf{S} b^\tau_{1,\mathcal{M}} + \hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp) b^\tau_{2,\mathcal{M}} + \hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{z}) b^\tau_{3,\mathcal{M}} + \hat{z}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp) b^\tau_{4,\mathcal{M}} + \hat{z}(\mathbf{S}\cdot\hat{z}) b^\tau_{5,\mathcal{M}}. \tag{16}$$

The functions $b^{\tau}_{i,\mathcal{M}}$, that depend upon x, $|\mathbf{k}_\perp|$, $(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2$, $(\mathbf{S}\cdot\hat{z})^2$, and $(\hat{\mathbf{k}}_\perp\times\hat{z})\cdot\mathbf{S}$, are integrals over the energy ϵ of the functions $\mathcal{B}^{\tau}_{i,\mathcal{M}}$ [see Eq. (14)], viz.,

$$b^{\tau}_{i,\mathcal{M}}[x,|\mathbf{k}_\perp|,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2,(\mathbf{S}\cdot\hat{z})^2,(\hat{\mathbf{k}}_\perp\times\hat{z})\cdot\mathbf{S}] = \frac{\pi}{4m}\int\frac{dp^+dp^-}{(2\pi)^4}\delta[p^+-xP^+]P^+\frac{E_S}{\kappa^+}\mathcal{B}^{\tau}_{i,\mathcal{M}}[x,|\mathbf{k}_\perp|,\epsilon,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2,(\mathbf{S}\cdot\hat{z})^2,(\hat{\mathbf{k}}_\perp\times\hat{z})\cdot\mathbf{S}]. \quad (17)$$

In Appendix C, for a three-nucleon system of total angular momentum $\mathcal{J}=1/2$ explicit expressions for the quantities $b^{\tau}_{i,\mathcal{M}}$ $(i=0,1,\ldots,5)$ are obtained in terms of the wave function of the three-nucleon system, according to the BT procedure. It is also shown that these functions do not depend on $\mathbf{S}$, while they do depend on $|\mathbf{k}_\perp|$ and x. Moreover, the quantity b_0 is independent of $\mathcal{M}$, while for $i=1,\ldots,5$ the dependence on $\mathcal{M}$ is through the factor $(-1)^{\mathcal{M}+1/2}$.

From the actual expressions of the quantities $b^{\tau}_{i,\mathcal{M}}$, the essential role of the Melosh matrices to generate the six different quantities $b^{\tau}_{i,\mathcal{M}}$ clearly emerges. Their effect is parametrized by the angle φ present in the expression of the Melosh rotations, $\mathcal{R}_M(\tilde{\mathbf{k}})$, given in Appendix D, viz.,

$$D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'} = \left[\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\right]_{\sigma\sigma'}, \quad (18)$$

where

$$\varphi = 2\ \arctan\frac{|\mathbf{k}_\perp|}{k^+ + m}. \quad (19)$$

It has to be pointed out that φ is small if the relevant values of $|\mathbf{k}_\perp|/m$ are too, making the effect of the Melosh rotations small.

Indeed, in absence of the Melosh matrices, the quantities $b^{\tau}_{2,\mathcal{M}}$, $b^{\tau}_{3,\mathcal{M}}$, $b^{\tau}_{4,\mathcal{M}}$, and $b^{\tau}_{5,\mathcal{M}}$ are related to each other by factors as $\cos^2\theta$, $\sin^2\theta$, and $\cos\theta\sin\theta$, with θ the angle between the momentum $\mathbf{k}$ and the z axis. Then in this case the spin-dependent momentum distribution can be expressed in terms of only three independent scalar quantities, b^{τ}_0, $b^{\tau}_{1,\mathcal{M}}$, and $b^{\tau}_{2,\mathcal{M}}/\sin^2\theta = b^{\tau}_{3,\mathcal{M}}/\cos\theta\sin\theta = b^{\tau}_{4,\mathcal{M}}/\cos\theta\sin\theta = b^{\tau}_{5,\mathcal{M}}/\cos^2\theta$, as in the nonrelativistic approximation.

Our aim is to obtain an expression of TMDs from the functions $b^{\tau}_{i,\mathcal{M}}$, just introduced. To accomplish this task, another ingredient, the fermion correlator for a semi-inclusive process, has to be added, along with its relation to the LF spectral function. This is detailed in the following section.

III. THE SEMI-INCLUSIVE CORRELATOR FOR A $\mathcal{J}=1/2$ BOUND SYSTEM AND THE LF SPECTRAL FUNCTION

Let p be the momentum in the laboratory frame of an off-mass-shell spin-1/2 particle, with isospin τ, inside a bound system of A spin-1/2 particles with total momentum P and spin S. The semi-inclusive fermion correlator in terms of the LF coordinates is [8]

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\zeta^-d\zeta^+d\boldsymbol{\zeta}_\perp e^{ip^-\zeta^+/2}e^{ip^+\zeta^-/2}e^{-i\mathbf{p}_\perp\cdot\boldsymbol{\zeta}_\perp} \times\langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)\mathcal{W}(\hat{n}\cdot\mathcal{A})\psi^{\tau}_{\alpha}(\zeta)|A,S,P\rangle, \quad (20)$$

where $|A,S,P\rangle$ is the A-particle state (e.g., a nucleus or a nucleon), $\psi^{\tau}_{\alpha}(\zeta)$ the particle field (e.g., a nucleon of isospin τ if the system is a nucleus, or a quark if the system is a nucleon), and $\mathcal{W}(\hat{n}\cdot\mathcal{A})$ is a link operator which makes $\Phi^{\tau}_{\alpha,\beta}(p,P,S)$ gauge invariant. By working in the $\mathcal{A}^+=0$ gauge, $\mathcal{W}$ can be reduced to unity. Let us notice that the $\mathcal{A}^+$ gauge condition is preserved under Lorentz transformation in the LF dynamics [59,60]. Hereafter, we assume that the link operator is the unity operator. Using the translation invariance relation

$$\psi^{\tau}_{\alpha}(\zeta) = e^{i\widehat{P}\cdot\zeta}\,\psi^{\tau}_{\alpha}(0)\,e^{-i\widehat{P}\cdot\zeta}, \quad (21)$$

one can rewrite the correlator as

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\zeta^-d\zeta^+d\boldsymbol{\zeta}_\perp e^{i(p-P)\cdot\zeta} \times\langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)e^{i\zeta\cdot\widehat{P}}\psi^{\tau}_{\alpha}(0)|A,S,P\rangle, \quad (22)$$

where the explicit expression for the fermion field in $\zeta^\mu=0$ is [60]

$$\psi^{\tau}_{\alpha}(0) = \int\frac{d\tilde{\mathbf{p}}}{2p^+(2\pi)^3} \times\sum_{\sigma}\left[b^{\tau}_{\sigma}(\tilde{\mathbf{p}})u_{\alpha}(\tilde{\mathbf{p}},\sigma)+d^{\tau\dagger}_{\sigma}(\tilde{\mathbf{p}})v_{\alpha}(\tilde{\mathbf{p}},\sigma)\right], \quad (23)$$

with $\tilde{\mathbf{p}}=(p^+,\mathbf{p}_\perp)$ the LF particle momentum in the laboratory frame and $u(\tilde{\mathbf{p}},\sigma)$ and $v(\tilde{\mathbf{p}},\sigma)$ the particle and antiparticle LF spinors [60,61]. The following spinor normalization is adopted:

$$\bar{u}(\tilde{\mathbf{p}},\sigma)u(\tilde{\mathbf{p}},\sigma') = 2\,m\,\delta_{\sigma\sigma'}. \quad (24)$$

The Fock operators satisfy the canonical anticommutation relations,

$$\left\{b^{\tau}_{\sigma}(\tilde{\mathbf{p}}),b^{\tau'\dagger}_{\sigma'}(\tilde{\mathbf{p}}')\right\} = \left\{d^{\tau}_{\sigma}(\tilde{\mathbf{p}}),d^{\tau'\dagger}_{\sigma'}(\tilde{\mathbf{p}}')\right\} = 2p^+(2\pi)^3\delta_{\sigma\sigma'}\delta_{\tau\tau'}\delta(\tilde{\mathbf{p}}-\tilde{\mathbf{p}}'). \quad (25)$$

By inserting in Eq. (22) for the correlator $\Phi^{\tau}_{\alpha,\beta}(p,P,S)$ the fermionic field given in Eq. (23), the particle correlator reads

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \int d\zeta^+ \int d\bar{\boldsymbol{\zeta}} e^{i(p^- - P^-)\zeta^+/2} e^{i(\tilde{\mathbf{p}}-\tilde{\mathbf{P}})\cdot\bar{\zeta}} \int \frac{d\tilde{\mathbf{p}}'}{2p'^+(2\pi)^3} \int \frac{d\tilde{\mathbf{p}}''}{2p''^+(2\pi)^3} \langle P,S,A| \times \sum_{\sigma''} \left[b^{\tau\dagger}_{\sigma''}(\tilde{\mathbf{p}}'')\bar{u}_\beta(\tilde{\mathbf{p}}'',\sigma'') + d^{\tau}_{\sigma''}(\tilde{\mathbf{p}}'')\bar{v}_\beta(\tilde{\mathbf{p}}'',\sigma'')\right] e^{i\hat{P}^-\zeta^+/2} \times \sum_{\sigma'} [e^{i(\tilde{\mathbf{P}}-\tilde{\mathbf{p}}')\cdot\bar{\zeta}} b^{\tau}_{\sigma'}(\tilde{\mathbf{p}}')u_\alpha(\tilde{\mathbf{p}}',\sigma') + e^{i(\tilde{\mathbf{P}}+\tilde{\mathbf{p}}')\cdot\bar{\zeta}} d^{\tau\dagger}_{\sigma'}(\tilde{\mathbf{p}}')v_\alpha(\tilde{\mathbf{p}}',\sigma')]|A,S,P\rangle, \tag{26}$$

with $\bar{\boldsymbol{\zeta}} = (\zeta^-/2, \boldsymbol{\zeta}_\perp)$. Recall that $b^{\tau}_{\sigma'}(\tilde{\mathbf{p}}')|A,S,P\rangle$ and $d^{\tau\dagger}_{\sigma'}(\tilde{\mathbf{p}}')|A,S,P\rangle$ are eigenvectors of the LF momentum operator (it is a kinematical one) with eigenvalues $\tilde{P} - \tilde{p}'$ and $\tilde{P} + \tilde{p}'$, respectively.

By performing the integration on $\{\zeta^+/2, \bar{\boldsymbol{\zeta}}\}$, the term in Eq. (26) with $v_\alpha(\tilde{\mathbf{p}}',\sigma')$ vanishes, since p'^+ cannot be negative. Then one has

$$\begin{aligned}\Phi^{\tau}_{\alpha,\beta}(p,P,S) &= \sum_{\sigma'} u_\alpha(\tilde{\mathbf{p}},\sigma')\frac{2\pi}{p^+}\int \frac{d\tilde{\mathbf{p}}''}{2p''^+(2\pi)^3}\langle P,S,A|\sum_{\sigma''}[b^{\tau\dagger}_{\sigma''}(\tilde{\mathbf{p}}'')\bar{u}_\beta(\tilde{\mathbf{p}}'',\sigma'') + d^{\tau}_{\sigma''}(\tilde{\mathbf{p}}'')\bar{v}_\beta(\tilde{\mathbf{p}}'',\sigma'')] \\ &\quad \times \delta(\hat{P}^- + p^- - P^-) b^{\tau}_{\sigma'}(\tilde{\mathbf{p}})|A,S,P\rangle \\ &= \frac{2\pi}{p^+}\sum_{\sigma}\sum_{\sigma'} u_\alpha(\tilde{\mathbf{p}},\sigma')\int \frac{\tilde{\mathbf{p}}''}{2p''^+(2\pi)^3}\langle P,S,A|b^{\tau\dagger}_{\sigma}(\tilde{\mathbf{p}}'')\delta(\hat{P}^- + p^- - P^-)b^{\tau}_{\sigma'}(\tilde{\mathbf{p}})|A,S,P\rangle \bar{u}_\beta(\tilde{\mathbf{p}}'',\sigma),\end{aligned} \tag{27}$$

where in the last step the antiparticle contribution has been eliminated, since the LF momentum has to be conserved and p''^+ cannot be negative.

In the previous equation, let us introduce the completeness for the states of $(A-1)$ particles in valence approximation (see also Ref. [1]):

$$\sum_{JJ_z\alpha}\sum_{T_S\tau_S} \sum\!\!\!\!\!\!\int \rho(\epsilon)d\epsilon \int \frac{d\tilde{\mathbf{P}}_S}{(2\pi)^3 2P^+_S}|\tilde{\mathbf{P}}_S; JJ_z\epsilon,\alpha; T_S\tau_S\rangle_{LF\,LF}\langle\tau_S T_S;\alpha,\epsilon J_z J;\tilde{\mathbf{P}}_S| = 1. \tag{28}$$

In Eq. (28) $\tilde{\mathbf{P}}_S$ is the total LF momentum of the fully interacting $(A-1)$-particle system. The symbol $\sum\!\!\!\!\!\int$ means a sum over the bound states of the $(A-1)$ system and an integration over the continuum.

For the bound states $|\tilde{\mathbf{P}}_S; JJ_z\epsilon,\alpha; T_S\tau_S\rangle_{LF}$, the normalization adopted is

$${}_{LF}\langle T'\tau'; J'J'_z\epsilon'\alpha', \tilde{\mathbf{P}}'_S|\tilde{\mathbf{P}}_S; JJ_z\epsilon\alpha; T\tau\rangle_{LF} = 2P^+_S(2\pi)^3\delta^3(\tilde{\mathbf{P}}'_S - \tilde{\mathbf{P}}_S)\delta_{T',T}\delta_{\tau',\tau}\delta_{\alpha',\alpha}\delta_{J',J}\delta_{J'_z,J_z}\delta_{\epsilon',\epsilon}, \tag{29}$$

while for the LF continuum states the orthogonality reads (see Appendix A of Ref. [1])

$${}_{LF}\langle T'\tau';\alpha'\epsilon' J'_z J'; \tilde{\mathbf{P}}'_S|\tilde{\mathbf{P}}_S; JJ_z\epsilon\alpha; T\tau\rangle_{LF} = 2P^+_S(2\pi)^3\delta^3(\tilde{\mathbf{P}}'_S - \tilde{\mathbf{P}}_S)\delta_{T',T}\delta_{\tau',\tau}\delta_{\alpha',\alpha}\delta_{J',J}\delta_{J'_z,J_z}\frac{\delta(\epsilon'-\epsilon)}{\rho(\epsilon)}. \tag{30}$$

Then, one can define as follows the valence contribution to the particle correlator:

$$\begin{aligned}\left[\Phi^{\tau}_V(p,P,S)\right]_{\alpha,\beta} &= \frac{2\pi}{p^+}\int\frac{d\tilde{\mathbf{p}}''}{(2\pi)^3 2p''^+}\sum_{JJ_z\alpha}\sum_{T_S\tau_S}\sum_{J'J'_z\alpha'}\sum_{T'_S\tau'_S}\sum\!\!\!\!\!\!\int\rho(\epsilon)d\epsilon\sum\!\!\!\!\!\!\int\rho(\epsilon')d\epsilon'\int\frac{d\tilde{\mathbf{P}}_S}{(2\pi)^3 2P^+_S}\int\frac{d\tilde{\mathbf{P}}'_S}{(2\pi)^3 2P'^+_S} \\ &\quad\times\sum_{\sigma\sigma'}[u_\alpha(\tilde{\mathbf{p}},\sigma')\langle P,S,A|\tilde{\mathbf{p}}''\sigma\tau;\tilde{\mathbf{P}}'_S;J'J'_z\epsilon',\alpha';T'_S\tau'_S\rangle_{LF\,LF}\langle\tau'_S T'_S;\alpha',\epsilon' J'_z J';\tilde{\mathbf{P}}'_S| \\ &\quad\times\delta(\hat{P}^- + p^- - P^-)|\tilde{\mathbf{P}}_S;JJ_z\epsilon,\alpha;T_S\tau_S\rangle_{LF\,LF}\langle\tau_S T_S;\alpha,\epsilon J_z J;\tilde{\mathbf{P}}_S;\tilde{\mathbf{p}}\sigma'\tau|A,S,P\rangle\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)] \\ &= \frac{2\pi}{p^+}\int\frac{d\tilde{\mathbf{p}}''}{(2\pi)^3 2p''^+}\sum_{JJ_z\alpha}\sum_{T_S\tau_S}\sum\!\!\!\!\!\!\int\rho(\epsilon)d\epsilon\int\frac{d\tilde{\mathbf{P}}_S}{(2\pi)^3 2P^+_S}\sum_{\sigma\sigma'}[u_\alpha(\tilde{\mathbf{p}},\sigma')\langle P,S,A|\tilde{\mathbf{p}}''\sigma\tau;\tilde{\mathbf{P}}_S;JJ_z\epsilon,\alpha;T_S\tau_S\rangle_{LF} \\ &\quad\times\delta(P^-_S + p^- - P^-)_{LF}\langle\tau_S T_S;\alpha,\epsilon J_z J;\tilde{\mathbf{P}}_S;\tilde{\mathbf{p}}\sigma'\tau|A,S,P\rangle\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)],\end{aligned} \tag{31}$$

where the equality

$$\hat{b}^{\tau\dagger}_{\sigma}(\tilde{\mathbf{p}}'')|\tilde{\mathbf{P}}'_S;J'J'_z\epsilon',\alpha';T'_S\tau'_S\rangle_{LF} = |\tilde{\mathbf{p}}''\sigma\tau;\tilde{\mathbf{P}}'_S;J'J'_z\epsilon',\alpha';T'_S\tau'_S,\rangle_{LF} \tag{32}$$

has been used; i.e., a free particle $|\tilde{\mathbf{p}}''\sigma\tau\rangle$, with momentum $\tilde{\mathbf{p}}''$ in the laboratory frame, has been created. Moreover, it has been taken into account that the operator $\hat{P}^-$ acts on $|\tilde{\mathbf{P}}_S; JJ_z\epsilon, \alpha; T_S, \tau_S\rangle_{LF}$ as follows:

$$\begin{aligned}\hat{P}^-|\tilde{\mathbf{P}}_S; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF} &= P_S^-|\tilde{\mathbf{P}}_S; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF}\\ &= \frac{M_S^2+|\mathbf{P}_{S\perp}|^2}{P_S^+}|\tilde{\mathbf{P}}_S; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF},\end{aligned} \quad (33)$$

with M_S the mass of the interacting $(A-1)$-particle system (for $A-1=2$, one has $M_S^2 = 4m^2+4m\epsilon$).

By considering that the LF momentum is conserved (the interaction is contained only in the minus component of the momenta) and the kinematical nature of the LF boosts, one has the exact separation of the intrinsic DOF from the center of mass (CM) ones (see Appendix A of Ref. [45]), obtaining

$$\begin{aligned}&|\tilde{\mathbf{p}}\sigma\tau; \tilde{\mathbf{P}}_S; JJ_z\epsilon, \alpha; T_S, \tau_S\rangle_{LF}\\ &= \sqrt{\frac{E_S}{\mathcal{M}_0[1,(A-1)]}}|\tilde{\mathbf{p}}+\tilde{\mathbf{P}}_S\rangle_{LF}|\tilde{\boldsymbol{\kappa}}\sigma\tau; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF},\end{aligned} \quad (34)$$

where $|\tilde{\mathbf{p}}+\tilde{\mathbf{P}}_S\rangle_{LF}$ is the total LF momentum eigenstate of the cluster $[1,(A-1)]$. The intrinsic state $|\tilde{\boldsymbol{\kappa}}\sigma\tau; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF}$ is composed of a fully interacting intrinsic state of $(A-1)$ constituents and a plane wave, describing a constituent that freely moves in the *intrinsic frame of the whole cluster* $[1,(A-1)]$ with LF momentum $\tilde{\boldsymbol{\kappa}}$ [see Eq. (2)].

In Eq. (34) one has $E_S = \sqrt{M_S^2+|\boldsymbol{\kappa}|^2}$, and $\mathcal{M}_0[1,(A-1)]$ is defined by Eq. (3). The factor $\sqrt{E_S/\mathcal{M}_0[1,(A-1)]}$ takes care of the proper normalization of the momentum eigenstates $|\tilde{\mathbf{p}}\rangle_{LF}$, $|\tilde{\mathbf{P}}_S\rangle_{LF}$, and $|\tilde{\mathbf{p}}+\tilde{\mathbf{P}}_S\rangle_{LF}$ (see Ref. [45]).

Summarizing, the following overlap, present in Eq. (31), can be written as follows:

$$\begin{aligned}&\langle P, S, A|\tilde{\mathbf{p}}\sigma\tau; \tilde{\mathbf{P}}_S; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF}\\ &= 2P^+(2\pi)^3\sqrt{\frac{E_S}{\mathcal{M}_0[1,(A-1)]}}\delta^3(\tilde{\mathbf{P}}-\tilde{\mathbf{P}}_S-\tilde{\mathbf{p}})\\ &\quad\times\langle A, S, \text{int}|\tilde{\boldsymbol{\kappa}}\sigma\tau; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF},\end{aligned} \quad (35)$$

where the orthogonality of the plane waves is given by $\langle\tilde{\mathbf{P}}|\tilde{\mathbf{P}}'\rangle = 2P^+(2\pi)^3\delta(\tilde{\mathbf{P}}-\tilde{\mathbf{P}}')$ and $|\text{int}, S, A\rangle \equiv |\psi_{\mathcal{JM}}; S, TT_z\rangle$ is the intrinsic eigenstate of the system.

The normalization for the intrinsic overlaps ${}_{LF}\langle\tau_S T_S; \alpha, \epsilon; J_z J; \tilde{\boldsymbol{\kappa}}\sigma\tau|\text{int}, S, A\rangle$ [see Eq. (5)] reads

$$\begin{aligned}&\int\frac{d\tilde{\boldsymbol{\kappa}}}{2\kappa^+(2\pi)^3}\sum\!\!\!\!\!\!\int\rho(\epsilon)d\epsilon\sum_\sigma\sum_{T_S\tau_S}\\ &\quad\times\sum_{JJ_z\alpha}|{}_{LF}\langle\tau_S T_S; \alpha, \epsilon J_z J; \tilde{\boldsymbol{\kappa}}\sigma\tau|\text{int}, S, A\rangle|^2 = 1.\end{aligned} \quad (36)$$

Eventually, with the help of Eq. (35) the general expression for the valence contribution to the semi-inclusive fermion correlator becomes

$$\begin{aligned}\left[\Phi_V^\tau(p,P,S)\right]_{\alpha,\beta} &= 2\pi\left(\frac{P^+}{p^+}\right)^2\sum_{JJ_z\alpha}\sum_{T_S\tau_S}\sum\!\!\!\!\!\!\int\rho(\epsilon)d\epsilon\frac{\delta(P_S^-+p^--P^-)}{(P^+-p^+)}\frac{E_S}{\mathcal{M}_0[1,(A-1)]}\\ &\quad\times\sum_{\sigma\sigma'}[u_\alpha(\tilde{\mathbf{p}},\sigma')\langle A, S, \text{int}|\tilde{\boldsymbol{\kappa}}\sigma\tau; JJ_z\epsilon, \alpha; T_S\tau_S\rangle_{LF}\,{}_{LF}\langle\tau_S T_S; \alpha, \epsilon J_z J; \tilde{\boldsymbol{\kappa}}\sigma'\tau|\text{int}, S, A\rangle_{LF}\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)],\end{aligned} \quad (37)$$

where $P_S^- = (M_S^2+|\mathbf{P}_{S\perp}|^2)/P_S^+$ [see Eq. (33)]. Once the mass M_S is expressed in terms of the intrinsic energy, the integration on ϵ can be easily performed, obtaining a relation between the correlator and the spin-dependent LF spectral function defined in Eq. (1).

In the case where $(A-1)=2$ and in the reference frame where $\mathbf{P}_\perp = 0$, the δ function in Eq. (37) implies the equation

$$\frac{M^2}{P^+} = p^- + \frac{4m^2+4m\epsilon+|\mathbf{P}_{S\perp}|^2}{P^+-p^+}, \quad (38)$$

i.e.,

$$\epsilon = \frac{\left(\frac{M^2}{P^+}-p^-\right)(P^+-p^+)-|\mathbf{P}_{S\perp}|^2}{4m} - m \quad (39)$$

with $\mathbf{P}_{S\perp} = -\mathbf{p}_\perp = -\boldsymbol{\kappa}_\perp$. Then for $A=3$ one obtains

$$\begin{aligned}[\Phi_V^\tau(p,P,S)]_{\alpha,\beta} &= \frac{2\pi(P^+)^2}{(p^+)^2 4m}\frac{E_S}{\mathcal{M}_0[1,(23)]}\\ &\quad\times\sum_{\sigma\sigma'}\{u_\alpha(\tilde{\mathbf{p}},\sigma')\mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)\}.\end{aligned} \quad (40)$$

Due to Eq. (24), the following equation holds:

$$\begin{aligned}&\bar{u}(\tilde{\mathbf{p}},\sigma')\Phi_V^\tau(p,P,S)u(\tilde{\mathbf{p}},\sigma)\\ &= \frac{2\pi mE_S}{\mathcal{M}_0[1,(23)]}\left(\frac{P^+}{p^+}\right)^2\mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S).\end{aligned} \quad (41)$$

From Eq. (40), using the relation (see Appendix C of Ref. [61])

$$\bar{u}(\tilde{\mathbf{p}}',\sigma')\gamma^+u(\tilde{\mathbf{p}},\sigma) = \delta_{\sigma'\sigma}2\sqrt{p'^+p^+}, \quad (42)$$

one has

$$\begin{aligned}\text{Tr}[\gamma^+\Phi_V^\tau(p,P,S)] &= \frac{\pi(P^+)^2}{mp^+}\frac{E_S}{\mathcal{M}_0[1,(23)]}\\ &\quad\times\sum_\sigma\mathcal{P}^\tau_{\mathcal{M},\sigma\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S).\end{aligned} \quad (43)$$

Therefore, since (see Ref. [1])

$$\frac{\partial x}{\partial\kappa_z} = \frac{(1-x)\kappa^+}{E_S\,E(\kappa)} \quad (44)$$

and [see Eq. (38)]

$$\frac{\partial p^-}{\partial \epsilon} = -\frac{4m}{(P^+ - p^+)}, \tag{45}$$

one obtains (recall that $x = p^+/P^+$ and $\mathbf{p}_\perp = \boldsymbol{\kappa}_\perp$)

$$\begin{aligned}&\frac{1}{2(2\pi)^4}\int dp^- \int \frac{dp^+}{2P^+}\int d\mathbf{p}_\perp \mathrm{Tr}[\gamma^+ \Phi_V^\tau(p,P,S)]\\ &= \frac{1}{(2\pi)^4}\int d\epsilon \int d\boldsymbol{\kappa}\frac{\pi}{E(\kappa)}\sum_\sigma \mathcal{P}^\tau_{\mathcal{M},\sigma\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\\ &= 1,\end{aligned} \tag{46}$$

because of the spectral function normalization [see Eq. (5)].

Eventually we have the normalization condition for the particle correlator,

$$\begin{aligned}&\int \frac{d^4p}{(2\pi)^4}\frac{1}{2P^+}\mathrm{Tr}[\gamma^+\Phi_V^\tau(p,P,S)]\\ &= \frac{1}{2P^+}\frac{1}{(2\pi)^4}\frac{1}{2}\int dp^- \int dp^+ \int d\mathbf{p}_\perp \mathrm{Tr}[\gamma^+\Phi_V^\tau(p,P,S)]\\ &= 1.\end{aligned} \tag{47}$$

For a generic value of A, the above equations can be easily generalized.

IV. T-EVEN TWIST-TWO TRANSVERSE MOMENTUM DISTRIBUTIONS

As shown in Appendix B, in valence approximation the leading-twist TMDs are related to the scalar functions $b_{i,\mathcal{M}}$, that contain the relevant information on the dynamics inside the bound system, by the equations

$$f(x,|\mathbf{p}_\perp|^2) = b_0, \tag{48}$$

$$\begin{aligned}S_z\,\Delta f + \frac{1}{M}\,\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp\, g_{1T} = [&S_z b_{1,\mathcal{M}} + (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)b_{4,\mathcal{M}}\\ &+ (\mathbf{S}\cdot\hat{z})b_{5,\mathcal{M}}],\end{aligned} \tag{49}$$

$$\begin{aligned}&S_x h_{1T} + \frac{S_z}{M}\,p_x\,h_{1L}^\perp + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\,p_x\,h_{1T}^\perp\\ &= \left[S_x b_{1,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)b_{2,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{z})b_{3,\mathcal{M}}\right],\end{aligned} \tag{50}$$

$$\begin{aligned}&S_y h_{1T} + \frac{S_z}{M}\,p_y\,h_{1L}^\perp + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\,p_y\,h_{1T}^\perp\\ &= \left[S_y b_{1,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)b_{2,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{z})b_{3,\mathcal{M}}\right],\end{aligned} \tag{51}$$

where (see Ref. [62])

$$h_{1T} = \int [dp^+dp^-]A_3^V. \tag{52}$$

Let us recall that for a three-body system with total angular momentum $\mathcal{J} = 1/2$, the dependence of $b_{i,\mathcal{M}}$ $(i = 0,\dots,5)$ on $\mathbf{S}$ is absent and that the $b_{i,\mathcal{M}}$ are invariant for rotations of $\mathbf{k}_\perp$ around the z axis (see Appendix C). Then any dependence on $\mathbf{S}$ and on the direction of $\mathbf{k}_\perp$ in the right-hand sides of Eqs. (49)–(51) is explicitly written down.

To obtain the explicit expressions of the TMDs in terms of the scalar functions $b_{i,\mathcal{M}}$ from Eqs. (49)–(51), we consider specific orientations of the target spin. From Eq. (49) one obtains the following:

(a) For $\mathbf{S} = (0,0,1)$,

$$\Delta f = b_{1,\mathcal{M}} + b_{5,\mathcal{M}}. \tag{53}$$

(b) For $\mathbf{S} = (1,0,0)$ and $\mathbf{S} = (0,1,0)$,

$$g_{1T} = \frac{M}{|\mathbf{p}_\perp|}b_{4,\mathcal{M}}. \tag{54}$$

From Eq. (50) one obtains the following:

(a) For $\mathbf{S} = (0,0,1)$,

$$h_{1L}^\perp = \frac{M}{|\mathbf{p}_\perp|}b_{3,\mathcal{M}}. \tag{55}$$

(b) For $\mathbf{S} = (1,0,0)$,

$$\begin{aligned}&h_{1T}(S_x = 1) + \frac{|\mathbf{p}_\perp|^2\cos^2\phi}{M^2}h_{1T}^\perp\\ &= b_{1,\mathcal{M}} + \left(\frac{k_x}{k_\perp}\right)^2 b_{2,\mathcal{M}}.\end{aligned} \tag{56}$$

(c) For $\mathbf{S} = (0,1,0)$,

$$h_{1T}^\perp = \frac{M^2}{|\mathbf{p}_\perp|^2}b_{2,\mathcal{M}}. \tag{57}$$

Eventually, from Eq. (51) one has the following:

(a) For $\mathbf{S} = (0,1,0)$,

$$\begin{aligned}&h_{1T}(S_y = 1) + \frac{|\mathbf{p}_\perp|^2\sin^2\phi}{M^2}h_{1T}^\perp\\ &= b_{1,\mathcal{M}} + \left(\frac{k_y}{k_\perp}\right)^2 b_{2,\mathcal{M}}.\end{aligned} \tag{58}$$

(b) For $\mathbf{S} = (1,0,0)$, an equation identical to Eq. (57) is obtained.

(c) For $\mathbf{S} = (0,0,1)$, an equation identical to Eq. (55) is obtained.

If Eq. (57) is inserted in Eqs. (56) and (58) one obtains

$$h_{1T} = b_{1,\mathcal{M}}. \tag{59}$$

The sum of Eqs. (56) and (58) gives

$$h_{1T} + \frac{|\mathbf{p}_\perp|^2}{2M^2}h_{1T}^\perp = \Delta_T' f = \frac{1}{2}(2b_{1,\mathcal{M}} + b_{2,\mathcal{M}}). \tag{60}$$

In conclusion, from Eqs. (49)–(51) and the three possible directions of the polarization vector $\mathbf{S}$, nine equations are obtained. However, only five out of these nine equations are independent and allow one to determine the five TMDs, Δf, g_{1T}, $h_{1L}^\perp$, $h_{1T,}^\perp$, and $\Delta_T' f$. Summarizing our results, we can write the following expressions for the leading-twist TMDs in valence approximation (recall $\mathbf{p}_\perp = \mathbf{k}_\perp = \boldsymbol{\kappa}_\perp$):

$$f^\tau(x,|\mathbf{p}_\perp|^2) = b_0^\tau, \tag{61}$$

$$\Delta f^\tau(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}}^\tau + b_{5,\mathcal{M}}^\tau, \tag{62}$$

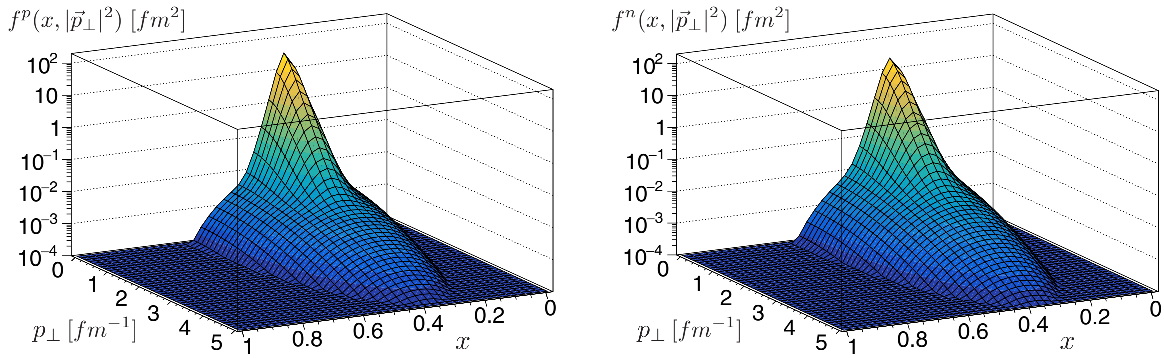

FIG. 1. Nucleon momentum distribution $f^\tau(x, |\mathbf{p}_\perp|^2)$ in an unpolarized ^{3}He for the proton (left panel) and the neutron (right panel).

$$g_{1T}^\tau(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|} b_{4,\mathcal{M}}^\tau, \tag{63}$$

$$\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2) = \frac{1}{2}\left(2b_{1,\mathcal{M}}^\tau + b_{2,\mathcal{M}}^\tau\right), \tag{64}$$

$$h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|} b_{3,\mathcal{M}}^\tau, \tag{65}$$

$$h_{1T}^{\perp\tau}(x, |\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2} b_{2,\mathcal{M}}^\tau, \tag{66}$$

where the isospin index τ has been reintroduced.

Since for a three-body system with total angular momentum $\mathcal{J} = 1/2$ the dependence of $b_{i,\mathcal{M}}$ on $\mathbf{S}$ is absent and the $b_{i,\mathcal{M}}$ are invariant for rotations of $\mathbf{k}_\perp$ around the z axis, the transverse momentum distributions $\Delta f(x, |\mathbf{p}_\perp|^2)$, $g_{1T}(x, |\mathbf{p}_\perp|^2)$, $\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$, $h_{1L}^\perp(x, |\mathbf{p}_\perp|^2)$, and $h_{1T}^\perp(x, |\mathbf{p}_\perp|^2)$ do not depend on the direction of $\mathbf{k}_\perp$, as expected.

In Appendix C 4, explicit expressions for the functions $b_{i,\mathcal{M}}^\tau$ are obtained in terms of the three-body wave function. From these expressions, according to Eqs. (61)–(66), the twist-two T-even transverse momentum distributions can be evaluated, directly from the wave function, without a cumbersome analysis of the spectral properties of the system, described by the spectral function.

V. APPLICATIONS

In this section, the above formalism developed for the valence contribution to the leading-twist TMDs for a $\mathcal{J} = 1/2$ system is applied to the proton and neutron inside ^{3}He. The quantitative analysis allows one to clearly show the impact of the inner dynamics on the evaluation of the TMDs, confirming the expectation of reaching a detailed 3D picture of the investigated system in momentum space. Moreover, the numerical information we have obtained could be exploited for motivating further experimental efforts for measuring the ^{3}He TMDs.

A. TMDs of the ^{3}He nucleus

The T-even TMDs for ^{3}He are evaluated by using the ^{3}He wave function of Refs. [63,64] with the realistic nuclear interaction of Ref. [65], but neglecting the small effect of the Coulomb repulsion between the protons. The results are shown in Figs. 1–6.

As a first observation one can notice that all of the TMDs are distributed around $x = 1/3$, as expected. The structure of $f^\tau(x, |\mathbf{p}_\perp|^2)$, presented in Fig. 1, is very smooth, while the other five distributions have a rich structure, as a function both of x and $|\mathbf{p}_\perp|$. Another relevant observation arises from the comparison of the TMDs $\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$, Eq. (62), with $\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$, Eq. (64), shown in Figs. 2 and 3, respectively. As is well known, these distributions should be equal

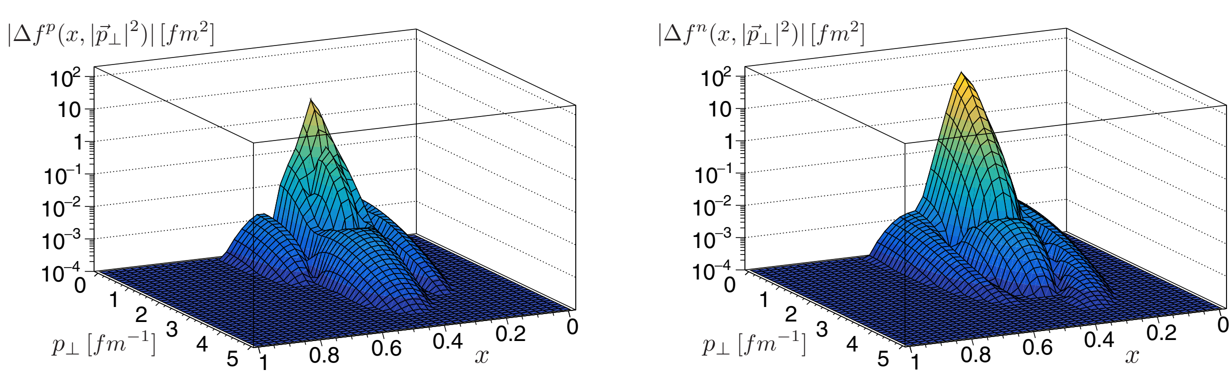

FIG. 2. Absolute value of the nucleon longitudinal-polarization distribution, $\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$, in a longitudinally polarized ^{3}He for the proton (left panel) and the neutron (right panel).

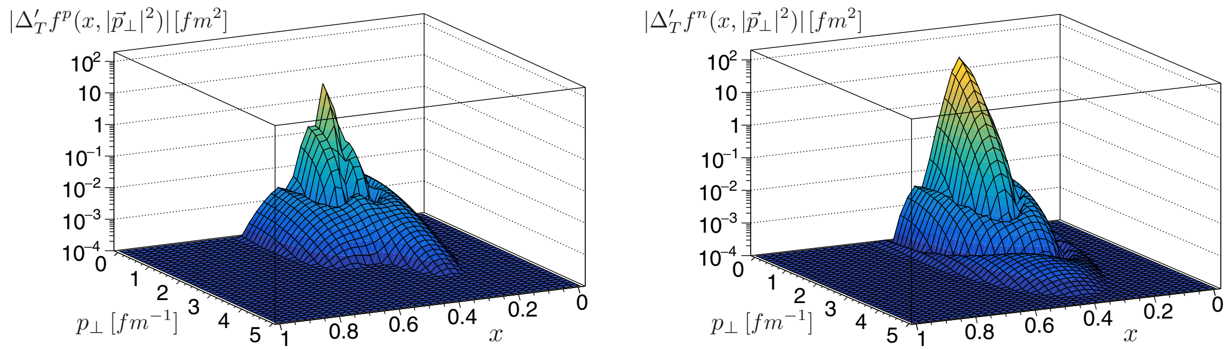

FIG. 3. Absolute value of the nucleon transverse-polarization distribution, $\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$, in a ^{3}He nucleus transversely polarized in the same direction of the nucleon polarization, for the proton (left panel) and the neutron (right panel).

in a nonrelativistic framework, where boosts and transverse rotations commute [66–69]. The same does not hold in a relativistic treatment, as the LFHD one adopted here. Indeed, the comparison shows that the two distributions are actually different, a signature of remarkable relativistic corrections, even in a system, the ^{3}He nucleus, where the ratio of the average nucleon momentum to the nucleon mass is rather small.

The TMDs presented in Figs. 4–6, namely, $g^\tau_{1T}(x, |\mathbf{p}_\perp|^2)$, $h^{\perp\tau}_{1L}(x, |\mathbf{p}_\perp|^2)$, and $h^{\perp\tau}_{1T}(x, |\mathbf{p}_\perp|^2)$, respectively, have relevant peaks at low values of $|\mathbf{p}_\perp|$. Interestingly, $h^{\perp\tau}_{1T}(x, |\mathbf{p}_\perp|^2)$ shows a sizable secondary bump at $|\mathbf{p}_\perp| \sim 2.5$ fm^{-1}, due to the presence of the squared transverse momentum in $b^\tau_{2,\mathcal{M}}$, differently from the linear power occurring in the other functions $b^\tau_{i,\mathcal{M}}$. It is very important to notice that in valence approximation $|g^\tau_{1T}(x, |\mathbf{p}_\perp|^2)|$, shown in Fig. 4, and $|h^{\perp\tau}_{1L}(x, |\mathbf{p}_\perp|^2)|$, shown in Fig. 5, are very similar, as expected by inspecting the two scalar functions, $b^\tau_{3,\mathcal{M}}$ and $b^\tau_{4,\mathcal{M}}$ given in Eqs. (C37)–(C40), and recalling that the effect of the Melosh rotations, parametrized through the angle φ, is small [see below Eq. (19)].

The shape of the presented distributions demonstrates that a comparison of these results with the TMDs extracted from future measurements of appropriate spin asymmetries in $^3\vec{\mathrm{He}}(\vec{e}, e'p)X$ experiments [55] could give very detailed information on the ^{3} He wave function, on the validity of the LF description, and consequently on the nuclear interaction, once the possible final state interaction is properly taken care of.

B. Effective polarizations

As a first application of our results, let us evaluate the LF longitudinal, $p^\tau_{||}$, and transverse, $p^\tau_\perp$, effective polarizations for the proton and for the neutron, viz.,

$$p^\tau_{||} = \int_0^1 dx \int d\mathbf{p}_\perp\, \Delta f^\tau(x, |\mathbf{p}_\perp|^2), \tag{67}$$

$$p^\tau_\perp = \int_0^1 dx \int d\mathbf{p}_\perp\, \Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2). \tag{68}$$

They are used in the extraction of neutron polarized structure functions and of neutron Collins and Sivers single spin asymmetries, respectively, from the corresponding quantities measured for ^{3}He (see, e.g., Refs. [70,71] and Refs. [72–74], respectively). This kind of extraction is based on the validity of the impulse approximation (IA), i.e., no final state interaction between the struck particle and the interacting spectator system, in the kinematics of the corresponding experiments. In the SIDIS case and in the kinematics of both Jefferson Lab and the future EIC, it has been shown [73] that effective polarizations can be used for the single spin asymmetries, even beyond the IA, including final state interaction effects.

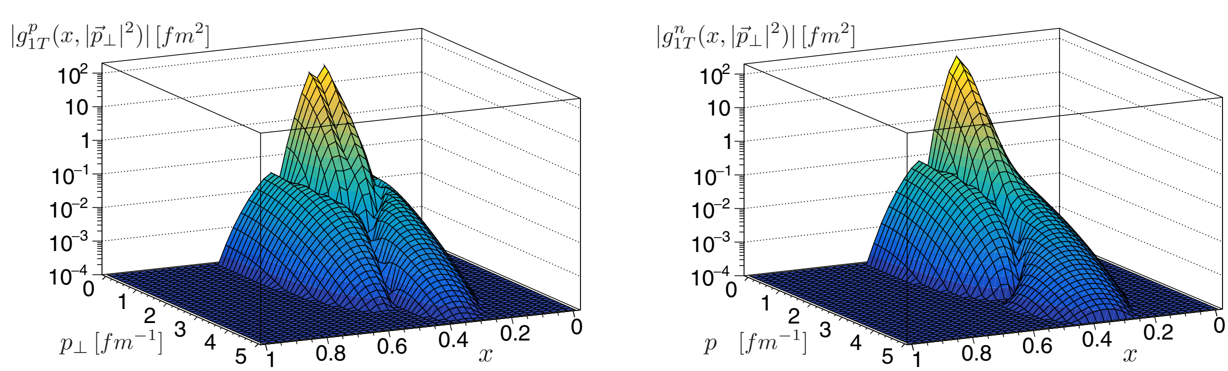

FIG. 4. Absolute value of the nucleon longitudinal-polarization distribution, $g^\tau_{1T}(x, |\mathbf{p}_\perp|^2)$, in a transversely polarized ^{3}He for the proton (left panel) and the neutron (right panel).

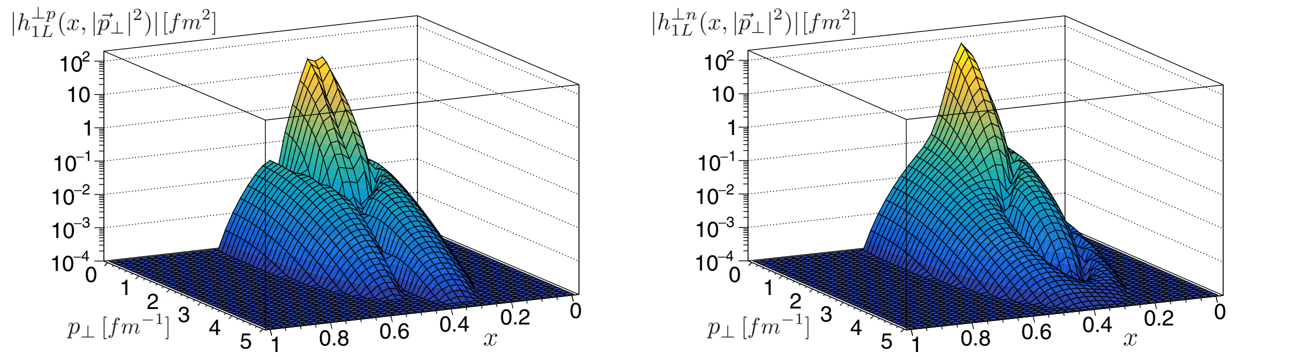

FIG. 5. Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)$, in a longitudinally polarized ^{3}He for the proton (left panel) and the neutron (right panel).

From the discussion in the previous section, and from the expressions of Eqs. (C44) and (C45), it is clear that the longitudinal and the transverse effective polarizations are not anymore equal, as it occurs in the nonrelativistic approximation. The difference between the two effective polarizations is due to the effect of the Melosh rotations for the spin. Indeed, without this effect one has [see Appendix (C 5)]

$$p_{||}^\tau = p_\perp^\tau = (-1)^{\mathcal{M}+1/2}\sqrt{3}\int dk_{23}k_{23}^2 \times \int_0^\infty k^2 dk 2\mathcal{H}^\tau(0,1,k_{23},k), \tag{69}$$

where the function $\mathcal{H}^\tau(0,1,k_{23},k)$ can be obtained from Eq. (C21). The LF results obtained for the effective polarizations of proton and neutron in ^{3}He with the nuclear interaction AV18 of Ref. [65], without the Coulomb repulsion, are shown in Table I, together with the corresponding normalizations, and compared with the nonrelativistic result. In the first and second lines the normalizations obtained from the proton and neutron spectral functions through Eq. (5) and directly from the wave function are shown, respectively. In the following three lines the LF calculations for the longitudinal and transverse polarizations and for the polarizations without the Melosh rotations are presented. In the last line, the results for the nonrelativistic polarizations with the same wave function are shown. The comparison between the two normalizations allows one to assess the numerical accuracy, which can be estimated of the order of a few parts in 10^4. Therefore the difference of a few parts in 10^3 between longitudinal and transverse polarizations is a meaningful result. However, this small difference indicates that the effects of the Melosh rotations are tiny [see the result in the second-to-last line and the comment below Eq. (19)], although for the proton it becomes sizable in percentage. Interestingly, the difference of the LF polarizations with respect to the nonrelativistic results is larger, up to 2% in the neutron case, and should be ascribed to the transformations performed between the symmetric and nonsymmetric intrinsic coordinate systems, not to the Melosh rotations involving the spins. In any case, the important point is that the relativistic results do not differ too much from the nonrelativistic ones currently used by the experimental collaborations. Therefore, the values of the effective polarizations used to extract the neutron polarized structure functions and the Collins and Sivers asymmetries from ^{3}He data can be considered reliable also from a Poincaré-covariant point of view, although for a more precise determination the new values for the effective polarizations should be adopted. In closing this section, we observe that the longitudinal and transverse polarizations of the nucleons in ^{3}He are analogous to the axial and tensor charges of the nucleon in terms of its constituent quarks, in valence approximation. Since the beginning of the transversity studies, their difference has been

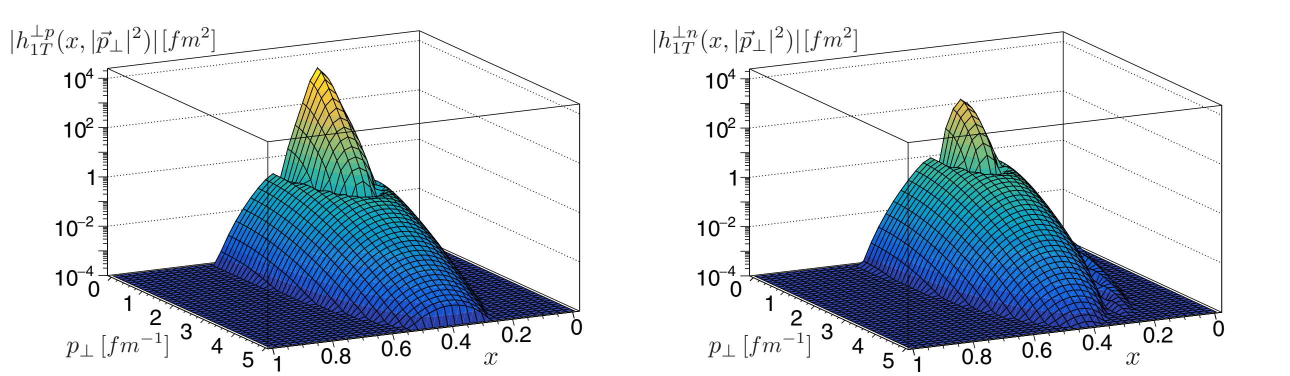

FIG. 6. Absolute value of the nucleon transverse-polarization distribution, $h_{1T}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)$, in a ^{3}He nucleus transversely polarized in a direction orthogonal to the direction of the nucleon polarization, for the proton (left panel) and the neutron (right panel).

TABLE I. Normalization and effective longitudinal and transverse polarizations for the proton and the neutron in ^{3}He.

Normalization and effective polarizations	Proton	Neutron
Normalization from the spectral function	0.99915	0.99885
Normalization from the wave function	0.99929	0.99897
LF longitudinal polarization	−0.02299	0.87261
LF transverse polarization	−0.02446	0.87314
LF polarization without Melosh rotations	−0.02407	0.87698
Nonrelativistic polarization	−0.02118	0.89337

always considered a signature of the relativistic content of the system [33,66–69].

C. Approximate relations

In Refs. [13,33], which define the TMDs as in Eqs. (B2)–(B4), approximate relations between the TMDs were discussed, i.e.

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \Delta'_T f(x, |\mathbf{p}_\perp|^2) + \frac{|\mathbf{p}_\perp|^2}{2M^2} h_{1T}^\perp(x, |\mathbf{p}_\perp|^2), \quad (70)$$

and

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = -h_{1L}^\perp(x, |\mathbf{p}_\perp|^2). \quad (71)$$

In principle, these relations put clear-cut phenomenological constraints on the number of independent T-even twist-two TMDs. Therefore, it is interesting to raise the question to what extent the above relations are valid. To attempt a realistic answer, we tested Eqs. (70) and (71) in the case with a refined dynamical content as the ^{3}He nucleus, assuming the nucleons as constituents. The first relation, Eq. (70), is not exactly satisfied, since the equality should hold if $b^\tau_{2,\mathcal{M}} = b^\tau_{5,\mathcal{M}}$, as it follows from Eqs. (62), (64), and (66). By inspecting Eqs. (C35), (C36), (C41), and (C42), one gets $b^{\tau(0)}_{2,\mathcal{M}} = b^{\tau(0)}_{5,\mathcal{M}}$, while $b^{\tau(2)}_{2,\mathcal{M}}$ and $b^{\tau(2)}_{5,\mathcal{M}}$ have not the same expressions. The quantitative difference between the left- and right-hand sides of Eq. (70) is quite small for the neutron, while it is not negligible for the proton as shown in Fig. 7.

From the evaluation of the effective polarizations we learned that the effects of the Melosh rotations are tiny, and if these effects are neglected in $b_{i,\mathcal{M}}$, i.e., $\sin(\varphi/2) \to 0$ in the expressions presented at the end of Appendix C 4, one finds that the second relation, Eq. (71), holds, but with the opposite sign. It is very interesting to analyze such a different sign, which could have far reaching consequences. The functions $b^\tau_{4,\mathcal{M}}$ and $b^\tau_{3,\mathcal{M}}$ determine g_{1T} and $h_{1L}^\perp$, respectively, as shown in Eqs. (63) and (65). From Eqs. (C37) and (C39), where the contribution with $L = 0$ is considered, one has $b^{\tau(0)}_{3,\mathcal{M}} = -b^{\tau(0)}_{4,\mathcal{M}}$, but the two functions are of the order $\sin(\varphi)$ and therefore very small. The contribution with $L = 2$ leads to $b^{\tau(2)}_{3,\mathcal{M}} \sim b^{\tau(2)}_{4,\mathcal{M}}$ [see Eqs. (C38) and (C40)] and generates the leading term, proportional to $\cos^2(\varphi)$, of the two TMDs. Therefore, the sign in Eq. (71) has to be considered as a signature of the bound-state orbital content that prevails in the actual value of g_{1T} and $h_{1L}^\perp$. As is clear from Appendix C (where it is shown that only the values $L = 0$ and $L = 2$ are possible), if only the contribution from $L = 0$ is present, this implies a vanishing value of the active fermion orbital angular momentum L_ρ in the bound-system wave function [see Eqs. (C9)–(C19)]. On the other hand, a nonvanishing value of L_ρ implies a nonvanishing value of the contribution from $L = 2$ to the momentum distribution.

For ^{3}He, Eq. (71) is fulfilled in modulus, as pictorially shown in Figs. 4 and 5.

For a system with a high average value of the ratio $|\mathbf{p}_\perp|/m$, as occurs when we shift from the ^{3}He case to the nucleon one, where the constituents are the quarks with $m_q < m_N$, (see Refs. [75,76]), Melosh rotations become relevant. If this is the case, from the previous discussion, one expects that relations (70) and (71) hold exactly in valence approximation when the contribution to the transverse momentum distributions from $L = 2$ is very small or totally absent.

Therefore, the validity of equalities (70) and (71) in a bound system with a high average value of $|\mathbf{p}_\perp|/m$ represents a well-defined constraint on the allowed dynamics inside the three-body bound system.

Another relation proposed in Refs. [13,33], i.e.,

$$[g_{1T}(x, |\mathbf{p}_\perp|^2)]^2 = -2\Delta'_T f(x, |\mathbf{p}_\perp|^2) h_{1T}^\perp(x, |\mathbf{p}_\perp|^2), \quad (72)$$

does not hold in our approach, even if the contribution from the angular momentum $L = 2$ is absent, because of the presence of the integration on k_{23} in the expressions of the transverse momentum distributions [see Eq. (C30)]. Namely, a

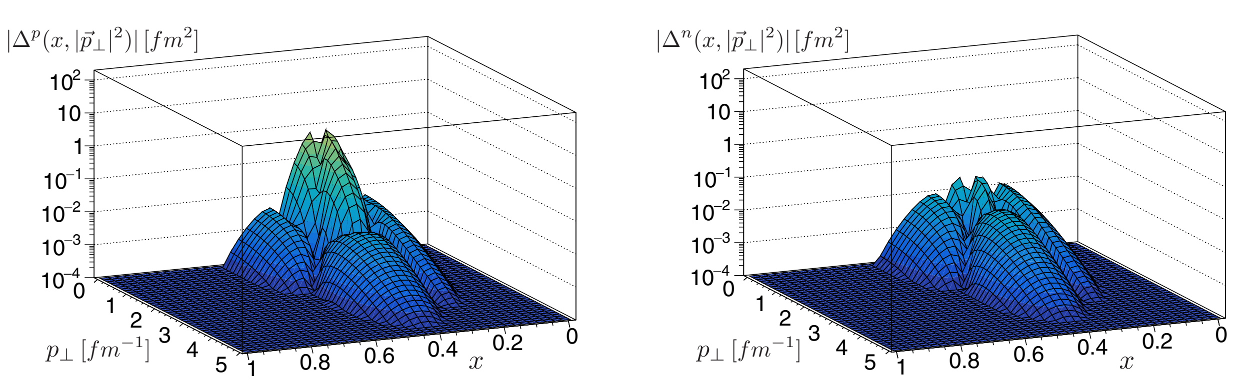

FIG. 7. Absolute value of the difference $\Delta(x, |\mathbf{p}_\perp|^2) = \Delta'_T f(x, |\mathbf{p}_\perp|^2) + \frac{|\mathbf{p}_\perp|^2}{2M^2} h_{1T}^\perp(x, |\mathbf{p}_\perp|^2) - \Delta f(x, |\mathbf{p}_\perp|^2)$ in ^{3}He for the proton (left panel) and the neutron (right panel).

genuine relativistic approach, related to the implementation of the macroscopic locality, spoils the above relation. Therefore, a signature of such a relativistic effect is given by the violation of Eq. (72).

VI. CONCLUSIONS

A Poincaré-covariant description of bound systems, with A constituents of spin $1/2$, was developed within the light-front Hamiltonian dynamics in Ref. [1] and applied in the present work to the T-even twist-two transverse momentum distributions of $\mathcal{J} = 1/2$ systems. The explicit expressions for the TMDs are obtained in terms of LF overlaps, between the bound state and states described by tensor products of a constituent plane wave and an $(A-1)$ fully interacting system. These LF overlaps are the basic ingredients of the spin-dependent spectral function [1], that is a 2×2 matrix with the main-diagonal terms yielding the distribution probability to find a constituent with given spin and LF momentum, once the $(A-1)$-spectator system has an assigned mass. Indeed, the leading-twist TMDs, in valence approximation, are linked to proper traces of the valence contribution to the semi-inclusive correlation function, which is linearly related to the spin-dependent spectral function. The formalism was applied to the ^{3}He nucleus, keeping a twofold aim in mind: (i) illustrating a realistic case, where a theoretical description of the bound state takes into account a highly nontrivial dynamics, and consequently assessing the impact of dynamics on the actual TMDs, and (ii) cumulating quantitative analyses for supporting future experimental efforts dedicated to achieve a detailed 3D picture of ^{3}He. As a matter of fact, for the ^{3}He nucleus there exists a reliable LF spin-dependent spectral function, obtained within the Bakamjian-Thomas framework [42], suitable for embedding the wide knowledge on the nuclear interaction and the successful phenomenology developed for few-nucleon systems in a Poincaré-covariant approach (preliminary calculations of the EMC effect in ^{3}He were presented in Refs. [44–47] and a full calculation will be soon available [43]).

In addition to the Melosh rotations, the peculiar feature of our LF approach is the macroscopic locality, implemented through the use of nonsymmetric intrinsic variables, i.e., intrinsic internal variables for the $A-1$ system and the LF momentum of the active fermion in the intrinsic reference frame of the $[1, (A-1)]$ cluster [1]. Their impact on the ^{3}He TMDs was discussed, with particular attention to the relevance from the experimental point of view. To this end, we have evaluated, as a first application, the LF longitudinal and transverse effective polarizations for the proton and the neutron in ^{3}He. These two quantities are widely used to extract the neutron polarized structure functions and the neutron Collins and Sivers asymmetries from the corresponding quantities measured in deep-inelastic scattering (DIS) and SIDIS off ^{3}He, respectively. Although this procedure is generally used assuming the validity of the impulse approximation, in Ref. [73] it was shown how it can be applied even including the final state interaction.

We found that, for a system with a small average value of $p_\perp/m$, as the three-nucleon system, the effect of the spin Melosh rotation is tiny and the longitudinal and the transverse effective polarizations differ very little from each other, both for the proton and the neutron. On the contrary, the difference with respect to the nonrelativistic result is not negligible and this effect, ascribed to the use of LF intrinsic variables, has to be considered for a more accurate extraction of neutron properties from ^{3}He data.

A second important result we discussed is the validation of the linear relations proposed in Refs. [13,35] between the T-even twist-two TMDs. Although those relations were introduced in the context of the nucleon studies in valence approximation, we investigated their validity for the ^{3}He case to show how a rich dynamics actually impacts the extraction of important information on the orbital momentum decomposition of the three-body bound system. Namely, one recovers the above-mentioned relations if the state has a simple S-wave structure, while in presence of a D wave and a small effect of the Melosh rotations, i.e., a small average value of $p_\perp/m$, the second relation even changes sign. The other relation proposed in Refs. [13,35], between the TMDs, a quadratic one, does not hold in any case, since the TMDs are given by integrals over the relative momentum of the spectator interacting pair, which represent an unavoidable feature of a genuinely Poincaré-covariant framework. Hence, an experimental investigation, performed at high luminosity facilities, could open a window directly on the orbital momentum content of the bound state and the relativistic regime of the inner dynamics.

The twist-three TMDs can be also evaluated in our scheme and will be the object of another study.

APPENDIX A: DEPENDENCE OF THE SPECTRAL FUNCTION ON THE POLARIZATION VECTOR S

The LF spin-dependent spectral function for any direction of the polarization vector $\mathbf{S}$ can be written in terms of the Wigner rotation matrices $D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha,\beta,\gamma)$ [see Eqs. (1) and (4)]:

$$\mathcal{P}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) = \sum_{m,m'} D^{\mathcal{J}*}_{m,\mathcal{M}}(\alpha,\beta,\gamma) D^{\mathcal{J}}_{m',\mathcal{M}}(\alpha,\beta,\gamma) \times \mathcal{P}^{m',m}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S_z), \tag{A1}$$

where $S_z = (0,0,0,1)$ and

$$\mathcal{P}^{m',m}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S_z) = \rho(\epsilon)\sum_{JJ_z\alpha}\sum_{T_S\tau_S} {}_{LF}\langle \tau_S T_S;\alpha,\epsilon;JJ_z;\tau\sigma',\tilde{\boldsymbol{\kappa}}|\mathcal{J}m';\epsilon^A,\Pi;TT_z\rangle_{zz}\langle TT_z;\Pi,\epsilon^A;\mathcal{J}m|\tilde{\boldsymbol{\kappa}},\sigma\tau;JJ_z;\epsilon,\alpha;T_S\tau_S\rangle_{LF}, \tag{A2}$$

with $|\mathcal{J}m;\epsilon^A,\Pi;TT_z\rangle_z$ the system ground state with energy ϵ^A, parity Π, and isospin T, T_z, polarized along $\hat{z}$.

For a system with $\mathcal{J} = 1/2$, Eq. (A1) can be written as follows:

$$\mathcal{P}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \sum_{m,m'} D^{1/2}_{-m,-\mathcal{M}}(\alpha, \beta, \gamma)(-1)^{m-\mathcal{M}} \times D^{1/2}_{m',\mathcal{M}}(\alpha, \beta, \gamma)\mathcal{P}^{m',m}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S_z). \quad \text{(A3)}$$

Let us take advantage of the following property for the product of two Wigner rotation matrices with the same arguments (see Eq. (1) at p. 84 of Ref. [77]):

$$D^j_{m,k}(\alpha, \beta, \gamma)D^{j'}_{m',k'}(\alpha, \beta, \gamma) = \sum_{J=|j-j'|}^{j+j'} \langle jm, j'm'|J(m+m')\rangle \times \langle jk, j'k'|J(k+k')\rangle D^J_{(m+m'),(k+k')}(\alpha, \beta, \gamma). \quad \text{(A4)}$$

In our case it reads

$$D^{1/2}_{-m,-\mathcal{M}}(\alpha, \beta, \gamma)D^{1/2}_{m',\mathcal{M}}(\alpha, \beta, \gamma) = \sum_{J=0}^{1} \left\langle \frac{1}{2} - m, \frac{1}{2}m' \middle| J(m'-m) \right\rangle \left\langle \frac{1}{2} - \mathcal{M}, \frac{1}{2}\mathcal{M} \middle| J0 \right\rangle \times D^J_{(m'-m),0}(\alpha, \beta, \gamma)$$
$$= \sum_{J=0}^{1} \left\langle \frac{1}{2} - m, \frac{1}{2}m' \middle| J(m'-m) \right\rangle \left\langle \frac{1}{2} - \mathcal{M}, \frac{1}{2}\mathcal{M} \middle| J0 \right\rangle \times (-1)^{m'-m}\sqrt{\frac{4\pi}{2J+1}} Y_{J(m-m')}(\beta, \alpha). \quad \text{(A5)}$$

Then the product of the two $D^j_{m,k}(\alpha, \beta, \gamma)$ matrices is a sum of two terms. The first one, with $J = 0$, is independent of β, α (i.e., of $\mathbf{S}$). In the second one, with $J = 1$, the spherical harmonics $Y_{1(m-m')}(\beta, \alpha)$ can be replaced by the spherical components S_i of the polarization vector [77]:

$$Y_{J(m-m')}(\beta, \alpha) = \sqrt{\frac{3}{4\pi}}\, S_{(m-m')}. \quad \text{(A6)}$$

Hence the spectral function depends linearly on $\mathbf{S}$. Since in expansion (10) of the pseudovector $\mathcal{F}^\tau_{\mathcal{M}}$ a linear dependence on $\mathbf{S}$ already explicitly appears in the five pseudovectors $\mathbf{S}$, $\hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)$, $\hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{z})$, $\hat{z}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)$, and $\hat{z}(\mathbf{S}\cdot\hat{z})$, as a consequence for a system with total angular momentum $\mathcal{J} = 1/2$ the quantities $\mathcal{B}_i$ for $i = 1, \ldots, 5$ can depend only on $|\mathbf{k}_\perp|$, x, and ϵ.

APPENDIX B: SEMI-INCLUSIVE FERMION CORRELATOR, TWIST-TWO TRANSVERSE MOMENTUM DISTRIBUTIONS, AND LF SPECTRAL FUNCTION

In this Appendix equations that link in valence approximation the leading-twist TMDs to the scalar functions $b_{i,\mathcal{M}}$, which contain the relevant information on the dynamics inside the bound system, are obtained.

For the sake of completeness, let us summarize the decomposition of the semi-inclusive fermion correlator in terms of the twist-two T-even TMDs as presented in Ref. [8]. Let us recall that $\mathbf{p}_\perp = \mathbf{k}_\perp = \boldsymbol{\kappa}_\perp$, and to simplify the notation the isospin index τ is omitted in what follows. The semi-inclusive correlator at leading twist is given by

$$\Phi(p, P, S) = \frac{1}{2}\not{P} A_1 + \frac{1}{2}\gamma_5\not{P}\left[A_2 S_z + \frac{1}{M}\widetilde{A}_1\, \mathbf{p}_\perp\cdot\mathbf{S}_\perp\right] + \frac{1}{2}\gamma_5\not{P}\left[A_3\not{S}_\perp + \widetilde{A}_2\frac{S_z}{M}\not{p}_\perp + \widetilde{A}_3\frac{\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\not{p}_\perp}{M^2}\right], \quad \text{(B1)}$$

where M is the mass of the system, and the scalar functions, A_l and $\widetilde{A}_l$, contain the information on the inner dynamics ($l = 1, 2, 3$). By performing the traces of the correlator with suitable combinations of Dirac matrices one has

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi) = A_1, \quad \text{(B2)}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi) = S_z A_2 + \frac{1}{M}\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\widetilde{A}_1, \quad \text{(B3)}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(i\sigma^{j+}\gamma_5\Phi) = -\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^j\gamma^+\gamma_5\Phi) = S^j_\perp A_3 + \frac{S_z}{M}\,p^j_\perp\widetilde{A}_2 + \frac{1}{M^2}\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,p^j_\perp\widetilde{A}_3, \quad \text{(B4)}$$

where $j = 1, 2$. Finally, by integrating proper combinations of A_l, and $\widetilde{A}_l$ on p^+ and p^-, one gets the TMDs as follows [8]:

$$f(x, |\mathbf{p}_\perp|^2) = \int [dp^+dp^-]A_1, \quad \text{(B5)}$$

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \int [dp^+dp^-]A_2, \quad \text{(B6)}$$

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = \int [dp^+dp^-]\widetilde{A}_1, \quad \text{(B7)}$$

$$\Delta'_T f(x, |\mathbf{p}_\perp|^2) = \int [dp^+dp^-]\left(A_3 + \frac{|\mathbf{p}_\perp|^2}{2M^2}\widetilde{A}_3\right), \quad \text{(B8)}$$

$$h^\perp_{1L}(x, |\mathbf{p}_\perp|^2) = \int [dp^+dp^-]\widetilde{A}_2, \quad \text{(B9)}$$

$$h^\perp_{1T}(x, |\mathbf{p}_\perp|^2) = \int [dp^+dp^-]\widetilde{A}_3, \quad \text{(B10)}$$

with

$$\int [dp^+dp^-] = \frac{1}{2}\int \frac{dp^+dp^-}{(2\pi)^4}\delta[p^+ - xP^+]P^+. \quad \text{(B11)}$$

Notice that on the right-hand side of the above equations, a factor of 2 is missing with respect to the expressions in Ref. [8], because of the different definitions of the variables $v^\pm$. If only the valence contribution to the correlator is retained, the full $\Phi(p, P, S)$ is approximated by $\Phi_V(p, P, S)$, and in turn $\Phi_V(p, P, S)$ is expanded in analogy with Eq. (B1) in terms of A^V_l and $\widetilde{A}^V_l$. Hence, instead of Eqs. (B2)–(B4), one can write

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi) \sim \frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi_V) = A_1^V, \tag{B12}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi) \sim \frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi_V) = S_z A_2^V + \frac{1}{M}\,\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp\,\widetilde{A}_1^V, \tag{B13}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(i\sigma^{j+}\gamma_5\Phi) \sim -\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^j\gamma^+\gamma_5\Phi_V) = S_\perp^j A_3^V + \frac{S_z}{M}\,p_\perp^j\,\widetilde{A}_2^V + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\,p_\perp^j\widetilde{A}_3^V, \tag{B14}$$

where A_l^V and $\widetilde{A}_l^V$ are the valence approximations for A_l and $\widetilde{A}_l$, respectively. Because of Eqs. (40) and (41), the above traces of Φ_V can be expressed by means of traces of the spectral function, as shown in Appendix E [see also Eq. (42)]:

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi_V) = \frac{P^+}{p^+}\frac{2\pi}{4m}\frac{E_S}{\mathcal{M}_0[1,(23)]}\times \mathrm{Tr}[\widehat{\mathcal{P}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)], \tag{B15}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi_V) = \frac{P^+}{p^+}\frac{2\pi}{4m}\frac{E_S}{\mathcal{M}_0[1,(23)]}\times \mathrm{Tr}[\sigma_z\widehat{\mathcal{P}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)], \tag{B16}$$

$$-\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^j\gamma^+\gamma_5\Phi_V) = \frac{P^+}{p^+}\frac{2\pi}{4m}\frac{E_S}{\mathcal{M}_0[1,(23)]}\times \mathrm{Tr}[\sigma^j\widehat{\mathcal{P}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)]. \tag{B17}$$

As noted in Sec. II, the LF spectral function can be written in terms of six scalar quantities, $\mathcal{B}_{i,\mathcal{M}}$.

Therefore, the following equations hold:

$$\mathrm{Tr}(\widehat{\mathcal{P}}_{\mathcal{M}}I) = \mathcal{B}_{0,\mathcal{M}}, \tag{B18}$$

$$\mathrm{Tr}(\widehat{\mathcal{P}}_{\mathcal{M}}\sigma_x) = S_x\mathcal{B}_{1,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}_{2,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{z})\mathcal{B}_{3,\mathcal{M}}, \tag{B19}$$

$$\mathrm{Tr}(\widehat{\mathcal{P}}_{\mathcal{M}}\sigma_y) = S_y\mathcal{B}_{1,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}_{2,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{z})\mathcal{B}_{3,\mathcal{M}}, \tag{B20}$$

$$\mathrm{Tr}(\widehat{\mathcal{P}}_{\mathcal{M}}\sigma_z) = S_z\mathcal{B}_{1,\mathcal{M}} + (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}_{4,\mathcal{M}} + (\mathbf{S}\cdot\hat{z})\mathcal{B}_{5,\mathcal{M}}. \tag{B21}$$

From Eqs. (B12)–(B21) one obtains

$$A_1^V = \frac{\pi}{2m}\frac{E_S}{\kappa^+}\,\mathcal{B}_{0,\mathcal{M}}, \tag{B22}$$

$$S_z A_2^V + \frac{1}{M}\,\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp\,\widetilde{A}_1^V = \frac{\pi}{2m}\frac{E_S}{\kappa^+}[S_z\mathcal{B}_{1,\mathcal{M}} + (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}_{4,\mathcal{M}} + (\mathbf{S}\cdot\hat{z})\mathcal{B}_{5,\mathcal{M}}], \tag{B23}$$

$$S_xA_3^V + \frac{S_z}{M}\,p_x\widetilde{A}_2^V + \frac{\mathbf{p}_\perp\cdot\mathbf{S}_\perp}{M^2}\,p_x\widetilde{A}_3^V = \frac{\pi}{2m}\frac{E_S}{\kappa^+}\left[S_x\mathcal{B}_{1,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}_{2,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{z})\mathcal{B}_{3,\mathcal{M}}\right], \tag{B24}$$

$$S_yA_3^V + \frac{S_z}{M}\,p_y\widetilde{A}_2^V + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\,p_y\widetilde{A}_3^V = \frac{\pi}{2m}\frac{E_S}{\kappa^+}\left[S_y\mathcal{B}_{1,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}_{2,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{z})\mathcal{B}_{3,\mathcal{M}}\right]. \tag{B25}$$

Let us integrate Eqs. (B22)–(B25) on p^+ and p^- as in Eqs. (B5)–(B10). Then in valence approximation one has

$$f(x,|\mathbf{p}_\perp|^2) = b_0, \tag{B26}$$

$$S_z\,\Delta f + \frac{1}{M}\,\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp\,g_{1T} = [S_zb_{1,\mathcal{M}} + (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)b_{4,\mathcal{M}} + (\mathbf{S}\cdot\hat{z})b_{5,\mathcal{M}}], \tag{B27}$$

$$S_xh_{1T} + \frac{S_z}{M}\,p_x\,h_{1L}^\perp + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\,p_x\,h_{1T}^\perp = \left[S_xb_{1,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)b_{2,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{z})b_{3,\mathcal{M}}\right], \tag{B28}$$

$$S_yh_{1T} + \frac{S_z}{M}\,p_y\,h_{1L}^\perp + \frac{\mathbf{p}_\perp\cdot\mathbf{S}_\perp}{M^2}\,p_y\,h_{1T}^\perp = \left[S_yb_{1,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)b_{2,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{z})b_{3,\mathcal{M}}\right], \tag{B29}$$

where (see Ref. [62])

$$h_{1T} = \int [dp^+dp^-]A_3^V. \tag{B30}$$

APPENDIX C: THE SPIN-DEPENDENT MOMENTUM DISTRIBUTION AND THE THREE-BODY WAVE FUNCTION

One can obtain the LF momentum distribution dependent upon the spin directions, $[\mathcal{N}^\tau_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})]_{\sigma\sigma'}$, Eq. (14), for any direction of the polarization vector $\mathbf{S}$ of the system, using Eqs. (A1) and (A2):

$$[\mathcal{N}^\tau_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})]_{\sigma\sigma'} = \sum\!\!\!\!\!\!\int d\epsilon\,\frac{1}{2(2\pi)^3}\frac{E_S}{(1-x)\kappa^+}\rho(\epsilon)\sum_{JJ_z\alpha}\sum_{T_S\tau_S}\sum_m D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha,\beta,\gamma)_{LF}\langle\tau_ST_S;\alpha,\epsilon;JJ_z;\tau\sigma,\tilde{\boldsymbol{\kappa}}|\mathcal{J}j_z=m;\epsilon^3,\Pi;TT_z\rangle$$
$$\times\sum_{m'}[D^{\mathcal{J}}_{m',\mathcal{M}}(\alpha,\beta,\gamma)]^*\langle TT_z;\Pi,\epsilon^3;\mathcal{J}j_z=m';|\tilde{\boldsymbol{\kappa}},\sigma'\tau;JJ_z;\epsilon,\alpha;T_S\tau_S\rangle_{LF}, \tag{C1}$$

where $|\mathcal{J} j_z = m; \epsilon^3, \Pi; TT_z\rangle$ is the three-body ground state, polarized along $\hat{z}$. Using the explicit expression for the overlaps given by Eq. (62) of Ref. [1], the two-body completeness

$$\sum_{J,J_z\alpha}\sum_{TT_z}\sum\!\!\!\!\!\!\int d\epsilon\,\rho(\epsilon)\langle\mathbf{k}'|JJ_z;\epsilon,\alpha;T_S\tau_S\rangle\langle\tau_S T_S;\alpha,\epsilon;J_zJ|\mathbf{k}\rangle = \delta^3(\mathbf{k}'-\mathbf{k}), \tag{C2}$$

and the unitarity of the D matrices, we have

$$\left[\mathcal{N}^{\tau}_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})\right]_{\sigma\sigma'} = \sum_m D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha,\beta,\gamma)\sum_{m'}\left[D^{\mathcal{J}}_{m',\mathcal{M}}(\alpha,\beta,\gamma)\right]^*\mathcal{F}^{\tau mm'}_{\sigma\sigma'}(x,\mathbf{k}_\perp), \tag{C3}$$

where

$$\mathcal{F}^{\tau mm'}_{\sigma\sigma'}(x,\mathbf{k}_\perp) = \frac{1}{(1-x)}\sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}E(\mathbf{k})\frac{E_{23}}{k^+}\sum_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}$$
$$\times\sum_{\sigma_2',\sigma_3'}\langle\sigma_3',\sigma_2',\sigma_1;\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}|j,j_z=m;\epsilon^3_{\rm int},\Pi;TT_z\rangle\langle\sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}|j,j_z=m';\epsilon^3_{\rm int},\Pi;TT_z\rangle^*, \tag{C4}$$

with $\mathcal{R}_M$ the Melosh rotation (see Appendix D) and $\langle\sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}|j,j_z=m;\epsilon^3_{\rm int},\Pi;TT_z\rangle$ the momentum-space instant-form wave function.

The quantities $\mathbf{k}$, $E(\mathbf{k})$, and E_{23} in Eq. (C4) are easily determined from the variables x, $\mathbf{k}_\perp$, and $\mathbf{k}_{23}$ [1]:

$$k^+ = x\,M_0(1,2,3) \tag{C5}$$

with

$$M_0^2(1,2,3) = \frac{m^2+\mathbf{k}_\perp^2}{x}+\frac{M_{23}^2+\mathbf{k}_\perp^2}{(1-x)},\qquad M_{23} = 2\sqrt{m^2+|\mathbf{k}_{23}|^2}, \tag{C6}$$

and eventually

$$k_z = \frac{1}{2}(k^+-k^-) = \frac{1}{2}\left(k^+-\frac{m^2+\mathbf{k}_\perp^2}{k^+}\right), \tag{C7}$$

while

$$E(\mathbf{k}) = \sqrt{m^2+|\mathbf{k}|^2}\,,\quad E_{23} = \sqrt{M_{23}^2+\mathbf{k}^2}. \tag{C8}$$

Let us recall that in the system rest frame one has $\boldsymbol{P}_\perp = 0$.

1. Evaluation of $\mathcal{F}^{\tau mm'}_{\sigma\sigma'}$

Let us take the instant-form three-body wave function as the ^{3}He nonrelativistic wave function [1]. The ^{3}He wave function in momentum space can be written as follows from the wave function in coordinate space [63,64]:

$$\left\langle\sigma_1,\sigma_2,\sigma_3;\tau_1,\tau_2,\tau_3;\mathbf{k}_{23},\mathbf{k}\middle|{}^3\text{He};\frac{1}{2}m;\frac{1}{2}T_z\right\rangle = \sum_{l_{23}\mu_{23}}\sum_{L_\rho M_\rho}Y_{l_{23}\mu_{23}}(\hat{\mathbf{k}}_{23})Y_{L_\rho M_\rho}(\hat{\mathbf{k}})\sum_{T_{23},\tau_{23}}\left\langle\frac{1}{2}\tau_2\frac{1}{2}\tau_3\middle|T_{23}\tau_{23}\right\rangle\left\langle T_{23}\tau_{23}\frac{1}{2}\tau_1\middle|\frac{1}{2}T_z\right\rangle$$
$$\times\sum_{XM_X}\sum_{j_{23}m_{23}}\left\langle XM_XL_\rho M_\rho\middle|\frac{1}{2}m\right\rangle\left\langle j_{23}\,m_{23}\frac{1}{2}\sigma_1\middle|XM_X\right\rangle\sum_{s_{23}\sigma_{23}}\left\langle\frac{1}{2}\sigma_2\frac{1}{2}\sigma_3\middle|s_{23}\sigma_{23}\right\rangle$$
$$\times\langle l_{23}\mu_{23}s_{23}\sigma_{23}|j_{23}\,m_{23}\rangle\mathcal{G}^{j_{23}l_{23}s_{23}}_{L_\rho X}(k_{23},k) \tag{C9}$$

with

$$\mathcal{G}^{j_{23}l_{23}s_{23}}_{L_\rho X}(k_{23},k) = \frac{2(-1)^{\frac{l_{23}+L_\rho}{2}}}{\pi}\int r^2dr\,j_{l_{23}}(k_{23}r)\int\rho^2d\rho\,j_{L_\rho}(k\rho)\phi^{j_{23}l_{23}s_{23}}_{L_\rho X}(|\mathbf{r}|,|\boldsymbol{\rho}|). \tag{C10}$$

The antisymmetrization of the wave function requires $l_{23}+s_{23}+T_{23}$, where T_{23} is the isospin of the pair 23, to be odd. In addition, $l_{23}+L_\rho$ has to be even, due to the parity of ^{3}He.

Then, inserting Eq. (C9) in Eq. (C4), the function $\mathcal{F}^{\tau mm'}_{\sigma\sigma'}(x,\mathbf{k}_\perp)$ becomes

$$\mathcal{F}^{\tau mm'}_{\sigma\sigma'}(x,\mathbf{k}_\perp) = \frac{1}{(1-x)}\sum_{\tau_2\tau_3}\sum_{\sigma_2'\sigma_3'}\int d\mathbf{k}_{23}\frac{E_{23}E(\mathbf{k})}{k_1^+}\sum_{\sigma_1}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{l_{23}\mu_{23}}\sum_{L_\rho M_\rho}Y_{l_{23}\mu_{23}}(\hat{\mathbf{k}}_{23})Y_{L_\rho M_\rho}(\hat{\mathbf{k}})$$
$$\times\sum_{T_{23},\tau_{23}}\left\langle\frac{1}{2}\tau_2\frac{1}{2}\tau_3\middle|T_{23}\tau_{23}\right\rangle\left\langle T_{23}\tau_{23}\frac{1}{2}\tau\middle|\frac{1}{2}T_z\right\rangle\sum_{XM_X}\sum_{j_{23}m_{23}}\left\langle XM_XL_\rho M_\rho\middle|\frac{1}{2}m\right\rangle\left\langle j_{23}\,m_{23}\frac{1}{2}\sigma_1\middle|XM_X\right\rangle\sum_{s_{23}\sigma_{23}}\left\langle\frac{1}{2}\sigma_2'\frac{1}{2}\sigma_3'\middle|s_{23}\sigma_{23}\right\rangle$$

$$\times \langle l_{23}\mu_{23}s_{23}\sigma_{23}|j_{23}\ m_{23}\rangle \mathcal{G}^{j_{23}l_{23}s_{23}}_{L_\rho X}(k_{23},k)\sum_{\sigma'_1} D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}}]_{\sigma'\sigma'_1}\sum_{l'_{23}\mu'_{23}}\sum_{L'_\rho M'_\rho} Y^*_{l'_{23}\mu'_{23}}(\hat{\mathbf{k}}_{23})Y^*_{L'_\rho M'_\rho}(\hat{\mathbf{k}})$$
$$\times \sum_{T'_{23},\tau'_{23}}\left\langle\frac{1}{2}\tau_2\frac{1}{2}\tau_3\middle|T'_{23}\tau'_{23}\right\rangle\left\langle T'_{23}\tau'_{23}\frac{1}{2}\tau\middle|\frac{1}{2}T_z\right\rangle\sum_{X'M'_X}\sum_{j'_{23}m'_{23}}\left\langle X'M'_XL'_\rho M'_\rho\middle|\frac{1}{2}m'\right\rangle\left\langle j'_{23}\ m'_{23}\frac{1}{2}\sigma'_1\middle|X'M'_X\right\rangle$$
$$\times \sum_{s'_{23}\sigma'_{23}}\left\langle\frac{1}{2}\sigma'_2\frac{1}{2}\sigma'_3\middle|s'_{23}\sigma'_{23}\right\rangle\langle l'_{23}\mu'_{23}s'_{23}\sigma'_{23}|j'_{23}\ m'_{23}\rangle \mathcal{G}^{*j'_{23}l'_{23}s'_{23}}_{L'_\rho X'}(k_{23},k). \tag{C11}$$

Since $\mathbf{k}$ is only a function of $|\mathbf{k}_{23}|$, then we are allowed to integrate the spherical harmonics in Eq. (C11) over $d\Omega_{\hat{\mathbf{k}}_{23}}$. Therefore, taking care of the orthogonality of the Clebsch-Gordan coefficients, we can write

$$\mathcal{F}^{\tau mm'}_{\sigma\sigma'} = \frac{1}{(1-x)}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sum_{\sigma_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma'_1} D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}}]_{\sigma'\sigma'_1}\sum_{s_{23}}\sum_{j_{23}}\sum_{l_{23}}\sum_{T_{23},\tau_{23}}\left\langle T_{23}\tau_{23}\frac{1}{2}\tau\middle|\frac{1}{2}T_z\right\rangle\left\langle T_{23}\tau_{23}\frac{1}{2}\tau\middle|\frac{1}{2}T_z\right\rangle$$
$$\times \sum_{L_\rho}\sum_{L'_\rho}\sum_X\sum_{X'} \mathcal{R}^{j_{23},mm',\sigma_1\sigma'_1}_{L_\rho L'_\rho XX'}(k_{23},\mathbf{k})\mathcal{G}^{j_{23}l_{23}s_{23}}_{L_\rho X}(k_{23},k)\mathcal{G}^{*j_{23}l_{23}s_{23}}_{L'_\rho X'}(k_{23},k), \tag{C12}$$

where

$$\mathcal{R}^{j_{23},mm',\sigma_1\sigma'_1}_{L_\rho L'_\rho XX'}(k_{23},\mathbf{k}) = \sum_{M_\rho} Y_{L_\rho M_\rho}(\hat{\mathbf{k}})\sum_{M_X}\sum_{m_{23}}\left\langle XM_XL_\rho M_\rho\middle|\frac{1}{2}m\right\rangle\left\langle j_{23}\ m_{23}\frac{1}{2}\sigma_1\middle|XM_X\right\rangle\sum_{M'_\rho}(-1)^{-M'_\rho}Y_{L'_\rho -M'_\rho}(\hat{\mathbf{k}})$$
$$\times \sum_{M'_X}\left\langle X'M'_XL'_\rho M'_\rho\middle|\frac{1}{2}m'\right\rangle\left\langle j_{23}\ m_{23}\frac{1}{2}\sigma'_1\middle|X'M'_X\right\rangle. \tag{C13}$$

The quantity $\mathcal{R}^{j_{23},mm',\sigma_1\sigma'_1}_{L_\rho L'_\rho XX'}(k_{23},\mathbf{k})$ is invariant for parity, since L_ρ and L'_ρ have the same parity.

2. Sum of products of five 3j symbols

By using the properties of the product of two spherical harmonics with the same argument [77] and 3j symbols, Eq. (C13) becomes

$$\mathcal{R}^{j_{23},mm',\sigma_1\sigma'_1}_{L_\rho L'_\rho XX'}(k_{23},\mathbf{k}) = \sum_{LM}\sqrt{\frac{(2L_\rho+1)(2L'_\rho+1)}{4\pi(2L+1)}}\langle L_\rho 0L'_\rho 0|L0\rangle Y_{LM}(\theta,\phi)(-1)^{(L_\rho-L'_\rho+M)}$$
$$\times \sum_{M_\rho M'_\rho}(-1)^{-M'_\rho}\sqrt{2L+1}\begin{pmatrix} L_\rho & L'_\rho & L\\ M_\rho & -M'_\rho & -M\end{pmatrix}\sum_{M_XM'_X}\sum_{m_{23}}(-1)^{(X-L_\rho+m)}(-1)^{(j_{23}-\frac{1}{2}+M_X)}(-1)^{(X'-L'_\rho+m')}$$
$$\times (-1)^{(j_{23}-\frac{1}{2}+M'_X)}\sqrt{2}\sqrt{2}\sqrt{2X+1}\sqrt{2X'+1}\begin{pmatrix} X & L_\rho & \frac{1}{2}\\ M_X & M_\rho & -m\end{pmatrix}\begin{pmatrix} j_{23} & \frac{1}{2} & X\\ m_{23} & \sigma_1 & -M_X\end{pmatrix}$$
$$\times \begin{pmatrix} X' & L'_\rho & \frac{1}{2}\\ M'_X & M'_\rho & -m'\end{pmatrix}\begin{pmatrix} j_{23} & \frac{1}{2} & X'\\ m_{23} & \sigma'_1 & -M'_X\end{pmatrix}, \tag{C14}$$

where the angles θ and ϕ define the direction of $\hat{\mathbf{k}}$. Only even values of L are allowed to satisfy the parity invariance of $\mathcal{R}^{j_{23},mm',\sigma_1\sigma'_1}_{L_\rho L'_\rho XX'}(k_{23},\mathbf{k})$.

Through permutations of the columns in the 3j symbols, to have the indices m and m' in the middle, and changing the sign of the third momentum components in the 3j symbols where X appears, we obtain (see Eq. (16) at p. 457 of Ref. [77])

$$\mathcal{R}^{j_{23},mm',\sigma_1\sigma'_1}_{L_\rho L'_\rho XX'}(k_{23},\mathbf{k}) = -2(-1)^{\sigma'_1}(-1)^m(-1)^{(X+m)}(-1)^{(X'+m')}(-1)^{j_{23}}\sqrt{\frac{(2L_\rho+1)(2L'_\rho+1)}{4\pi}}\langle L_\rho 0L'_\rho 0|L0\rangle\sqrt{2X+1}\sqrt{2X'+1}$$
$$\times \sum_{LM}(-1)^{(L+M)}Y_{LM}(\theta,\phi)(-1)^{(X'-L'_\rho-1/2-1/2-L-1/2)}\sum_{a\alpha y\eta}(-1)^{(a-\alpha+y-\eta)}\Pi^2_{ay}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & a\\ \sigma'_1 & -\sigma_1 & \alpha\end{pmatrix}$$
$$\times \begin{pmatrix} a & L & y\\ -\alpha & -M & -\eta\end{pmatrix}\begin{pmatrix} y & \frac{1}{2} & \frac{1}{2}\\ \eta & m & -m'\end{pmatrix}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & a\\ X & X' & j_{23}\end{Bmatrix}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & y\\ X & X' & a\\ L_\rho & L'_\rho & L\end{Bmatrix}. \tag{C15}$$

where $\Pi^2_{ay} = (2a+1)(2y+1)$.

3. Sums involving the D matrices for the system polarization and the Melosh factors

Let us consider the following combination of Wigner functions and Melosh rotations:

$$D^{j\mathcal{M},ay}_{\sigma\sigma'}(\mathbf{S},\tilde{\mathbf{k}}) = -2\sum_{LM}(-1)^M\, Y_{LM}(\theta,\phi)\sum_m D^j_{m,\mathcal{M}}(\alpha,\beta,\gamma)\sum_{m'}\left[D^j_{m',\mathcal{M}}(\alpha,\beta,\gamma)\right]^*\sum_{\sigma_1} D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma_1'} D^{\frac12 *}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}$$
$$\times\sum_{\mu,\eta}(-1)^{-\mu-\eta}\begin{pmatrix}\frac12 & \frac12 & a\\ \sigma_1' & -\sigma_1 & \mu\end{pmatrix}\begin{pmatrix}a & L & y\\ -\mu & -M & -\eta\end{pmatrix}\begin{pmatrix}y & \frac12 & \frac12\\ \eta & m & -m'\end{pmatrix}(-1)^{\sigma_1}(-1)^{2m+m'}. \tag{C16}$$

If one applies Eq. (A4) to the product $D^j_{m,\mathcal{M}}(\alpha,\beta,\gamma)\, D^j_{-m',-\mathcal{M}}(\alpha,\beta,\gamma)$, then Eq. (C16) becomes

$$D^{j\mathcal{M},ay}_{\sigma\sigma'}(\mathbf{S},\tilde{\mathbf{k}}) = \sqrt{2}\frac{(-1)^{a+1}}{\sqrt{2a+1}}\sum_{LM}(-1)^M Y_{LM}(\theta,\phi)\sum_{J=0}^{2j}\langle j\mathcal{M}, j-\mathcal{M}|J0\rangle\sum_{\mu,\eta}(-1)^{-\eta}\sum_{mm'}(-1)^{m'-\mathcal{M}}\langle jm, j-m'|J(m-m')\rangle$$
$$\times D^J_{(m-m'),0}(\alpha,\beta,\gamma)\sum_{\sigma_1}D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma_1'}D^{\frac12 *}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}(-1)^{\sigma_1}(-1)^{(1-a)}\left\langle\frac12\sigma_1,\frac12-\sigma_1'\middle|a\mu\right\rangle$$
$$\times\begin{pmatrix}a & L & y\\ -\mu & -M & -\eta\end{pmatrix}(-1)^{1/2+2y}(-1)^{1/2-m}\sqrt{\frac{2}{2y+1}}\left\langle\frac12 m,\frac12-m'|y-\eta\right\rangle. \tag{C17}$$

One has to recall that $j=1/2$ and that $(m-m')$ has to be equal to $-\eta$. Then we obtain

$$D^{1/2\mathcal{M},ay}_{\sigma\sigma'}(\mathbf{S},\tilde{\mathbf{k}}) = -2\frac{1}{\sqrt{2a+1}}\sum_{LM}(-1)^M\, Y_{LM}(\theta,\phi)\left\langle\frac12\mathcal{M},\frac12-\mathcal{M}\middle|y0\right\rangle\sum_{\mu,\eta}(-1)^{-\mathcal{M}}Y^*_{y-\eta}(\beta,\alpha)\sqrt{\frac{4\pi}{2y+1}}\sum_{\sigma_1}D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}$$
$$\times\sum_{\sigma_1'}D^{\frac12 *}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}\left\langle\frac12\sigma_1,\frac12-\sigma_1'\middle|a\mu\right\rangle(-1)^{\sigma_1}\begin{pmatrix}a & L & y\\ -\mu & -M & -\eta\end{pmatrix}\frac{1}{\sqrt{2y+1}}. \tag{C18}$$

4. Spin-dependent momentum distribution

Making use of Eq. (C18) to express the quantity $D^{j\mathcal{M},ay}_{\sigma\sigma'}(\mathbf{S},\tilde{\mathbf{k}})$, we can now summarize our results for the spin-dependent momentum distribution, Eq. (14), as follows:

$$\left[\boldsymbol{\mathcal{N}}^\tau_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})\right]_{\sigma\sigma'} = \frac{(-1)^{\mathcal{M}}2}{(1-x)}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sum_{s_{23}}\sum_{j_{23}}\sum_{l_{23}}\sum_{T_{23},\tau_{23}}\left\langle T_{23}\tau_{23}\frac12\tau\middle|\frac12 T_z\right\rangle\left\langle T_{23}\tau_{23}\frac12\tau\middle|\frac12 T_z\right\rangle(-1)^{j_{23}}$$
$$\times\sum_{L_\rho}\sum_{L_\rho'}\sum_X\sum_{X'}\sqrt{(2L_\rho+1)(2L_\rho'+1)}\langle L_\rho 0L_\rho' 0|L0\rangle\sqrt{2X+1}\sqrt{2X'+1}(-1)^{(-L_\rho'-1/2)}$$
$$\times\boldsymbol{\mathcal{G}}^{j_{23}l_{23}s_{23}}_{L_\rho X}(k_{23},k)\boldsymbol{\mathcal{G}}^{*j_{23}l_{23}s_{23}}_{L_\rho' X'}(k_{23},k)(-1)^X\sum_{ay}(-1)^a\Pi^2_{ay}\sum_{LM}Y_{LM}(\theta,\phi)\frac{1}{\sqrt{2a+1}}\langle\frac12\mathcal{M},\frac12-\mathcal{M}|y0\rangle$$
$$\times\sum_{\mu,\eta}Y^*_{y-\eta}(\beta,\alpha)\frac{1}{\sqrt{2y+1}}\sum_{\sigma_1}D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma_1'}D^{\frac12 *}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}\left\langle\frac12\sigma_1,\frac12-\sigma_1'\middle|a\mu\right\rangle(-1)^{\sigma_1}$$
$$\times\langle a\mu LM|y-\eta\rangle(-1)^\mu\frac{1}{2y+1}\begin{Bmatrix}\frac12 & \frac12 & a\\ X & X' & j_{23}\end{Bmatrix}\begin{Bmatrix}\frac12 & \frac12 & y\\ X & X' & a\\ L_\rho & L_\rho' & L\end{Bmatrix}. \tag{C19}$$

It has to be noticed that $(a+y)$ is an even number, as can be easily shown exchanging X with X' and L_ρ with L_ρ'. Furthermore, both a and y can be only zero or 1.

Then, if $L=0$, one has $a=y=0$ or $a=y=1$. If $L=2$, only $a=y=1$ is possible.

Let us define the quantity

$$\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},a,y,L) = \Pi^2_{ay}\frac{1}{\sqrt{2a+1}}\frac{1}{(2y+1)^{3/2}}\left\langle\frac12\mathcal{M},\frac12-\mathcal{M}\middle|y0\right\rangle\mathcal{H}^\tau(L,a,k_{23},k)\sum_M Y_{LM}(\theta,\phi)\sum_{\sigma_1}D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}$$
$$\times\sum_{\sigma_1'}D^{\frac12 *}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}\sum_{\mu\eta}Y^*_{y-\eta}(\beta,\alpha)\langle a\mu,LM|y-\eta\rangle(-1)^\mu\left\langle\frac12\sigma_1,\frac12-\sigma_1'\middle|a\mu\right\rangle(-1)^{\sigma_1-1/2}, \tag{C20}$$

where

$$\mathcal{H}^\tau(L,a,k_{23},k) = (-1)^a \sum_{j_{23}}\sum_{l_{23}}\sum_{s_{23}}\sum_{T_{23},\tau_{23}} \left\langle T_{23}\tau_{23}\frac{1}{2}\tau\middle|\frac{1}{2}T_z\right\rangle\left\langle T_{23}\tau_{23}\frac{1}{2}\tau\middle|\frac{1}{2}T_z\right\rangle(-1)^{j_{23}}$$
$$\times \sum_{L_\rho}\sum_{L'_\rho}\sum_{X}\sum_{X'}(-1)^{(X+1/2)}\sqrt{(2L_\rho+1)(2L'_\rho+1)}\langle L_\rho 0 L'_\rho 0|L0\rangle\sqrt{2X+1}\sqrt{2X'+1}(-1)^{X+X'}$$
$$\times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & a \\ X & X' & j_{23}\end{Bmatrix}\begin{Bmatrix} L_\rho & X & \frac{1}{2} \\ L'_\rho & X' & \frac{1}{2} \\ L & a & y\end{Bmatrix}(-1)^{l_{23}}\mathcal{G}^{j_{23}l_{23}s_{23}}_{L_\rho X}(k_{23},k)\mathcal{G}^{*j_{23}l_{23}s_{23}}_{L'_\rho X'}(k_{23},k). \tag{C21}$$

Then the momentum distribution can be written as a function of the three independent quantities $\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},0,0,0)$, $\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,0)$, and $\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,2)$ that notably depend on $\mathcal{H}^\tau(0,0,k_{23},k)$, $\mathcal{H}^\tau(0,1,k_{23},k)$, and $\mathcal{H}^\tau(2,1,k_{23},k)$, viz.,

$$\left[\boldsymbol{\mathcal{N}}^\tau_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})\right]_{\sigma\sigma'} = (-1)^{\mathcal{M}+1/2}\frac{2}{(1-x)}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\{\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},0,0,0)$$
$$+\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,0)+\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,2)\}. \tag{C22}$$

We evaluate separately $\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},a,y,L)$ for the three possible cases of the variables L,a,y. The first two quantities to evaluate are

$$\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},0,0,0) = \Pi^2_{00}\left\langle\frac{1}{2}\mathcal{M},\frac{1}{2}-\mathcal{M}\middle|00\right\rangle\mathcal{H}^\tau(0,0,k_{23},k)\sum_{\sigma_1}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}$$
$$\times\sum_{\sigma'_1}D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma'_1}Y_{00}(\theta,\phi)Y^*_{00}(\beta,\alpha)\langle 00,00|00\rangle\left\langle\frac{1}{2}\sigma_1,\frac{1}{2}-\sigma'_1\middle|00\right\rangle(-1)^{\sigma_1-1/2}$$
$$=\delta_{\sigma\sigma'}(-1)^{1/2-\mathcal{M}}\mathcal{H}^\tau(0,0,k_{23},k)\frac{1}{8\pi} \tag{C23}$$

and

$$\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,0) = \Pi^2_{1,1}\frac{1}{\sqrt{3}}\frac{1}{(3)^{3/2}}\left\langle\frac{1}{2}\mathcal{M},\frac{1}{2}-\mathcal{M}\middle|10\right\rangle\mathcal{H}^\tau(0,1,k_{23},k)\sum_{\sigma_1}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma'_1}D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma'_1}$$
$$\times\sum_{\mu}Y_{00}(\theta,\phi)\sum_{\eta}Y^*_{1-\eta}(\beta,\alpha)\langle 1\mu,00|1-\eta\rangle(-1)^\mu\left\langle\frac{1}{2}\sigma_1,\frac{1}{2}-\sigma'_1\middle|1\mu\right\rangle(-1)^{\sigma_1-1/2}$$
$$=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\mathcal{H}^\tau(0,1,k_{23},k)\sum_{\sigma_1}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma'_1}D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma'_1}\frac{1}{2}\sqrt{\frac{3}{2\pi}}[\boldsymbol{\sigma}\cdot\hat{\mathbf{S}}]_{\sigma_1\sigma'_1}, \tag{C24}$$

where the identity (see Ref. [77])

$$\sum_\mu Y_{1-\mu}(\beta,\alpha)(-1)^\mu\left\langle\frac{1}{2}\sigma_1,\frac{1}{2}-\sigma'_1\middle|1\mu\right\rangle(-1)^{\sigma'_1} = \frac{1}{2}(-1)^{1/2}\sqrt{\frac{3}{2\pi}}[\boldsymbol{\sigma}\cdot\hat{\mathbf{S}}]_{\sigma_1\sigma'_1} \tag{C25}$$

is used.

Through the actual expressions of the Melosh rotations [see Appendix D and in particular Eqs. (D5) and (D7)], Eq. (C24) becomes

$$\mathcal{Z}^\tau_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,0) = \frac{1}{2}\frac{1}{\sqrt{8\pi}}\sqrt{\frac{3}{2\pi}}\mathcal{H}^\tau(0,1,k_{23},k)\left[\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\right]_{\sigma\sigma_1}[\boldsymbol{\sigma}\cdot\hat{\mathbf{S}}]_{\sigma_1\sigma'_1}\left[\cos\frac{\varphi}{2}-i\sin\frac{\varphi}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\right]_{\sigma'_1\sigma'}$$
$$=\frac{\sqrt{3}}{8\pi}\mathcal{H}^\tau(0,1,k_{23},k)\left\{[\boldsymbol{\sigma}\cdot\mathbf{S}]_{\sigma\sigma'}-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\{[(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{z}-(\mathbf{S}\cdot\hat{z})\hat{\mathbf{k}}_\perp]\cdot\boldsymbol{\sigma}\}_{\sigma\sigma'}-2\sin^2\frac{\varphi}{2}\right.$$
$$\left.\times\{[(\mathbf{S}\cdot\hat{z})\hat{z}-(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{\mathbf{k}}_\perp]\cdot\sigma\}_{\sigma\sigma'}\right\}, \tag{C26}$$

where $\sin\varphi/2$ and $\cos\varphi/2$ are defined in Eq. (D3).

The last quantity, $\mathcal{Z}^{\tau}_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,2)$, is

$$\mathcal{Z}^{\tau}_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,2)=\Pi^2_{1,1}\frac{1}{\sqrt{3}}\frac{1}{(3)^{3/2}}\left\langle\frac{1}{2}\mathcal{M},\frac{1}{2}-\mathcal{M}\middle|10\right\rangle\mathcal{H}^{\tau}(2,1,k_{23},k)\sum_{\sigma_1}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma_1'}D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}$$
$$\times\sum_M Y_{2M}(\theta,\phi)\sum_{\mu\eta}Y^*_{1-\eta}(\beta,\alpha)\langle 1\mu,2M|1-\eta\rangle(-1)^{\mu}\left\langle\frac{1}{2}\sigma_1,\frac{1}{2}-\sigma_1'\middle|1\mu\right\rangle(-1)^{\sigma_1-1/2}$$
$$=-\frac{1}{2}\mathcal{H}^{\tau}(2,1,k_{23},k)\sum_{\sigma_1}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1}\sum_{\sigma_1'}D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma'\sigma_1'}\sum_{\mu}(-1)^{\mu}\left\langle\frac{1}{2}\sigma_1,\frac{1}{2}-\sigma_1'\middle|1\mu\right\rangle(-1)^{\sigma_1-1/2}$$
$$\times\left[Y^*_{1\mu}(\theta,\phi)\sqrt{3}Y_{1,0}(\hat{\mathbf{k}}\cdot\hat{\mathbf{S}})-Y^*_{1\mu}(\hat{\mathbf{S}})\frac{1}{\sqrt{4\pi}}\right].\tag{C27}$$

In the last step the first of the bipolar harmonics in Eq. (A5) of Ref. [58] was used. Therefore, using again Eq. (C25) and the results of Appendix D [see Eq. (D5)] we have

$$\mathcal{Z}^{\tau}_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,2)=-\frac{1}{4}\sqrt{\frac{3}{8}}\frac{1}{\pi}\mathcal{H}^{\tau}(2,1,k_{23},k)\Bigg[3\hat{\mathbf{k}}\cdot\hat{\mathbf{S}}\sum_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(k^+,\mathbf{k}_\perp)]_{\sigma\sigma_1}[\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}]_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(k^+,-\mathbf{k}_\perp)]_{\sigma_1'\sigma'}$$
$$-\sum_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(k^+,\mathbf{k}_\perp)]_{\sigma\sigma_1}[\boldsymbol{\sigma}\cdot\hat{\mathbf{S}}]_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(k_1^+,-\mathbf{k}_\perp)]_{\sigma_1'\sigma'}\Bigg].\tag{C28}$$

Eventually from Eqs. (D7) and (D8) we have

$$\mathcal{Z}^{\tau}_{\sigma\sigma'}(k_{23},\mathbf{k},\mathbf{S},1,1,2)=-\frac{1}{4}\sqrt{\frac{3}{8}}\frac{1}{\pi}\mathcal{H}^{\tau}(2,1,k_{23},k)\Bigg\{\frac{3}{k}\hat{\mathbf{k}}\cdot\hat{\mathbf{S}}\Big[\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)[\boldsymbol{\sigma}\cdot\mathbf{k}]_{\sigma\sigma'}$$
$$-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}[(k_\perp\hat{z}-k_z\hat{\mathbf{k}}_\perp)\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}\Big]-\Big\{[\boldsymbol{\sigma}\cdot\mathbf{S}]_{\sigma\sigma'}-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\Big[\big((\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{z}-(\mathbf{S}\cdot\hat{z})\hat{\mathbf{k}}_\perp\big)\cdot\boldsymbol{\sigma}\Big]_{\sigma\sigma'}$$
$$-2\sin^2\frac{\varphi}{2}[((\mathbf{S}\cdot\hat{z})\hat{z}+(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{\mathbf{k}}_\perp)\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}\Big\}\Bigg\}.\tag{C29}$$

Summarizing our results, the spin-dependent momentum distribution in terms of the pseudovectors $\mathbf{S}$, $\hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)$, $\hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{z})$, $\hat{z}\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)$, and $\hat{z}\,(\mathbf{S}\cdot\hat{z})$ is

$$\big[\boldsymbol{\mathcal{N}}^{\tau}_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})\big]_{\sigma\sigma'}=\frac{(-1)^{\mathcal{M}+1/2}}{4\pi(1-x)}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\big\{\delta_{\sigma\sigma'}(-1)^{1/2-\mathcal{M}}\mathcal{H}^{\tau}(0,0,k_{23},k)$$
$$+\sqrt{3}\Bigg[\mathcal{H}^{\tau}(0,1,k_{23},k)+\sqrt{\frac{1}{2}}\mathcal{H}^{\tau}(2,1,k_{23},k)\Bigg]\Big[[\boldsymbol{\sigma}\cdot\mathbf{S}]_{\sigma\sigma'}-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}[((\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{z}-(\mathbf{S}\cdot\hat{z})\hat{\mathbf{k}}_\perp)\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}$$
$$-2\sin^2\frac{\varphi}{2}[((\mathbf{S}\cdot\hat{z})\hat{z}+(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{\mathbf{k}}_\perp)\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}\Big]-\sqrt{\frac{3}{2}}\mathcal{H}^{\tau}(2,1,k_{23},k)\frac{3}{k^2}\Big[\mathbf{k}_\perp\cdot\mathbf{S}\Big[\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)(\mathbf{k}_\perp\cdot\boldsymbol{\sigma})_{\sigma\sigma'}$$
$$+\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)k_z(\hat{z}\cdot\boldsymbol{\sigma})_{\sigma\sigma'}-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}[(k_\perp\hat{z}-k_z\hat{\mathbf{k}}_\perp)\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}+k_z(\hat{z}\cdot\mathbf{S})\Big[\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)$$
$$\times(\mathbf{k}_\perp\cdot\boldsymbol{\sigma})_{\sigma\sigma'}+\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)k_z(\hat{z}\cdot\boldsymbol{\sigma})_{\sigma\sigma'}-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}[(k_\perp\hat{z}-k_z\hat{\mathbf{k}}_\perp)\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}\Big]\Big]\Big\}.\tag{C30}$$

From a comparison of Eqs. (15), (16), and (C30) one can immediately obtain explicit expressions for the functions $b^{\tau}_{i,\mathcal{M}}[|\mathbf{k}_\perp|,x,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2,(\mathbf{S}\cdot\hat{z})^2,(\hat{\mathbf{k}}_\perp\times\hat{z})\cdot\mathbf{S}]$ $(i=0,1,\dots,5)$ and verify that actually they do not depend on $\mathbf{S}$ and then they do not depend on the direction of $\mathbf{k}_\perp$. For $i=0$ one has

$$b^{\tau}_{0,\mathcal{M}}[x,|\mathbf{k}_\perp|]=\frac{(-1)}{2\pi(1-x)}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\mathcal{H}^{\tau}(0,0,k_{23},k).\tag{C31}$$

It can be useful to decompose the functions $b^{\tau}_{i,\mathcal{M}}$ $(i=1,\dots,5)$ according to the values zero or 2 of the momentum L,

$$b^{\tau}_{i,\mathcal{M}}=b^{\tau(L=0)}_{i,\mathcal{M}}+b^{\tau(L=2)}_{i,\mathcal{M}},\tag{C32}$$

and one obtains

$$b^{\tau(0)}_{1,\mathcal{M}}(x,|\mathbf{k}_\perp|) = \frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\mathcal{H}^\tau(0,1,k_{23},k), \tag{C33}$$

$$b^{\tau(2)}_{1,\mathcal{M}}(x,|\mathbf{k}_\perp|) = \frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k), \tag{C34}$$

$$b^{\tau(0)}_{2,\mathcal{M}}(x,|\mathbf{k}_\perp|) = -\frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\mathcal{H}^\tau(0,1,k_{23},k)2\sin^2\frac{\varphi}{2}, \tag{C35}$$

$$b^{\tau(2)}_{2,\mathcal{M}}(x,|\mathbf{k}_\perp|) = -\frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k)$$
$$\times\left\{2\sin^2\frac{\varphi}{2}+3\frac{1}{k^2}\left[\left(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\right)k_\perp^2+2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}k_\perp k_z\right]\right\}, \tag{C36}$$

$$b^{\tau(0)}_{3,\mathcal{M}}(x,|\mathbf{k}_\perp|) = \frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\mathcal{H}^\tau(0,1,k_{23},k), \tag{C37}$$

$$b^{\tau(2)}_{3,\mathcal{M}}(x,|\mathbf{k}_\perp|) = \frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k)$$
$$\times\left\{2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}-3\frac{1}{k^2}\left[k_\perp k_z\left(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\right)+k_z^2 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\right]\right\}, \tag{C38}$$

$$b^{\tau(0)}_{4,\mathcal{M}}(x,|\mathbf{k}_\perp|) = -b^{\tau(0)}_{3,\mathcal{M}}(x,|\mathbf{k}_\perp|), \tag{C39}$$

$$b^{\tau(2)}_{4,\mathcal{M}}(x,|\mathbf{k}_\perp|) = -\frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k)$$
$$\times\left\{2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}+3\frac{1}{k^2}\left[k_\perp k_z\left(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\right)-k_\perp^2 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\right]\right\}, \tag{C40}$$

$$b^{\tau(0)}_{5,\mathcal{M}}(x,|\mathbf{k}_\perp|) = b^{\tau(0)}_{2,\mathcal{M}}(x,|\mathbf{k}_\perp|), \tag{C41}$$

$$b^{\tau(2)}_{5,\mathcal{M}}(x,|\mathbf{k}_\perp|) = -\frac{(-1)^{\mathcal{M}+1/2}}{2\pi(1-x)}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\frac{E_{23}E(\mathbf{k})}{k^+}\sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k)$$
$$\times\left\{2\sin^2\frac{\varphi}{2}+3\frac{1}{k^2}\left[\left(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\right)k_z^2-2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}k_\perp k_z\right]\right\}, \tag{C42}$$

where the dependence upon φ is generated by the Melosh rotations (see Appendix D). It has to be pointed out that, in the case of a three-nucleon bound system, φ is small, as one can deduce from its definition in Eq. (D3), and therefore $\sin(\varphi/2)/\cos(\varphi/2) \ll 1$.

One can immediately recognize that the quantities $b^\tau_{i,\mathcal{M}}$ actually are invariant for rotations around the z axis, while they do depend on $|\mathbf{k}_\perp|$ and x. The quantity b^τ_0 is independent of $\mathcal{M}$. For $i = 1,\dots,5$, the dependence on $\mathcal{M}$ is through the factor $(-1)^{\mathcal{M}+1/2}$.

5. Effective polarizations

Using the equations (see Appendix B of Ref. [1])

$$\frac{dx}{dk^+} = \frac{1-x}{E_{23}},\qquad \frac{dk^+}{dk_z} = \frac{k^+}{E(\mathbf{k})}, \tag{C43}$$

the longitudinal and transverse effective polarizations of Eqs. (67) and (68), respectively, are given by

$$p^\tau_{||} = (-1)^{\mathcal{M}+1/2}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\int_0^\infty k^2dk \times\int_{-1}^1 d\cos\theta\, f_{||}(\mathbf{k},k_{23}) \tag{C44}$$

and

$$p^\tau_\perp = (-1)^{\mathcal{M}+1/2}\sqrt{3}\int_0^\infty dk_{23}k_{23}^2\int_0^\infty k^2dk \times\int_{-1}^1 d\cos\theta\, f_\perp(\mathbf{k},k_{23}), \tag{C45}$$

where

$$f_{||}(\mathbf{k},k_{23}) = \mathcal{H}^\tau(0,1,k_{23},k)\left(1-2\sin^2\frac{\varphi}{2}\right) + \sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k)\Big\{1-2\sin^2\frac{\varphi}{2}$$

$$-3\Big[\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)\cos^2\theta - 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\sin\theta\cos\theta\Big]\Big\} \tag{C46}$$

and

$$f_\perp(\mathbf{k},k_{23}) = \mathcal{H}^\tau(0,1,k_{23},k)\Big(1-\sin^2\frac{\varphi}{2}\Big) + \sqrt{\frac{1}{2}}\mathcal{H}^\tau(2,1,k_{23},k)\Big\{1-\sin^2\frac{\varphi}{2} - \frac{3}{2}\Big[\Big(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big)\sin^2\theta + 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\sin\theta\cos\theta\Big]\Big\}. \tag{C47}$$

In Eqs. (C46) and (C47) we defined $\cos\theta = k_z/k$ and $\sin\theta = k_\perp/k$.

Let us emphasize that without the effect of the Melosh rotations one has $\sin\frac{\varphi}{2}=0$ and $\cos\frac{\varphi}{2}=1$. Then the two polarizations become equal, viz.,

$$p^\tau_{||} = p^\tau_\perp = (-1)^{\mathcal{M}+1/2}2\sqrt{3}\int_0^\infty dk_{23}k_{23}^2 \times \int_0^\infty k^2 dk \mathcal{H}^\tau(0,1,k_{23},k). \tag{C48}$$

APPENDIX D: PRODUCTS OF MELOSH AND PAULI MATRICES

The Melosh matrix

$$D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'} = \chi^\dagger_\sigma \frac{m+k^+ - \imath\boldsymbol{\sigma}\cdot(\hat{z}\times\mathbf{k}_\perp)}{\sqrt{(m+k^+)^2+|\mathbf{k}_\perp|^2}}\chi_{\sigma'} \tag{D1}$$

can be rewritten as follows:

$$D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'} = \Big[\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\Big]_{\sigma\sigma'}, \tag{D2}$$

where

$$\varphi = 2\arctan\frac{|\mathbf{k}_\perp|}{k^++m},\quad \cos\frac{\varphi}{2} = \frac{k^++m}{\sqrt{(k^++m)^2+\mathbf{k}_\perp^2}},\quad \sin\frac{\varphi}{2} = \frac{|\mathbf{k}_\perp|}{\sqrt{(k^++m)^2+\mathbf{k}_\perp^2}}, \tag{D3}$$

and

$$\hat{\mathbf{n}} = -\frac{\hat{z}\times\mathbf{k}_\perp}{|\mathbf{k}_\perp|}. \tag{D4}$$

Then the equality

$$D^{\frac12 *}[\mathcal{R}_M(k^+,\mathbf{k}_\perp)]_{\sigma\sigma'} = D^{\frac12}[\mathcal{R}_M(k^+,-\mathbf{k}_\perp)]_{\sigma'\sigma} = \Big[\cos\frac{\varphi}{2} - i\sin\frac{\varphi}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\Big]_{\sigma'\sigma} \tag{D5}$$

holds.

Let us now evaluate the sandwich of $[\boldsymbol{\sigma}\cdot\hat{\mathbf{e}}]$ between two Melosh matrices [see Eqs. (C24), (C26), and (C28)], with $\hat{\mathbf{e}}$ a unit vector. One gets

$$\begin{aligned}\mathcal{D}_{\sigma\sigma'}(k^+,\mathbf{k}_\perp,\hat{\mathbf{e}}) &= \sum_{\sigma_1'\tilde{\sigma}_1'} D^{\frac12}[\mathcal{R}_M(k^+,\mathbf{k}_\perp)]_{\sigma\sigma_1'}[\boldsymbol{\sigma}\cdot\hat{\mathbf{e}}]_{\sigma_1'\tilde{\sigma}_1'} \times D^{\frac12}[\mathcal{R}_M(k^+,-\mathbf{k}_\perp)]_{\tilde{\sigma}_1'\sigma'} \\ &= \Big[\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big][\boldsymbol{\sigma}\cdot\hat{\mathbf{e}}]_{\sigma\sigma'} - 2\sin\frac{\varphi}{2} \times \cos\frac{\varphi}{2}\{[(\hat{\mathbf{e}}\cdot\hat{\mathbf{k}}_\perp)\hat{z} - (\hat{\mathbf{e}}\cdot\hat{z})\hat{\mathbf{k}}_\perp]\cdot\boldsymbol{\sigma}\}_{\sigma\sigma'} \\ &\quad + 2\sin^2\frac{\varphi}{2}\{(\hat{\mathbf{n}}\cdot\hat{\mathbf{e}})(\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})\}_{\sigma\sigma'}.\end{aligned} \tag{D6}$$

For $\hat{\mathbf{e}} = \mathbf{S}$ one has

$$\begin{aligned}\mathcal{D}_{\sigma\sigma'}(k^+,\mathbf{k}_\perp,\mathbf{S}) &= [\boldsymbol{\sigma}\cdot\mathbf{S}]_{\sigma\sigma'} - 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2} \times \{[(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{z} - (\mathbf{S}\cdot\hat{z})\hat{\mathbf{k}}_\perp]\cdot\boldsymbol{\sigma}\}_{\sigma\sigma'} \\ &\quad - 2\sin^2\frac{\varphi}{2}\{[(\mathbf{S}\cdot\hat{z})\hat{z} + (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\hat{\mathbf{k}}_\perp]\cdot\boldsymbol{\sigma}\}_{\sigma\sigma'}.\end{aligned} \tag{D7}$$

For $\hat{\mathbf{e}} = \hat{\mathbf{k}}$, one obtains

$$\mathcal{D}_{\sigma\sigma'}(k^+,\mathbf{k}_\perp,\hat{\mathbf{k}}) = \Big[\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}\Big][\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}]_{\sigma\sigma'} - 2\sin\frac{\varphi}{2} \times \cos\frac{\varphi}{2}\frac{1}{k}[[k_\perp\hat{z} - k_z\hat{\mathbf{k}}_\perp]\cdot\boldsymbol{\sigma}]_{\sigma\sigma'}, \tag{D8}$$

where $k_\perp = \sqrt{k_x^2+k_y^2}$ and $k_z = \mathbf{k}\cdot\hat{z} = \frac12(k^+-k^-) = \frac12(k^+ - \frac{m^2+\mathbf{k}_\perp^2}{k^+})$.

APPENDIX E: TRACES OF THE VALENCE CORRELATOR AND OF THE LIGHT-FRONT SPECTRAL FUNCTION

Let us denote by Γ a generic 4×4 matrix. The traces of $[\Gamma\,\Phi_V]$ can be expressed through traces of the spectral function or of the spectral function times $\boldsymbol{\sigma}$ matrices. Indeed with the help of Eqs. (40) and (41) one has

$$\frac{1}{2P^+}\mathrm{Tr}[\Gamma\Phi_V] = \frac12\frac{1}{2m}\sum_\sigma\sum_{\sigma'}\bar{u}(\tilde{\mathbf{p}},\sigma)\Gamma u(\tilde{\mathbf{p}},\sigma') \times \frac{1}{p^+}\frac{\pi E_S}{\xi\mathcal{M}_0[1,(23)]}\mathcal{P}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S). \tag{E1}$$

As in Appendix B, to simplify the notation the isospin index τ is understood.

The traces of Φ_V in Eqs. (B12)–(B14), i.e., the traces needed when the correlator is expanded at twist-two level considering only the T-even terms, can be expressed by traces of the spectral function with the help of Eq. (E1) and of the following equalities for the matrix elements of γ matrices between LF spinors (see Ref. [61]):

$$\bar{u}(\tilde{\mathbf{p}}', \sigma')\gamma^+ u(\tilde{\mathbf{p}}, \sigma) = \delta_{\sigma'\sigma} 2\sqrt{p'^+ p^+}, \tag{E2}$$

$$\bar{u}(\tilde{\mathbf{p}}', \sigma')\gamma^+ \gamma_5 u(\tilde{\mathbf{p}}, \sigma) = 2\sqrt{p'^+ p^+}\, \chi^\dagger_{\sigma'} \sigma_z \chi_\sigma, \tag{E3}$$

$$\bar{u}(\tilde{\mathbf{p}}', \sigma')\gamma^+ \gamma_5 \gamma_x u(\tilde{\mathbf{p}}, \sigma) = -2\sqrt{p'^+ p^+}\, \chi^\dagger_{\sigma'} \sigma_x \chi_\sigma, \tag{E4}$$

where χ_σ is the spin eigenfunction. Then one obtains

$$\frac{1}{2P^+} \mathrm{Tr}(\gamma^+ \Phi_V) = c \mathrm{Tr}[\mathcal{P}_{\mathcal{M}}(\tilde{\kappa}, \epsilon, S)], \tag{E5}$$

$$\frac{1}{2P^+} \mathrm{Tr}(\gamma^+ \gamma_5 \Phi_V) = c \mathrm{Tr}[\sigma_z \mathcal{P}_{\mathcal{M}}(\tilde{\kappa}, \epsilon, S)] \tag{E6}$$

$$-\frac{1}{2P^+} \mathrm{Tr}(\gamma^i \gamma^+ \gamma_5 \Phi_V) = c \mathrm{Tr}[\sigma^i \mathcal{P}_{\mathcal{M}}(\tilde{\kappa}, \epsilon, S)], \tag{E7}$$

where $c = \pi\, E_S/(2m\, \kappa^+)$ and $i = 1, 2$.

[1] A. Del Dotto, E. Pace, G. Salmè, and S. Scopetta, Light-front spin-dependent spectral function and nucleon momentum distributions for a three-body system, Phys. Rev. C **95**, 014001 (2017).

[2] P. A. M. Dirac, Forms of relativistic dynamics, Rev. Mod. Phys. **21**, 392 (1949).

[3] B. D. Keister and W. N. Polyzou, Relativistic Hamiltonian dynamics in nuclear and particle physics, Adv. Nucl. Phys. **20**, 225 (1991).

[4] F. Coester, Null plane dynamics of particles and fields, Prog. Part. Nucl. Phys. **29**, 1 (1992).

[5] F. M. Lev, Relativistic quantum mechanics and its applications to few-nucleon systems, Riv. Nuovo Cim. **16**, 1 (1993).

[6] W. N. Polyzou, W. Glöckle, and H. Witala, Spin in relativistic quantum theory, Few Body Syst. **54**, 1667 (2013).

[7] W. N. Polyzou, Representations of relativistic particles of arbitrary spin in Poincaré, Lorentz, and Euclidean covariant formulations of relativistic quantum mechanics, Phys. Rev. C **99**, 025202 (2019).

[8] V. Barone, A. Drago, and P. G. Ratcliffe, Transverse polarisation of quarks in hadrons, Phys. Rep. **359**, 1 (2002).

[9] D. W. Sivers, Single spin production asymmetries from the hard scattering of point-like constituents, Phys. Rev. D **41**, 83 (1990).

[10] D. W. Sivers, Hard scattering scaling laws for single spin production asymmetries, Phys. Rev. D **43**, 261 (1991).

[11] J. C. Collins, Fragmentation of transversely polarized quarks probed in transverse momentum distributions, Nucl. Phys. B **396**, 161 (1993).

[12] P. J. Mulders and R. D. Tangerman, The complete tree level result up to order 1/Q for polarized deep inelastic leptoproduction, Nucl. Phys. B **461**, 197 (1996); **484**, 538(E) (1997).

[13] R. Jakob, P. J. Mulders, and J. Rodrigues, Modeling quark distribution and fragmentation functions, Nucl. Phys. A **626**, 937 (1997).

[14] D. Boer and P. J. Mulders, Time reversal odd distribution functions in leptoproduction, Phys. Rev. D **57**, 5780 (1998).

[15] A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Single-spin asymmetries: The Trento conventions, Phys. Rev. D **70**, 117504 (2004).

[16] J. C. Collins and T. C. Rogers, Equality of two definitions for transverse momentum dependent parton distribution functions, Phys. Rev. D **87**, 034018 (2013).

[17] R. Angeles-Martinez *et al.*, Transverse momentum dependent (TMD) parton distribution functions: Status and prospects, Acta Phys. Polon. B **46**, 2501 (2015).

[18] J. Collins, L. Gamberg, A. Prokudin, T. C. Rogers, N. Sato, and B. Wang, Relating transverse momentum dependent and collinear factorization theorems in a generalized formalism, Phys. Rev. D **94**, 034014 (2016).

[19] H. Avakian, B. Parsamyan, and A. Prokudin, Spin orbit correlations and the structure of the nucleon, Riv. Nuovo Cim. **42**, 1 (2019).

[20] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, and A. Prokudin, Extracting the Sivers function from polarized SIDIS data and making predictions, Phys. Rev. D **72**, 094007 (2005); **72**, 099903(E) (2005).

[21] Z.-B. Kang, A. Prokudin, P. Sun, and F. Yuan, Extraction of quark transversity distribution and Collins fragmentation functions with QCD evolution, Phys. Rev. D **93**, 014009 (2016).

[22] M. Anselmino, R. Kishore, and A. Mukherjee, Extraction of the valence transversity distributions from SIDIS data, Phys. Rev. D **102**, 096012 (2020).

[23] J.-P. Chen *et al.*, Asymmetries in semi-inclusive deep-inelastic $(e, e'\pi^\pm)$ reactions on a longitudinally polarized ^{3}He target at 8.8 and 11 GeV, approved experiment E12-11-007 (unpublished).

[24] G. Cates *et al.*, Measurement of the semi-inclusive π and K electro-production in DIS regime from transversely polarized ^{3}He target with the SBS and BB spectrometers in Hall A, approved experiment E12-09-018 (unpublished).

[25] X. Jiang *et al.*, Target single spin asymmetry in semi-inclusive deep-inelastic $(e, e'\pi^\pm)$ reaction on a transversely polarized ^{3}He target at 8.8 and 11 GeV, approved experiment E12-10-006 (unpublished).

[26] E. Cisbani *et al.*, π^+ transversity, approved experiment E-06-011 (unpublished).

[27] Y. Zhang *et al.* (Jefferson Lab Hall A Collaboration), Measurement of pretzelosity asymmetry of charged pion production in semi-inclusive deep inelastic scattering on a polarized ^{3}He target, Phys. Rev. C **90**, 055209 (2014).

[28] A. Airapetian *et al.* (HERMES Collaboration), Single-Spin Asymmetries in Semi-Inclusive Deep-Inelastic Scattering on a Transversely Polarized Hydrogen Target, Phys. Rev. Lett. **94**, 012002 (2005).

[29] M. G. Alekseev *et al.* (COMPASS Collaboration), Measurement of the Collins and Sivers asymmetries on transversely polarised protons, Phys. Lett. B **692**, 240 (2010).

[30] A. Martin, F. Bradamante, and V. Barone, Extracting the transversity distributions from single-hadron and dihadron production, Phys. Rev. D **91**, 014034 (2015).

[31] M. Aghasyan *et al.* (COMPASS Collaboration), First Measurement of Transverse-Spin-Dependent Azimuthal Asymmetries in the Drell-Yan Process, Phys. Rev. Lett. **119**, 112002 (2017).

[32] A. Bacchetta, F. Conti, and M. Radici, Transverse-momentum distributions in a diquark spectator model, Phys. Rev. D **78**, 074010 (2008).
[33] B. Pasquini, S. Cazzaniga, and S. Boffi, Transverse momentum dependent parton distributions in a light-cone quark model, Phys. Rev. D **78**, 034025 (2008).
[34] S. Boffi, A. V. Efremov, B. Pasquini, and P. Schweitzer, Azimuthal spin asymmetries in light-cone constituent quark models, Phys. Rev. D **79**, 094012 (2009).
[35] C. Lorcé and B. Pasquini, On the origin of model relations among transverse-momentum dependent parton distributions, Phys. Rev. D **84**, 034039 (2011).
[36] M. Burkardt and B. Pasquini, Modelling the nucleon structure, Eur. Phys. J. A **52**, 161 (2016).
[37] A. Bacchetta, Where do we stand with a 3-D picture of the proton? Eur. Phys. J. A **52**, 163 (2016).
[38] A. Bacchetta, F. Delcarro, C. Pisano, and M. Radici, The three-dimensional distribution of quarks in momentum space, arXiv:2004.14278.
[39] C. Lorcé, L. Mantovani, and B. Pasquini, Spatial distribution of angular momentum inside the nucleon, Phys. Lett. B **776**, 38 (2018).
[40] X.-d. Ji, J.-P. Ma, and F. Yuan, Three quark light cone amplitudes of the proton and quark orbital motion dependent observables, Nucl. Phys. B **652**, 383 (2003).
[41] X. Ji, X. Xiong, and F. Yuan, Proton Spin Structure from Measurable Parton Distributions, Phys. Rev. Lett. **109**, 152005 (2012).
[42] B. Bakamjian and L. H. Thomas, Relativistic particle dynamics. 2, Phys. Rev. **92**, 1300 (1953).
[43] E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta (unpublished).
[44] E. Pace, G. Salmè, and S. Scopetta, Poincaré covariant light-front spectral function and transverse momentum distributions, Springer Proc. Phys. **238**, 691 (2020).
[45] E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta, EMC effect, few-nucleon systems and Poincaré covariance, Phys. Scr. **95**, 064008 (2020).
[46] E. Pace, A. Del Dotto, L. Kaptari, M. Rinaldi, G. Salmé, and S. Scopetta, Light-front dynamics and the ^{3}He spectral function, Few Body Syst. **57**, 601 (2016).
[47] E. Pace, A. Del Dotto, L. Kaptari, M. Rinaldi, G. Salmè, and S. Scopetta, Neutron parton structure and the light-front spectral function of ^{3}He, Proc. Sci. **INPC2016**, 313 (2017).
[48] U. Oelfke, P. U. Sauer, and F. Coester, Convolution models of deep inelastic scattering: The three nucleon bound states as a test case, Nucl. Phys. A **518**, 593 (1990).
[49] S. Scopetta, A. Del Dotto, E. Pace, and G. Salmè, Transversity studies with a polarized ^{3}He target, Nuovo Cim. C **035N2**, 101 (2012).
[50] E. Pace, G. Salmè, S. Scopetta, and A. Del Dotto, Quark transverse momentum distributions inside a nucleon: A light-front Hamiltonian dynamics study, Acta Phys. Polon. Supp. **6**, 103 (2013).
[51] E. Pace, G. Salmè, S. Scopetta, A. Del Dotto, and M. Rinaldi, Neutron transverse-momentum distributions and polarized ^{3}He within light-front Hamiltonian dynamics, Few Body Syst. **54**, 1079 (2013).
[52] E. Pace, A. Del Dotto, L. Kaptari, M. Rinaldi, G. Salmè, and S. Scopetta, Transverse momentum distributions and nuclear effects, EPJ Web Conf. **85**, 02027 (2015).
[53] M. Rinaldi, A. Del Dotto, L. Kaptari, E. Pace, G. Salmè, and S. Scopetta, The ^{3}He spectral function in light-front dynamics, EPJ Web Conf. **113**, 05010 (2016).
[54] R. Abdul Khalek *et al.*, Science requirements and detector concepts for the electron-ion collider: EIC yellow report, arXiv:2103.05419.
[55] E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta (unpublished).
[56] H. J. Melosh, Quarks: Currents and constituents, Phys. Rev. D **9**, 1095 (1974).
[57] M. V. Terentev, On the structure of wave functions of mesons considered as bound states of relativistic quarks, Sov. J. Nucl. Phys. **24**, 106 (1976).
[58] C. Ciofi degli Atti, E. Pace, and G. Salmè, Investigation of the neutron form factors by inclusive quasielastic scattering of polarized electrons off polarized ^{3}He: A theoretical overview, Phys. Rev. C **51**, 1108 (1995).
[59] Kelly Yu-Ju Chiu and S. J. Brodsky, Angular momentum conservation law in light-front quantum field theory, Phys. Rev. D **95**, 065035 (2017).
[60] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Quantum chromodynamics and other field theories on the light cone, Phys. Rep. **301**, 299 (1998).
[61] F. M. Lev, E. Pace, and G. Salmè, Poincaré covariant current operator and elastic electron deuteron scattering in the front form Hamiltonian dynamics, Phys. Rev. C **62**, 064004 (2000).
[62] K. Goeke, A. Metz, and M. Schlegel, Parameterization of the quark-quark correlator of a spin-1/2 hadron, Phys. Lett. B **618**, 90 (2005).
[63] A. Kievsky, M. Viviani, and S. Rosati, Study of bound and scattering states of three nucleon systems, Nucl. Phys. A **577**, 511 (1994).
[64] A. Kievsky, M. Viviani, and S. Rosati, Cross-section, polarization observables, and phase shift parameters in proton deuteron and neutron deuteron elastic scattering, Phys. Rev. C **52**, R15 (1995).
[65] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, An accurate nucleon-nucleon potential with charge independence breaking, Phys. Rev. C **51**, 38 (1995).
[66] R. L. Jaffe and X.-D. Ji, Chiral Odd Parton Distributions and Polarized Drell-Yan, Phys. Rev. Lett. **67**, 552 (1991).
[67] R. L. Jaffe, Spin, twist and hadron structure in deep inelastic processes, arXiv:hep-ph/9602236.
[68] V. Barone, T. Calarco, and A. Drago, A confinement model calculation of $h_1(x)$, Phys. Lett. B **390**, 287 (1997).
[69] S. Scopetta and V. Vento, A quark model analysis of the transversity distribution, Phys. Lett. B **424**, 25 (1998).
[70] C. Ciofi degli Atti, S. Scopetta, E. Pace, and G. Salmè, Nuclear effects in deep inelastic scattering of polarized electrons off polarized ^{3}He and the neutron spin structure functions, Phys. Rev. C **48**, R968 (1993).
[71] K. Abe *et al.* (E154 Collaboration), Precision Determination of the Neutron Spin Structure Function g_1^n, Phys. Rev. Lett. **79**, 26 (1997).
[72] S. Scopetta, Neutron single spin asymmetries from semi-inclusive deep inelastic scattering off transversely polarized ^{3}He, Phys. Rev. D **75**, 054005 (2007).
[73] A. Del Dotto, L. P. Kaptari, E. Pace, G. Salmè, and S. Scopetta, Final state interactions and the extraction of neutron single spin asymmetries from semi-inclusive deep-inelastic scattering by a transversely polarized ^{3}He target, Phys. Rev. C **96**, 065203 (2017).

[74] X. Qian *et al.* (Jefferson Lab Hall A Collaboration), Single Spin Asymmetries in Charged Pion Production from Semi-Inclusive Deep Inelastic Scattering on a Transversely Polarized ^{3}He Target, Phys. Rev. Lett. **107**, 072003 (2011).
[75] F. Cardarelli, E. Pace, G. Salmè, and S. Simula, Nucleon and pion electromagnetic form factors in a light-front constituent quark model, Phys. Lett. B **357**, 267 (1995).
[76] F. Cardarelli, E. Pace, G. Salmè, and S. Simula, Electromagnetic $N - \Delta$ transition form factors in a light-front constituent quark model, Phys. Lett. B **371**, 7 (1996).
[77] D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonsky, *Quantum Theory of Angular Momentum* (World Scientific, Singapore, 1988).