Quark matter and quark stars within the quasiparticle model under magnetic fields

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We investigate the properties of the equation of state (EOS), the quark fraction, and the isospin asymmetry for strange quark matter (SQM) under constant magnetic field within the quasiparticle model. Our results indicate that the effect of the magnetic field is important for the properties of strange quark matter, and the recently discovered heavy compact stars PSR J0348+0432, MSR J0740+6620, PSR J2215+5135, and especially the GW190814s secondary component m_2 can be well described as quark stars within the quasiparticle model under magnetic fields by approximately using spherically symmetric Tolman-Oppenheimer-Volkov (TOV) equations.

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I. INTRODUCTION

The properties of the equation of state (EOS) of strongly interacting matter have attracted more and more attention in the field of nuclear physics and astrophysics [1–4]; such perties can be explored through heavy ion collisions (HICs) in terrestrial laboratories. The features of strongly interacting matter at zero baryon density and high temperature have been revealed by the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN, while the properties of strongly interacting matter at higher baryon density regions can be explored by the beam-energy scan program at RHIC. In nature, the properties of compact stars may provide another way of exploring the thermodynamical properties of strongly interacting matter at low temperature and finite chemical potential, and the appearance of quark matter in neutron stars (NSs) is generally considered one of the hottest in compact star physics. Quark stars (QSs), whose possible existence is still of great importance for modern nuclear physics and astrophysics [5–11], are totally made up of absolutely stable deconfined u, d, an s quarks and leptons (e and μ), i.e., strange quark matter (SQM) [7,8,12–16]. In recent reports, a heavy pulsar PSR J0348+0432 with a mass of $2.01 \pm 0.04\, M_\odot$ [17] was discovered in 2013, and then a more massive compact star PSR J2215+5135, whose mass reaches $2.27^{+0.17}_{-0.15}\, M_\odot$, was detected by fitting the three-band light curves and the radial velocity lines in the irradiated compact stars model [18]. In Ref. [19], MSR J0740+6620 ($2.14 \pm^{0.10}_{0.09}\, M_\odot$ with 68.3% credibility interval and $2.14 \pm^{0.20}_{0.18}\, M_\odot$ with 95.4% credibility interval) was observed by using the data of relativistic Shapiro delay from the Green Bank Telescope. Last year, the LIGO and Virgo Collaborations reported the newly discovered compact binary merger GW190814 [20], whose secondary component m_2 has a mass of $2.50M_\odot$–$2.67M_\odot$ at 90% credibility level, and in Ref. [21] the authors investigated the possibility that the low mass companion of the black hole in the source of GW190814 was a strange quark star. The candidate for the secondary component of GW190814 can set very strict constraints on the equation of state (EOS) of strongly interacting matter if we consider the candidate as a QS or hybrid star, which may rule out most of the conventional phenomenological models of quark matter, whereas there still exist some other models describing heavy quark stars with strong isospin interaction inside the star matter [22–34].

For compact stars, an important aspect of the physics is that they may be endowed with magnetic fields. In the works [35–37], the strength of the magnetic field at the surface of magnetars is estimated about $B = 10^{14}$–10^{15} G. For the strongly interacting matter under strong magnetic fields, the spatial rotational [O(3)] symmetry will break and one must consider the anisotropy of the pressure [38–42]. Furthermore, in order to describe the strength distribution of the magnetic fields from the surface to the core of magnetars, researchers usually introduce density-dependent magnetic fields [43–46]. We should mention that the ideal way to solve the structure of magnetars is to combine the Einstein, Maxwell, and equilibrium equations with the EOS under magnetic fields, which is quite complicated because the spherically symmetric Tolman-Oppenheimer-Volkov (TOV) equations are not applicable under strong magnetic fields. The so-called universal magnetic field profile is proposed as one solution in the context of NSs by Chatterjee in [47,48]; it can be used to calculate the properties of NSs under strong magnetic fields. On the other hand, from the research of [49], the authors show that the magnetic field might not increase exponentially inside the magnetars, which means the magnetic field in the

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core of the magnetars might be only several times the magnetic field of the surface. Then the magnetic field inside the magnetars in this case would be relatively not strong and the pressure anisotropy inside the star matter might be very small. Therefore, it is of interest and importance to investigate the magnetic field effects on the EOS of star matter and to explore under what conditions the pressure anisotropy inside the magnetars can be negligible so that the Tolman-Oppenheimer-Volkoff (TOV) equations can be approximately used to calculate the structure of magnetars.

The paper is organized as follows. In Sec. II, we introduce the models and methods for the quark matter under magnetic fields within the quasiparticle model, and the properties of EOS, the quark fraction, the isospin asymmetry for SQM, and the maximum mass of QSs under magnetic fields within quasiparticle model are studied in Sec. III. Finally, a conclusion is given in Sec. IV.

II. MODELS AND METHODS

A. The quasiparticle model

For the conventional phenomenological quark mass models, the quark-quark effective interaction inside the quark matter is included in the equivalent mass term [50–82]. In [78], the equivalent mass of the quasiparticle model was derived at the zero-momentum limit of the dispersion relations by resumming one-loop self-energy diagrams in the hard dense loop approximation, which is derived as [78,83,84]

$$m_q = \frac{m_{q_0}}{2} + \sqrt{\frac{m_{q_0}^2}{4} + \frac{g^2\mu_q^2}{6\pi^2}}, \tag{1}$$

where m_{q0} ($m_{u0} = 5.5$ MeV, $m_{d0} = 5.5$ MeV, and $m_{s0} = 95$ MeV) is the quark current mass, μ_q represents the quark chemical potential, and g means the strongly interacting coupling constant which is considered as a free input parameter in the present work.

The total thermodynamic potential density for SQM within quasiparticle model can be written as

$$\Omega = \sum_i [\Omega_i + B_i(\mu_i)] + B_m, \tag{2}$$

where $B_i(\mu_i)$ comes from the chemical dependence of the mass term, B_m is the negative vacuum pressure term for confinement [84,85], and Ω_i represents the quasiparticle contribution to the thermodynamic potential density for u, d, and s quarks and leptons. The contribution to the thermodynamic potential Ω_i from each particle under a magnetic field can be written as

$$\Omega_i = -\sum_{\nu=0}^{\nu_{\max}^i} \frac{g_i(|q_i|B)}{2\pi^2}\alpha_\nu \left\{ \frac{1}{2}\mu_i\sqrt{{\mu_i}^2 - s_i(\nu,B)^2} - \frac{s_i(\nu,B)^2}{2}\ln\left(\frac{\mu_i + \sqrt{{\mu_i}^2 - s_i(\nu,B)^2}}{s_i(\nu,B)}\right)\right\}. \tag{3}$$

In the above, i in the sum is for all flavors of quarks and leptons, and $\alpha_\nu = 2 - \delta_{\nu,0}$. Here we have assumed that the direction of the magnetic field is along z axis, which is also defined in the previous works [42,43,86–88]. The degeneracy factor g_i is considered to be 3 for quarks and 1 for leptons, and the Fermi energy for quarks and leptons is

$$\mu_i^* = \sqrt{{k_{F,\nu}^i}^2 + s_i(\nu,B)^2} \tag{4}$$

with $k_{F,\nu}^i$ being the Fermi momentum and $s_i(\nu,B) = \sqrt{m_i^2 + 2\nu|q_i|B}$. The upper Landau level is defined as

$$\nu_{\max}^i \equiv \text{int}\left[\frac{\mu_i^{*2} - m_i^2}{2|q_i|B}\right], \tag{5}$$

where int[$\cdots$] is the integer function. The medium-dependent term $B_i(\mu_i)$ is determined by using the integration formula

$$B_i(\mu_i) = -\int_{s_i}^{\mu_i} \frac{\partial\Omega_i}{\partial\mu_i}\frac{\partial m_i}{\partial\mu_i}d\mu_i. \tag{6}$$

B. Properties of strange quark matter at zero temperature

Strange quark matter is composed of u, d, and s quarks and leptons (e and μ) with electric charge neutrality in beta equilibrium. The weak beta-equilibrium condition at zero temperature can be written as

$$\mu_d = \mu_s = \mu_u + \mu_e \quad \text{and} \quad \mu_\mu = \mu_e. \tag{7}$$

The electric charge neutrality condition can be expressed as

$$\tfrac{2}{3}n_u = \tfrac{1}{3}n_d + \tfrac{1}{3}n_s + n_e. \tag{8}$$

The total energy density $\mathcal{E}$ is written as

$$\mathcal{E}_{\text{tot}} = \sum_i \mathcal{E}_i = \sum_i (\Omega_i + B_i(\mu_i) + \mu_i n_i) + B_m + \frac{B^2}{2}, \tag{9}$$

where the term $B^2/2$ comes from the magnetic field contribution. For SQM under a magnetic field, the O(3) rotational symmetry is broken and the pressure becomes anisotropic. The anisotropic pressure is defined as the longitudinal pressure $P_\parallel$ which is parallel to the magnetic field and the transverse pressure $P_\perp$ which is perpendicular to the magnetic field. The analytic forms of $P_\parallel$ and $P_\perp$ for SQM can be derived as [39]

$$P_\parallel = \sum_i \mu_i n_i - \mathcal{E}_{\text{tot}}, \tag{10}$$

$$P_\perp = \sum_i \mu_i n_i - \mathcal{E}_{\text{tot}} + B^2 - MB, \tag{11}$$

where M is the system magnetization. One can find that only the longitudinal pressure $P_\parallel$ can satisfy the Hugenholtz–Van Hove theorem.

III. RESULTS AND DISCUSSIONS

A. Properties of SQM under magnetic fields

The absolute stability condition for SQM proposed in Ref. [9] requires a minimum energy per baryon of SQM less than 930 MeV [the minimum value of energy per baryon of the observed nuclei, $M(^{56}\text{Fe})/56$], which usually sets strong constraints on the chosen parameter sets for most phenomenological quark models.

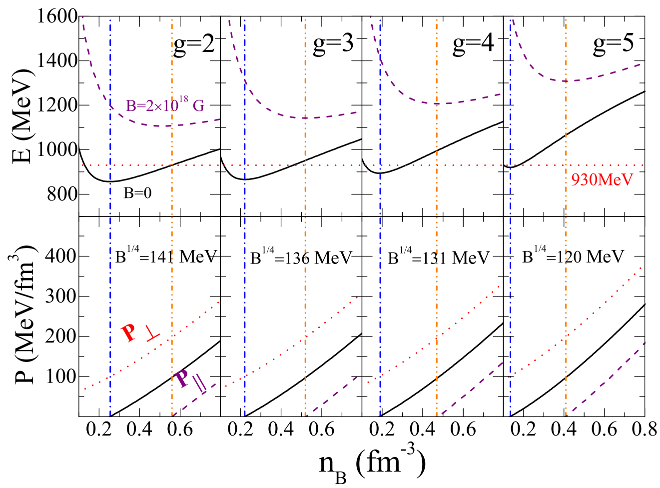

FIG. 1. Energy per baryon, longitudinal pressure, and transverse pressure for SQM as functions of baryon density within the quasiparticle model under zero magnetic field and $B = 2 \times 10^{18}$ G with different sets of parameters.

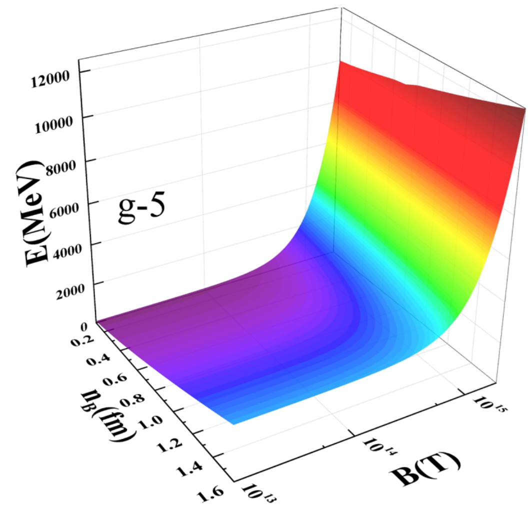

FIG. 2. The energy density of SQM as functions of baryon number density and magnetic fields B with g-5.

In Fig. 1, we calculate the energy per baryon, longitudinal pressure, and transverse pressure for SQM as functions of baryon density within the quasiparticle model under zero magnetic field and $B = 2 \times 10^{18}$ G with four sets of parameters, i.e., g-2 ($g = 2$, $B_m^{1/4} = 141$ MeV), g-3 ($g = 3$, $B_m^{1/4} = 136$ MeV), g-4 ($g = 4$, $B_m^{1/4} = 131$ MeV), and g-5 ($g = 5$, $B_m^{1/4} = 120$ MeV). From the calculation of the four parameter sets, we can obtain that all the minimum values of the energy per baryon for SQM from these four cases are smaller than 930 MeV, which satisfies the absolutely stable condition for SQM. One can find in Fig. 1 that the baryon number density of the minimum energy per baryon for all the cases is exactly the density of zero pressure for $B = 0$, while the density of the minimum energy per baryon is identical to the density of zero longitudinal pressure for $B = 2 \times 10^{18}$ G, which is consistent with thermodynamics. It can also be found that the energy per baryon and the transverse pressure for SQM increase with magnetic field due to Eqs. (9) and (10), while the longitudinal pressure decreases with magnetic field because of the negative term $-B^2/2$ in Eq. (11). Furthermore, one can also see that both the energy per baryon and pressure of SQM at $B = 0$ and $B = 2 \times 10^{18}$ G increase with the constant g, which indicates that the EOS of SQM within the quasiparticle model is stiffened by increasing the constant g.

To further investigate the effects of magnetic fields on SQM within the quasiparticle model, we calculate the energy density and the anisotropic pressures of SQM as functions of baryon number density and magnetic fields B with g-5 in Figs. 2 and 3, respectively. It can be seen in Fig. 2 that the energy density of SQM increases with the increment of baryon density at a fixed magnetic field, and the energy density can also be enhanced by $B^2/2$ from the magnetic field contribution at even low baryon density once the magnetic field is larger than 10^{14} T.

Figure 3 shows the transverse pressure and longitudinal pressure as functions of the baryon density and magnetic fields. One can obtain that the two pressures are almost identical and increase with the baryon density when $B < 10^{13}$ T, which shows almost no magnetic field effects. As the magnetic field increasing, the split between $P_{||}$ and $P_{\perp}$ becomes larger due to the decrement (increment) of $P_{||}$ ($P_{\perp}$) with magnetic fields, and one can find that $P_{||}$ at a certain baryon density may decrease to zero when B increases to the critical value B_c (e.g., $P_{||} = 0$ at $n_B = 1.5$ fm^{-3} with $B_c = 4.5 \times 10^{14}$ T for g-5), which indicates that the magnetic field in the quark matter of the whole magnetized star should satisfy the condition $B < B_c$ in order to maintain a stable magnetized star.

As shown in Fig. 4, we calculate the quark fraction for u, d, and s quarks of SQM as functions of baryon density with different sets of parameters in the quasiparticle model when $B = 0$ and $B = 2 \times 10^{18}$ G. For all the cases under zero and 2×10^{18} G magnetic fields, one can see that the fraction of d quarks at low baryon density is twice the fraction of u quarks due to the charge neutrality, while the s quark fraction is zero because the corresponding chemical potential is less than the

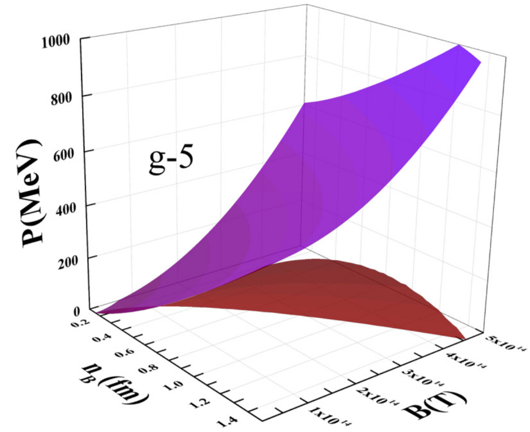

FIG. 3. The transverse pressure and longitudinal pressure as functions of the baryon density and magnetic fields with g-5 at zero temperature.

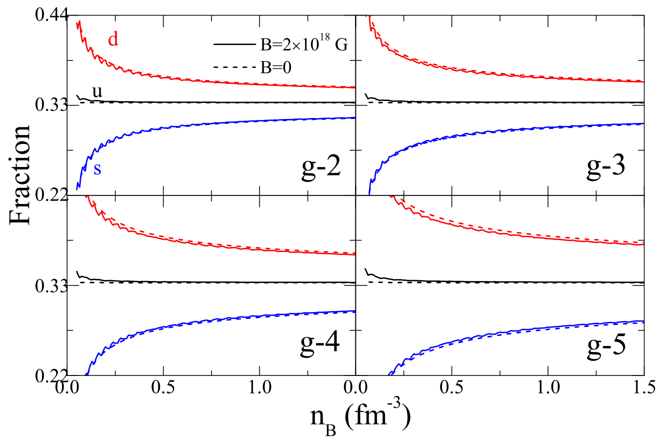

FIG. 4. Quark fraction for u, d, and s quarks of SQM as functions of baryon density with different sets of parameters in the quasiparticle model when $B = 0$ and $B = 2 \times 10^{18}$ G.

quark mass of the s quark when baryon density is small. As baryon density increases, the fraction of s quarks increases and the difference of the fractions among the three flavors of quarks decreases at high baryon density. Furthermore, one can find that the quark fractions begin oscillating when $B = 2 \times 10^{18}$ G, and we also calculate the quark fractions as functions of the magnetic fields in Fig. 5 when $n_B = 0.8$ fm^{-3} in order to investigate the effects of magnetic fields on the quark fraction at a certain baryon density. The baryon density $n_B = 0.8$ fm^{-3} is usually obtained as the central density of the maximum mass of QSs for conventional phenomenological quark models, which is the upper bound of the baryon density for the star matter. One can find in Fig. 5 that the difference among different flavors of quarks increases with g, and the obvious oscillation of quark fractions approximately takes place when $B > 2 \times 10^{18}$ G. It can also be seen in Fig. 5 that the amplitude of the oscillation caused by the strong magnetic field gets even larger with the increment of B. The reason for the oscillation of the physical quantities under strong magnetic fields is mainly dependence on the upper Landau level $\nu^i_{\max}$ in Eq. (5), which is small under strong magnetic field. Additionally, we calculate the isospin asymmetry

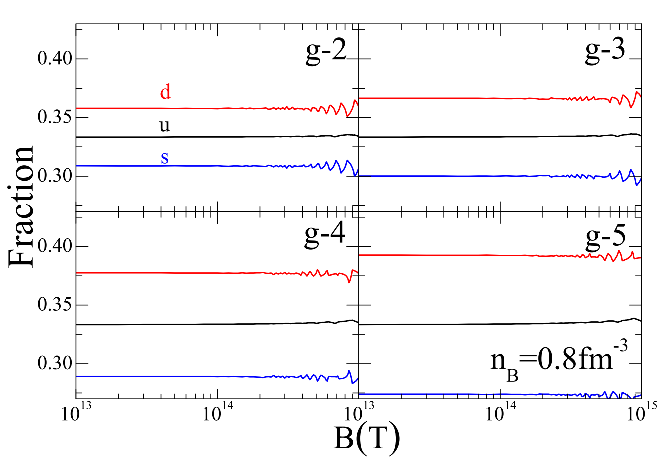

FIG. 5. Quark fraction as functions of the magnetic fields when $n_B = 0.8$ fm^{-3}.

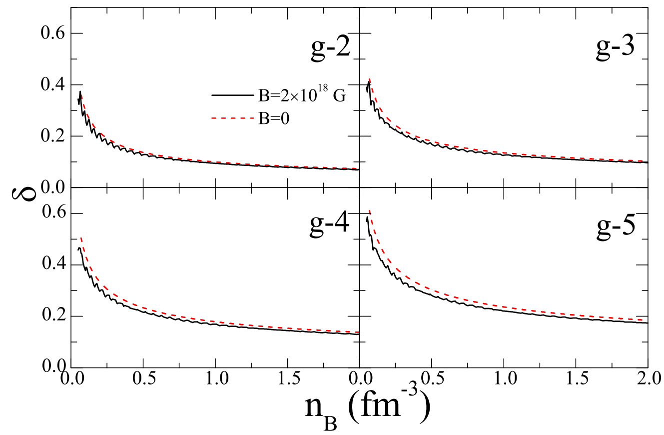

FIG. 6. Isospin asymmetry as functions of baryon density with $B = 0$ and $B = 2 \times 10^{18}$ G.

($\delta = \frac{3(n_d - n_u)}{n_d + n_u}$) with different sets of parameters as functions of baryon density when $B = 0$ and $B = 2 \times 10^{18}$ G. One can find that the isospin asymmetry for all cases decreases with the baryon density, which agrees with the results in Fig. 4, and the oscillation of δ also appears when B is larger than 2×10^{18} G. Furthermore, it can be seen intuitively in Fig. 6 that the value of δ at $B = 2 \times 10^{18}$ G is smaller than that at zero magnetic fields. We can also find the similar result in Fig. 4 that the difference between the fractions of d and u quarks gets smaller with B, which implies that the quark matter symmetry energy within the quasiparticle model increases with the magnetic field and then reduces the difference among u and d quark fractions. One can also observe a similar symmetry energy effect in neutron star matter, where the large nuclear matter symmetry energy can reduce the difference between neutron and proton fractions (see, e.g., [89]).

B. Quark stars under weak magnetic fields

The magnetic field strength is generally considered as varying along the radial orientation inside the magnetars, which is conventionally defined by using a density-dependent magnetic field, and the popular parametrization for the analytic expression for the density-dependent magnetic field we used in QSs for this work is written as [43,45,46,86]

$$B = B_{\text{surf}} + B_0[1 - \exp\{-\beta_0(n_B/n_0)^\gamma\}], \qquad (12)$$

where B_{surf} is the magnetic field strength at the star surface (B_{surf} is fixed at 1×10^{11} T in this work), $n_0 = 0.16$ fm^{-3} is the normal nuclear matter density, B_0 is a parameter adjusting the total magnetic field inside the magnetars (the total magnetic field obviously increases with B_0 while other parameters are fixed), and β_0 and γ are the parameters which are capable of controlling the density-dependence of B inside the stars to mimic the magnetic field strength distribution decaying from center to surface. In this work, we employ the fast-B profile from [53], which demonstrates a strong density dependence of magnetic field strength with $\gamma = 3$ and $\beta_0 = 0.001$. On the other hand, the orientation distribution of the magnetic fields is also significant for the structure of the magnetars since the pressure might be anisotropic inside the stars under strong magnetic fields. In this work, we employ the two extremely special cases for the orientation distribution of the magnetars from [53]: one is denoted as "radial orientation" whose local magnetic fields are along the radial direction, while the other orientation distribution denoted as "transverse orientation" contains magnetic fields perpendicular to the radial direction but randomly oriented in the plane which is perpendicular to the radial orientation. Using these two extreme cases, the pressure distribution inside the magnetars can be considered to be spherically symmetric, and then one can use $P_{||}$ and $P_{\perp}$ to provide the radial pressure in TOV for the radial orientation case and the transverse orientation case, respectively. We can calculate the upper and lower limits of the maximum mass of magnetars by using these two extreme cases [the mass for the radial orientation shows the lower limit of the star mass under magnetic field due to the "$-B^2/2$" term in pressure from Eq. (10) while the transverse orientation sets the upper limit]. We must point out that these two extreme cases for the orientation distribution of magnetars are not perfect: (1) there exists some research showing that the magnetic field might not increase exponentially inside the magnetars [49]—the increment might be several times and cannot be orders of magnitude from surface to the core—and (2) the presence of strong magnetic field can break the spherical symmetry of the star matter, and thus the deformation of the magnetars must be considered in extended nonspherically symmetric TOV equations if the magnetic strength is large enough. Since the magnetic field strength around the surface of magnetars is relatively weak (around 10^{11}–10^{12} T) and the increased magnetic field in the center of the magnetars might be only several times larger than the magnetic field at the surface, from the works [49,90], the pressure anisotropy inside the magnetars may not be very large and then one can approximately use the TOV equation to calculate the properties of QSs under magnetic fields. In this subsection, we intend to investigate the properties of the maximum mass of QSs under density-dependent magnetic fields by considering the two extreme cases above in the fast-B profile when the magnetic field inside the magnetars is not very strong, and the study of QSs under strong magnetic fields requires a much more complicated system of equations in general relativity [48], which is far beyond the scope of the present work and will be studied in future works.

In Fig. 7, we calculate the maximum mass as functions of the magnetic field parameter B_0 in Eq. (12) with different sets of parameters by considering the radial orientation (case with $P_{||}$ as the radial pressure) and transverse orientation ($P_{\perp}$ as the radial pressure), and the maximum masses at zero magnetic fields are also included as solid lines. For all the star masses in Fig. 7, we consider $B_0 < 4 \times 10^{18}$ G because a strong magnetic field can remarkably break the spherical symmetry of the star matter, which is not suitable for approximate calculations of the maximum mass of QSs by using the TOV equation, and the magnetic field might not increase exponentially inside the magnetars [49], whose increment might be several times and cannot be orders of magnitude from the surface (about 10^{11} T) to the core. One can find in Fig. 7 that the maximum mass of QSs decreases in the radial orientation case with B_0

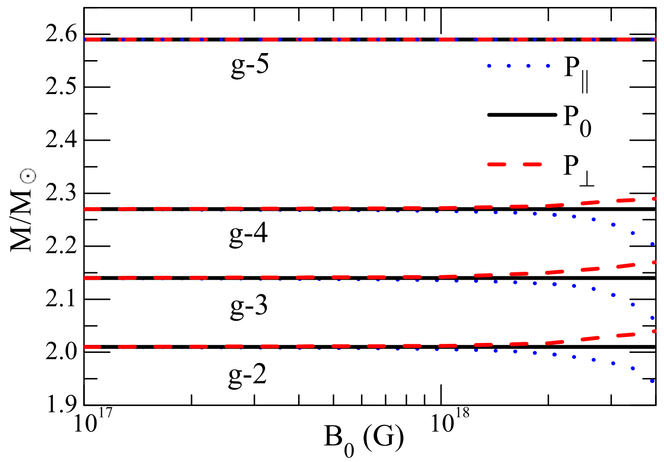

FIG. 7. Maximum star mass for anisotropic pressures as functions of B_0 with different g.

while it increases in the transverse orientation case with B_0 for g-2, g-3, and g-4. In order to see the effects of the magnetic field orientation distribution on the maximum mass of QSs, we use the normalized mass asymmetry from [53] as $\delta_m = \frac{M_{\perp}-M_{||}}{(M_{\perp}+M_{||})/2}$, where $M_{\perp}$ and $M_{||}$ stand for the maximum masses of magnetars using transverse orientation and radial orientation. It is seen from Fig. 7 that the maximum masses of QSs with radial orientation and transverse orientation are almost identical when $B_0 < 1 \times 10^{18}$ G. The result of the maximum mass of QSs with g-2 can describe PSR J0348+ 0432 with the mass of $2.01 \pm 0.04 M_{\odot}$ [17] as QSs, while the recently discovered massive pulsar MSR J0740+6620 ($2.14 \pm^{0.10}_{0.09} M_{\odot}$ with 68.3% credibility interval and $2.14 \pm^{0.20}_{0.18} M_{\odot}$ with 95.4% credibility interval) [19] and PSR J2215+5135 with the mass of $2.27 \pm^{0.10}_{0.09} M_{\odot}$ can be described as QSs with g-3 and g-4, respectively. For g-5, the result shows that the maximum mass of the QS is $2.59 M_{\odot}$, which is able to describe GW190814's secondary component as a QS within the quasiparticle model. As the magnetic field increases, the maximum mass of QSs with the radial/transverse orientation can reach $1.94 M_{\odot}/2.04 M_{\odot}$, $2.06 M_{\odot}/2.17 M_{\odot}$, and, $2.20 M_{\odot}/2.29 M_{\odot}$ for g-2, g-3, and g-4 at $B_0 = 4 \times 10^{18}$ G, respectively. The corresponding normalized mass asymmetries of the results above are obtained as 0.05, 0.05, and 0.04, which indicates that the difference from the maximum mass of QSs is very small by considering the radial orientation and transverse orientation cases when B_0 is less than 4×10^{18} G within the quasiparticle model. One can find for the g-5 case that the difference of the maximum masses of QSs for these two extreme cases is almost zero when $B_0 < 4 \times 10^{18}$ G, which implies that the maximum mass of magnetars for g-5 within the quasiparticle model changes very little in this magnetic field region. We would like to point out that the reason why the star mass under the two extreme magnetic field orientation cases changes little in our results is that we use small B_0 in the calculations, which is set in order to guarantee that the magnetic field strength in the core of the magnetar can only be several times that at the surface [49]. Then the lower limit of the maximum mass of the magnetar for all magnetic field orientation distributions is

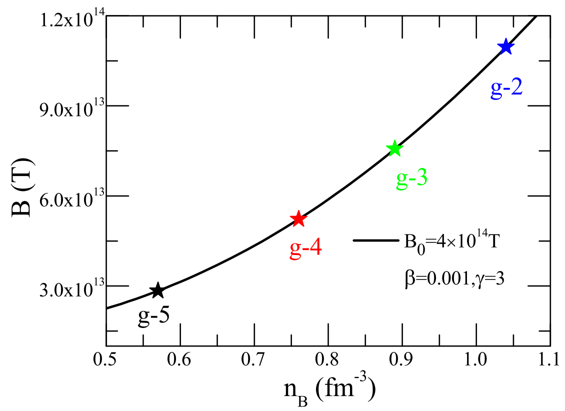

FIG. 8. Density-dependent magnetic field as a function of the baryon density with different sets of parameters.

the star mass of the longitudinal orientation case, while the upper limit of the maximum magnetar mass is the star mass of the transverse orientation case, which implies that all the maximum magnetar masses in other magnetic field orientation cases can all be calculated between the lower and upper limits with small B_0. Hence we approximately use the spherical TOV equation to calculate the star mass with small B_0, and the results indicate that the difference between the lower and upper limits of the star mass for g-2, g-3, and g-4 is very small. Moreover, the ratios of the star mass [for longitudinal (transverse) orientation] at $B_0 = 4 \times 10^{18}$ G to the star mass at zero magnetic field are 0.965 (1.02), 0.962 (1.01), and 0.969 (1.01) for g-2, g-3, and g-4, respectively. Especially for the g-5 case, we can find that the maximum mass of the magnetar within quasiparticle model changes little with the magnetic field when $B_0 = 4 \times 10^{18}$, which implies a very tiny effect of the magnetic fields on very massive stars.

From the results in Fig. 2, we can obtain that the magnetic field effect on SQM is not large until $B > 1 \times 10^{18}$ G, which is not believed to be created in usual compact stars. In Fig. 8, we calculate the density-dependent magnetic field as a function of the baryon density with different sets of parameters. One can find the central densities of the maximum mass of QSs at zero magnetic field with g-2, g-3, g-4, and g-5 are 1.04, 0.89, 0.79, and 0.57 fm^{-3}, respectively. For g-2, g-3, and g-4, we obtain the density-dependent magnetic field at the central density of the magnetars when $B_0 = 4 \times 10^{18}$ G and $B = 1.1 \times 10^{18}$, 7.6×10^{17}, and 5.2×10^{17} G. One can find from these results that the maximum value of the magnetic field inside the magnetars for each case is not large, and the average value of the density dependent magnetic field is even smaller than the maximum value, which indicates that the magnetic field effect on the maximum mass of magnetars is small due to the issue that magnetic field might not increase exponentially inside the magnetars. For the central density of the maximum star mass for g-5, the corresponding magnetic field is as small as $B = 2.8 \times 10^{17}$ G, which contributes very tiny effects on the properties, and this is the reason why the maximum mass of the magnetars for g-5 barely changes with the density-dependent magnetic field of the fast profile in this work. In our further calculation, the maximum star mass for g-5 does not change much until we increases B_0 to 1.7×10^{19} G ($B = 8.8 \times 10^{17}$ G) with the star mass changing to $2.58 M_\odot$ and $2.61 M_\odot$ for $P_{||}$ and $P_\perp$, whose δ_m is only 0.01 at such large B_0. Since the magnetic field strength around the surface of magnetars is relatively weak (10^{15}–10^{16} G) and the increment of the magnetic field may not be orders of magnitude in the center of the magnetars [49,90], the magnetic field strength in the central region of the massive stars cannot be very strong. Then our results indicate that the effects of the magnetic fields on the properties of QSs (like PSR J0348+ 0432 and other pulsars whose star mass is larger than 2 solar masses) might not be large, and the pressure anisotropy inside the magnetars is also very small. In particular, we also find the effects of the magnetic field on the maximum star mass is almost zero once we describe GW190814's secondary component as a QS. Combined with the maximum star mass cases with g-2, g-3, and g-4, we can obtain that the effects of the density dependent magnetic field on the maximum mass of QSs decreases when we describe more massive compact stars as QSs within the quasiparticle model.

IV. CONCLUSION AND DISCUSSION

In this work, we have investigated the properties of EOS, the quark fraction, and the isospin asymmetry for SQM under constant magnetic field within the quasiparticle model. The energy density and the anisotropic pressure have been calculated self-consistently as functions of baryon density and magnetic field, and the results show that the pressure of SQM becomes anisotropic when magnetic field gets strong. The longitudinal pressure and the transverse pressure decreases and increases with the magnetic field, respectively.

Furthermore, considering the the magnetic field of the surface and the core of the magnetars is not very strong, we have investigated the maximum mass of QSs under magnetic fields by using density-dependent magnetic field and radial/transverse orientations for the strength distribution and orientation distribution of the magnetic fields inside the magnetars. PSR J0348+0432, MSR J0740+6620, and PSR J2215+5135 can be described as QSs within quasiparticle model, and the normalized mass asymmetry for the maximum mass of QSs of the the radial/transverse orientation case is merely around 0.05 when $B_0 = 4 \times 10^{18}$ G. We have also described GW190814's secondary component as a QS, and the star mass does not change much with the magnetic fields, because the central density of the magnetar is small and the corresponding magnetic field is weak even when B_0 increases to 1.7×10^{19} G.

Therefore our present results have shown that the effect of the magnetic fields is important for the properties of strange quark matter, and the recent large mass compact stars can be described as QSs within the quasiparticle model under magnetic fields by approximately using TOV equations.

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[1] N. K. Glendenning, *Compact Stars*, 2nd ed. (Spinger- Verlag, New York, 2000).
[2] F. Weber, *Pulsars as Astrophysical Laboratories for Nuclear and Particle Physics* (IOP, London, 1999).
[3] J. M. Lattimer and M. Prakash, Science **304**, 536 (2004).
[4] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. J. Ellis, Phys. Rep. **411**, 325 (2005).
[5] D. Ivanenko and D. F. Kurdgelaidze, Lett. Nuovo Cimento **2**, 13 (1969).
[6] N. Itoh, Prog. Theor. Phys. **44**, 291 (1970).
[7] A. R. Bodmer, Phys. Rev. D **4**, 1601 (1971).
[8] E. Witten, Phys. Rev. D **30**, 272 (1984).
[9] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984).
[10] C. Alcock, E. Farhi, and A. Olinto, Astrophy. J. **310**, 261 (1986).
[11] F. Weber, Prog. Part. Nucl. Phys. **54**, 193 (2005).
[12] I. Bombaci, I. Parenti, and I. Vidana, Astrophy. J. **614**, 314 (2004).
[13] J. Staff, R. Ouyed, and M. Bagchi, Astrophy. J. **667**, 340 (2007).
[14] M. Herzog and F. K. Röpke, Phys. Rev. D **84**, 083002 (2011).
[15] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).
[16] H. Terazawa, University of Tokyo INS Report No, 336, 1979 (unpublished).
[17] J. Antoniadis *et al.*, Science **340**, 6131 (2013).
[18] M. Linares, T. Shahbaz, and J. Casares, Astrophys. J. **859**, 54 (2018).
[19] H. T. Cromartie *et al.*, Nat. Astron. Lett. **4**, 72 (2020).
[20] R. Abbott *et al.*, Astrophys. J. Lett. **896**, L44 (2020).
[21] I. Bombaci, A. Drago, D. Logoteta, G. Pagliara, and I. Vidaña, Phys. Rev. Lett. **126**, 162702 (2021).
[22] M. Alford and S. Reddy, Phys. Rev. D **67**, 074024 (2003).
[23] M. Alford, P. Jotwani, C. Kouvaris, J. Kundu, and K. Rajagopal, Phys. Rev. D **71**, 114011 (2005).
[24] M. Baldo, Phys. Lett. B **562**, 153 (2003).
[25] N. D. Ippolito, M. Ruggieri, D. H. Rischke, A. Sedrakian, and F. Weber, Phys. Rev. D **77**, 023004 (2008).
[26] X. Y. Lai and R. X. Xu, Research Astron. Astrophys. **11**, 687 (2011).
[27] M. G. B. de Avellar, J. E. Horvath, and L. Paulucci, Phys. Rev. D **84**, 043004 (2011).
[28] L. Bonanno and A. Sedrakian, Astron. Astrophys. **539**, A16 (2012).
[29] P.-C. Chu, B. Wang, Y.-Y. Jia, Y.-M. Dong, S.-M. Wang, X.-H. Li, L. Zhang, X.-M. Zhang, and H.-Y. Ma, Phys. Rev. D **94**, 123014 (2016).
[30] P. C. Chu *et al.*, Eur. Phys. J. C **77**, 512 (2017).
[31] P. C. Chu *et al.*, J. Phys. G: Nucl. Part. Phys. **47**, 085201 (2020).
[32] P. C. Chu *et al.*, Eur. Phys. J. C **81**, 93 (2021).
[33] Z. Zhang, P.-C. Chu, X.-H. Li, H. Liu, and X.-M. Zhang, Phys. Rev. D **103**, 103021 (2021).
[34] P. C. Chu *et al.*, Eur. Phys. J. C **81**, 569 (2021).
[35] L. Woltjer, Astrophys. J. **140**, 1309 (1964).
[36] T. A. Mihara, Nature (London) **346**, 250 (1990).
[37] G. Chanmugam, Annu. Rev. Astron. Astrophys. **30**, 143 (1992).
[38] D. Lai and S. L. Shapiro, Astrophys. J. **383**, 745 (1991).
[39] E. J. Ferrer, V. de la Incera, J. P. Keith, I. Portillo, and P. L. Springsteen, Phys. Rev. C **82**, 065802 (2010); E. J. Ferrer and V. de la Incera, in *Strongly Interacting Matter in Magnetic Fields*, Lecture Notes in Physics Vol. 871 (Springer, Berlin, 2013), p. 399.
[40] A. A. Isayev and J. Yang, Phys. Rev. C **84**, 065802 (2011).
[41] A. A. Isayev and J. Yang, Phys. lett. B **707**, 163 (2012).
[42] A. A. Isayev and J. Yang, J. Phys. G **40**, 035105 (2013).
[43] D. Bandyopadhyay, S. Pal, and S. Chakrabarty, J. Phys. G **24**, 1647 (1998).
[44] D. P. Menezes, M. B. Pinto, S. S. Avancini, A. P. Martínez, and C. Providência, Phys. Rev. C **79**, 035807 (2009); D. P. Menezes, M. B. Pinto, S. S. Avancini, and C. Providência, *ibid.* **80**, 065805 (2009).
[45] C. Y. Ryu, K. S. Kim, and M. K. Cheoun, Phys. Rev. C **82**, 025804 (2010).
[46] C. Y. Ryu, M. K. Cheoun, T. Kajino, T. Maruyama, and G. J. Mathews, Astropart. Phys. **38**, 25 (2012).
[47] D. Chatterjee, T. Elghozi, J. Novak, and M. Oertel, Mon. Not. R. Astron. Soc. **447**, 3785 (2015).
[48] D. Chatterjee, J. Novak, and M. Oertel, Phys. Rev. C **99**, 055811 (2019).
[49] V. Dexheimer *et al.*, Phys. Lett. B **773**, 487 (2017).
[50] A. Chodos, R. L. Jaffe, K. Ohnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974).
[51] M. Alford, M. Braby, M. Paris, and S. Reddy, Astrophy. J. **629**, 969 (2005).
[52] J. Y. Chao, P. Chu, and M. Huang, Phys. Rev. D **88**, 054009 (2013).
[53] P. C. Chu, L. W. Chen, and X. Wang, Phys. Rev. D **90**, 063013 (2014).
[54] P.-C. Chu, X. Wang, L.-W. Chen, and M. Huang, Phys. Rev. D **91**, 023003 (2015).
[55] P.-C. Chu, B. Wang, H.-Y. Ma, Y.-M. Dong, S.-L. Chang, C.-H. Zheng, J.-T. Liu, and X.-M. Zhang, Phys. Rev. D **93**, 094032 (2016).
[56] P. Rehberg, S. P. Klevansky, and J. Hüfner, Phys. Rev. C **53**, 410 (1996).
[57] M. Hanauske, L. M. Satarov, I. N. Mishustin, H. Stocker, and W. Greiner, Phys. Rev. D **64**, 043005 (2001).
[58] S. B. Rüster and D. H. Rischke, Phys. Rev. D **69**, 045011 (2004).
[59] D. P. Menezes, C. Providencia, and D. B. Melrose, J. Phys. G **32**, 1081 (2006).
[60] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994) and references therein.
[61] H. S. Zong, L. Chang, F. Y. Hou, W. M. Sun, and Y. X. Liu, Phys. Rev. C **71**, 015205 (2005).
[62] S. X. Qin, L. Chang, H. Chen, Y. X. Liu, and C. D. Roberts, Phys. Rev. Lett. **106**, 172301 (2011).
[63] B. A. Freedman and L. D. Mclerran, Phys. Rev. D **16**, 1169 (1977).
[64] E. S. Fraga, R. D. Pisarski, and J. Schaffner-Bielich, Phys. Rev. D **63**, 121702(R) (2001).
[65] E. S. Fraga and P. Romatschke, Phys. Rev. D **71**, 105014 (2005).
[66] A. Kurkela, P. Romatschke, and A. Vuorinen, Phys. Rev. D **81**, 105021 (2010).
[67] G. N. Fowler, S. Raha, and R. M. Weiner, Z. Phys. C **9**, 271 (1981).
[68] S. Chakrabarty, S. Raha, and B. Sinha, Phys. Lett. B **229**, 112 (1989).
[69] S. Chakrabarty, Phys. Rev. D **43**, 627 (1991); **48**, 1409 (1993); **54**, 1306 (1996).
[70] P.-C. Chu and L.-W. Chen, Phys. Rev. D **96**, 083019 (2017).
[71] P.-C. Chu, Y. Zhou, X. -Qi, X. H. Li, Z. Zhang, and Y. Zhou, Phys. Rev. C **99**, 035802 (2019).

[72] P.-C. Chu, Y. Zhou, X.-H. Li, and Z. Zhang, Phys. Rev. D **100**, 103012 (2019).
[73] O. G. Benvenuto and G. Lugones, Phys. Rev. D **51**, 1989 (1995).
[74] G. X. Peng, H. C. Chiang, J. J. Yang, L. Li, and B. Liu, Phys. Rev. C **61**, 015201 (1999).
[75] G. X. Peng, H. C. Chiang, B. S. Zou, P. Z. Ning, and S. J. Luo, Phys. Rev. C **62**, 025801 (2000).
[76] G. X. Peng, A. Li, and U. Lombardo, Phys. Rev. C **77**, 065807 (2008).
[77] A. Li, G. X. Peng, and J. F. Lu, Res. Astron. Astrophys. **11**, 482 (2011).
[78] K. Schertler, C. Greiner, and M. H. Thoma, Nucl. Phys. A **616**, 659 (1997).
[79] K. Schertler, C. Greiner, P. K. Sahu, and M. H. Thoma, Nucl. Phys. A **637**, 451 (1998).
[80] P. C. Chu and L. W. Chen, Astrophys. J. **780**, 135 (2014).
[81] P. C. Chu *et al.*, Phys. Lett. B **778**, 447 (2018).
[82] P. C. Chu and L. W. Chen, Phys. Rev. D **96**, 103001 (2017).
[83] R. D. Pisarski, Nucl. Phys. A **498**, 423 (1989).
[84] X. J. Wen *et al.*, J. Phys. G: Nucl. Part. Phys. **36**, 025011 (2009).
[85] B. K. Patra and C. P. Singh, Phys. Rev. D **54**, 3551 (1996).
[86] D. Bandyopadhyay, S. Chakrabarty, and S. Pal, Phys. Rev. Lett. **79**, 2176 (1997).
[87] X.-J. Wen, S.-Z. Su, D.-H. Yang, and G.-X. Peng, Phys. Rev. D **86**, 034006 (2012).
[88] X. J. Wen, Physica A **392**, 4388 (2013).
[89] J. Xu *et al.*, Astrophys. J. **697**, 1549 (2009).
[90] K. Makishima, T. Enoto, J. S. Hiraga, T. Nakano, K. Nakazawa, S. Sakurai, M. Sasano, and H. Murakami, Phys. Rev. Lett. **112**, 171102 (2014).