R-matrix theory with level-dependent boundary condition parameters

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I present a new formalism of the *R*-matrix theory where the formal parameters for the resonance energies and widths are identical to the observed values. By allowing the boundary condition parameters to vary from level to level, the freedom required to adjust the formal parameters for the pole positions to the observed values is obtained. The basis of the resulting theory becomes nonorthogonal, and I describe the procedure to construct a consistent *R*-matrix theory with such a nonorthogonal basis. And by adjusting the normalization of the states that form the basis, the formal parameters for the reduced decay widths also become the same as those observed, leaving no formal parameters that are different from the observed ones. A demonstration of the developed theory to the elastic $^{12}\mathrm{C}+p$ scattering data is presented.

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I. INTRODUCTION

The R-matrix theory [1,2] introduced by Wigner and Eisenbud [1] and by Kapur and Peierls [2] is an extremely powerful and indispensable tool for describing nuclear reactions [3,4], in which collision matrices are described in terms of the positions and widths of the resonances. The values of the resonance parameters used in the *R*-matrix theory (which are referred to as the *formal* parameters) are, however, different from the *observed* values, and the conversion between the two sets of parameters is nonlinear.

Having two different sets of parameters complicates both theoretical and computational studies of the theory quite substantially, see Ref. [4] for a detailed discussion. A potential improvement is to derive an *R*-matrix theory where the formal parameters coincide with the observed values by a reformulation of the theory, such that unnecessary confusion and complications can be removed.

In this paper, I address this issue by presenting a new *R*-matrix theory where there is no distinction between the two sets of parameters. The key steps of the developed formulation can be summarized as follows. When constructing the conventional *R*-matrix theory, the basis for the Hilbert space defined in the *internal* region of the theory is required to be orthogonal. Since the domain of the basis is not the whole configuration space but limited only to the internal region, the seemingly innocent orthogonality condition imposes quite a strict constraint on the states that form the basis: The boundary condition parameters that define the basis states should be level-independent in order to satisfy the orthogonality condition (see the text for more detailed explanation). This constraint is in fact responsible for the discrepancies between the formal parameters and the observed ones. However, it is possible to construct a consistent *R*-matrix theory with a nonorthogonal basis. It turns out that releasing the orthogonality condition provides the freedom to have level-dependent boundary conditions with which the *formal* parameters can be equivalent to the *observed* values. To be more specific, if one adjusts the boundary condition parameters $B_{\lambda c}$ to the shift factor S_c at E_λ (the pole-position of level λ), the formal parameters for the pole positions coincide with the observed ones. Furthermore, there is additional freedom associated with the normalization of the basis states. By selecting the normalization given in Eq. (28), the formal parameters for reduced widths also become the same as observed. As a result, there are no formal parameters that are different from the observed values in the present theory, achieving the desired goal.

The new formalism shares many features in common with previous studies. For example, the very original work by Kapur and Peierls [2] has an energy-dependent boundary condition. Schemes with level-dependent boundary condition parameters were introduced by Barker [5] and by Azuma *et al.* [6]. Angulo and Descouvemont developed a formalism [7] where there are no level shifts, which is however applicable only to single-channel cases. In particular, this work is rather similar to the work of Brune [8], where an alternative parametrization was introduced to use the observed pole locations as inputs of the *R*-matrix theory, developing a transformation scheme between the formal and the observed parameters. In this work, the theory is formulated in such a way that all the formal parameters are directly equivalent to the observed values.

In Sec. II, I briefly review the *R*-matrix theory, explaining how the orthogonality condition can be released in a consistent manner to allow the boundary condition parameters to be level-dependent. In Sec. III, I then describe the procedure to align the formal parameters to the observed values. In Sec. VI, a demonstration of the developed theory is made for the $^{12}\mathrm{C}+p$ elastic scattering reaction. Section V is devoted to discussions.

II. FORMALISM FOR THE EXTENDED BOUNDARY CONDITION

I begin with a brief review of the R-matrix theory described in Lane and Thomas (LT) [3]. In the R-matrix theory, a nuclear system is described in terms of channels that consist of two subsystems, $\alpha_1 = \{Z_1, A_1, I_1\}$ and $\alpha_2 = \{Z_2, A_2, I_2\}$, where Z_i, A_i, and I_i are the proton number, mass number, and spin of the ith subsystem, respectively. The quantum state of a channel c might be denoted as $c = \{\alpha, (I_1, I_2)s, \ell; JJ_z\}$, where $\alpha = \alpha_1 \otimes \alpha_2$ is the partition index, s is the channel spin ($\vec{s} = \vec{I}_1 + \vec{I}_2$), ℓ is the relative orbital angular momentum, J is the total angular momentum ($\vec{J} = \vec{s} + \vec{\ell}$), and J_z is its projection [4]. The Hamiltonian of the system in the center-of-mass frame then reads

$$H = -\frac{\hbar^2}{2M_c}\nabla^2_{\vec{r}_c} + V_c(\vec{r}_c) + H_{\alpha_1} + H_{\alpha_2}, \tag{1}$$

where $\vec{r}_c$ and M_c are the relative position vector and the reduced mass of channel c, respectively, and H_{α_i} is the Hamiltonian for the internal energy of the ith subsystem. The wave function of the system that satisfies the Schrödinger equation at energy E, $H\Psi = E\Psi$ can be written as

$$\Psi = \sum_c \varphi_c\, u_c(r_c), \tag{2}$$

where $u_c(r_c)$ is the radial function of channel c. The spinors and the angular dependence of the subsystems are embodied in $\varphi_c = \frac{1}{r_c}[[\varphi_{\alpha_1} \otimes \varphi_{\alpha_2}] \otimes i^\ell Y_\ell(\hat{r}_c)]_{JJ_z}$ [4]. The radial space of each channel is divided into two parts: the internal region ($r_c \leqslant a_c$) and the external region ($r_c > a_c$), where a_c is the *channel radius* that defines the surface between the two regions. φ_c is assumed to be orthonormal when integrated on the channel surface,

$$\int \varphi_c^* \varphi_{c'}\, d\mathcal{S} = \delta_{cc'}, \tag{3}$$

where $\mathcal{S}$ is the surface defined by $r_c = a_c$. Here and hereafter, unless stated otherwise, I follow the notation given in LT.

The wave function in the internal region is expanded as a linear combination of the basis states X_λ,

$$\Psi = \sum_\lambda C_\lambda X_\lambda, \tag{4}$$

where C_λ are E-dependent coefficients. The basis states

$$X_\lambda = \sum_c \varphi_c\, u_{\lambda c}(r_c) \tag{5}$$

are defined by the eigenvalue equation

$$HX_\lambda = E_\lambda X_\lambda \tag{6}$$

with a boundary condition imposed at the channel surface, which will be discussed soon.

Since the basis states are relevant only in the internal region, I define their inner products as[1]

$$J_{\lambda\lambda'} \equiv \int_\tau X_\lambda^* X_{\lambda'}\, d\tau, \tag{7}$$

where $\int_\tau d\tau$ denotes the volume integral limited only to the internal region. The orthogonality condition of the basis then corresponds to have $J_{\lambda\lambda'} = 0$ for $\lambda \neq \lambda'$. $J_{\lambda\lambda'}$ can be evaluated by the following steps: If one multiplies Eq. (6) by $X^*_{\lambda'}$ from the left and integrate it in the internal region, and then subtract it with interchanging λ' and λ, one obtains

$$\begin{aligned}(E_\lambda - E_{\lambda'}) &\int_\tau X_\lambda^* X_{\lambda'}\, d\tau \\ &= \int_\tau [(HX_\lambda)^* X_{\lambda'} - X_\lambda^*(HX_{\lambda'})]\, d\tau \\ &= -\sum_c \frac{\hbar^2}{2M_c}\left(u_{\lambda' c}\frac{du_{\lambda c}}{dr} - u_{\lambda c}\frac{du_{\lambda' c}}{dr}\right)_{r=a_c},\end{aligned} \tag{8}$$

where I have inserted the Hamilton given in Eq. (1) at the last step. Here and hereafter, I limit myself to the cases where the nuclear potential is hermitian and the radial functions are real, $u_{\lambda c}(r)^* = u_{\lambda c}(r)$. Diving the above equation by $E_\lambda - E_{\lambda'}$, one is then led to[2]

$$J_{\lambda\lambda'} = -\frac{1}{E_\lambda - E_{\lambda'}}\sum_c \gamma_{\lambda c}(B_{\lambda c} - B_{\lambda' c})\gamma_{\lambda' c}, \text{ for } \lambda \neq \lambda', \tag{9}$$

where

$$\gamma_{\lambda c} \equiv \sqrt{\frac{\hbar^2}{2M_c a_c}}\, u_\lambda(a_c), \tag{10}$$

$$B_{\lambda c} \equiv \frac{a_c}{u_{\lambda c}(a_c)}\frac{du_{\lambda c}(r)}{dr}\bigg|_{r=a_c}. \tag{11}$$

From Eq. (9), it is clear that the orthogonality condition for a general multilevel and multichannel case can be guaranteed only when the boundary condition is level-independent, $B_{\lambda c} = B_c$, as is demanded in the conventional R-matrix theory. However, the orthogonality is not a necessary condition for the basis of a consistent R-matrix theory. If one does not adhere to it, as I explain below, one is granted additional freedom to have level-dependent boundary condition parameters that can be used to remove the gap between the formal parameter set and the observed parameter set.

I now describe how a new R-matrix theory can be built with a nonorthogonal basis. From Eqs. (9) and (4), the coefficients

[1] By making use of Eqs. (5) and (3), $J_{\lambda\lambda'}$ can also be represented as

$$J_{\lambda\lambda'} = \sum_c \int_0^{a_c} dr_c\, u^*_{\lambda c}(r_c) u_{\lambda' c}(r_c),$$

[2] Here, I assume that for a given spin and parity, X_λ is nondegenerate, and thus $\lambda \neq \lambda'$ implies $E_\lambda \neq E_{\lambda'}$. The degenerate levels with the same level-energy, if any, can be merged into a single level, as discussed in Ref. [8].

C_λ read

$$C_{\lambda'} = \sum_\lambda \left(J^{-1}\right)_{\lambda'\lambda} \int_\tau X_\lambda \Psi \, d\tau, \tag{12}$$

and the integral in the above equation can be evaluated by inserting Ψ in place of $X_{\lambda'}$ in Eq. (8),

$$\int_\tau X_\lambda \Psi \, d\tau = \frac{1}{E_\lambda - E} \sum_c \gamma_{\lambda c} \sqrt{\frac{\hbar^2}{2M_c a_c}} \left(a_c \frac{du_c(r)}{dr} - B_{\lambda c} u_c(a_c) \right)_{r=a_c}. \tag{13}$$

Insertion of the resulting coefficients into Eq. (4) gives the following equation:

$$u_{c'}(a_{c'}) = \sum_{\lambda'} C_{\lambda'} u_{\lambda' c'}(a_{c'}) = \sum_c \sqrt{\frac{M_{c'} a_{c'}}{M_c a_c}} \left(\mathcal{R}_{c'c} \, a_c \frac{du_c(r)}{dr} - \mathcal{R}^B_{c'c} u_c(a_c) \right)_{r=a_c} \tag{14}$$

with

$$\mathcal{R}_{c'c} \equiv \sum_{\lambda',\lambda} \gamma_{\lambda' c'} \left(J^{-1}\right)_{\lambda'\lambda} \frac{1}{E_\lambda - E} \gamma_{\lambda c}, \tag{15}$$

$$\mathcal{R}^B_{c'c} \equiv \sum_{\lambda',\lambda} \gamma_{\lambda' c'} \left(J^{-1}\right)_{\lambda'\lambda} \frac{1}{E_\lambda - E} \gamma_{\lambda c} B_{\lambda c}. \tag{16}$$

On the other hand, the radial wave functions in the external region can be written analytically, because the channel radius a_c is assumed to be large enough so that all the nuclear forces between the two subsystems vanish and only the Coulomb interaction remains,

$$u_{c'}(r) = \frac{1}{\sqrt{v_{c'}}} \sum_c [I_{c'}(r)\delta_{c'c} - O_{c'}(r) U_{c'c}] y_c, \quad r_c \geqslant a_c, \tag{17}$$

where y_c are coefficients, $I_c(r)$ and $O_c(r)$ are the incoming and outgoing radial wave functions, respectively, U is the collision matrix, and $v_c = \sqrt{2|E_c|/M_c}$ are the relative velocities. The collision matrix can be obtained by requiring that the logarithmic derivatives of the radial functions on the channel surface resulting from Eq. (17) should be equal to the derivatives derived from Eq. (14),

$$U_{c'c} = \Omega_{c'} (\delta_{c'c} + 2i\sqrt{P_{c'}} ([1 - \mathcal{R}(S_c + iP_c) + \mathcal{R}^B]^{-1} R)_{c'c} \sqrt{P_c}) \Omega_c, \tag{18}$$

where $\Omega_c = \sqrt{I_c/O_c}$, and the shift ($S_c$) and penetration ($P_c$) factors are the real and imaginary parts of the logarithmic derivative of the outgoing wave function on the channel surface, respectively,

$$r(\partial O_c/\partial r)/O_c|_{r=a_c} = S_c + iP_c. \tag{19}$$

In this context, it is convenient to represent the collision matrix in terms of the so-called A matrix, which is defined by

$$([1 - \mathcal{R}(S + iP) + \mathcal{R}^B]^{-1} R)_{c'c} = \sum_{\lambda',\lambda} \gamma_{\lambda' c'} A_{\lambda'\lambda} \gamma_{\lambda c} \tag{20}$$

or, equivalently,

$$(A^{-1})_{\lambda\lambda'} = \bar{\mathcal{E}}(E)_{\lambda\lambda'} - i \sum_c \gamma_{\lambda c} \gamma_{\lambda' c} P_c(E), \tag{21}$$

where $\bar{\mathcal{E}}(E)$ is the real part of $A(E)^{-1}$,

$$\bar{\mathcal{E}}(E)_{\lambda\lambda'} = (E_\lambda - E) J_{\lambda\lambda'} - \sum_c \gamma_{\lambda c} [S_c(E) - B_{\lambda c}] \gamma_{\lambda' c}. \tag{22}$$

The collision matrix with this A matrix reads

$$U_{c'c} = \Omega_{c'} \left(\delta_{c'c} + 2i\sqrt{P_{c'}} \sum_{\lambda',\lambda} \gamma_{\lambda' c'} A_{\lambda'\lambda} \gamma_{\lambda c} \sqrt{P_c} \right) \Omega_c. \tag{23}$$

Using the above Eqs. (21), (22), (23), one can thus construct the collision matrix with the general inner products of the basis states given in Eq. (9), and the basis no longer needs to be orthogonal. The values of the level-dependent boundary condition parameters $B_{\lambda c}$ and the diagonal elements $J_{\lambda\lambda}$ should then be determined, which will be discussed in the next section.

III. DETERMINATION OF THE BOUNDARY CONDITION PARAMETERS

So far, I have shown that releasing the orthogonality condition of the basis states allows the boundary condition parameters $B_{\lambda c}$ to depend on the level. This section describes how to utilize this additional freedom associated with the level-dependence to make the *formal* parameters coincide with the observed ones.

Consider first the *observed* pole-positions of resonances, $E^{\rm obs}_\lambda$. The precise definition of the pole-position may be ambiguous, and I adopt the convention of Ref. [8], where $E^{\rm obs}_\lambda$ are defined as the zeros of the determinant of the real part of the inverse of the A matrix, or, equivalently, the solutions of the secular equation

$$\det \bar{\mathcal{E}}(E) = 0. \tag{24}$$

The aim of equalizing the observed pole positions with the formal parameters

$$E^{\rm obs}_\lambda = E_\lambda \tag{25}$$

can be achieved if one sets the boundary condition parameters $B_{\lambda c}$ to be the shift factor at $E = E_\lambda$,

$$B_{\lambda c} = S_c(E_\lambda). \tag{26}$$

This can be seen by simply noting that $\bar{\mathcal{E}}(E_\lambda)_{\lambda\lambda'}$ vanishes if one inserts Eq. (26) into Eq. (22). That is, for any λ, the entire λth row of the matrix $\bar{\mathcal{E}}(E_\lambda)$ vanishes, which in turn makes E_λ the solution of Eq. (24). This proves that the formal parameter E_λ is equal to the *observed* $E^{\rm obs}_\lambda$.

From Eqs. (21), (22), (23), it is not difficult to see that the collision matrix U is invariant under the following

transformation:

$$J_{\lambda\lambda} \to J_{\lambda\lambda}^{\rm new},$$
$$\gamma_{\lambda c} \to \gamma_{\lambda c}^{\rm new} = \sqrt{\frac{J_{\lambda\lambda}^{\rm new}}{J_{\lambda\lambda}}}\,\gamma_{\lambda c}. \tag{27}$$

The normalization of $\gamma_{\lambda c}$ is thus determined by the values of $J_{\lambda\lambda}$, which are not yet determined. The off-diagonal elements of J are given in Eq. (9). A natural extrapolation to the diagonal cases would be to take the limit $E_{\lambda'} \to E_\lambda$ of the equation, which results in, with Eq. (26),

$$J_{\lambda\lambda} = 1 - \sum_c \gamma_{\lambda c}^2 \left.\frac{dS_c(E)}{dE}\right|_{E=E_\lambda}. \tag{28}$$

Insertion of the above equation into Eq. (22) yields

$$\bar{\mathcal{E}}(E)_{\lambda\lambda'} = \begin{cases} E_\lambda - E - \sum_c \gamma_{\lambda c}^2 [S_c(E) - S_{\lambda c} + (E_\lambda - E) S'_{\lambda c}], & \text{for } \lambda' = \lambda, \\ -\sum_c \gamma_{\lambda c}\gamma_{\lambda' c}\left[S_c(E) + \frac{(E_{\lambda'}-E)S_{\lambda c} - (E_\lambda - E)S_{\lambda' c})}{E_\lambda - E_{\lambda'}}\right], & \text{for } \lambda' \neq \lambda, \end{cases} \tag{29}$$

where $S_{\lambda c} \equiv S_c(E_\lambda)$.

I now consider the consequence of Eq. (28) on the *observed* widths of the resonances, which are usually defined with the "one-level approximation". With this approximation, the collision matrix reads (LT)

$$U_{c'c} \simeq \Omega_{c'}\left(\delta_{c'c} + i\frac{\sqrt{\Gamma_{\lambda c'}}\sqrt{\Gamma_{\lambda c}}}{E_\lambda - E - \frac{i}{2}\sum_{c''}\Gamma_{\lambda c''}}\right)\Omega_c. \tag{30}$$

And the *observed* reduced widths $\gamma_{\lambda c}^{\rm obs}$ are defined by [4]

$$\Gamma_{\lambda c}(E_\lambda) = 2P_c(E_\lambda)\gamma_{\lambda c}^{\rm obs\,2}. \tag{31}$$

On the other hand, if one inserts Eq. (29) into Eq. (23) and then takes the one-level approximation, the widths are given as

$$\Gamma_{\lambda c}(E) = 2P_c(E)\gamma_{\lambda c}^2. \tag{32}$$

Thus, the formal parameters for the reduced widths of the present formalism are the same as the observed ones,

$$\gamma_{\lambda c}^{\rm obs} = \gamma_{\lambda c}. \tag{33}$$

This simple relation should be compared with

$$\gamma_{\lambda c}^{\rm obs} = \check{\gamma}_{\lambda c}/\sqrt{1 + \sum_{c'}\check{\gamma}_{\lambda c'}^2 S'_{\lambda c'}}, \tag{34}$$

where $\check{\gamma}_{\lambda c}$ are the formal reduced width parameters in the conventional R-matrix theory.

I note that the alternate parametrization obtained by Brune [8] has the same off-diagonal terms of A^{-1} with Eq. (29). But the diagonal elements are different, and the values corresponding to $J_{\lambda\lambda}$ used in Ref. [8] are 1 instead of those from Eq. (28). Consequently, Brune's reduced width parameters are the same as those of the conventional R-matrix theory, $\check{\gamma}_{\lambda c}$, which are subject to the nonlinear relation described in Eq. (34).

IV. ^{12}C $+p$ ELASTIC SCATTERING

To demonstrate the performance of the new R-matrix theory developed here, I developed a simple Mathematica code [9] which calculates the collision matrices and the differential cross-sections of nuclear reactions based on Eqs. (26), (28), (29). Since there is no need for conversions between the formal and the observed parameters, a substantial simplification could be achieved. The code is then applied to describe ^{12}C $+p$ elastic scattering, for which accurate experimental data are available [10]. For a detailed discussion of the process in connection with the (conventional) R-matrix theory, see Ref. [4].

At low energy, this process is dominated by the three low-lying resonances of ^{13}N: $J^\pi = 1/2^+$ at 0.421 MeV, $3/2^-$ at 1.559 MeV, and $5/2^+$ at 1.604 MeV, where the resonance energies are the center-of-mass energies of the ^{12}C $+p$ system. I use the values of the *observed* parameters given in Ref. [4] for my formal parameters, which are listed in Table I. There is little dependence on the channel radius, which was chosen as $a_c = 5$ fm for calculations.

For two particular center-of-mass angles, $\theta = 89.1^\circ$ and 146.9°, the resulting differential cross sections with respect to the proton energy in the laboratory frame are drawn in Fig. 1. The figure shows that the code with the newly developed R-matrix theory reproduces nicely the experimental data [10].

In Table II, the parameters of the conventional R-matrix theory are compared with those of the present theory for the first $J^\pi = 1/2^+$ resonance in ^{12}C $+p$, varying the channel radius from 4 fm to 7 fm. The observed position and the width of the resonance are set to $E_R = 0.42$ MeV and $\Gamma_R = 32$ keV [4]. While the E_1 of the present theory is the same as the input value for the observed pole position E_R by construction, the formal parameter $E_{1,\rm formal}$ in the conventional theory has a strong dependence on the channel radius. The table also shows that γ_1^2 of the present theory agrees well with $\gamma_{1,\rm observed}^2$ in Ref. [4], which is the intended outcome of this work. The table also shows that these observed parameters are less dependent on a_c than the formal parameters.

TABLE I. R-matrix parameters for ^{12}C $+p$ elastic scattering with $a_c = 5$ fm. E_λ are the center-of-mass energies of the ^{12}C $+p$ system. E_λ and $\Gamma_{\lambda c}$ are from Ref. [4], and $\gamma_{\lambda c}$ are the corresponding reduced width parameters.

J^π	E_λ [MeV]	$\Gamma_{\lambda c}$ [keV]	$\gamma_{\lambda c}^2$ [MeV]
$1/2^+$	0.427	32.5	0.569
$3/2^-$	1.559	51.4	0.0835
$5/2^+$	1.604	48.1	0.414

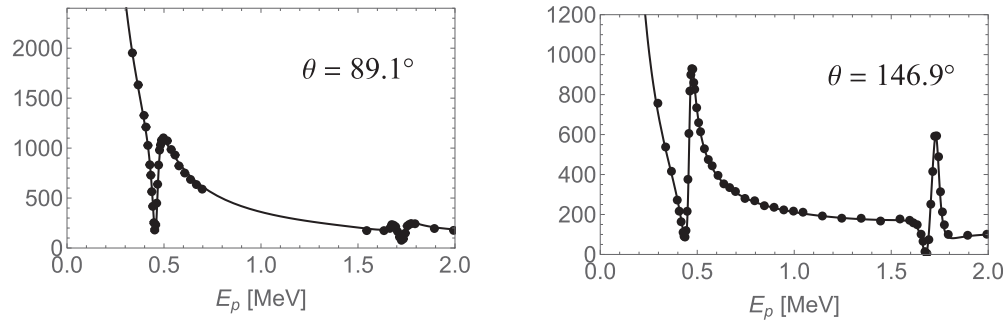

FIG. 1. ^{12}C $+p$ elastic scattering cross-sections (in mb/sr) with respect to the incident proton energy in the laboratory frame for the center-of-mass angle $\theta = 89.1^\circ$ (left) and 146.9° (right). The experimental data are from Ref. [10] (closed circles).

V. DISCUSSIONS

The conventional *R*-matrix theory has the problem of having *formal* parameters that are different from the observed ones, and thus requiring nontrivial conversions between the two sets of parameters. As discussed in the text, this drawback is a consequence of requiring orthogonality of the basis states for the Hilbert space in the internal region of the *R*-matrix theory. However, the orthogonality is not a necessary condition for the basis. By exploiting the additional freedom that can be achieved when the orthogonality condition is released, I have developed a new *R*-matrix theory that has no distinction between the two sets of parameters.

In this theory, the boundary condition parameters are allowed to be level-dependent and adjusted to make the *formal* parameters E_λ identical to the *observed* pole positions. That is, I assigned $B_{\lambda c}$ to the shift factor of channel c at E_λ, see Eq. (26), which makes the secular equation Eq. (24) vanish at that energy. Recalling that the observed pole-positions $E_\lambda^{\rm obs}$ are defined to be the zeros of the secular equation, one sees that the imposed boundary condition leads to $E_\lambda = E_\lambda^{\rm obs}$.

In addition, there is another freedom in the normalization of the diagonal elements $J_{\lambda\lambda}$, which correspond to the square of the norm of the basis states. By selecting the normalization factor of the basis states as given in Eq. (28), which can be viewed as a natural extrapolation of the off-diagonal elements, I could derive the *formal* reduced width parameters to be the same as the *observed* ones as well, $\gamma_{\lambda c}^{\rm obs} = \gamma_{\lambda c}$. As a result, there are no *formal* parameters which are different from the *observed* ones in the present formalism.

TABLE II. *R*-matrix parameters for the first $1/2^+$ resonance ($E_R = 0.42$ MeV and $\Gamma_R = 32$ keV) in ^{12}C $+p$ elastic scattering (in MeV). The parameters of the conventional *R*-matrix theory are from Table 10 of Ref. [4].

a_c [fm]	4	5	6	7
$\gamma^2_{1,\rm observed}$ (Ref. [4])	1.089	0.592	0.353	0.227
$\gamma^2_{1,\rm formal}$ (Ref. [4])	3.083	1.157	0.569	0.323
γ_1^2 (this work)	1.087	0.591	0.353	0.226
$E_{1,\rm formal}$(Ref. [4])	−2.152	−0.614	−0.110	0.113
E_1 (this work)	0.42	0.42	0.42	0.42

As a demonstration, I tested a computation code based on the developed *R*-matrix theory, where the trial case was the elastic scattering of protons on ^{12}C. The code required only the resonance data as input and did not invoke any transformations of parameters. The code was able to reproduce the experimental differential cross sections quite well.

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