Comment on "Reexamining the relation between the binding energy of finite nuclei and the equation of state of infinite nuclear matter"

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The article [1] presents a new method to estimate the binding energy of nuclear matter based on Green's function theory and the Galitskii-Migdal-Koltun sum rule [2,3]. The abstract asserts that there is a significant difference between the extracted nuclear matter binding energy B ("13–14 MeV") and the accepted value ("16 MeV"). This may be seen in their Fig. 6. We wish to point out that the discrepancy between theories is much smaller. Also, one of the necessary approximations may cause a significant error in the estimate. The article [1] focuses on the energy density in the interior of the ^{208}Pb nucleus. As the authors note, to extract B, that energy density has to be corrected for the isospin asymmetry of the nucleus. Using the liquid drop asymmetry term a_A as the authors do gives a symmetry correction of 1.04 MeV, yielding a corrected energy per particle of 14.6 MeV, which lies within the band at the center of the nucleus as displayed in the article's Fig. 5. (The dashed line in Fig. 5 is obtained by adding the liquid drop asymmetry term to the canonical 16-MeV binding energy, rather than the corresponding liquid drop volume energy).

Furthermore, the liquid drop model should not be directly extrapolated to the infinite system for asymmetric matter because the liquid drop asymmetry term implicitly includes both bulk and surface asymmetry effects [4,5]. The binding energy of nuclear matter at saturation density is expressed as

$$B(\delta) = B(0) - S_v\delta^2 + O(\delta^4),$$

where $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the asymmetry and S_v is the symmetry energy. To take a typical set of values, Ref. [6] reports $B(0) = 15.8 \pm 0.3$ and $S_v = 32 \pm 2$ MeV. For the central region of ^{208}Pb with an asymmetry $\delta = (N - Z)/(N + Z)$ the extracted value is, thus, $B = 14.4 \pm 0.4$ MeV, again overlapping well the uncertainty band in Fig. 5.

There is a another point regarding the importance of $3N$ forces. As the authors note, the Galitskii-Migdal-Koltun sum rule in their Eq. (3) assumes a two-body interaction. To justify neglecting three-body interactions, they argue that the three-body potential-energy density $\mathcal{U}$ is much smaller than the two-body potential-energy density $\mathcal{V}$. Consequently, they assert that $\mathcal{U}$ cannot meaningfully change the shape of the energy density. However, the substantial cancellation between kinetic and potential energies, evident in their Fig. 3, means that the three-body potential contributes non-negligibly to the total energy. The contribution is on the order of 1 MeV$/A$ for ^{12}C in Fig. 4 and presumably greater for heavier nuclei, so this is comparable with the size of the discrepancy in question.

Finally, nuclear matter calculations starting from realistic two-body interactions with three-body terms report sizable contributions from the latter [7–13]. Reference [7] estimates that the three-contribution is 2.74 MeV, based on the A18 two-body interaction and the UIX three-body interaction. The Hamiltonian in Ref. [9] is based on effective field theory with chiral interactions; the nuclear matter binding results in their Fig. 2 show a three-body contribution in the range 6.5–7.9 MeV at nuclear matter density ($k_F \approx 1.3$ fm^{-1}).

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