Robustness of "noncollective" rotational behavior for nuclei in the presence of random interactions

J. J. Shen,[1] H. Jiang,[1,*] and G. J. Fu[2,†]
[1]School of Arts and Sciences, Shanghai Maritime University, Shanghai 201306, China
[2]School of Physics Science and Engineering, Tongji University, Shanghai 200092, China

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We investigate the robustness of "noncollective" rotational behavior for nuclei with random two-body interactions in the framework of nuclear shell model. The normal $I(I+1)$ behavior of average excitation energy of yrast states is universal for both even-nucleon and odd-nucleon systems by two-body random ensemble. In some special random samplings, especially the samplings with spin-$I_{\max}$ ground state, the inverse noncollective rotational behavior is found for both even-nucleon and odd-nucleon systems in this paper. Interestingly, above rotational behaviors are still robust in the presence of random two-body interactions plus realistic single-particle energies.

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I. INTRODUCTION

In 1997, the dominance of the spin-zero ground state for even-even nuclei in the presence of random interactions was found [1]. This discovery provides us with a new clue to study the low-lying states of quantum many-body systems in the presence of random interactions (see the reviews in Refs. [2–5]).

The two-body random ensemble (TBRE) model, i.e., the embedded Gaussian orthogonal ensemble (EGOE) [6] of random matrices with two-body interactions [7–9], is one of the important approaches to study quantum many-body systems. Many efforts have been made to understand the dominance of the 0^+ ground state and ordered yrast spectra via the TBRE [10–21]. It has been found that the dominance of the spin-zero ground state is attributed to some two-body matrix elements (TBME) [10,11]. The dominance of the spin-zero ground state is also explained by the geometric chaoticity of the spin coupling of individual particles [15,16] and group symmetries of the TBRE [13,14]. The phenomenon of positive-parity ground states being dominant have been observed and explained by the different dimensions of Hilbert spaces of either parity [17,18]. For some simple systems, the probability of spin-I to be the ground state is related to the geometry of the eigenvalues [12]. It is also found that the energy of yrast states is related to the energy centroid and spectral widths [19–21].

In recent years, the structure of yrast states via the TBRE Hamiltonian has been studied based on random samplings with spin-zero ground state. For *sd*-boson systems, both vibrational and rotational band structures [22,23] are found in the framework of the interacting boson model (IBM) [24]. However, for fermion systems, there is no obvious collective behavior of vibration and rotation among the yrast states in the full space of the nuclear shell model [25]. Even in the truncated space of the nucleon pair approximation of the shell model, only in the Hamiltonian containing the strong quadrupole-quadrupole components will there be a rotational collectivity [26]. Another interesting phenomenon has been found in the full shell-model space; that is, for even-even nuclei, the energy of the yrast state by averaging over the spin-zero ground state subset of the TBRE is approximately proportional to $I(I+1)$ [1]. This $I(I+1)$ behavior is called the "noncollective" rotation behavior in nuclear spectroscopy.

In the random samplings of TBRE, about 40%–70% of the samples have a spin-zero ground state, and there are still many samples with a spin-nonzero ground state. Recently, it was observed that the yrast states are highly correlated in random samplings with a spin-nonzero ground state for *sd* bosons in the IBM model [27]. On the other hand, the probability that the state with the maximum spin $I_{\max}$ is the ground state is considerably large for identical-nucleon systems in a single-j shell model [10,15]. It is therefore interesting to study whether the noncollective rotational behavior is still right in random samples where the spin-nonzero (or spin-$I_{\max}$) state is the ground state. We will prove in this paper that this noncollective rotational behavior is robust by using the TBRE in the full shell-model space, regardless of whether the ground-state spin is zero or not. In addition, we show that the average energy of yrast state decreases with $I(I+1)$ for some special random samplings of the TBRE, which is called the inverse noncollective rotational behavior in this paper.

This paper is organized as follows: In Sec. II we introduce briefly the theoretical framework of the TBRE in the shell model, including the cases of the single-j shell and the many-j shell. In Sec. III, we calculate energy levels of both even-nucleon and odd-nucleon systems. We classify the random samplings of the TBRE (without and with realistic single-particle energies) into four different cases, and discuss the noncollective rotational behavior and its inverse behavior for the yrast states. In Sec. IV we summarize this paper.

*Corresponding author: huijiang@shmtu.edu.cn
†Corresponding author: gjfu@tongji.edu.cn

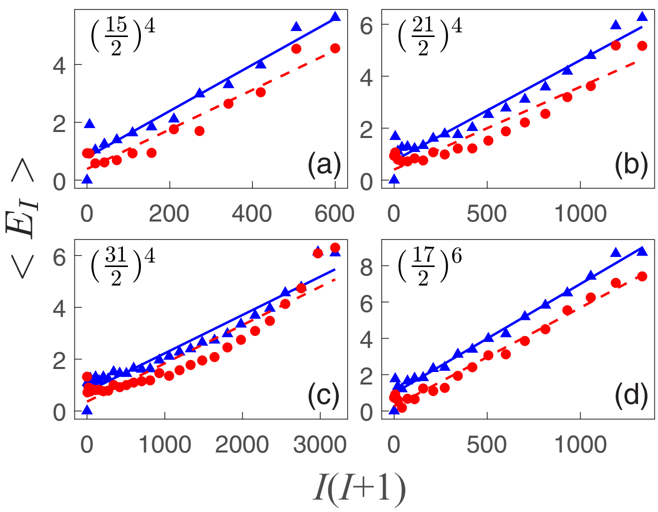

FIG. 1. Average excitation energy of yrdast states (the lowest state for a given I) $\langle E_I \rangle$ (in MeV) as a function of $I(I+1)$ for (a) four fermions in $j = 15/2$ shell, (b) four fermions in $j = 21/2$ shell, (c) four fermions in $j = 31/2$ shell and (d) six fermions in $j = 17/2$ shell. Results with $E_{\rm g.s.} = E_0 < E_{I_{\max}}$ (Case 1) and $E_{\rm g.s.} < E_0 < E_{I_{\max}}$ (Case 2) are shown in blue triangles and red circles, respectively. The lines are plotted by using linear fitting for different cases.

II. TWO-BODY RANDOM ENSEMBLE

In this paper, we carry out the shell-model calculations using the shell-model code developed by the Kyushu group [28–30] under the TBRE. For a single-j shell, the Hamiltonian with angular moment I can be written as follows:

$$\widehat{H}_I = \sum_{J=0,\rm even}^{2j-1} \sqrt{2J+1} G_J [A^{J\dagger} \times \widetilde{A}^J]^0, \quad (1)$$

where $A^{J\dagger} = \frac{1}{\sqrt{2}}[a_j^\dagger a_j^\dagger]^J$ and $\widetilde{A}^J = -\frac{1}{\sqrt{2}}[\widetilde{a}_j \widetilde{a}_j]^J$. The parameters G_J are random values which follow the standard normal distribution, i.e.,

$$\rho(G_J) = \frac{1}{\sqrt{2\pi}} \exp\left(-G_J^2/2\right), \quad J = 0, 2, \ldots, 2j-1. \quad (2)$$

The shell-model Hamiltonian with realistic interactions for a multi-j shell consists of a one-body term and a two-body-term, which are derived from the average shell-model potential and the residual two-body interaction, respectively. Its expression is

$$H = \sum_{jmm_t} \varepsilon_{jm_t} a^\dagger_{jm,m_t} a_{jm,m_t} + \frac{1}{4} \sum_{j_1 j_2 j_3 j_4, JT} G_{JT}(j_1 j_2; j_3 j_4) A^\dagger(j_1 j_2)^{JT}_{M_J M_T} A(j_3 j_4)^{JT}_{M_J M_T}, \quad (3)$$

TABLE I. The probabilities P (in %) of four cases in the random sampling of TBRE for different many-nucleon systems. The subscripts 1, 2, 3, and 4 of the symbol P correspond to even-nucleon systems, and the subscripts in parentheses ($1'$, $2'$, $3'$, and $4'$) correspond to odd-nucleon systems.

Single j^n	P_1	P_2	P_3	P_4
$(15/2)^4$	48.51	33.13	11.99	6.37
$(21/2)^4$	44.93	39.80	8.52	6.75
$(31/2)^4$	27.47	58.35	6.36	7.83
$(17/2)^6$	62.67	22.77	11.96	2.59
Even-even nuclei	P_1	P_2	P_3	P_4
^{22}Ne	34.84	62.03	0.77	2.36
^{26}Si	51.45	46.18	0.74	1.63
^{44}Ti	52.79	46.17	0.54	0.51
^{46}Cr	37.28	62.17	0.11	0.44
Odd-mass nuclei	$P_{1'}$	$P_{2'}$	$P_{3'}$	$P_{4'}$
^{23}Mg	20.01	78.32	0.24	1.44
^{23}Si	27.11	72.69	0.00	0.20
^{45}Sc	21.00	78.59	0.04	0.37
^{45}Ti	18.18	81.48	0.03	0.31

with

$$A^\dagger(j_1 j_2)^{JT}_{M_J M_T} = \sum_{m_1 m_2 m_{t_1} m_{t_2}} (j_1 m_1, j_2 m_2 | J M_J) \times \left(\frac{1}{2} m_{t_1}, \frac{1}{2} m_{t_2} | T M_T\right) a^\dagger_{j_1 m_1, m_{t_1}} a^\dagger_{j_2 m_2, m_{t_2}},$$
$$A(j_3 j_4)^{JT}_{M_J M_T} = \left(A^\dagger(j_1 j_2)^{JT}_{M_J M_T}\right)^\dagger. \quad (4)$$

Here, $(j_1 m_1, j_2 m_2 | J M_J)$ is the Clebsch-Gordan coefficient. The operators J and T denote the total spin and isospin for one particle in the j_1 orbit and another one in the j_2 orbit. The coefficient $G_{JT}(j_1 j_2; j_3 j_4)$ refers to the two-body matrix element (TBME).

In the TBRE, the single-particle energies ε_{jm_t} are set to be zero, and the two-body matrix elements $G_{JT}(j_1 j_2; j_3 j_4)$ belong to Gaussian orthogonal ensemble (GOE) and satisfy the following condition:

$$\rho(G_{JT}(j_1 j_2; j_3 j_4)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{G_{JT}(j_1 j_2; j_3 j_4)^2}{2\sigma^2}\right), \quad (5)$$

where σ is equal to 1 for diagonal two-body matrix elements and $\sqrt{1/2}$ for off-diagonal ones, i.e.,

$$\sigma^2 = \tfrac{1}{2}(1 + \delta_{j_1 j_2; j_3 j_4}). \quad (6)$$

III. RESULTS AND DISCUSSIONS

In this section, based on the TBRE in the full shell-model space, we study the yrast band structure of many-nucleon systems. The many-nucleon systems studied in this paper include the even-nucleon systems of single-j shell (that is, four fermions in the $j = 15/2$, $j = 21/2$, and $j = 31/2$ shell, and six fermions in the $j = 17/2$ shell), the even-even nuclei (^{22}Ne, ^{26}Si, ^{44}Ti, and ^{46}Cr) and the odd-mass nuclei (^{23}Mg,

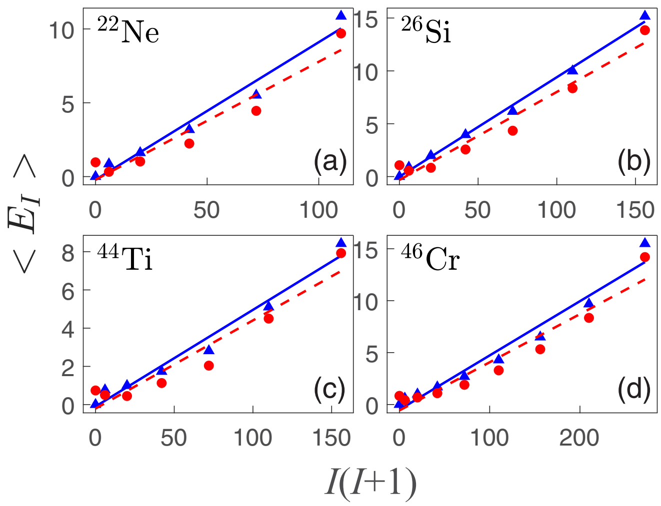

FIG. 2. Same as Fig. 1 except for even-even nuclei (a)^{22}Ne, (b)^{26}Si, (c)^{44}Ti, and (d)^{46}Cr.

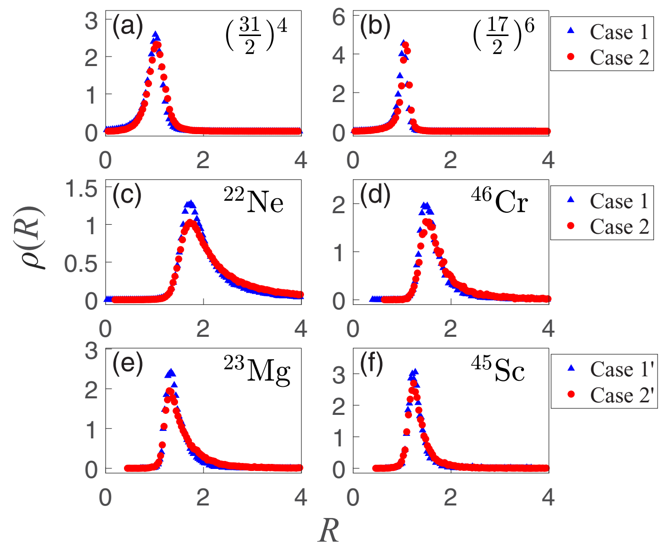

FIG. 4. The probability density distribution of R for (a) four fermions in $j = 31/2$ shell, (b) six fermions in $j = 17/2$ shell, (c)^{22}Ne, (d)^{46}Cr, (e)^{23}Mg, and (f)^{45}Sc in Case 1 (1′) and Case 2 (2′), respectively.

^{23}Si, ^{45}Sc, and ^{45}Ti) in the *sd* and *pf* shell. We perform more than 10 000 sets of a TBRE Hamiltonian for each many-nucleon system.

The energy of yrast states monotonically increases with spin for the noncollective rotational behavior on average, which means normal ordering of spin in the yrast band [31]. Based on this point, we classify the random samplings of the TBRE of even-nucleon system (or odd-nucleon system) into four different cases: in Case 1 (1′), the ground-state energy $E_{\rm g.s.}$, the lowest energy of the spin-zero (or spin-$\frac{1}{2}$) state E_0 ($E_{\frac{1}{2}}$), and the lowest energy of spin-$I_{\max}$ state satisfy the following relationship: $E_{\rm g.s.} = E_0 < E_{I_{\max}}$ ($E_{\rm g.s.} = E_{\frac{1}{2}} < E_{I_{\max}}$); in Case 2 (2′), $E_{\rm g.s.} < E_0 < E_{I_{\max}}$ ($E_{\rm g.s.} < E_{\frac{1}{2}} < E_{I_{\max}}$); in Case 3 (3′), $E_{\rm g.s.} = E_{I_{\max}} < E_0$ ($E_{\rm g.s.} = E_{I_{\max}} < E_{\frac{1}{2}}$); and in Case 4 (4′), $E_{\rm g.s.} < E_{I_{\max}} < E_0$ ($E_{\rm g.s.} < E_{I_{\max}} < E_{\frac{1}{2}}$). The probability of these cases in the random samplings of the TBRE is denoted by $P_{1(1')}$, $P_{2(2')}$, $P_{3(3')}$, and $P_{4(4')}$, respectively. And their values are summarized in Table I for the even-nucleon systems of single-j shell and realistic nuclei of a multi-j shell. One can see in this table that the sum of $P_{1(1')}$ and $P_{2(2')}$ is greater than 81% for single-j shell systems (e.g., $P_1 + P_2 = 81.64\%$ for four fermions in the $j = 15/2$ shell) and 96% for the realistic nuclei (e.g., $P_1 + P_2 = 96.87\%$ for ^{22}Ne and $P_{1'} + P_{2'} = 98.33\%$ for ^{23}Mg). This indicates that the random samplings with $E_{0(\frac{1}{2})} < E_{I_{\max}}$ [i.e., Case 1 (1′) and Case 2 (2′)] are dominant for all the many-nucleon systems. The probability of random samplings with $E_{0(\frac{1}{2})} > E_{I_{\max}}$

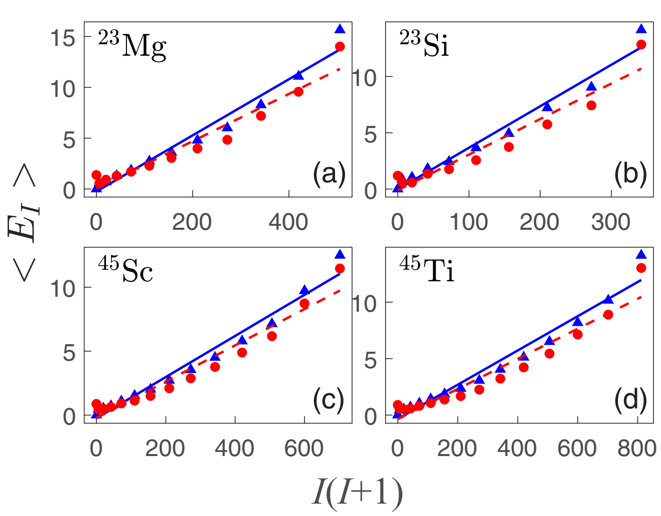

FIG. 3. Same as Fig. 1 except for odd-mass nuclei (a)^{23}Mg, (b)^{23}Si, (c)^{45}Sc, and (d)^{45}Ti. Results with $E_{\rm g.s.} = E_{\frac{1}{2}} < E_{I_{\max}}$ (Case 1′) and $E_{\rm g.s.} < E_{\frac{1}{2}} < E_{I_{\max}}$ (Case 2′) are shown in blue and red, respectively.

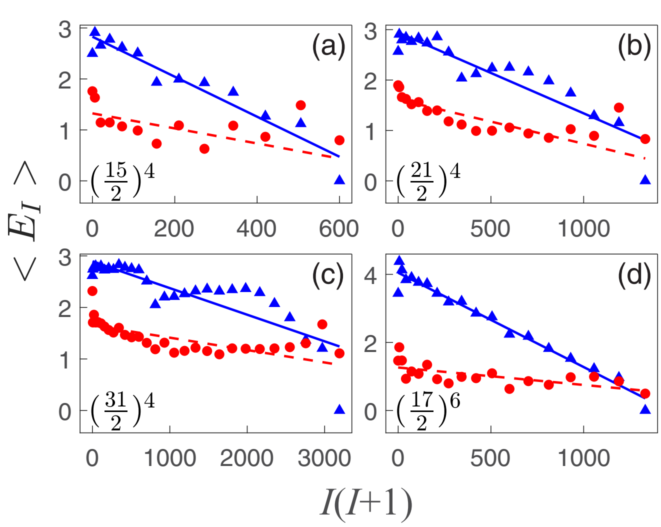

FIG. 5. Same as Fig. 1 except for the random samplings based on Case 3 (in blue) and Case 4 (in red).

TABLE II. The Pearson product-moment correlation coefficients (denoted by C) of Case 1 (1′) and Case 2 (2′) in the random sampling of TBRE for different many-nucleon systems. The subscripts 1 and 2 of the symbol C correspond to even-nucleon systems, and the subscripts in parentheses (1′ and 2′) correspond to odd-nucleon systems.

Single-j^n	C_1	C_2
$(15/2)^4$	0.966	0.964
$(21/2)^4$	0.974	0.953
$(31/2)^4$	0.970	0.940
$(17/2)^6$	0.991	0.992
Even-even nuclei	C_1	C_2
^{22}Ne	0.986	0.961
^{26}Si	0.998	0.979
^{44}Ti	0.986	0.960
^{46}Cr	0.982	0.964
Odd-mass nuclei	$C_{1'}$	$C_{2'}$
^{23}Mg	0.984	0.969
^{23}Si	0.986	0.962
^{45}Sc	0.986	0.972
^{45}Ti	0.980	0.962

TABLE III. The average ground-state gaps [ΔE_1 ($\Delta E_{1'}$) and ΔE_2 ($\Delta E_{2'}$) in MeV] of different many-nucleon systems. The subscripts 1 and 2 of the symbol ΔE correspond to even-nucleon systems, and the subscripts in parentheses (1′ and 2′) correspond to odd-nucleon systems.

Single-j^n	ΔE_1	ΔE_2
$(15/2)^4$	0.720	0.251
$(21/2)^4$	0.678	0.219
$(31/2)^4$	0.601	0.199
$(17/2)^6$	0.863	0.248
Even-even nuclei	ΔE_1	ΔE_2
^{22}Ne	0.792	0.571
^{26}Si	0.836	0.523
^{44}Ti	0.656	0.432
^{46}Cr	0.542	0.386
Odd-mass nuclei	$\Delta E_{1'}$	$\Delta E_{2'}$
^{23}Mg	0.424	0.646
^{23}Si	0.650	0.629
^{45}Sc	0.323	0.348
^{45}Ti	0.312	0.297

[i.e., Case 3 (3′) and Case 4 (4′)] is small. And, as we will see later, since Case 3 (3′) and Case 4 (4′) do not satisfy the normal ordering of spins, they do not satisfy the noncollective rotational behavior.

Below we first discuss the robustness of noncollective rotational behavior based on the random samplings of Case 1 (1′) and Case 2 (2′). Then we go to discuss the inverse noncollective rotational behavior based on the random samplings of Case 3 (3′) and Case 4 (4′). Finally, we discuss the robustness of above rotational behavior in the framework of the TBRE with realistic single-particle energies.

A. The noncollective rotational behavior

In Ref. [1], the noncollective rotational behavior of the yrast band was studied in the TBRE of even-even nuclei with spin-zero ground state. In this section, we study the robustness of this behavior for both even-nucleon and odd-nucleon systems, based on the random samplings of ground states with the lowest spin [i.e., Case 1 (1′)] and the nonlowest spin [Case 2 (2′)]. Here, for even-nucleon system, the lowest spin is 0; for odd-nucleon system, the lowest spin is $\frac{1}{2}$.

The average energies $\langle E_I\rangle$ of the yrast states as a function of $I(I+1)$ for even-nucleon systems in the single-j shell, even-even nuclei, and odd-mass nuclei are shown in Figs. 1–3, respectively. The blue and red lines in the figures are linear fitting lines for different cases. It can be seen that $\langle E_I\rangle$ increases with the increase of $I(I+1)$ and has a good linear correlation for both Case 1 (1′) and Case 2 (2′). The line of Case 1 (1′) lies overall above that of Case 2 (2′). This indicates that the average energy of Case 1 (1′) is generally higher for the same spin state.

To investigate the degree of linear dependence between $\langle E_I\rangle$ and $I(I+1)$, we calculate their Pearson product-moment correlation coefficients [32] (denoted by C), shown in Table II. The absolute value of C ranges from 0 to 1, where 0.8–1.0 is a very strong correlation, 0.6–0.8 is a strong correlation, 0.4–0.6 is a moderate correlation, and 0.2–0.4 is a weak correlation. According to our calculations in Table II, C values between $\langle E_I\rangle$ and $I(I+1)$ are close to 1 for both Case 1 (1′) and Case 2 (2′). This demonstrates that the very strong linear correlation (that is, the robustness of noncollective rotational behavior) between $\langle E_I\rangle$ and $I(I+1)$ for both even-nucleon and odd-nucleon systems, based on the random samplings of ground states with the lowest spin [i.e., Case 1 (1′)] and the nonlowest spin [Case 2 (2′)]. It is interesting that C_1 ($C_{1'}$) is always a little bigger than C_2 ($C_{2'}$), which is attributed to the disordering of spin for ground state in Case 2 (2′).

In Table III, we compare the average ground-state gaps of the first-excited state of Case 1 (1′) and Case 2 (2′) in the random sampling of TBRE for different many-nucleon systems, which are denoted ΔE_1 ($\Delta E_{1'}$) and ΔE_2 ($\Delta E_{2'}$). One sees that the gaps with spin-zero ground state (ΔE_1) are always larger than the gaps with spin-nonzero ground state (ΔE_2) for even-nucleon systems, which is in accordance with the result of Ref. [1]. However, the relationship of the average ground-state gaps is unstable for $\Delta E_{1'}$ and $\Delta E_{2'}$ of odd-nucleon systems.

For fermion systems, it is reported in the literature [25] that there is no obvious collective behavior of vibration and rotation among the yrast states in the nuclear shell model, by using the traditional energy-level ratio $R_{42} = (E_4 - E_{\rm g.s.})/(E_2 - E_{\rm g.s.})$. The value of R_{42} is sensitive to the spin of the ground state and the ground-state gap. According to Figs. 2 and 3 and Table III, one can see that there is a slight difference in the slope between the blue lines [Case 1 (1′)] and the red lines [Case 2 (2′)], which is mainly due to the difference in the ground-state gap and ground-state spin. This indicates that the yrast structures of these cases are different for low-spin states

TABLE IV. Same as Table II except for the random samplings based on Case 3 (3′) and Case 4 (4′).

Single-j^n	C_3	C_4
$(15/2)^4$	−0.960	−0.381
$(21/2)^4$	−0.925	−0.732
$(31/2)^4$	−0.845	−0.615
$(17/2)^6$	−0.985	−0.658
Even-even nuclei	C_3	C_4
^{22}Ne	−0.990	−0.592
^{26}Si	−0.995	−0.635
^{44}Ti	−0.899	−0.568
^{46}Cr	−0.993	−0.683
Odd-mass nuclei	$C_{3'}$	$C_{4'}$
^{23}Mg	−0.990	−0.698
^{23}Si	−0.961	−0.570
^{45}Sc	−0.987	−0.458
^{45}Ti	−0.886	−0.731

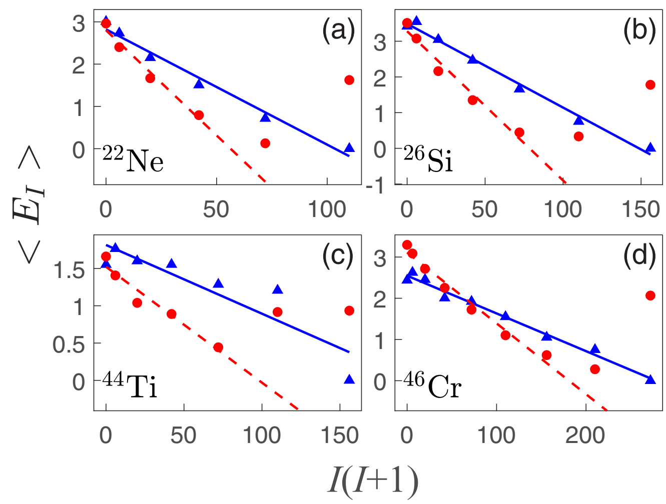

FIG. 6. Same as Fig. 2 except for the random samplings based on Case 3 (in blue) and Case 4 (in red).

and similar for high-spin states. Therefore, we define a new kind of energy-level ratio R here as an indicator to study the yrast structure of high-spin states as follows:

$$R = \frac{E_{I_{\max}} - E_{\text{g.s.}}}{E_{I_{\max}-2} - E_{\text{g.s.}}}. \tag{7}$$

The probability density distribution of R for both even-nucleon and odd-nucleon systems are presented and compared in Fig. 4. As you can see, the distribution peaks of random sampling in Case 1 (1′) almost coincide with those in Case 2 (2′). This indicates that high-spin states in Case 1 (1′) and Case 2 (2′) have similar yrast structures. Therefore, the results in Fig. 4 show the robustness of noncollective rotational behavior from another perspective.

B. The inverse noncollective rotational behavior

The probability of random samplings with $E_{I_{\max}} < E_0$ ($E_{I_{\max}} < E_{\frac{1}{2}}$) [i.e., $E_{\text{g.s.}} = E_{I_{\max}} < E_0$ ($E_{\text{g.s.}} = E_{I_{\max}} < E_{\frac{1}{2}}$) in Case 3 (3′) and $E_{\text{g.s.}} < E_{I_{\max}} < E_0$ ($E_{\text{g.s.}} < E_{I_{\max}} < E_{\frac{1}{2}}$) in Case 4 (4′)] is small (see Table I). However, the average energy $\langle E_I \rangle$ based on these random samples presents another interesting phenomenon with respect to $I(I+1)$. Our results of even-nucleon systems in single-j shell, even-even nuclei and odd-mass nuclei are presented in Fig. 5, Figs. 6 and 7, respectively. As you can see, $\langle E_I \rangle$ decreases as $I(I+1)$ increases in general. This behavior is called the inverse noncollective rotational behavior in this paper. The corresponding Pearson coefficients C are shown in Table IV. One sees that the Pearson correlation coefficients C_3 ($C_{3'}$) are around −0.84 to −0.99 in Case 3 (3′). This indicates a very strong negative linear correlation (i.e., the inverse noncollective rotational behavior) between $\langle E_I \rangle$ and $I(I+1)$ for both even-nucleon and odd-nucleon systems in Case 3 (3′). However, in Case 4 (4′), this linear correlation is unstable, with coefficients C_4 ($C_{4'}$) ranging from −0.38 to −0.73. Since the ground-state spin of Case 4 (4′) is between the maximum and minimum, the lowest

TABLE V. The Pearson product-moment correlation coefficients (denoted by C) of Case 1 (1′), Case 2 (2′), Case 3 (3′), and Case 4 (4′) in the random sampling of TBRE with realistic single-particle energies.

Even-even nuclei	C_1	C_2	C_3	C_4
^{22}Ne	0.952	0.898	−0.996	−0.507
^{26}Si	0.983	0.973	−0.994	−0.637
^{44}Ti	0.989	0.949	−0.979	−0.552
^{46}Cr	0.950	0.908	−0.984	−0.619
Odd-mass nuclei	$C_{1'}$	$C_{2'}$	$C_{3'}$	$C_{4'}$
^{23}Mg	0.939	0.906	−0.989	−0.642
^{23}Si	0.971	0.952	−0.943	−0.572
^{45}Sc	0.953	0.901	−0.900	−0.356
^{45}Ti	0.944	0.875	−0.926	−0.667

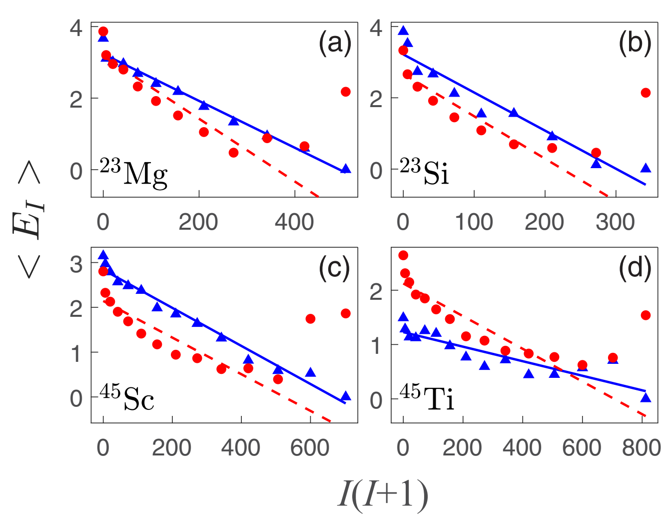

FIG. 7. Same as Fig. 3 except for the random samplings based on Case 3′ (in blue) and Case 4′ (in red).

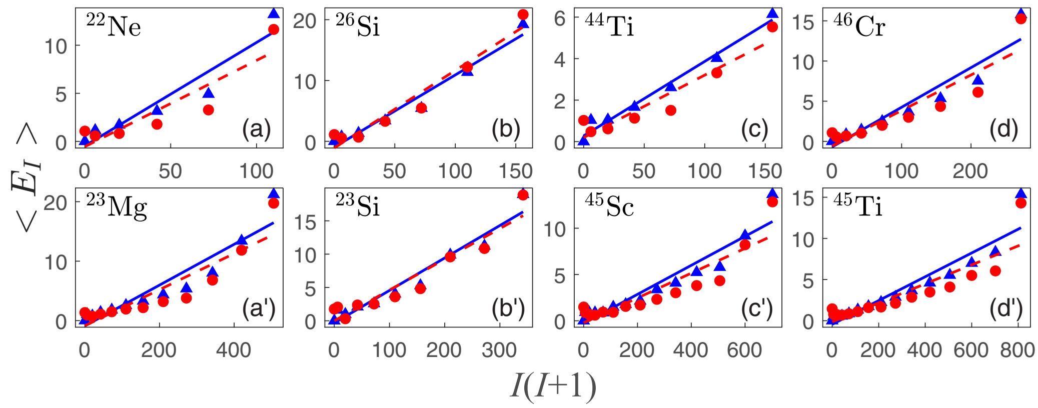

FIG. 8. Same as Figs. 2 and 3 except for the results of the TBRE with realistic single-particle energies. Here, panels (a)^{22}Ne, (b)^{26}Si, (c)^{44}Ti, and (d)^{46}Cr present results of Case 1 (in blue) and Case 2 (in red); panels (a′) ^{23}Mg, (b′) ^{23}Si, (c′) ^{45}Sc, and (d′) ^{45}Ti correspond to Case 1′ (in blue) and Case 2′ (in red).

value of $\langle E_I \rangle$ usually occurs in the state with medium spin. From Figs. 5–7, the inverse noncollective rotational behavior for Case 4 (4′) is robust from low spin state to medium spin state.

C. The rotational behaviors in the presence of the two-body random ensemble plus realistic single-particle energies

In the above calculations of the TBRE, the single-particle energies are set to zero. Then, are the noncollective rotational behavior and its inverse behavior still robust under the TBRE plus realistic single-particle energies? To answer this question, we recalculate the average energies $\langle E_I \rangle$ of the yrast states as a function of $I(I+1)$ for even-even nuclei and odd-mass nuclei by using the realistic single-particle energies of USDB [33] interactions for the *sd* shell nuclei and the GXPF1 [34] interactions for the *pf* shell nuclei. Our results of random samplings in Case 1 (1′) and Case 2 (2′) are shown in Fig. 8, and those in Case 3 (3′) and Case 4 (4′) are in Fig. 9. One sees that the $I(I+1)$ regularities are still robust in the presence of realistic single-particle energies plus random two-body interactions. The corresponding linear dependence between $\langle E_I \rangle$ and $I(I+1)$ are quite good in Case 1 (1′), Case 2 (2′), and Case 3 (3′) (see the Pearson coefficients C in Table V).

IV. SUMMARY

To summarize, in this paper we study the noncollective rotational behavior and its inverse behavior of the yrast bands via the TBRE Hamiltonian in the full shell-model space (without and with realistic single-particle energies) for both even-nucleon and odd-nucleon systems. We classify the random samplings of the TBRE into four different cases: In Case 1 (1′), $E_{\rm g.s.} = E_0 < E_{I_{\rm max}}$ ($E_{\rm g.s.} = E_{\frac{1}{2}} < E_{I_{\rm max}}$); in Case 2 (2′), $E_{\rm g.s.} < E_0 < E_{I_{\rm max}}$ ($E_{\rm g.s.} < E_{\frac{1}{2}} < E_{I_{\rm max}}$); in Case 3 (3′), $E_{\rm g.s.} = E_{I_{\rm max}} < E_0$ ($E_{\rm g.s.} = E_{I_{\rm max}} < E_{\frac{1}{2}}$); and in Case 4 (4′), $E_{\rm g.s.} < E_{I_{\rm max}} < E_0$ ($E_{\rm g.s.} < E_{I_{\rm max}} < E_{\frac{1}{2}}$).

The random samplings with $E_0 < E_{I_{\rm max}}$ ($E_{\frac{1}{2}} < E_{I_{\rm max}}$) [i.e., Case 1 (1′) and Case 2 (2′)] are dominant for all the many-nucleon systems. Based on random sampling of these two cases, it is found that there is a very strong positive linear correlation between $\langle E_I \rangle$ and $I(I+1)$ for both even-nucleon and odd-nucleon systems. That is to say, the noncollective rotation behavior is robust regardless of whether the ground-state spin is the lowest angular momentum. We define a new kind of energy-level ratio R and compare its probability density distribution between Case 1 (1′) and Case 2 (2′). It can be seen from the distribution peaks of random sampling that

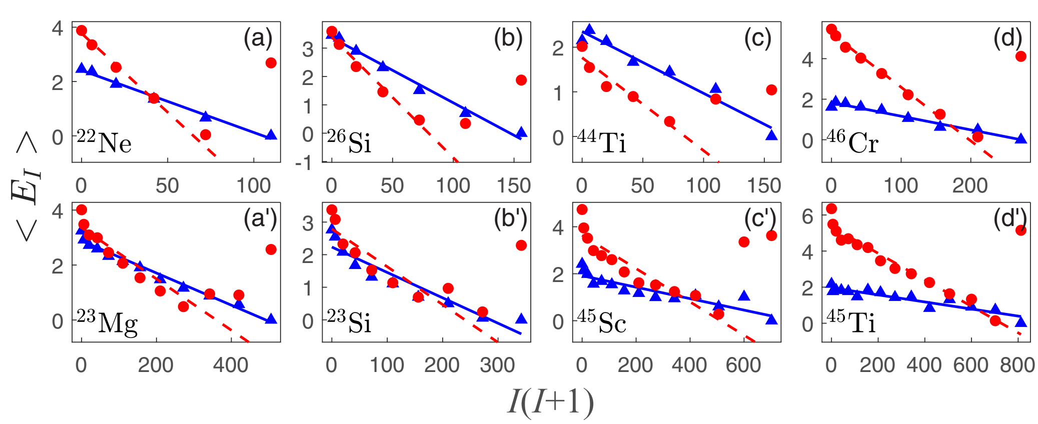

FIG. 9. Same as Fig. 8 except for the random samplings based on Case 3 (3′) (in blue) and Case 4 (4′) (in red).

the high-spin states in Case 1 (1′) and Case 2 (2′) have similar yrast structures.

Case 3 (3′) and Case 4 (4′) with $E_0 > E_{I_{\max}}$ ($E_{\frac{1}{2}} > E_{I_{\max}}$) do not satisfy the normal ordering of spins. The probability of random samplings in these two cases is small. It is interesting that the average excitation energies of yrast states $\langle E_I \rangle$ in these two cases basically decrease with the increase of $I(I+1)$. In particular, $\langle E_I \rangle$ and $I(I+1)$ have a very strong negative linear correlation in Case 3 (3′). We call this behavior the inverse noncollective rotational behavior.

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