Detailed theoretical study of the decay properties of states in the ^{7}He nucleus within an *ab initio* approach

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(Received 22 April 2021; accepted 6 October 2021; published 20 October 2021)

The energies and decay widths of the states of the exotic ^{7}He nucleus are studied in an *ab initio* approach. The spectrum of these states is calculated using the no-core shell model (NCSM) and a corresponding extrapolation procedure. The Daejeon16 potential, well proved on a large amount of data, is used in the calculations. The previously developed NCSM-based approach, which includes a method for constructing the basis of functions of cluster channels and a procedure for matching the cluster form factors obtained within this method with the corresponding asymptotic wave functions, is applied to compute the decay widths of the levels. The possibilities of the approach are demonstrated for calculating the partial decay widths of nuclear states into various channels that strongly differ in type of fragmentation, spin, angular momentum of relative motion, and amplitude. The results obtained are compared with the results of other microscopic calculations.

DOI: 10.1103/PhysRevC.104.044323

I. INTRODUCTION

In recent decades, research related to low-energy nuclear physics has shown two basic trends.

In experimental research, there has been a rapid transition from the study of stable and neighboring long-lived nuclei to the study of exotic systems, including nucleon-unstable ones. The most important factor in the development of such a research is the upgrade of existing radioactive beam facilities and construction of new ones. Programs for upgrading existing facilities are being implemented in RIKEN (creation of RIBF), GANIL (SPIRAL2), GSI (FAIR), and NSCL (FRIB). Building of new centers, HIAF and RAON, for the study of radioactive isotopes is in progress.

In theoretical research, largely due to the rapid progress of computer technologies, an increasingly significant place is occupied by high-precision microscopic approaches, in particular *ab initio* (from first principles) methods of describing nuclear systems.

An important place among *ab initio* methods is taken by different versions of the no-core shell model (NCSM) [1–5], the Gamov shell model (GSM) [6–8], the Green's function Monte Carlo method [9–12], the coupled cluster method [13,14], the hyperspherical harmonic approach [15,16], the in-medium similarity renormalization group [17], the self-consistent Green's function method [18–20], and the lattice effective field theory for multinucleon systems [21,22]. These methods are based on realistic two-nucleon (NN) and three-nucleon (NNN) potentials. These potentials could be derived from chiral effective field theory [23–25] or from nucleon scattering data by the use of the J-matrix inverse scattering method [26].

Recently, there has been a sharp increase in the attention of various theoretical groups to the problems of describing unbound nuclear states within the framework of *ab initio* approaches or methods approaching them in terms of their theoretical level. This trend is clearly reflected in the review [27].

In view of the circumstances mentioned above, the choice of the object of research and the methodology for solving the problem posed in this work seem to be timely.

It should be noted that *ab initio* approaches focused on the discussed problem are already present in the literature. Among them the methods which combine NCSM and the resonating group model (RGM), namely the no-core shell model/resonating group model (NCSM/RGM) [28] and the no-core shell model with continuum (NCSMC) [29–35], seem to be the most versatile. To describe resonances (including those undergoing multiparticle decay) using calculations of scattering phase shifts, the NCSM-based SS-HORSE method was proposed [36,37]. Some nuclear resonances can be also studied with the use of GSM [8], mentioned above.

A significant place among the methods of high-precision description of unbound states of nuclei is occupied by *ab initio* microscopic approaches, which are not, in strict definition, *ab initio*, but accurately take into account almost all dynamic properties of unbound systems. Among such approaches are, in particular, the microscopic cluster model (MCM) [38–41], the version of RGM [42] which exploits realistic NN potentials, and the fermionic molecular dynamics (FMD) [43–45]. Admittedly, all the above-mentioned approaches aimed at describing unbound states can be applied to a very limited number of nuclear resonances compare to the list of bound states whose total binding energies (TBEs)

and electromagnetic properties are described by conventional *ab initio* methods, NCSM, for example.

In our previous papers [46–48], an *ab initio* approach was developed. It allows one to calculate the asymptotic characteristics of the real decay of unbound and virtual splitting of bound states of nuclei, namely the total and partial decay widths and the asymptotic normalization coefficients (ANCs) of certain channels, respectively. This approach includes NCSM as a basic building block.

Both in the previous works and in the present one, at the first stage the A-nucleon Schrödinger equation with realistic NN potentials is solved using the basis of totally antisymmetric A-nucleon oscillator wave functions (WFs). The M-scheme, in which one-nucleon functions are characterized by the angular momentum j_i and its projection m_i, is exploited. The discussed basis is built to be complete up to the maximal total number of oscillator quanta $N_{\rm tot}^{\rm max}$. The size of the used basis in the M-scheme in our calculations is usually in the range 10^8–10^9 components. The binding energies and the WFs of the ground, excited, and resonance states of nuclei are computed. In the current paper we use the Daejeon16 potential [23], which is based on chiral effective field Theory and is well proved for calculating the spectra of nuclei with mass $A \leqslant 16$, their sizes, and other characteristics. Next, the so-called cluster channel orthogonalized functions method (CCOFM) [49–51] is applied. The procedure, as a whole, looks as follows. A basis of the orthogonalized cluster-channel WFs is built by this method, and the eigenfunctions computed in the NCSM are projected onto the functions of this basis. At the third stage, the functions obtained within the projecting procedure—so-called cluster form factors (CFFs)—are matched with the asymptotic WFs of the corresponding channels.

The most important problem determining the prospects for the application of the discussed approach as a whole is that the range of distances where solutions of the Schrödinger equation are correctly described by NCSM A-nucleon WFs expands proportionally to the size of the classically allowed area of the harmonic-oscillator potential and therefore proportionally to $[N_{\rm max}^{\rm tot}]^{1/2}$. So this range is somewhat limited. Does the CFF obtained in these calculations reproduce the shape of the asymptotic WF at intercluster distances where the nuclear interaction is negligible? The results of our studies presented in Refs. [47,48] give the answer "yes" to this question in a certain formulation of the problem of asymptotics for nuclei ^{7}Li and ^{8}Be, which are close to ^{7}He in mass, and not only for one-nucleon but even for cluster channels.

We also point out that the methodological scheme of this work includes a new element in comparison with those discussed above. For a more precise determination of the energies of the levels calculated in NCSM, the extrapolation procedure presented in Ref. [7] is used.

In Secs. II and III, we give a brief outline of the just described methodology.

In this paper we present the results of its application to the detailed theoretical study of characteristics of the ^{7}He nucleus spectrum. The choice of this object is determined by the following motivations. Despite the fact that the ^{7}He system has no bound states, there are experimental data on some of its states. The prospects for a more detailed experimental study of this object appear to be good. We have successful experience in studying nuclei of the same and close mass. The literature contains theoretical works devoted to *ab initio* studies of this object, which makes it possible to compare the results of the approaches used in them with the results of this one.

II. OUTLINE OF METHODOLOGY OF THE APPROACH

A. Total binding energy extrapolation procedure

In our previous works [46–48], it was shown that even a not too large deviation of the calculated resonance energy from the experimental one leads to a noticeable change in the calculated value of the decay width. For this reason, the use of experimental resonance energies in calculating decay widths is preferable. In the framework of theoretical study of exotic nuclei such as ^{7}He, it is much more difficult to rely on experimental results due to their incompleteness, unreliability, or absence. So, the requirements for the accuracy of calculating the level energies for this kind of problems are stringent. At present, despite broad capabilities of the shell model calculations, for neutron-rich nuclei such as ^{6}He and ^{7}He they are not fully converged. The only way out of this situation is the use of one of the extrapolation procedures. There are three widely known techniques, namely, the deep learning extrapolation tool based on artificial neural networks (ANNs) [52], or the five-parameter "Extrapolation A5" [7] or three-parameter "Extrapolation B" [53] methods. The "Extrapolation B" method adopts a three-parameter extrapolation function that contains a term that is exponential in total number of quanta above the minimum harmonic oscillator energy configuration (cutoff parameter), $N_{\rm tot}^{*\rm max}$. The "Extrapolation A5" method adopts a five-parameter extrapolation function that contains a term proportional to $\exp(-N_{\rm tot}^{*\rm max\,1/2})$ in addition to the single exponential term $\exp(-N_{\rm tot}^{*\rm max})$ used in the "Extrapolation B" method. According to paper [52] both these methods give comparable results with the ANN deep learning tool, and at the same time they are much easier. The "Extrapolation A5" method demonstrates slightly smaller deviation values, so in our work we used this method for obtaining total binding energies of ^{7}He and ^{6}He states in "infinite" shell-model basis. The extrapolation function proposed in Ref. [7] depends on the five free parameters $E_\infty, a, c, d, k_\infty$ and also on parameters $b = \sqrt{\hbar/m\omega}$, $\Lambda_i = b^{-1}\sqrt{2(N_{{\rm tot},i}^{*\rm max} + 3/2)}$, $L_i = b\sqrt{2(N_{{\rm tot},i}^{*\rm max} + 3/2)}$, $L_t = L_i + 0.54437b(L_{i=0}/b)^{-1/3}$. It is written in the following form:

$$E_{\rm state}\left(N_{{\rm tot},i}^{*\rm max}, \hbar\omega\right) = E_\infty + a\exp\left(-c\Lambda_i^2\right) + d\exp(-2k_\infty L_t). \tag{1}$$

The values of free parameters are determined for each level independently by fitting this function to theoretically calculated total binding energies. For the ^{6}He nucleus states we fit these parameters in the $\hbar\omega$ range from 10 to 25 MeV with 2.5 MeV steps and $N_{{\rm tot},i}^{*\rm max} = 4, 6, 8, 10, 12, 14$. For the ^{7}He nucleus states the $\hbar\omega$ range is from 2.5 to 22.5 MeV with 2.5 MeV steps and $N_{{\rm tot},i}^{*\rm max} = 5, 7, 9, 11, 13$. Changes in energy values resulting from this fit are presented below in Tables I–III.

TABLE I. TBEs and excitation energies (MeV) of the lowest ^{6}He nucleus states ($T = 1$).

J^π	$E_{\text{exp.}}$ [72]	$E_{\text{th.}}^{\text{lim.}}$	$E_{\text{th.}}^{\text{extr.}}$	$E_{\text{exp.}}^{*}$	$E_{\text{th.}}^{*\text{lim.}}$	$E_{\text{th.}}^{*\text{extr.}}$
0_1^+	29.269	29.239	29.397	0	0	0
2_1^+	27.472	27.199	27.533	1.797	2.040	1.864
2_2^+	23.7	24.161	25.856	5.6	5.087	3.541

B. Cluster channel orthogonalized basis constructing

The method of construction of the basis of the WFs of cluster channels is described in detail in our previous works [46–51], so here we demonstrate only its key points.

Let us consider translationally invariant A-nucleon WFs of arbitrary two-fragment decay channel basis corresponding to the separation $A = A_1 + A_2$:

$$\Psi_{A,nlm}^{\{k_1,k_2\}} = \hat{A}\left\{\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}\varphi_{nlm}(\vec{\rho})\right\}_{J_c,M_{J_c},M_J T}, \tag{2}$$

where $\hat{A}$ is the antisymmetrizer, $\Psi_{A_i}^{\{k_i\}}$ is a translationally invariant internal WF of the fragment labeled by a set of quantum numbers $\{k_i\}$, and $\varphi_{nlm}(\vec{\rho})$ is the WF of the relative motion. The channel WF as a whole is labeled by the set of quantum numbers c_κ that includes $\{k_1\}$, $\{k_2\}$, l, J_c, M_{J_c}, M_J, T, where J_c is the channel spin. The basic idea of the method is to represent each function of the cluster basis as a linear combination of the functions of the M-scheme. To do that, function (2) is multiplied by the function of the center-of-mass (CM) zero vibrations $\Phi_{000}(\vec{R})$. Then the transformation of WFs caused by changing from $\vec{R}, \vec{\rho}$ to $\vec{R}_1, \vec{R}_2$ coordinates—the different-mass Talmi-Moshinsky-Smirnov transformation [54]—is performed and WF (2) takes the form

$$\begin{aligned}&\Phi_{000}(\vec{R})\Psi_{A,nlm}^{\{k_1,k_2\}}\\ &\quad= \sum_{N_i,L_i,M_i}\left\langle\begin{matrix}000\\ nlm\end{matrix}\middle|\begin{matrix}N_1,L_1,M_1\\ N_2,L_2,M_2\end{matrix}\right\rangle\\ &\qquad\times\hat{A}\left\{\Phi_{N_1,L_1,M_1}^{A_1}(\vec{R}_1)\Psi_{A_1}^{\{k_1\}}\Phi_{N_2,L_2,M_2}^{A_2}(\vec{R}_2)\Psi_{A_2}^{\{k_2\}}\right\}_{J_c,M_{J_c},M_J T}.\end{aligned} \tag{3}$$

The key technical procedure of the method is to transform each of the two products of the internal WF of the fragment by the function of nonzero oscillations of its CM into a superposition of Slater determinants (SDs),

$$\Phi_{N_i,L_i,M_i}^{A_i}(\vec{R}_i)\Psi_{A_i}^{\{k_i\}} = \sum_k X_{N_i,L_i,M_i}^{A_i(k)}\Psi_{A_i(k)}^{\text{SD}}. \tag{4}$$

It is the possibility of implementing this procedure that imposes restrictions on the list of cluster channels available for research within the framework of this method. Quantity $X_{N_i,L_i,M_i}^{A_i(k)}$ is called a cluster coefficient (CC). The technique of using these objects is presented in detail in [55]. Various approaches to calculating these coefficients, convenient in various special cases, are presented in papers [56–61] In the present work we use the formalism based on the method of the second quantization of the oscillator quanta described in detail in [47,48]. As a result of these transformations, the antisymmetrized product of functions in the right-hand side of expression (3) also turns out to be a superposition of SDs.

It should be noted that WFs of cluster-channel basis terms (2) of one and the same channel c_κ characterized by the pair of internal functions $\Psi_{A_1}^{\{k_1\}}$, $\Psi_{A_2}^{\{k_2\}}$ and extra quantum numbers l, J_c, J, M_J, T (i.e., the vector coupling is meant here), briefly denoted as $\Psi_{A,n}^{c_\kappa}$, are non-normalized due to the properties of the antisymmetrization operator and, with rare exceptions, nonorthogonal. Creation of orthonormalized basis functions of a separate channel c_κ is performed by the diagonalization of the overlap kernel matrix

$$\begin{aligned}||N_{nn'}|| &\equiv \left\langle\Psi_{A,n'}^{c_\kappa}|\Psi_{A,n}^{c_\kappa}\right\rangle\\ &= \langle\Phi_{00}(\vec{R})\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}\varphi_{nl}(\rho)\\ &\quad\times|\hat{A}^2|\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}\varphi_{n'l}(\rho)\Phi_{00}(\vec{R})\rangle.\end{aligned} \tag{5}$$

The eigenvalues and eigenvectors of this overlap kernel are the same in the shell-model and translationally invariant representations and can be written as

$$\varepsilon_{\kappa,k} = \left\langle\hat{A}\left\{\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}f_l^k(\rho)\right\}\middle|\hat{1}|\hat{A}\left\{\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}f_l^k(\rho)\right\}\right\rangle, \tag{6}$$

$$f_l^k(\rho) = \sum_n B_{nl}^k\varphi_{nl}(\rho). \tag{7}$$

On the other hand, the WFs of the orthonormalized channel basis c_κ

$$\Psi_{A,kl}^{\text{SD},c_\kappa} = \varepsilon_{\kappa,k}^{-1/2}\left|\Phi_{00}(\vec{R})\hat{A}\left\{\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}f_l^k(\rho)\right\}\right\rangle. \tag{8}$$

turn out to be represented in the form of the superposition of the SDs. The basis of such functions is complete in the sense that a function of this channel,

$$\Psi^{\text{SD},c_\kappa} = \Phi_{00}(\vec{R})\hat{A}\{\Psi_{A_1}^{\{k_1\}}\Psi_{A_2}^{\{k_2\}}\Phi(\rho)Y_l(\Omega)\}_{J_c,M_J T}, \tag{9}$$

including arbitrary WF $\Phi(\rho)$ can be represented as a superposition of such WFs.

These functions can be introduced into the basis of the NCSM calculations for its expansion towards large $N_{\max}^{\text{tot}}$ or used to find the cluster characteristics of nuclear states calculated in the NCSM basis, both extended due to their inclusion, and in the traditional one.

TABLE II. TBEs (MeV) and resonance energies (keV) of ^{7}He nucleus states ($T = 3/2$).

J^π	$E_{\text{exp.}}$ [72]	$E_{\text{th.}}^{\text{lim.}}$	$E_{\text{th.}}^{\text{extr.}}$	$E_n^{\text{extr.}}$	E_n [30]	E_n [8]	E_n [37]
$3/2_1^-$	28.83[a]	28.625	28.850	547	710	390	240
$1/2_1^+$		26.057	27.701	1696			
$1/2_1^-$		25.864	27.079	2318	2390		2700
$5/2_1^-$	25.91[b]	24.743	25.960	3437	3130	3470	3630
$3/2_1^+$		24.115	25.905	3492			4100
$5/2_1^+$		23.937	25.833	3564			4200
$3/2_2^-$		23.966	25.455	3921			

[a] $E_n^{\text{exp.}} = 430$ keV [73].
[b] $E_n^{\text{exp.}} = 3360$ keV [72].

C. Cluster characteristics in *ab initio* calculations

In this work, the energies of the levels of the initial and resulting nuclear states are calculated in the usual M-scheme

TABLE III. Resonance energies, decay widths of open channels (keV), and channel spectroscopic factors of ^{7}He nucleus states for "new" definitions of the CFF and SF. Numeric subscript denotes the value $N_{\rm tot}^{*\rm max}$.

J^π (^{7}He)	J^π (^{6}He)	$E_n^{\rm lim.}$	$E_n^{\rm extr.}$	$l(S)$	SF	$\Gamma^{\rm lim.}$	$\Gamma_{11}^{\rm extr.}$	$\Gamma_{13}^{\rm extr.}$	$\Gamma_{\rm tot}$ [30]	$\Gamma_{\rm tot}$ [8]	$\Gamma_{\rm tot}$ [37]	$\Gamma_{\rm tot}^{\rm exp.}$
$3/2_1^-$	0_1^+	614	547	1(1/2)	0.730	387	336	334[a]	300	178	110	182
$1/2_1^+$	0_1^+	3184	1696	0(1/2)	0.844	3670	2670	2670				
$1/2_1^-$	0_1^+	3375	2318	1(1/2)	0.814	2440	1940	1850	2890		4300	750[b]
	2_1^+	1335	454	1(3/2)	0.509		253	221				
				3(5/2)	0.21×10^{-3}	812	0.47 eV	0.46 eV				
$5/2_1^-$	0_1^+	4496	3437	3(1/2)	0.37×10^{-3}	110 eV	56 eV	52 eV				
	2_1^+	2606	1573	1(3/2)	0.420	1366	980	881				
				1(5/2)	0.758	1783	1200	1060	1070	2300	1360	1990
$3/2_1^+$	0_1^+	5124	3492	2(1/2)	0.041	125	84.4	83.4				
	2_1^+	3084	1628	0(3/2)	0.752	3430	2590	2490				
				2(3/2)	0.051	58	22.3	17.9			4400	
$5/2_1^+$	0_1^+	5302	3564	2(1/2)	0.126	382	285	258				
	2_1^+	3262	1700	0(5/2)	0.704	3100	2210	2240				
				2(5/2)	0.019	29	13.7	11.1			5000	
	2_2^+	224	23	0(5/2)	0.001	9.3	3.33	2.98				
				2(3/2)	0.012	38.6 eV	0.15 eV	0.14 eV				
$3/2_2^-$	0_1^+	5273	3921	1(1/2)	0.069	276	249	229				
	2_1^+	3233	2057	1(3/2)	0.190	556	408	399				
				1(5/2)	0.470	1486	1150	1060				
	2_2^+	196	380	1(3/2)	0.268	44	126	108				

[a]For the experimental value of $3/2_1^-$ state resonance energy 430 keV, computed value $\Gamma = 250$ keV.
[b]There are other experimental results: 2.0 MeV [74], 10.0 MeV [75], or 1.0 MeV [76].

NCSM model, and the basis described above is used to calculate the cluster characteristics of decay channels: the cluster form factor and the spectroscopic factor (SF).

The CFF $\Phi_A^{c_\kappa}(\rho)$ describes the relative motion of subsystems in A-nucleon configuration space. In the modern (so-called "new") definition, CFF is the following overlap:

$$\Phi_A^{c_\kappa}(\rho) = \left\langle \Psi_A | \hat{A}\left\{ \Psi_{A_1}^{\{k_1\}} \Psi_{A_2}^{\{k_2\}} \hat{N}^{-1/2} \frac{\delta(\rho-\rho')}{\rho'^2} Y_l(\Omega) \right\}_{J_c,M_J T} \right\rangle, \tag{10}$$

where Ψ_A is the WF of the initial nucleus—the NCSM solution of the A-nucleon Schrödinger equation—and $\hat{N}$ is the norm operator of the generalized function of the cluster channel, which takes the form:

$$\hat{N}(\rho',\rho'') = \left\langle \hat{A}\left\{ \Psi_{A_1}^{\{k_1\}} \Psi_{A_2}^{\{k_2\}} \frac{\delta(\rho-\rho')}{\rho'^2} Y_l(\Omega) \right\}_{J_c,M_J T} \right| \times \left| \hat{A}\left\{ \Psi_{A_1}^{\{k_1\}} \Psi_{A_2}^{\{k_2\}} \frac{\delta(\rho-\rho'')}{\rho''^2} Y_l(\Omega) \right\}_{J_c,M_J T} \right\rangle. \tag{11}$$

Representation of the generalized function of the relative motion in the form of an expansion in terms of oscillator functions,

$$[\delta(\rho-\rho')/\rho'^2] Y_{lm}(\Omega) = \sum_n \varphi_{nlm}(\vec{\rho})\varphi_{nlm}(\vec{\rho}'), \tag{12}$$

first, reduces the norm operator to the overlap kernel matrix (5) and, second, makes it possible to write the CFF in the form

$$\Phi_A^{c_\kappa}(\rho) = \sum_k \varepsilon_{\kappa,k}^{-1/2} \langle \Psi_A | \hat{A}\{\Psi_{A_1}^{\{k_1\}} \Psi_{A_2}^{\{k_2\}} f_l^k(\rho')\}\rangle f_l^k(\rho). \tag{13}$$

After that the CCF can be expressed in the form of an expansion in the oscillator basis using the techniques presented above:

$$\Phi_A^{c_\kappa}(\rho) = \sum_k \varepsilon_{\kappa,k}^{-1/2} \sum_{n,n'} B_{nl}^k B_{n'l}^k C_{AA_1A_2}^{n'l} \varphi_{nl}(\rho). \tag{14}$$

The coefficient contained in this expression has the form

$$C_{AA_1A_2}^{nl} = \left\langle \hat{A}\left\{ \Psi_{A_1}^{\{k_1\}} \Psi_{A_2}^{\{k_2\}} \varphi_{nl}(\rho) \right\} | \Psi_A \right\rangle = \left\langle \Psi_{A,nl}^{\rm SD,c_\kappa} \middle| \Phi_{000}(R) | \Psi_A \right\rangle = \left\langle \Psi_{A,nl}^{\rm SD,c_\kappa} \middle| \Psi_A^{\rm SM} \right\rangle. \tag{15}$$

This coefficient is traditionally called the spectroscopic amplitude (SA). A number of very diverse methods of its calculation depending on the masses of the initial nuclei and fragments were described in [55,58–60]. All of them are based on the CCs formalism.

The SF is defined as the norm of CFF; for the discussed channel c_κ it can be written as

$$S_l^{c_\kappa} = \int |\Phi_A^{c_\kappa}(\rho)|^2 \rho^2 d\rho = \sum_k \varepsilon_k^{-1} \sum_{nn'} C_{AA_1A_2}^{nl} C_{AA_1A_2}^{n'l} B_{nl}^k B_{n'l}^k. \tag{16}$$

The "old" definition of CFF, commonly used for many years, is similar to the definition of the "new" one (10) but its expression does not contain the normalization operator $\hat{N}^{-1/2}$:

$$\tilde{\Phi}_A^{c_\kappa}(\rho) = \left\langle \Psi_A | \hat{A}\left\{ \Psi_{A_1}^{\{k_1\}} \Psi_{A_2}^{\{k_2\}} \frac{\delta(\rho-\rho')}{\rho'^2} Y_l(\Omega) \right\}_{J_c,M_J T} \right\rangle, \tag{17}$$

Its expression through the spectroscopic amplitudes looks simpler than expression (14):

$$\tilde{\Phi}_A^{c_\kappa}(\rho) = \sum_n C^{nl}_{AA_1A_2}\varphi_{nl}(\rho). \tag{18}$$

The "old" SF is defined by the first equality of expression (16), and its calculation formula reads

$$\tilde{S}_l^{c_\kappa} = \sum_n \left(C^{nl}_{AA_1A_2}\right)^2. \tag{19}$$

We note that an alternative method for calculating CFFs in the NCSM (and obviously SFs, if necessary) that does not use the formalism of cluster coefficients is presented in the literature [62]. In this paper we use the "old" definitions of CFF and SF.

D. About the "old" and "new" definitions of CFF

The definitions of the CFF (10) and SF (16) are completely equivalent to those proposed in [63] (in this work they were called "new" spectroscopic factor and CFF as opposed to "old" ones). In contrast to the traditional definition, the new CFF and SF characterize the total contribution of orthonormalized cluster components to the solution of the Schrödinger equation describing an A-nucleon system. Reasons for the necessity of their use to describe nuclear decay and reactions can be found in [64,65]. In [66,67], it was shown that the correct definitions eliminate a sharp contradiction between theoretically calculated values of the cross sections for reactions of knockout and transfer of α clusters and the experimental data. For example the use of the "old" definition of the SF, that is, the loss of normalization of the asymptotic wave function, leads to an underestimation of the cross section of knockout reaction ^{40}Ca$(p, p'\alpha)\,^{36}$Ar by more than 20 times, while using the "new" definition gives a result close to the measured one. Using this fact, one can talk about "experimental" confirmation of validity of the latter definition.

Nevertheless it is pertinent to note one significant circumstance. For the decay of the ^{7}He nucleus studied here, only neutron channels are relevant. In contrast to cluster channels, the "old" definition is still often used for form factors and SFs of single-nucleon channels. This is due to a well-established tradition and the fact that the numerical differences in the results of calculations employing the "old" and "new" definitions are usually not large for single-nucleon channels, in contrast to cluster channels. In some instances, the calculations within the framework of the "old" approach may turn out to be closer to the experimental ones because of the influence of some other used approximations. There were cases when the same research team in earlier works had preferred to use the "old" [40] definition and in later works preferred to use the "new" [41] definition. Taking into account all these facts, it seems reasonable to calculate the widths of one-nucleon decay of the states of the ^{7}He nucleus, using both the "new" and "old" system of the definitions of the CFF and SF, to compare the results and to give a detailed comment on this problem as a whole.

In principle, within the framework of the procedure for matching the CFF with the asymptotic wave function of the channel at large distances, both the "new" and "old" versions of CFF can be used, since they coincide at rather large distances (see below). However, within the framework of NCSM, it is difficult to achieve a correct description of the wave functions in the region where nontrivial permutations contained in the antisymmetrization operator do not play an appreciable role. For this reason, it becomes necessary to use more accurate (i.e., correctly describing the properties of the channel at distances which are well described by means of NCSM) multinucleon functions as the asymptotic wave functions of the channel. The most convenient approach to solving the problem of "short-distance asymptotics" would be a direct use of the RGM wave functions as asymptotic ones. Obviously, this conclusion applies to any approach in which functions—solutions of a multinucleon problem—need to be combined with asymptotic ones. The RGM wave function has the form

$$F_l^{A_1+A_2} = |\hat{A}\{\Psi_{A_1}\Psi_{A_2}\phi_l(\rho)Y_{lm}(\Omega_\rho)\}_{J_c,M_JT}\rangle. \tag{20}$$

where $\phi_l(\rho)$ is the sought-for function. It is important to note that the condition of normalization of the RGM multinucleon wave function of the channel (for example let us consider one-open-channel RGM problem)

$$\left\langle F_l^{A_1+A_2} \mid F_l^{A_1+A_2}\right\rangle = \begin{pmatrix} 1 \\ \delta(E-E'),\ \delta(k-k'),\ \text{etc.}\end{pmatrix} \tag{21}$$

implies the condition for the relative motion function contained in the RGM multi-nucleon wave function (20):

$$\left\langle \hat{N}_\rho^{1/2}\phi_l(\rho) \mid \hat{N}_\rho^{1/2}\phi_l(\rho)\right\rangle = \begin{pmatrix} 1 \\ \delta(E-E'),\ \delta(k-k'),\ \text{etc.}\end{pmatrix}. \tag{22}$$

As a result, it is not function $\phi_l(\rho)$ but function

$$\chi_l(\rho) = \hat{N}_\rho^{1/2}\phi_l(\rho) \tag{23}$$

that has the correct two-body asymptotics, and the RGM function can be written in the form

$$F_l^{A_1+A_2} = |\hat{A}\{\Psi_{A_1}\Psi_{A_2}N^{-1/2}\chi_l(\rho)Y_{lm}(\Omega_{\rho'})\}_{J_c,M_JT}\rangle. \tag{24}$$

To compare multinucleon wave functions in the region where both NCSM and RGM descriptions are valid, it is required to compare quantities that have the same meaning, for example, their projections on a certain hypersurface. This hypersurface may be defined in the usual form

$$|\hat{A}\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_{\rho'})\}_{J_c,M_JT}\rangle. \tag{25}$$

As a result, the expression of the "old" CFF (17) is obtained.

The same projecting procedure for the RGM wave function leads to a nontrivial result:

$$\left\langle F_l^{A_1+A_2}\middle|\hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_{\rho'})\right\}_{J_c,M_JT}\right\rangle$$
$$= \hat{N}_\rho\phi_l(\rho) = \hat{N}^{1/2}\chi_l(\rho). \tag{26}$$

Comparison of these two projections at "short-range asymptotic" (sas) distances leads either to the appearance of the operator in the function describing this asymptotics,

$$\tilde{\Phi}_A^{c_\kappa}(\rho_{\text{sas}}) = \hat{N}^{1/2}\chi_l(\rho_{\text{sas}}), \tag{27}$$

or to the appearance of the operator in the expression for the projection of the NCSM solution [see expression (10)],

$$\Phi_A^{c_\kappa}(\rho_{\rm sas}) = \chi_l(\rho_{\rm sas}). \quad (28)$$

Evidently the results of two versions of comparison are equivalent. Within the framework of the just presented procedure of description of the internal and asymptotic solutions, it is impossible to eliminate the indicated integral transformation using any identical rearrangement of the formulas.

It must be pointed out that the approach we use includes not RGM wave functions (20) but simpler, approximate, asymptotic ones. The cause is that, in the case where NCSM solutions are used as the functions of fragments, the calculation of RGM wave functions turns out to be even more difficult than the direct NCSM calculations. Therefore, the following approximations are introduced into the original RGM-based formalism:

(1) As is usual for the description of asymptotics, it is assumed that the effect of strong interaction on the wave function in a given distance range is negligible.
(2) The Coulomb interaction between the protons of the fragments is replaced by the Coulomb interaction of point charges. Numerous studies demonstrate the high reliability of this approximation.

The antisymmetric A-nucleon function built in this way is projected using procedure (26) with $\chi_l(\rho)$ replaced by $G_l(\eta, \rho)$ and compared with the "old" CFF; or, when projecting using a procedure similar to (26), but onto normalized function

$$\left|\hat{A}\left\{\Psi_{A_1}\Psi_{A_2}N^{-1/2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_{\rho'})\right\}_{J_c,M_JT}\right\rangle, \quad (29)$$

a comparison is made with the "new" CFF. The results of this comparison, as we indicated above, are equivalent. It should be emphasized that, within the framework of the indicated approximations, the integral transformation $N^{1/2}$ or $N^{-1/2}$ does not disappear.

Exchange kernel $\hat{N}(\rho', \rho'')$ degenerates into unity at large distances; therefore, the replacement $\hat{N}(\rho', \rho'') \to 1$ resulting in the "old" CFF formula at such distances is quite adequate. But this replacement is an approximation to the just presented "new" version for any example and for any area of the space anyway. On the other hand, the "new" approach is also approximate.

Nevertheless, it should be noted in advance that, with regard to the predictions of the decay widths of ^{7}He states presented below, we give preference to those obtained within the framework of the "new" scheme. The issue is that, in the standard two-body collision theory, asymptotic regular and irregular wave functions [Bessel $j_l(\rho)$, Coulomb $G_l(\eta, \rho)$, etc.] are given over the entire range range of ρ. The "new" approach allows one to construct multinucleon asymptotic functions with the same properties. At the same time, an attempt to describe the multinucleon asymptotic wave function

$$F_l^{as} = |\hat{A}\{\Psi_{A_1}\Psi_{A_2}[j_l(\rho), \text{ or } G_l(\eta, \rho), \text{ etc.}]Y_{lm}(\Omega_{\rho'})\}\rangle \quad (30)$$

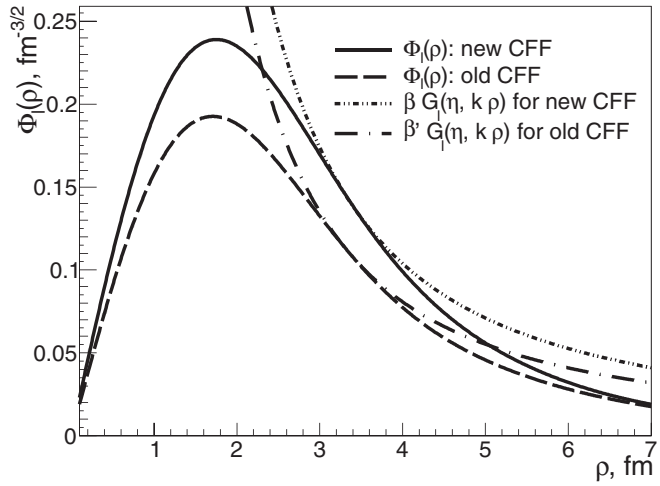

FIG. 1. Illustration of the conditions for matching CFFs in the "new" and "old" definitions with an asymptotic function.

over the entire range of ρ leads to violation of the relation (21). Because of this, one should be treated with caution the results obtained in the framework of "old" matching procedure even at not very large distances.

E. Application of CFFs for studies of decay characteristics of nuclear states

Both SFs and CFFs are objects used in theoretical studies on nuclear decays and reactions. Evidently CFF is the more informative characteristic of a cluster channel. In the current work the obtained CFFs (both "new" and "old") are exploited for computing the widths of resonances. The norms of these values—SFs—are used to distinguish the main channels against the background of a multitude of other ones, existence of which practically do not affect the results of experiments.

As in our previous works [47,48], we use the procedure of matching the CFF with the asymptotic wave function of the corresponding channel. The results of our studies presented in these works demonstrate that the CFF in its new definition allows matching with the asymptotic wave function at relatively small distances, where the nuclear interaction is negligibly weak, but exchange effects generated by the antisymmetry of the total channel wave function and manifested through the exchange terms of the overlap kernel are still not negligible. "Switch off" the effects of antisymmetrization—the vanishing of the matrix elements of the permutation operators included in the antisymmetrizer—occurs at larger distances. Here we demonstrate one more illustration of the indicated behavior of CFFs and asymptotic wave functions.

In Fig. 1 the pattern of matching "old" and "new" CFFs of the decay channel of the ground state of the ^{7}He nucleus into channel ^{6}He $+n$ with an irregular function $G_l(\eta, \rho)$ is presented. This figure clearly shows the range of distances $3.1 \leqslant \rho \leqslant 3.7$ fm at which NCSM calculations reproduce well the Coulomb function $G_l(\eta, \rho)$. Thus, on the one hand, the model adequately describes this area of space and, on the other hand, the nuclear interaction is negligibly weak: CFFs behave almost the same as the asymptotic function. It

TABLE IV. Resonance energies, decay widths of open channels (keV), and channel spectroscopic factors of ^{7}He nucleus states for "old" definitions of the CFF and SF. Numeric subscript denotes the value $N_{\rm tot}^{*\rm max}$.

J^π (^{7}He)	J^π (^{6}He)	$E_n^{\rm lim.}$	$E_n^{\rm extr.}$	$l(S)$	SF	$\Gamma^{\rm lim.}$	$\Gamma_{11}^{\rm extr.}$	$\Gamma_{13}^{\rm extr.}$
$3/2_1^-$	0_1^+	614	547	1(1/2)	0.472	234	202	202[a]
$1/2_1^+$	0_1^+	3184	1696	0(1/2)	0.882	3860	2860	2810
$1/2_1^-$	0_1^+	3375	2318	1(1/2)	0.814	2420	1920	1820
	2_1^+	1335	454	1(3/2)	185	59.8	52.2	
			3(5/2)	0.129	0.21×10^{-3}		0.47 eV	0.46 eV
$5/2_1^-$	0_1^+	4496	3437	3(1/2)	0.37×10^{-3}	114 eV	54.9 eV	51.3 eV
	2_1^+	2606	1573	1(3/2)	0.089	297	204	190
				1(5/2)	0.828	1910	1315	1198
$3/2_1^+$	0_1^+	5124	3492	2(1/2)	0.042	127	86.6	84.5
	2_1^+	3084	1628	0(3/2)	0.773	3520	2660	2560
				2(3/2)	0.046	51.2	19.7	16.0
$5/2_1^+$	0_1^+	5302	3564	2(1/2)	0.137	403	304	270
	2_1^+	3262	1700	0(5/2)	0.736	3360	2450	2430
				2(5/2)	0.019	30.0	13.8	11.2
	2_2^+	224	23	0(5/2)	0.003	7.09	2.61	2.27
				2(3/2)	0.012	40.68 eV	0.16 eV	0.14 eV
$3/2_2^-$	0_1^+	5273	3921	1(1/2)	0.060	257	213	215
	2_1^+	3233	2057	1(3/2)	0.311	856	648	593
				1(5/2)	0.512	1570	1240	1110
	2_2^+	196	380	1(3/2)	0.213	32.6	94.5	81.1

[a]For the experimental value of $3/2_1^-$ state resonance energy 430 keV, computed value $\Gamma = 146$ keV.

is interesting that in the discussed example for the "new" and "old" CFFs the range of reproduction of the asymptotic function is practically the same. At the same time, the absolute values of the the matching parameter β differ markedly. This leads to a significant difference in the decay widths of a given state, presented below in Tables III and IV.

The figure also shows the influence of the exchange terms of the overlap kernel. Indeed, in the region where this influence turns out to be negligible, the "new" and "old" CFFs for the same solution to the multinucleon problem coincide. One may see that this area begins at about the value $\rho \simeq 7.0$ fm. To obtain, within the framework of the "old" approach, results coinciding with those obtained in the "new" one, it would be necessary to achieve a satisfactory description of this spatial domain using NCSM. This is precisely what constitutes the advantages of the "new" definition as applied to the study of nuclear decays. This property is very important for dealing with NCSM CFFs.

It must be emphasized that the argument just given is purely mathematical in nature. It only demonstrates the area where the influence of the overlap kernel ends, but evidently in this area the results obtained in NCSM do not correspond to physical reality.

So, in the discussed approach, a direct matching procedure described in classical textbooks of quantum mechanics is applied to calculate the widths of narrow resonances. For such resonances or, more precisely, for those of them whose small width is due to a low penetrability of the potential barrier, we used a very compact procedure proposed in [68]. This procedure is applicable because for such resonances there is a sufficiently wide range of distances in which the nuclear attraction is already switched off and at the same time the potential barrier is high enough. At any inner point ρ_{in} of this area, the relationship between the regular and irregular solutions of the two-body Schrödinger equation in the Wentzel-Kramers-Brillouin (WKB) approximation has the form

$$F_l(\rho_{\rm in})/G_l(\rho_{\rm in}) = P_l(\rho_{\rm in}) \ll 1, \tag{31}$$

where $P_l(\rho_{\rm in})$ is the penetrability of the part of the potential barrier that is located between the point $\rho_{\rm in}$ and the outer turning point. The smallness of this penetrability is the condition of applicability of the approximation in which the contribution of the regular solution can be neglected. To determine the position of the matching point ρ_m of the CFF (either "old" or "new") and the irregular wave function in this range, we use the condition of equality of the logarithmic derivatives,

$$\frac{d\Phi_A^{c_\kappa}(\rho)/d\rho}{\Phi_A^{c_\kappa}(\rho)} = \frac{dG_l(\rho)/d\rho}{G_l(\rho)}. \tag{32}$$

Comparison of the values of the CFF and function $G_l(\rho)$ in the matching point allows one to determine the amplitude of the channel wave function in the asymptotic region, which takes the form $\beta G_l(\rho)$, where

$$\beta = \Phi_A^{c_\kappa}(\rho_m)/G_l(\rho_m). \tag{33}$$

As a result, the decay width is given by the expression

$$\Gamma = \frac{\hbar^2}{\mu k}\left[\frac{\Phi_A^{c_\kappa}(\rho_m)}{G_l(\rho_m)}\right]^2. \tag{34}$$

To make the list of the properties of the states of a certain nucleus broader, large-width resonances are considered too. If the resonance is wide and so the penetrability $P_l(\rho_{\rm in})$ (31) is not small, the width of this resonance is calculated using the

simple version of the conventional R-matrix theory:

$$\Gamma = \frac{\hbar^2}{\mu k_0}\left[F_l^2(\rho_m) + G_l^2(\rho_m)\right]^{-1}\left[\Phi_A^{c_\kappa}(\rho_m)\right]^2. \quad (35)$$

Naturally the use of this version leads to reduction in accuracy of calculation results. However, this technique seems natural for describing experimental decay widths, since, when extracting their values from the cross sections of resonance processes, different versions of the R-matrix theory of nuclear reactions are also used. Besides that, the accuracy of the data concerning large-width resonances, both decay widths and excitation energies, extracted from various experiments, is also very limited. It is important to note that, in contrast to calculating the total fragmentation width of any resonant state, finding the partial widths of its decay into various channels requires calculating the amplitudes of channel wave functions in the asymptotic region.

In this paper the proposed approach is utilized to study the spectrum of resonance states of the ^{7}He nucleus and the partial decay widths of these states.

An important point is that the Daejeon16 potential [23] is exploited as a model of NN interaction in the current work. It is built using the next-to-next-to-next-to leading order (N3LO) limitation of chiral effective field theory [69] softened by a similarity renormalization group (SRG) transformation [70]. This potential is designed to calculate all kinds of characteristics of nuclei with masses $A \leqslant 16$. It was tested in the framework of large-scale computations of the total binding energies, excitation energies, radii, moments of nuclear states, and the reduced probabilities of electromagnetic transitions. These tests demonstrated that such characteristics are, in general, reproduced well. Besides that, this choice is supported by our previous studies of the asymptotic characteristics of the cluster channels of light nuclei, in which other NN potentials were also involved in the analysis. The results of these studies are presented in Refs. [46,49,51].

III. RESULTS OF THE CALCULATIONS AND DISCUSSION

In this work, we calculate the total binding energies (TBEs), excitation energies, as well as decay energies and widths of the levels of the ^{7}He system; and also calculate TBEs of the lower levels of the fragment ^{6}He nucleus necessary for that. The NCSM calculations were carried out with the use of newest version of BIGSTICK shell model code [71]. The basis is limited by the value of cutoff parameter $N_{\rm tot}^{*\rm max} = 13$, i.e., maximal total number of oscillator quanta $\mathrm{N}_{\rm tot}^{\rm max} = 16$. For a limited basis, the optimal value of the oscillator parameter turns out to be $\hbar\omega = 12.5$ MeV.

Let us consider, first of all, the calculated values of the total binding energy and the excitation energy of the ^{6}He nucleus. They are presented in Table I. Despite the fact that the ground state of the ^{6}He nucleus is a classic example of a nuclear system with a two-neutron halo, the experimental value of its total binding energy-TBE ($E_{\rm exp.}$) is well reproduced in the shell-model calculations on a limited basis (values of such type are denoted by symbol $E_{\rm th.}^{\rm lim.}$). The extrapolated TBE value $E_{\rm th.}^{\rm extr.}$ deviates from the experimental result even somewhat more significantly, but, in any case, the deviation does not exceed 0.4%. The magnitude of the absolute deviation, which is equal to 128 keV, also appears to be satisfactory. This is not the case for the first excited state, for which the deviation of the TBE calculation result from the experimental one is 273 keV. This fact, and especially the significant overestimation of the excitation energy for this level, demonstrates the need to use an extrapolation procedure. As for the second excited state of the ^{6}He nucleus, its gigantic width ($\approx$12 MeV), obviously, does not allow simultaneously evaluating its excitation energy from the experiment with an accuracy better than several MeV.

An even more expressive pattern is observed for the lower resonance level $3/2_1^-$ of the ^{7}He nucleus and the higher resonance $5/2_1^-$, for which experimental data have been obtained so far. For the former one the calculations on the limited basis underestimate TBEs by 200 keV; for the latter case, this underestimation exceeds 1 MeV. In contrast, the extrapolated results are in good agreement with the measurement ones. The differences between the results obtained on the limited basis and the extrapolation results for other levels that have not yet been discovered also exceed 1 MeV.

In Table II and its caption, the energies of neutron decay of the ^{7}He nucleus into the $^6\mathrm{He}_{gs}$ channels presented in published papers are also given, both calculated and obtained in experiments. The experimental data are better described by the GSM [8], although the energy of higher lying level $5/2_1^-$ is reproduced rather well in all approaches. Our data correlate fairly well with the data obtained within the framework of NCSMC [30]. It is interesting to compare the resonance energies which are calculated in the current work with the same values obtained in the framework of approach SS-HORSE [37], since this approach, first, is also based on NCSM and, second, this work also uses the NN potential Daejeon16. This comparison shows that the use of the extrapolation procedure lowers the values of the resonance energies more than the simultaneous use of a higher value of the cutoff parameter $N_{\rm tot}^{*\rm max}$ and the inclusion of the continuous spectrum in the calculations performed by the SS-HORSE method for all states of the ^{7}He nucleus, excluding the ground state.

The results of our calculations of the decay widths of ^{7}He nucleus resonances are presented in Tabsles III and IV for "new" and "old" definitions of CFF and SF. A wide list of decay channels was analyzed, including decays into channels in which fragment ^{6}He is in 2_1^+ or 2_2^+ excited state. Because of the gigantic decay width of the latter state, the results of calculations of the characteristics of the channels in which it is contained are physically of little interest. By including the channels containing this state in the analysis, we, first, demonstrate the capabilities of the developed method and, second, we follow the tradition laid down in Ref. [30].

It is natural to start the discussion by assessing the accuracy of the method used. Comparison of the results contained in the third and fourth and, respectively, in the seventh and ninth columns of Tables III and IV demonstrates a strong dependence of the decay width of a certain state on its energy. Roughly assessing the tendency, one may say that the relative change in the decay energy leads to a proportional change in the width. From this point of view, the relative error in computing the decay energy value of a particular state is reproduced when calculating its width. The relative decay

energy errors of *ab initio* computations on a limited basis for highly excited levels are quite large because decay energy values themselves are the difference between TBEs. Therefore, the introduction of approaches that refine the energy values, in particular the extrapolation method, is extremely important.

The size of the used basis affects the behavior of the CFF curves, which also exerts some influence on the magnitude of the decay width; however, this influence is somewhat weaker in comparison with the effect produced by the change in the decay energy. The scale of the change in the decay widths with a change in the size of the basis from $N_{\text{tot}}^{*\max} = 11$ to $N_{\text{tot}}^{*\max} = 13$ is illustrated by columns 8 and 9 of Tables III and IV. The computation results show that convergence has actually been achieved for a significant part of the examples. Changes in decay widths for other examples are in the region of 10%. Only in two cases does this change exceed 20%. There is no doubt that this accuracy is sufficient for planning experiments aimed at studying the spectra of exotic nuclei.

It is interesting to compare the results of calculating the decay widths and SFs in the "new" and "old" approaches. It should be noted, first of all, that the decay width ratios $\Gamma_{new}/\Gamma_{old}$ are almost independent of the choice of the basis dimensionality and are close to the corresponding SF ratios. Large values of these ratios are typical for channels with the moment of relative motion $l = 1$. Already for the width of the lowest ^{7}He resonance the ratio resulting from these two different approaches to descriptions of CFF is about 1.7. For two examples of such channels, these ratios exceed 4. For $l = 2, 3$ these ratios are close to unity, and only in one such case is this ratio close to 1.1. *S*-wave channels occupy an intermediate position in this respect, the characteristic differences between the calculation results in the two discussed approaches amount to 10–30%. Such a relationship between the characteristic results for channels with different values of l looks natural, since the main components of the wave functions of the states of residual nucleus ^{6}He are those that, as a rule, contain single-nucleon p orbitals; the nonorthogonality of these orbitals to the wave function of the relative motion of neutron in the corresponding channel leads to the strong renormalization of the latter function by the overlap kernel.

Comparison of the obtained results with experimental data leads to the following conclusions. TBEs of the levels of the ^{7}He nucleus, values of which were measured, are very well reproduced in the framework of the calculations. Therefore the first source of the discrepancy between measured values of the decay width of the ground state of the ^{7}He nucleus and the ones calculated using the "new" approach is the overestimation of the binding energy of the ^{6}He nucleus. This overestimation, evidently, has nothing to do with the choice of the parameters of the NCSM basis and, very likely, with the peculiarities of the extrapolation procedure. Its causes are, most likely, the properties of the Daejeon16 potential. The second source of the 30% overestimation of the decay width, revealed by its calculation using the experimental decay energy, is not clear, since all the conditions listed above for correct matching in this example are satisfied, and the convergence of the result with respect to $N_{\text{tot}}^{*\max}$ is achieved. In principle, it can be assumed that this discrepancy is generated by the procedure of extracting the width from the experiment.

TABLE V. Channel spectroscopic factors of closed and strongly suppressed decay channels of the ^{7}He nucleus states for "old" and "new" definitions. Symbol * indicates closed channels

J^π (^{7}He)	J^π (^{6}He)	$l(S)$	"new" SF ("old" SF)
		1(3/2)	0.691 (1.599)
$3/2_1^-$	2_1^{+*}	1(5/2)	0.254 (0.305)
		3(3/2)	0.0006 (0.0006)
		3(5/2)	0.0046 (0.0046)
		1(3/2)	0.338 (0.269)
	2_2^{+*}	1(5/2)	0.0015 (0.0015)
		3(3/2)	0.0003 (0.0003)
		3(5/2)	0.0024 (0.0024)
$1/2_1^+$	2_1^{+*}	2(3/2)	0.0258 (0.0296)
		2(5/2)	0.0198 (0.0192)
	2_2^{+*}	2(3/2)	0.0117 (0.0115)
		2(5/2)	0.0017 (0.0014)
$1/2_1^-$	2_2^{+*}	1(3/2)	0.562 (0.664)
		3(5/2)	0.0004 (0.0004)
$5/2_1^-$	2_1^+	3(3/2)	0.0098 (0.0100)
		3(5/2)	0.0036 (0.0036)
		1(3/2)	0.641 (0.880)
		1(5/2)	0.604 (0.287)
	2_2^{+*}	3(3/2)	0.0008 (0.0007)
		3(5/2)	0.0001 (0.0001)
$3/2_1^+$	2_1^+	2(5/2)	0.0052 (0.0050)
		0(3/2)	0.124 (0.0076)
	2_2^{+*}	2(3/2)	0.0014 (0.0014)
		2(5/2)	0.0005 (0.0005)
		2(3/2)	0.0052 (0.0046)
$5/2_1^+$	2_1^+	4(3/2)	0.0006 (0.0006)
		4(5/2)	0.0000 (0.0000)
		2(5/2)	0.0079 (0.0087)
	2_2^+	4(3/2)	0.0001 (0.0001)
		4(5/2)	0.0002 (0.0002)
		1(5/2)	0.0025 (0.0036)
$3/2_2^-$	2_1^+	3(3/2)	0.0064 (0.0065)
		3(5/2)	0.0002 (0.0002)
		1(5/2)	0.0025 (0.0036)
	2_2^+	3(3/2)	0.0064 (0.0064)
		3(5/2)	0.0002 (0.0002)

Indeed, there are a variety of procedures used by evaluators to determine the decay energy and width, and these procedures contain various variation parameters. Thirty percent differences in the results of processing the same experiments can be found in the same databases (see, for example, [72]). On the other hand the GSM calculations [8] are in a good agreement with the experimental data, while other theoretical methods show overestimation or underestimation of $3/2_1^-$ resonance width. So, in our opinion, the issue remains open.

The calculated values of the decay energy of state $5/2_1^-$, which is equal to 3437 keV, and its total width 1941 keV are in a very good agreement with the experimental data. A good agreement was also achieved in the calculations based on the GSM [8].

As for the results of calculations of the decay widthes in the "old" approach, the situation is the opposite. They describe the decay width of the ^{7}He nucleus ground state somewhat better, which, quite possibly, may be an example of compensation of one approximation by another. On the other hand this approach greatly underestimates the partial decay width into the $l = 1(J = 3/2)$ channel, which affects the value of the total decay width of this state and makes it smaller in comparison with the experimental one.

The differences between the predicted values of the energies and decay widths of the ^{7}He nucleus states obtained by us and the results of other authors are large enough. That makes future experiments exciting.

Approaching the conclusion, it should be emphasized that one of the main goals of this work is to support experiments aimed at studying the spectrum of ^{7}He that are carried out or planned. We believe that the data presented in Tables III and IV can be used as preliminary results for the analysis of the decay properties of these states. For the same purposes, we consider it useful to supplement the tabular data presented above with one more table. Table V contains SFs of many channels not discussed above. The data presented in the table indicate that, in order to analyze the spectrum of the nucleus, one can restrict oneself to specific channels, the parameters of which are presented in Tables III and IV, since the other channels are either closed or their SFs are small.

Data from Table V confirm the patterns in the behavior of the relationship between the "new" and "old" SFs identified in the analysis of Tables III and IV. The only exception is the example of fragmentation of the $3/2^+_1$ state into a closed channel $l = 0(J = 3/2)$ for which the ratio $S_l^{c_\kappa}/\tilde{S}_l^{c_\kappa}$ exceeds 1.5.

In Table V one can also find an example for which the value of the "old" SF exceeds 1. This example demonstrates that it is impossible to ascribe a sense of probability to this quantity. That emphasizes the problems formulated above that may arise in the framework of the application of the "old" definitions of quantities characterizing the clustering of nuclei.

IV. CONCLUSIONS

The main results of this work are the following.

(I) Based on the successful application of the cluster channel orthogonalized functions method for describing ^{7}Li and ^{8}Be nuclei, total binding energies (TBEs), excitation energies, as well as decay energies and widths of the levels of ^{7}He nucleus are calculated in this NCSM-based *ab initio* scheme. The Extrapolation A5 method is used to obtain TBEs with better precision.

(II) The approach used allows one to solve multichannel problems, which made it possible for the first time to calculate in an *ab initio* scheme not only the total but the partial widths of decay of the ^{7}He nucleus into many exit channels.

(III) The studies carried out demonstrate good prospects of the method used for the theoretical study of neutron-rich nuclei, in particular, for predicting the results of planned experiments.

(IV) The prospects for using this approach in the field of study of the interaction of neutrons with light nuclei seem to be even wider.

ACKNOWLEDGMENTS

We are grateful to A. M. Shirokov, A. I. Mazur, and I. A. Mazur for fruitful discussions and providing us the Daejeon16 *NN*-potential matrices, as well as to C.W. Johnson for supporting our efforts to introduce high-performance code BIGSTICK for our NCSM calculations.

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