Structure of the correlation coefficients $S(E_e)$ and $U(E_e)$ of the neutron β decay

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In the standard effective $V-A$ theory of low-energy weak interactions [i.e., in the standard model (SM)] we analyze the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$, where E_e is the electron energy. These correlation coefficients were introduced to the electron-energy and angular distribution of the neutron beta decay by Ebel and Feldman [Nucl. Phys. **4**, 213 (1957)] in addition to the set of correlation coefficients proposed by Jackson *et al.* [Phys. Rev. **106**, 517 (1957)]. The correlation coefficients $S(E_e)$ and $U(E_e)$ are induced by simultaneous correlations of the neutron and electron spins and electron and antineutrino three-momenta. These correlation structures do no violate discrete P, C, and T symmetries. We analyze the contributions of the radiative corrections of order $O(\alpha/\pi)$, taken to leading order in the large nucleon mass m_N expansion, and corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil. In addition to the radiative $O(\alpha/\pi)$ and $O(E_e/m_N)$ corrections we take into account the contributions of the *second class* currents by Weinberg [Phys. Rev. **112**, 1375 (1958)]. The contributions of interactions beyond the SM (BSM) are calculated in terms of the phenomenological coupling constants of BSM interactions by Lee and Yang [Phys. Rev. **104**, 254 (1956)].

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I. INTRODUCTION

The general form of the electron-energy and angular distribution of the neutron beta decay for polarized neutrons, polarized electrons, and unpolarized protons were proposed by Jackson *et al.* [1] and Ebel and Feldman [2]. It can be written in the following form:

$$\begin{aligned}
&\frac{d^5\lambda_n(E_e,\vec{k}_e,\vec{k}_{\bar{\nu}},\vec{\xi}_n,\vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}}\\
&\propto \zeta(E_e)\Big\{1 + b(E_e)\frac{m_e}{E_e} + a(E_e)\frac{\vec{k}_e\cdot\vec{k}_{\bar{\nu}}}{E_eE_{\bar{\nu}}}\\
&+ A(E_e)\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + B(E_e)\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\\
&+ K_n(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{k}_{\bar{\nu}})}{E_e^2E_{\bar{\nu}}} + Q_n(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}})(\vec{k}_e\cdot\vec{k}_{\bar{\nu}})}{E_eE_{\bar{\nu}}^2}\\
&+ D(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{k}_{\bar{\nu}})}{E_eE_{\bar{\nu}}} + G(E_e)\frac{\vec{\xi}_e\cdot\vec{k}_e}{E_e}\\
&+ H(E_e)\frac{\vec{\xi}_e\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} + N(E_e)\vec{\xi}_n\cdot\vec{\xi}_e\\
&+ Q_e(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{(E_e+m_e)E_e} + K_e(E_e)\frac{(\vec{\xi}_e\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{k}_{\bar{\nu}})}{(E_e+m_e)E_eE_{\bar{\nu}}}\\
&+ R(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e} + L(E_e)\frac{\vec{\xi}_e\cdot(\vec{k}_e\times\vec{k}_{\bar{\nu}})}{E_eE_{\bar{\nu}}}\\
&+ S(E_e)\frac{(\vec{\xi}_n\cdot\vec{\xi}_e)(\vec{k}_e\cdot\vec{k}_{\bar{\nu}})}{E_eE_{\bar{\nu}}} + T(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}})(\vec{\xi}_e\cdot\vec{k}_e)}{E_eE_{\bar{\nu}}}\\
&+ U(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{\xi}_e\cdot\vec{k}_{\bar{\nu}})}{E_eE_{\bar{\nu}}} + V(E_e)\frac{\vec{\xi}_n\cdot(\vec{\xi}_e\times\vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}}\\
&+ W(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{k}_{\bar{\nu}})(\vec{\xi}_e\cdot\vec{k}_e)}{(E_e+m_e)E_eE_{\bar{\nu}}}\Big\}, \qquad (1)
\end{aligned}$$

where $\vec{\xi}_n$ and $\vec{\xi}_e$ are unit three-vectors of spin-polarizations of the neutron and electron, $(E_e,\vec{k}_e)$ and $(E_{\bar{\nu}},\vec{k}_{\bar{\nu}})$ are energies and three-momenta of the electron and antineutrino, $d\Omega_e$ and $d\Omega_{\bar{\nu}}$ are infinitesimal solid angles in directions of three-momenta of the electron and antineutrino, respectively.

The calculation of the correlation coefficients in Eq. (1) in terms of the phenomenological coupling constants, introduced

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by Lee and Yang [3], has been carried out by Jackson *et al.* [1] and by Ebel and Feldman [2] to leading order (LO) in the large nucleon mass m_N expansion. In turn, the analysis of the distribution in Eq. (1) within the standard effective $V-A$ theory of low-energy weak interactions [4–7] [i.e., within the standard model (SM)], carried out to LO in the large nucleon mass m_N expansion [8], has shown that, in addition to the well-known expressions for correlation coefficients $a(E_e)$, $A(E_e)$, $B(E_e)$, $G(E_e)$, $H(E_e)$, $N(E_e)$, $Q_e(E_e)$, and $K_e(E_e)$, the correlation coefficient $T(E_e)$, introduced by Ebel and Feldman [2], survives and depends on the axial coupling constant g_A only [9–11]. Recall that the axial coupling constant appears in the effective $V-A$ theory of low-energy weak interactions by renormalizing of the hadronic axial-vector current by strong low-energy interactions [5,12]. The function $\zeta(E_e)$ defines the contributions of different corrections to the neutron lifetime [13–18]. In the SM it is equal to unity in the LO of the large nucleon mass m_N expansion and at the neglect of radiative corrections [6,7] (see also Ref. [17]) and Wilkinson's corrections [15]. In Refs. [14,16,19,20] (see also Ref. [17]) the radiative corrections of order $O(\alpha/\pi)$ (or so-called *outer* model-independent radiative corrections [21]) were calculated to LO in the large nucleon mass m_N expansion to the neutron lifetime and correlation coefficients $a(E_e)$, caused by electron-antineutron three-momentum correlations, $A(E_e)$ and $B(E_e)$, defining the electron(beta)- and antineutrino asymmetries, respectively. In turn, the outer radiative corrections of order $O(\alpha/\pi)$ were calculated to LO in the large nucleon mass m_N expansion to the correlation coefficients $G(E_e)$, $H(E_e)$, $N(E_e)$, $Q_e(E_e)$, and $K_e(E_e)$ in Refs. [22,23] and to the correlation coefficient $T(E_e)$ in Ref. [8]. These correlation coefficients are induced by correlations of the electron spin with a neutron spin and three-momenta of the electron and antineutrino. The corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil, were calculated (i) to the neutron lifetime and correlation coefficients $a(E_e)$, $A(E_e)$, and $B(E_e)$ in Refs. [13,15] (see also Refs. [16,17,20]), (ii) to the correlation coefficients $G(E_e)$, $H(E_e)$, $N(E_e)$, $Q_e(E_e)$, and $K_e(E_e)$ in Refs. [22,23], and (iii) to the correlation coefficient $T(E_e)$ in Ref. [8]. The correlation coefficients $D(E_e)$, $R(E_e)$, and $L(E_e)$ characterize the strength of violation of time-reversal invariance (T-odd effect) [24,25]. According to Callan and Treiman [26], the correlation coefficient $D(E_e)$ is of order $O(\alpha E_e/m_N)$. It is induced by the weak magnetism, proton recoil, and the final-state electron-proton electromagnetic interaction. In Ref. [27] the correlation coefficient $D(E_e)$ has been calculated in the frame work of the heavy baryon chiral perturbation theory (HBχPT). The authors have reproduced the results obtained by Callan and Treiman [25] and analyzed the contributions of order $O(\alpha^2/2\pi)$ and $O(E_e^2/m_N^2)$, respectively. In turn, the correlation coefficients $R(E_e)$ and $L(E_e)$ are caused by the distortion of the Dirac wave function of the decay electron in the Coulomb field of the decay proton [28,29] (see also Ref. [23]). The correlation coefficient $b(E_e)$ is the Fierz interference term [30]. It is assumed that the Fierz interference term is caused by BSM interactions [30]. As regards the contemporary experimental and theoretical status of the Fierz interference term we refer to Refs. [31–37]. So one may conclude that the neutron lifetime and the correlation coefficients of the electron-energy and angular distribution of the neutron β decay proposed by Jackson *et al.* [1] are investigated theoretically well in the SM at the level of 10^{-4}–10^{-3} caused by the outer radiative corrections of order $O(\alpha/\pi)$ and the corrections of order $O(E_e/m_N)$ induced by weak magnetism and proton recoil.

This paper is addressed to the analysis of the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$ introduced by Ebel and Feldman [2]. As has been shown in Ref. [8] these correlation coefficients do not survive to leading order in the large nucleon mass m_N expansion in contrast to the correlation coefficient $T(E_e)$.

The paper is organized as follows: In Sec. II we adduce the analytical expressions for the correlation coefficients $S(E_e)$ and $U(E_e)$ in dependence of (i) the radiative corrections of order $O(\alpha/\pi)$, calculated to LO in the large nucleon mass m_N expansion, and (ii) the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil. In Sec. III we calculate the contributions of the *second-class* currents by Weinberg [38]. In Sec. IV we analyze the contributions of BSM interactions, expressed in terms of the phenomenological coupling constants of the effective phenomenological BSM interactions by Lee and Yang [3] (see also Jackson *et al.* [1] and Ebel and Feldman [2]). In Sec. V we give the total expressions for the $S(E_e)$ and $U(E_e)$. We discuss the results obtained and the use of these correlation coefficients for experimental searches of BSM interactions. We point out that the obtained SM theoretical background of the correlation coefficients $S(E_e)$ and $U(E_e)$, carried out at the level a few parts of 10^{-4}, should be very useful for experimental searches of contributions of BSM interactions in the experiments with transversally polarized decay electrons [39]. In Appendixes A and B we give in details the calculations of the correlation coefficients $S(E_e)$ and $U(E_e)$ and the analysis of the correlation structure of the neutron radiative beta decay for polarized neutrons, polarized electrons, unpolarized protons, and unpolarized photons.

II. CORRELATION COEFFICIENTS $S(E_e)$ AND $U(E_e)$ IN THE STANDARD MODEL

In the SM with the account for the contributions of the radiative corrections of order $O(\alpha/\pi)$ and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil, the neutron beta decay can be described by the standard effective $V-A$ low-energy weak interaction [4,5] and electromagnetic interaction with the Lagrangian

$$\mathcal{L}_{\mathrm{W}\gamma}(x) = \mathcal{L}_{\mathrm{W}}(x) + \mathcal{L}_{\mathrm{em}}(x), \tag{2}$$

where $\mathcal{L}_{\mathrm{W}}(x)$ and $\mathcal{L}_{\mathrm{em}}(x)$ are the Lagrangian of the standard effective $V-A$ low-energy weak interactions [4,5] (see also Ref. [17]):

$$\begin{aligned}\mathcal{L}_{\mathrm{W}}(x) = -G_V\{&[\bar{\psi}_p(x)\gamma_\mu(1-g_A\gamma^5)\psi_n(x)]\\ &+\frac{\kappa}{2m_N}\partial^\nu[\bar{\psi}_p(x)\sigma_{\mu\nu}\psi_n(x)]\}\\ &\times[\bar{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_\nu(x)],\end{aligned} \tag{3}$$

and the Lagrangian of electromagnetic interactions [25]

$$\mathcal{L}_{\mathrm{em}}(x) = -e\{[\bar{\psi}_p(x)\gamma_\mu\psi_p(x)] - [\bar{\psi}_e(x)\gamma^\mu\psi_e(x)]\}A_\mu(x), \tag{4}$$

respectively, where G_V is the vector weak-coupling constant, including the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud} [11], g_A is the real axial coupling constant [9,10], $\psi_p(x)$, $\psi_n(x)$, $\psi_e(x)$, and $\psi_\nu(x)$ are the field operators of the proton, neutron, electron, and antineutrino, respectively, $\gamma^\mu = (\gamma^0, \vec{\gamma})$, γ^5, and $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ are the Dirac matrices [25], $\kappa = \kappa_p - \kappa_n = 3.7059$ is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton $\kappa_p = 1.7929$ and the neutron $\kappa_n = -1.9130$ and measured in nuclear magneton [11], $m_N = (m_n + m_p)/2$ is the average nucleon mass, e is the electric charge of the proton, and $A_\mu(x)$ is a four-vector electromagnetic potential.

For the calculation of the correlation coefficients under consideration we use the amplitude of the neutron beta decay, calculated in Ref. [17] (see also Ref. [23] and the Appendix). The detailed calculation we have carried out in the Appendix. Below we adduce only the results obtained.

Analytical expressions for the correlation coefficients $S(E_e)$ and $U(E_e)$ in the standard model

In Eq. (A9) of Appendix A we have defined the general expression for the structure part of the electron-energy and angular distribution of the neutron beta decay for a polarized neutron, a polarized electron, and an unpolarized proton. According to this expression, we have shown that the contributions of the radiative corrections of order $O(\alpha/\pi)$, caused by one-virtual photon exchanges [14,16,19,20] (for the detailed calculations we refer to Ref. [17]) do not appear in the correlation coefficients $S(E_e)$ and $U(E_e)$, respectively. In Appendix B we show that the neutron radiative beta decay $n \to p + e^- + \bar{\nu}_e + \gamma$ does not contribute to the correlation coefficients $S(E_e)$ and $U(E_e)$. It is well known [40–44] (see also Refs. [14,17,19]) that the contribution of the neutron radiative beta decay is extremely needed for cancellation of infrared divergences in the radiative corrections of order $O(\alpha/\pi)$, caused by one-virtual photon exchanges.

Thus [see Eq. (A9)], the contributions caused by the SM interactions appear in the correlation coefficients $S(E_e)$ and $U(E_e)$ only due to weak magnetism and proton recoil. For the correlation coefficients $\zeta(E_e)^{(\mathrm{SM})}S(E_e)^{(\mathrm{SM})}$ and $\zeta(E_e)^{(\mathrm{SM})}U(E_e)^{(\mathrm{SM})}$ we have obtained the following analytical expressions:

$$\begin{aligned}\zeta(E_e)^{(\mathrm{SM})}S(E_e)^{(\mathrm{SM})} &= \frac{1}{1+3g_A^2}\frac{m_e}{m_N}\big[-5g_A^2 - g_A(\kappa-4)\\ &\quad + (\kappa+1)\big],\\ \zeta(E_e)^{(\mathrm{SM})}U(E_e)^{(\mathrm{SM})} &= 0,\end{aligned} \tag{5}$$

where for the calculation of the corrections of order $O(E_e/m_N)$, which cause weak magnetism and proton recoil, we have taken into account the contribution of the phase volume of the neutron β decay [see Eq. (A3)]. The correlation function $\zeta(E_e)^{(\mathrm{SM})}$ was calculated in Refs. [13–18]. It is equal to unity at the neglect of the contributions of radiative corrections and corrections, caused by weak magnetism and proton recoil, Wilkinson's corrections [15]. Hence, the correlation coefficients $S(E_e)^{(\mathrm{SM})}$ and $U(E_e)^{(\mathrm{SM})}$, including the SM contributions of order $O(E_e/m_N)$, are equal to

$$\begin{aligned}S(E_e)^{(\mathrm{SM})} &= \frac{1}{1+3g_A^2}\frac{m_e}{m_N}\big[-5g_A^2 - g_A(\kappa-4) + (\kappa+1)\big],\\ U(E_e)^{(\mathrm{SM})} &= 0.\end{aligned} \tag{6}$$

Now we may move on to calculating the contributions of the second class currents and BSM interactions.

III. CONTRIBUTIONS OF THE SECOND CLASS CURRENTS OR THE *G*-ODD CORRELATIONS

For the calculation of the contributions of the second class currents or the G-odd correlations (see Ref. [45]) we follow Weinberg [38], Holstein [46], Gardner and Zhang [47], and Gardner and Plaster [48] (see also Refs. [8,23,49]). Skipping intermediate calculations we give the results:

$$\begin{aligned}S(E_e)^{(\mathrm{SCC})} &= -[\mathrm{Re}g_2(0) - \mathrm{Re}f_3(0)]\frac{2g_A}{1+3g_A^2}\frac{m_e}{m_N},\\ U(E_e)^{(\mathrm{SCC})} &= +[\mathrm{Re}g_2(0) - g_A\mathrm{Re}f_3(0)]\frac{2}{1+3g_A^2}\frac{m_e}{m_N},\end{aligned} \tag{7}$$

where $\mathrm{Re}f_3(0)$ and $\mathrm{Re}g_2(0)$ are the phenomenological coupling constants of the induced scalar and tensor second class currents [38,46–48] (see also Ref. [31]), respectively.

IV. CONTRIBUTIONS OF INTERACTIONS BEYOND THE STANDARD MODEL

For the calculation of the contributions of interactions beyond the SM we use the effective phenomenological Lagrangian of BSM interactions proposed by Lee and Yang [3] (see also Refs. [1,50,51]). Skipping intermediate calculations we give the results:

$$S(E_e)^{(\mathrm{BSM})} = -U(E_e)^{(\mathrm{BSM})} = \frac{1}{1+3g_A^2}\mathrm{Re}(C_T - \bar{C}_T), \tag{8}$$

where C_T and $\bar{C}_T$ are the phenomenological tensor coupling constants of the effective phenomenological BSM interactions by Lee and Yang [3]. The contributions of the tensor BSM interactions are linear in the phenomenological tensor coupling constants C_T and $\bar{C}_T$. This agrees well with the result obtained by Ebel and Feldman [2]. However, in addition to the results obtained by Ebel and Feldman [2] we, following Refs. [52–56] (see also Refs. [8,17,22,23,49]), have taken the contributions of the phenomenological vector coupling constants C_V and $\bar{C}_V$ in the linear approximation, i.e., $C_V = 1 + \delta C_V$ and $\bar{C}_V = -1 + \delta\bar{C}_V$, where we have used the notations of Refs. [8,17,22,23,49].

V. DISCUSSION

We have analyzed the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$ introduced by Ebel and Feldman [2] in addition to the set of correlation coefficients proposed by Jackson *et al.* [1]. Summing up the SM contributions, caused by weak magnetism and proton recoil only, and contributions beyond the SM we obtain the following expressions:

$$
\begin{aligned}
S(E_e) &= \frac{1}{1+3g_A^2}\frac{m_e}{m_N}\big[-5g_A^2 - g_A(\kappa-4) + (\kappa+1)\big] \\
&\quad + \frac{1}{1+3g_A^2}\mathrm{Re}(C_T - \bar{C}_T) \\
&\quad - [\mathrm{Re}g_2(0) - \mathrm{Re}f_3(0)]\frac{2g_A}{1+3g_A^2}\frac{m_e}{m_N}, \\
U(E_e) &= -\frac{1}{1+3g_A^2}\mathrm{Re}(C_T - \bar{C}_T) \\
&\quad + [\mathrm{Re}g_2(0) - g_A\mathrm{Re}f_3(0)]\frac{2}{1+3g_A^2}\frac{m_e}{m_N}.
\end{aligned} \tag{9}
$$

For the axial coupling constant $g_A = 1.276\,41(45)_{\rm stat.}(33)_{\rm sys.}$ [9], measured with a relative experimental uncertainty of about a few parts of 10^{-4}, the correlation coefficients $S(E_e)$ and $U(E_e)$ are given by

$$
\begin{aligned}
S(E_e) &= -2.83\times 10^{-4} + 0.17(\mathrm{Re}(C_T - \bar{C}_T) \\
&\quad + 1.39\times 10^{-3}\mathrm{Re}f_3(0)) - 2.36\times 10^{-4}\mathrm{Re}g_2(0), \\
U(E_e) &= -0.17(\mathrm{Re}(C_T - \bar{C}_T) + 1.39\times 10^{-3}\mathrm{Re}f_3(0)) \\
&\quad + 1.85\times 10^{-4}\mathrm{Re}g_2(0),
\end{aligned} \tag{10}
$$

where we have also used $m_e = 0.5110$ MeV and $m_N = (m_n + m_p)/2 = 938.9188$ MeV [11].

We would like to notice that the correlation structures of the correlation coefficients $S(E_e)$ and $U(E_e)$ and as well as the correlation coefficients $T(E_e)$ are even with respect to parity transformation (P even), charge conjugation (C even), and time-reversal transformation (T even). However, in contrast with the correlation coefficient $T(E_e)$, the absolute value of which is of about $|T(E_e)| \approx 1$, the absolute values of the correlation coefficients $S(E_e)$ and $U(E_e)$ are of a few orders of magnitude smaller. It is also important to mention that, unlike the correlation coefficient $T(E_e)$, the correlation coefficients $S(E_e)$ and $U(E_e)$ do not depend on the electron energy E_e.

The correlation coefficients $S(E_e)$ and $U(E_e)$ can, in principle, be investigated in experiments with both longitudinally and transversally polarized decay electrons [39] (see also Ref. [8]). However, a successful result for searches of the contributions of BSM interactions one might expect only from experiments with experimental uncertainties of about a few parts of 10^{-5}. In this case, any deviation of the correlation coefficient $S(E_e)$ from -2.83×10^{-4}, caused by weak magnetism and proton recoil, should testify to a presence of the contributions of the second-class currents and BSM interactions.

The phenomenological tensor coupling constant $\mathrm{Re}g_2(0)$ can be measured in experiments on the β asymmetry [47,49] and in experiments with longitudinally polarized decay electrons from the measurements of the correlation coefficient $T(E_e)$ [8]. If in the neutron β decay the absolute value of the Fierz interference term b could be of order 10^{-2} (see, for example, Refs. [33,36]), after the measurement of the phenomenological tensor coupling constant $\mathrm{Re}g_2(0)$ the contribution of the scalar coupling constant $\mathrm{Re}f_3(0)$ to the correlation coefficients $S(E_e)$ and $U(E_e)$ could be screened by the contributions of the phenomenological tensor coupling constants $\mathrm{Re}(C_T - \bar{C}_T)$ of the phenomenological tensor BSM interactions.

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APPENDIX A: THE ELECTRON-ENERGY AND ANGULAR DISTRIBUTION OF THE NEUTRON β DECAY FOR POLARIZED NEUTRONS, POLARIZED ELECTRONS, AND UNPOLARIZED PROTONS

Following Refs. [17,22,23] (see also Ref. [8]), we define the electron-energy and angular distribution of the neutron β decay for a polarized neutron, a polarized electron, and an unpolarized proton as follows:

$$
\begin{aligned}
\frac{d^5\lambda_{\beta^-_c\gamma}(E_e,\vec{k}_e,\vec{k}_{\bar{\nu}},\vec{\xi}_n,\vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} &= \big(1+3g_A^2\big)\frac{|G_V|^2}{16\pi^5}(E_0-E_e)^2\sqrt{E_e^2-m_e^2}\,E_eF(E_e,Z=1) \\
&\quad \times \Phi_n(\vec{k}_e,\vec{k}_{\bar{\nu}})\sum_{\rm pol.}\frac{|M(n\to pe^-\bar{\nu}_e)|^2}{\big(1+3g_A^2\big)|G_V|^2 64 m_n^2 E_e E_{\bar{\nu}}},
\end{aligned} \tag{A1}
$$

where the sum is over polarizations of massive fermions. Then, $F(E_e, Z=1)$ is the relativistic Fermi function, describing the electron-proton final-state Coulomb interaction, is equal to (see, for example, Ref. [57], Ref. [15], and a discussion in Ref. [22])

$$
F(E_e,Z=1) = \left(1+\frac{1}{2}\gamma\right)\frac{4(2r_pm_e\beta)^{2\gamma}}{\Gamma^2(3+2\gamma)}\frac{e^{\pi\alpha/\beta}}{(1-\beta^2)^\gamma}\left|\Gamma\left(1+\gamma+i\frac{\alpha}{\beta}\right)\right|^2, \tag{A2}
$$

where $\beta = k_e/E_e = (E_e^2 - m_e^2)^{1/2}/E_e$ is the electron velocity, $\gamma = (1-\alpha^2)^{1/2} - 1$, and r_p is the electric radius of the proton [58]. The function $\Phi_n(\vec{k}_e, \vec{k}_{\bar{\nu}})$ defines the contribution of the phase-volume of the neutron β decay [17,59]. It is equal to [17,59]

$$\Phi_n(\vec{k}_e, \vec{k}_{\bar{\nu}}) = 1 + 3\frac{E_e}{m_N}\Big(1 - \frac{\vec{k}_e\cdot\vec{k}_{\bar{\nu}}}{E_eE_{\bar{\nu}}}\Big), \tag{A3}$$

taken to next-to-leading order in the large nucleon mass m_N expansion. The amplitude of the neutron β decay $M(n\to pe^-\bar{\nu}_e)$, taking into account the contribution of the corrections, caused by one-virtual photon exchanges, weak magnetism and proton recoil, was calculated in Ref. [17] (see also Ref. [23]). It is given by

$$\begin{aligned} M(n\to pe^-\bar{\nu}_e) = -2m_nG_V\Big\{&\Big(1+\frac{\alpha}{2\pi}f_{\beta_c^-}(E_e,\mu)\Big)\big[\varphi_p^\dagger\varphi_n\big]\big[\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}\big]\\ &+\tilde{g}_A\Big(1+\frac{\alpha}{2\pi}f_{\beta_c^-}(E_e,\mu)\Big)\big[\varphi_p^\dagger\vec{\sigma}\varphi_n\big]\cdot\big[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}\big] - \frac{\alpha}{2\pi}g_F(E_e)\big[\varphi_p^\dagger\varphi_n\big]\big[\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}\big]\\ &-\frac{\alpha}{2\pi}\tilde{g}_Ag_F(E_e)\big[\varphi_p^\dagger\vec{\sigma}\varphi_n\big]\cdot\big[\bar{u}_e\gamma^0\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}\big] - \frac{m_e}{2m_N}\big[\varphi_p^\dagger\varphi_n\big]\big[\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}\big]\\ &-\frac{\tilde{g}_A}{2m_N}\big[\varphi_p^\dagger(\vec{\sigma}\cdot\vec{k}_p)\varphi_n\big]\big[\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}\big] - i\frac{\kappa+1}{2m_N}\big[\varphi_p^\dagger(\vec{\sigma}\times\vec{k}_p)\varphi_n\big]\cdot\big[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}\big]\Big\}, \end{aligned} \tag{A4}$$

where φ_p and φ_n are Pauli spinorial wave functions of the proton and neutron, u_e and v_ν are Dirac wave functions of the electron and electron antineutrino, $\vec{\sigma}$ are the Pauli 2×2 matrices, and $\tilde{g}_A = g_A(1 - E_0/2m_N)$, $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2926$ MeV is the endpoint energy of the electron-energy spectrum of the neutron β decay [6,7,11], and $\vec{k}_p = -\vec{k}_e - \vec{k}_\nu$ is the proton three–momentum in the rest frame of the neutron. The functions $f_{\beta_c^-}(E_e,\mu)$ and $g_F(E_e)$ were calculated by Sirlin [14] (see also Eq. (D-51) of Ref. [17]), and μ is a covariant infrared cutoff introduced as a finite virtual photon mass [14] (see also Refs. [40–44]). The function $g_F(E_e)$ [see Eq. (D-44) of Ref. [17]] is equal to

$$g_F(E_e) = \frac{\sqrt{1-\beta^2}}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big). \tag{A5}$$

It is defined by the contributions of one-virtual-photon exchanges [14] (see also Ref. [17]). Using Eq. (A4) for the square of the absolute value of the amplitude $M(n\to pe^-\bar{\nu}_e)$, summed over polarizations of massive fermions, we obtain the following expression:

$$\begin{aligned} \sum_{\rm pol}\cdot\frac{|M(n\to pe^-\bar{\nu}_e)|^2}{(1+3g_A^2)|G_V|^2 64m_n^2E_eE_{\bar{\nu}}} = \frac{1}{(1+3g_A^2)8E_eE_{\bar{\nu}}}\Big\{&\Big(1+\frac{\alpha}{\pi}f_{\beta_c^-}(E_e,\mu)\Big)\\ &\times\big({\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\}{\rm tr}\{(\hat{k}_e + m_e\gamma^5\hat{\zeta}_e)\gamma^0\hat{k}_{\bar{\nu}}\gamma^0(1-\gamma^5)\}\\ &+\tilde{g}_A{\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\vec{\sigma}\}\cdot{\rm tr}\{(\hat{k}_e + m_e\gamma^5\hat{\zeta}_e)(\gamma^0\hat{k}_{\bar{\nu}}\vec{\gamma} + \vec{\gamma}\hat{k}_{\bar{\nu}}\gamma^0)(1-\gamma^5)\}\\ &+\tilde{g}_A^2{\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\sigma^a\sigma^b\}{\rm tr}\{(\hat{k}_e + m_e\gamma^5\hat{\zeta}_e)\gamma^b\hat{k}_{\bar{\nu}}\gamma^a(1-\gamma^5)\}\big)\\ &-\Big(\frac{\alpha}{2\pi}g_F(E_e) + \frac{m_e}{2m_N}\Big){\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\}\big({\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\hat{k}_{\bar{\nu}}\gamma^0(1-\gamma^5)\}\\ &+{\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\gamma^0\hat{k}_{\bar{\nu}}(1+\gamma^5)\}\big)\\ &-\tilde{g}_A\Big(\frac{\alpha}{2\pi}g_F(E_e) + \frac{m_e}{2m_N}\Big){\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\vec{\sigma}\}\cdot\big({\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\hat{k}_{\bar{\nu}}\vec{\gamma}(1-\gamma^5)\}\\ &+{\rm tr}\{(\hat{k}_e + m_e\gamma^5\hat{\zeta}_e)\vec{\gamma}\hat{k}_{\bar{\nu}}(1+\gamma^5)\}\big)\\ &-\tilde{g}_A\frac{\alpha}{2\pi}g_F(E_e){\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\vec{\sigma}\}\cdot\big({\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\gamma^0\vec{\gamma}\hat{k}_{\bar{\nu}}\gamma^0(1-\gamma^5)\}\\ &-{\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\gamma^0\hat{k}_{\bar{\nu}}\gamma^0\vec{\gamma}(1+\gamma^5)\}\big)\\ &-\tilde{g}_A^2\frac{\alpha}{2\pi}g_F(E_e){\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\sigma^a\sigma^b\}\big({\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\gamma^0\gamma^b\hat{k}_{\bar{\nu}}\gamma^a(1-\gamma^5)\}\\ &-{\rm tr}\{(m_e + \hat{k}_e\gamma^5\hat{\zeta}_e)\gamma^b\hat{k}_{\bar{\nu}}\gamma^0\gamma^a(1+\gamma^5)\}\big)\\ &-\frac{\tilde{g}_A}{m_N}{\rm tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})(\vec{\sigma}\cdot\vec{k}_p)\}{\rm tr}\{(\hat{k}_e + m_e\gamma^5\hat{\zeta}_e)\gamma^0\hat{k}_{\bar{\nu}}\gamma^0(1-\gamma^5)\} \end{aligned}$$

$$
\begin{aligned}
& -\frac{\tilde{g}_A^2}{2m_N}\Big(\mathrm{tr}\big\{\big(1+\vec{\xi}_n\cdot\vec{\sigma}\big)\vec{\sigma}\,(\vec{\sigma}\cdot\vec{k}_p)\big\}\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\gamma^0\hat{k}_{\bar{\nu}}\vec{\gamma}\,(1-\gamma^5)\big\}\\
& +\mathrm{tr}\big\{\big(1+\vec{\xi}_n\cdot\vec{\sigma}\big)(\vec{\sigma}\cdot\vec{k}_p)\vec{\sigma}\big\}\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\vec{\gamma}\hat{k}_{\bar{\nu}}\gamma^0(1-\gamma^5)\big\}\Big)\\
& -i\frac{\kappa+1}{2m_N}\mathrm{tr}\big\{\big(1+\vec{\xi}_n\cdot\vec{\sigma}\big)(\vec{\sigma}\times\vec{k}_p)\big\}\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\big(\vec{\gamma}\hat{k}_{\bar{\nu}}\gamma^0-\gamma^0\hat{k}_{\bar{\nu}}\vec{\gamma}\big)(1-\gamma^5)\big\}\\
& -i\tilde{g}_A\frac{\kappa+1}{2m_N}\Big(\mathrm{tr}\big\{\big(1+\vec{\xi}_n\cdot\vec{\sigma}\big)\big[\sigma^a(\vec{\sigma}\times\vec{k}_p)^b-(\vec{\sigma}\times\vec{k}_p)^a\sigma^b\big]\big\}\\
& \times\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\gamma^b\hat{k}_{\bar{\nu}}\gamma^a(1-\gamma^5)\big\}\Big),
\end{aligned}
\tag{A6}
$$

where ζ_e is the four-vector of the spin polarization of the electron. It is defined by [25]

$$
\zeta_e=\big(\zeta_e^0,\vec{\zeta}_e\big)=\left(\frac{\vec{k}_e\cdot\vec{\xi}_e}{m_e},\vec{\xi}_e+\frac{(\vec{k}_e\cdot\vec{\xi}_e)\vec{k}_e}{m_e(E_e+m_e)}\right).
\tag{A7}
$$

The four-vector ζ_e of the spin polarization of the electron is normalized by $\zeta_e^2=-1$ and obeys also the constraint $k_e\cdot\zeta_e=0$ [25]. Calculating the traces over the nucleon degrees of freedom and using the properties of the Dirac matrices [25] gives

$$
\gamma^\alpha\gamma^\nu\gamma^\mu=\gamma^\alpha\eta^{\nu\mu}-\gamma^\nu\eta^{\mu\alpha}+\gamma^\mu\eta^{\alpha\nu}+i\varepsilon^{\alpha\nu\mu\beta}\gamma_\beta\gamma^5,
\tag{A8}
$$

where $\eta^{\mu\nu}$ is the metric tensor of the Minkowski spacetime, $\varepsilon^{\alpha\nu\mu\beta}$ is the Levi–Civita tensor defined by $\varepsilon^{0123}=1$ and $\varepsilon_{\alpha\nu\mu\beta}=-\varepsilon^{\alpha\nu\mu\beta}$ [25], we transcribe the right-hand side (r.h.s.) of Eq. (A6) into the form [17,22,23] (see also Ref. [8])

$$
\begin{aligned}
\sum_{\mathrm{pol.}}\frac{|M(n\to pe^-\bar{\nu}_e)|^2}{(1+3g_A^2)|G_V|^2 64m_n^2E_eE_{\bar{\nu}}}
&=\frac{1+3\tilde{g}_A^2}{(1+3g_A^2)4E_e}\Big\{\Big(1+\frac{\alpha}{\pi}f_{\beta_c^-}(E_e,\mu)\Big)\Big[\Big(1+\tilde{B}_0\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\gamma^0(1-\gamma^5)\big\}\\
&+\Big(\tilde{A}_0\vec{\xi}_n+\tilde{a}_0\frac{\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\vec{\gamma}\,(1-\gamma^5)\big\}\Big]\\
&-\frac{1}{1+3\tilde{g}_A^2}\Big(\frac{\alpha}{\pi}g_F(E_e)+\frac{m_e}{m_N}\Big)\Big(\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\big\}\\
&-\frac{\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\cdot\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\gamma^0\vec{\gamma}\gamma^5\big\}\Big)-\frac{\tilde{g}_A}{1+3\tilde{g}_A^2}\Big(\frac{\alpha}{\pi}g_F(E_e)+\frac{m_e}{m_N}\Big)\Big(\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\big\}\\
&-\vec{\xi}_n\cdot\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\gamma^0\vec{\gamma}\gamma^5\big\}+i\frac{\vec{\xi}_n\times\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\cdot\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\gamma^0\vec{\gamma}\big\}\Big)-\frac{\tilde{g}_A}{1+3\tilde{g}_A^2}\frac{\alpha}{\pi}g_F(E_e)\\
&\times\Big(\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\big\}-\vec{\xi}_n\cdot\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\gamma^0\vec{\gamma}\gamma^5\big\}\\
&-i\frac{\vec{\xi}_n\times\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\cdot\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\gamma^0\vec{\gamma}\big\}\Big)-\frac{\tilde{g}_A^2}{1+3\tilde{g}_A^2}\frac{\alpha}{\pi}g_F(E_e)\Big[\Big(3+2\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\big\}\\
&+\Big(2\vec{\xi}_n+\frac{\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)\cdot\mathrm{tr}\big\{\big(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e\big)\gamma^0\vec{\gamma}\gamma^5\big\}\Big]\\
&-\frac{\tilde{g}_A}{1+3\tilde{g}_A^2}\frac{1}{m_N}\Big((\vec{\xi}_n\cdot\vec{k}_p)\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\gamma^0(1-\gamma^5)\big\}\\
&+(\vec{\xi}_n\cdot\vec{k}_p)\frac{\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\vec{\gamma}\,(1-\gamma^5)\big\}\Big)-\frac{\tilde{g}_A^2}{1+3\tilde{g}_A^2}\frac{1}{m_N}\\
&\times\Big[\frac{\vec{k}_p\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\gamma^0(1-\gamma^5)\big\}\\
&+\Big(\vec{k}_p+\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\vec{k}_p-\frac{\vec{k}_p\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\vec{\xi}_n\Big)\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\vec{\gamma}\,(1-\gamma^5)\big\}\Big]-\frac{\kappa+1}{1+3\tilde{g}_A^2}\frac{1}{m_N}\\
&\times\Big(\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\vec{k}_p-\frac{\vec{k}_p\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\vec{\xi}_n\Big)\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\vec{\gamma}\,(1-\gamma^5)\big\}
\end{aligned}
$$

$$+\frac{\tilde{g}_A}{1+3\tilde{g}_A^2}\frac{\kappa+1}{m_N}\Bigg[\Bigg(2\frac{\vec{k}_p\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}+2\big(\vec{\xi}_n\cdot\vec{k}_p\big)\Bigg)\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\gamma^0\big(1-\gamma^5\big)\big\}$$
$$+\Bigg(-2\vec{k}_p-\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\vec{k}_p-\frac{\vec{k}_p\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\vec{\xi}_n\Bigg)\cdot\mathrm{tr}\big\{\big(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e\big)\vec{\gamma}\big(1-\gamma^5\big)\big\}\Bigg]\Bigg\}, \tag{A9}$$

where $\tilde{a}_0$, $\tilde{A}_0$, and $\tilde{B}_0$ are defined in terms of the axial coupling constant $\tilde{g}_A$:

$$a_0=\frac{1-\tilde{g}_A^2}{1+3\tilde{g}_A^2},\quad A_0=2\frac{\tilde{g}_A(1-\tilde{g}_A)}{1+3\tilde{g}_A^2},\quad B_0=2\frac{\tilde{g}_A(1+\tilde{g}_A)}{1+3\tilde{g}_A^2}. \tag{A10}$$

Before the calculation of the traces over leptonic degrees of freedom one may see that the terms proportional two $g_F(E_e)i(\vec{\xi}_n\times\vec{k}_{\bar{\nu}})\cdot\mathrm{tr}\{(m_e+\hat{k}_e\gamma^5\hat{\zeta}_e)\gamma^0\vec{\gamma}\}$, which are responsible for contributions of the radiative corrections of order $O(\alpha/\pi)$ to the correlation coefficients $S(E_e)$ and $U(E_e)$, cancel each other out. Hence, there are no contributions of the radiative corrections of order $O(\alpha/\pi)$, caused by one-virtual-photon exchanges, to the correlation coefficients $S(E_e)$ and $U(E_e)$, respectively.

In Eq. (A9) the second term on the third line from above, proportional to m_e/m_N, and last four lines define the contributions of order $O(E_e/m_N)$ of weak magnetism and proton recoil to the correlation coefficients of the neutron β decay. Having calculated the traces over leptonic degrees of freedom, taking into account the contribution of the phase volume (A3) and keeping only the contributions with the correlation structures, inducing the correlation coefficients $S(E_e)$ and $U(E_e)$, we obtain the SM corrections, caused by weak magnetism and proton recoil only, which we give in Eq. (5).

APPENDIX B: ELECTRON-PHOTON ENERGY AND ANGULAR DISTRIBUTION OF NEUTRON RADIATIVE β DECAY FOR POLARIZED NEUTRONS, POLARIZED ELECTRONS, AND UNPOLARIZED PROTONS AND PHOTONS

Following Refs. [8,17,22,23] we define the electron-photon energy and angular distribution of the neutron radiative β decay for a polarized neutron, a polarized electron, a polarized photon, and an unpolarized proton as follows:

$$\frac{d^8\lambda_{\beta^-_c\gamma}\big(E_e,\omega,\vec{k}_e,\vec{k}_{\bar{\nu}},\vec{q},\vec{\xi}_n,\vec{\xi}_e\big)_{\lambda'\lambda}}{d\omega dE_e d\Omega_e d\Omega_{\bar{\nu}}\Omega_\gamma}=\big(1+3g_A^2\big)\frac{\alpha}{\pi}\frac{|G_V|^2}{(2\pi)^6}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)(E_0-E_e-\omega)^2\frac{1}{\omega}$$
$$\times\sum_{\rm pol.}\frac{|M(n\to pe^-\bar{\nu}_e\gamma)|^2_{\lambda'\lambda}\omega^2}{\big(1+3g_A^2\big)e^2|G_V|^2 64m_n^2E_eE_{\bar{\nu}}}, \tag{B1}$$

where we sum over polarizations of massive fermions. Since we calculate the contribution of the neutron radiative β decay to leading order in the large nucleon mass m_N expansion, the contribution of the phase volume of the decay is equal to unity. The photon state is determined by the four-momentum $q^\mu=(\omega,\vec{q})$ and the four-vector of polarization $\varepsilon^\mu(q)_\lambda$ with $\lambda=1,2$, obeying the constraints $\varepsilon^*(q)_{\lambda'}\cdot\varepsilon_\lambda(q)=-\delta_{\lambda'\lambda}$ and $q\cdot\varepsilon_\lambda(q)=0$. In the tree-approximation and to leading order in the large nucleon mass m_N expansion, the amplitude of the neutron radiative β decay is equal to [17]

$$M(n\to pe^-\bar{\nu}_e\gamma)_\lambda=eG_V\frac{m_n}{\omega}\frac{1}{E_e-\vec{n}\cdot\vec{k}_e}\{[\varphi_p^\dagger\varphi_n][\bar{u}_eQ_\lambda\gamma^0(1-\gamma^5)v_{\bar{\nu}}]+g_A[\varphi_p^\dagger\vec{\sigma}\varphi_n]\cdot[\bar{u}_eQ_\lambda\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}]\}. \tag{B2}$$

The Hermitian conjugate amplitude is determined by

$$M^\dagger(n\to pe^-\bar{\nu}_e\gamma)_{\lambda'}=eG_V\frac{m_n}{\omega}\frac{1}{E_e-\vec{n}\cdot\vec{k}_e}\{[\varphi_p^\dagger\varphi_n][\bar{u}_e\bar{Q}_{\lambda'}\gamma^0(1-\gamma^5)v_{\bar{\nu}}]+g_A[\varphi_p^\dagger\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\bar{Q}_{\lambda'}\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}]\}, \tag{B3}$$

where $\vec{n}=\vec{q}/\omega$, $Q=2(\varepsilon^*\cdot k_e)+\hat{\varepsilon}^*\hat{q}$, and $\bar{Q}=\gamma^0Q^\dagger\gamma^0=2(\varepsilon\cdot k_e)+\hat{q}\hat{\varepsilon}$. Then, φ_n and φ_p are the Pauli wave functions of the neutron and proton, u_e and $v_{\bar{\nu}}$ are the Dirac wave functions of the electron and antineutrino, respectively. The sum over polarizations of the massive fermions is equal to [17,22,23]

$$\sum_{\rm pol.}\frac{|M(n\to pe^-\bar{\nu}_e\gamma)|^2_{\lambda'\lambda}\omega^2}{\big(1+3g_A^2\big)e^2|G_V|^2 64m_n^2E_eE_{\bar{\nu}}}$$
$$=\frac{1}{(E_e-\vec{n}\cdot\vec{k}_e)^2}\frac{1}{\big(1+3g_A^2\big)32E_eE_{\bar{\nu}}}\big\{\mathrm{tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\}\mathrm{tr}\{(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e)Q_\lambda\gamma^0\hat{k}_{\bar{\nu}}\gamma^0\bar{Q}_{\lambda'}(1-\gamma^5)\}$$
$$+g_A\mathrm{tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\vec{\sigma}\}\cdot\mathrm{tr}\{(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e)Q_\lambda\gamma^0\hat{k}_{\bar{\nu}}\vec{\gamma}\bar{Q}_{\lambda'}(1-\gamma^5)\}+g_A\mathrm{tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\vec{\sigma}\}\cdot\mathrm{tr}\{(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e)$$
$$\times Q_\lambda\vec{\gamma}\hat{k}_{\bar{\nu}}\gamma^0\bar{Q}_{\lambda'}(1-\gamma^5)\}+g_A^2\mathrm{tr}\{(1+\vec{\xi}_n\cdot\vec{\sigma})\sigma^a\sigma^b\}\mathrm{tr}\{(\hat{k}_e+m_e\gamma^5\hat{\zeta}_e)Q_\lambda\gamma^b\hat{k}_{\bar{\nu}}\gamma^a\bar{Q}_{\lambda'}(1-\gamma^5)\}\big\}. \tag{B4}$$

Having calculated the traces over the nucleon degrees of freedom and using the properties of the Dirac matrices Eq. (A8) we transcribe the r.h.s. of Eq. (B4) into the form [17,22,23]

$$\sum_{\rm pol.}\frac{|M(n\to pe^-\bar\nu_e\gamma)|^2_{\lambda'\lambda}\omega^2}{(1+3g_A^2)e^2|G_V|^2 64 m_n^2E_eE_{\bar\nu}} = \frac{1}{(E_e-\vec n\cdot\vec k_e)^2}\frac{1}{16E_e}\Big\{\Big(1+B_0\frac{\vec\xi_n\cdot\vec k_{\bar\nu}}{E_{\bar\nu}}\Big){\rm tr}\{(\hat k_e+m_e\gamma^5\hat\zeta_e)Q_\lambda\gamma^0\bar Q_{\lambda'}(1-\gamma^5)\} + \Big(A_0\vec\xi_n + a_0\frac{\vec k_{\bar\nu}}{E_{\bar\nu}}\Big)\cdot{\rm tr}\{(\hat k_e+m_e\gamma^5\hat\zeta_e)Q_\lambda\vec\gamma\bar Q_{\lambda'}(1-\gamma^5)\}\Big\}. \quad \text{(B5)}$$

The traces over Dirac matrices in Eq. (B5) were calculated in the covariant form in Refs. [17,23]. The result is

$$\begin{aligned}&\tfrac{1}{16}{\rm tr}\{\hat a Q_\lambda\gamma^\mu\bar Q_{\lambda'}(1-\gamma^5)\}\\ &= (\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot k_e)a^\mu + \tfrac12[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot a) + (\varepsilon^*_\lambda\cdot a)(\varepsilon_{\lambda'}\cdot k_e) - (\varepsilon^*_\lambda\cdot\varepsilon_{\lambda'})(a\cdot q)]q^\mu\\ &\quad - \tfrac12[(\varepsilon^*_\lambda\cdot k_e)\varepsilon^\mu_{\lambda'} + \varepsilon^{*\mu}_\lambda(\varepsilon_{\lambda'}\cdot k_e)](a\cdot q) - i\tfrac12\varepsilon^{\mu\nu\alpha\beta}[(\varepsilon^*_\lambda\cdot k_e)\varepsilon_{\lambda'\nu} - \varepsilon^*_{\lambda\nu}(\varepsilon_{\lambda'}\cdot k_e)]a_\alpha q_\beta - i\tfrac12 q^\mu\varepsilon^{\rho\varphi\alpha\beta}\varepsilon^*_{\lambda\rho}\varepsilon_{\lambda'\varphi}a_\alpha q_\beta,\end{aligned} \quad \text{(B6)}$$

where $a = k_e$ or $a = \zeta_e$, respectively. As a result, for the r.h.s. of Eq. (B5) we obtain the following expression:

$$\begin{aligned}&\sum_{\rm pol.}\frac{|M(n\to pe^-\bar\nu_e\gamma)|^2_{\lambda'\lambda}\omega^2}{(1+3g_A^2)e^2|G_V|^2 64 m_n^2E_eE_{\bar\nu}}\\ &= \frac{1}{(E_e-\vec n\cdot\vec k_e)^2}\Big\{\Big(1+B_0\frac{\vec\xi_n\cdot\vec k_{\bar\nu}}{E_{\bar\nu}}\Big)\Big\{\Big[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot k_e)\Big(1+\frac{\omega}{E_e}\Big) - \frac12(\varepsilon^*_\lambda\cdot\varepsilon_{\lambda'})(k_e\cdot q)\frac{\omega}{E_e} - \frac12\big[(\varepsilon^*_\lambda\cdot k_e)\varepsilon^0_{\lambda'} + \varepsilon^{*0}_\lambda(\varepsilon_{\lambda'}\cdot k_e)\big]\\ &\times\frac{k_e\cdot q}{E_e}\Big] - \frac{m_e}{E_e}\Big[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot k_e)\zeta^0_e + \frac12\Big[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot\zeta_e) + (\varepsilon^*_\lambda\cdot\zeta_e)(\varepsilon_{\lambda'}\cdot k_e) - (\varepsilon^*_\lambda\cdot\varepsilon^*_{\lambda'})(\zeta_e\cdot q)\Big]\omega\\ &-\frac12\big[(\varepsilon^*_\lambda\cdot k_e)\varepsilon^0_{\lambda'} + \varepsilon^{*0}_\lambda(\varepsilon_{\lambda'}\cdot k_e)\big](\zeta_e\cdot q)\Big]\Big\} + \Big(A_0\vec\xi_n + a_0\frac{\vec k_{\bar\nu}}{E_{\bar\nu}}\Big)\cdot\Big\{\Big[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot k_e)\\ &\times\frac{\vec k_e}{E_e} + \Big((\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot k_e) - \frac12(\varepsilon^*_\lambda\cdot\varepsilon_{\lambda'})(k_e\cdot q)\Big)\vec n\frac{\omega}{E_e} - \frac12[(\varepsilon^*_\lambda\cdot k_e)\vec\varepsilon_{\lambda'} + \vec\varepsilon^{\,*}_\lambda(\varepsilon_{\lambda'}\cdot k_e)]\frac{k_e\cdot q}{E_e}\Big] - \frac{m_e}{E_e}\Big[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot k_e)\\ &\times\vec\zeta_e + \frac12[(\varepsilon^*_\lambda\cdot k_e)(\varepsilon_{\lambda'}\cdot\zeta_e) + (\varepsilon^*_\lambda\cdot\zeta_e)(\varepsilon_{\lambda'}\cdot k_e) - (\varepsilon^*_\lambda\cdot\varepsilon_{\lambda'})(\zeta_e\cdot q)]\omega\vec n - \frac12[(\varepsilon^*_\lambda\cdot k_e)\vec\varepsilon_{\lambda'} + \vec\varepsilon^{\,*}_\lambda(\varepsilon_{\lambda'}\cdot k_e)](\zeta_e\cdot q)\Big]\Big\}\Big\}.\end{aligned} \quad \text{(B7)}$$

The r.h.s. of Eq. (B7) we calculate in the physical gauge $\varepsilon_\lambda = (0,\vec\varepsilon_\lambda)$ [17,22,23,32,34,35,60], where the polarization vector $\vec\varepsilon_\lambda$ obeys the constraints

$$\vec q\cdot\vec\varepsilon^{\,*}_\lambda = \vec q\cdot\vec\varepsilon_{\lambda'} = 0,\quad \vec\varepsilon^{\,*}_\lambda\cdot\vec\varepsilon_{\lambda'} = \delta_{\lambda\lambda'},\quad \sum_{\lambda=1,2}\vec\varepsilon^{\,i*}_\lambda\vec\varepsilon^{\,j}_\lambda = \delta^{ij} - \frac{\vec k^i\vec k^j}{\omega^2} = \delta^{ij} - \vec n^i\vec n^j,\quad \sum_{j=1,2,3}\sum_{\lambda=1,2}\vec\varepsilon^{\,j*}_\lambda\vec\varepsilon^{\,j}_\lambda = 2. \quad \text{(B8)}$$

In the physical gauge $\varepsilon_\lambda = (0,\vec\varepsilon_\lambda)$ we obtain for the r.h.s. of Eq. (B7) the following expression:

$$\begin{aligned}&\sum_{\rm pol.}\frac{|M(n\to pe^-\bar\nu_e\gamma)|^2_{\lambda'\lambda}\omega^2}{(1+3g_A^2)e^2|G_V|^2 16 m_n^2E_eE_{\bar\nu}}\\ &= \frac{1}{(E_e-\vec n\cdot\vec k_e)^2}\Big\{\Big(1+B_0\frac{\vec\xi_n\cdot\vec k_{\bar\nu}}{E_{\bar\nu}}\Big)\Big\{\Big[(\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e)\Big(1+\frac{\omega}{E_e}\Big) + \frac12(\vec\varepsilon^{\,*}_\lambda\cdot\vec\varepsilon_{\lambda'})(k_e\cdot q)\frac{\omega}{E_e}\Big]\\ &-\frac{m_e}{E_e}\Big[(\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e)\zeta^0_e + \frac12\Big((\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec\zeta_e) + (\vec\varepsilon^{\,*}_\lambda\cdot\vec\zeta_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e) + (\vec\varepsilon^{\,*}_\lambda\cdot\vec\varepsilon_{\lambda'})(\zeta_e\cdot q)\Big)\omega\Big]\Big\}\\ &+\Big(A_0\vec\xi_n + a_0\frac{\vec k_{\bar\nu}}{E_{\bar\nu}}\Big)\cdot\Big\{\Big[(\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e)\frac{\vec k_e}{E_e} + \Big((\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e) + \frac12(\vec\varepsilon^{\,*}_\lambda\cdot\vec\varepsilon_{\lambda'})(k_e\cdot q)\Big)\vec n\frac{\omega}{E_e}\\ &+\frac12\Big[(\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)\vec\varepsilon_{\lambda'} + \vec\varepsilon^{\,*}_\lambda(\vec\varepsilon_{\lambda'}\cdot\vec k_e)\Big]\frac{k_e\cdot q}{E_e}\Big] - \frac{m_e}{E_e}\Big[(\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e)\vec\zeta_e\\ &+\frac12\big[(\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)(\vec\varepsilon_{\lambda'}\cdot\vec\zeta_e) + (\vec\varepsilon^{\,*}_\lambda\cdot\vec\zeta_e)(\vec\varepsilon_{\lambda'}\cdot\vec k_e) + (\vec\varepsilon^{\,*}_\lambda\cdot\vec\varepsilon_{\lambda'})(\zeta_e\cdot q)\big]\omega\vec n + \frac12\big((\vec\varepsilon^{\,*}_\lambda\cdot\vec k_e)\vec\varepsilon_{\lambda'} + \vec\varepsilon^{\,*}_\lambda(\vec\varepsilon_{\lambda'}\cdot\vec k_e)\big)(\zeta_e\cdot q)\Big]\Big\}\Big\}.\end{aligned} \quad \text{(B9)}$$

Plugging Eq. (B9) into Eq. (B1) we obtain the electron-energy and angular distribution for a polarized neutron, a polarized electron, an unpolarized proton, and a polarized photon:

$$\begin{aligned}
&\frac{d^8\lambda_{\beta^-_c\gamma}\big(E_e,\omega,\vec{k}_e,\vec{k}_{\bar\nu},\vec{q},\vec{\xi}_n,\vec{\xi}_e\big)_{\lambda'\lambda}}{d\omega dE_e d\Omega_e d\Omega_{\bar\nu}\Omega_\gamma}\\
&= \big(1+3g_A^2\big)\frac{\alpha}{\pi}\frac{|G_V|^2}{(2\pi)^6}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)(E_0-E_e-\omega)^2\\
&\quad\times\frac{1}{\omega}\frac{1}{\big(E_e-\vec{n}\cdot\vec{k}_e\big)^2}\Bigg\{\Bigg(1+B_0\frac{\vec{\xi}_n\cdot\vec{k}_{\bar\nu}}{E_{\bar\nu}}\Bigg)\bigg\{\bigg[\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)\Big(1+\frac{\omega}{E_e}\Big)+\frac{1}{2}(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{\varepsilon}_{\lambda'})(k_e\cdot q)\frac{\omega}{E_e}\bigg]\\
&\quad-\frac{m_e}{E_e}\Big[\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)\zeta_e^0+\frac{1}{2}\Big(\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{\zeta}_e\big)+\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{\zeta}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)+(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{\varepsilon}_{\lambda'})(\zeta_e\cdot q)\Big)\omega\Big]\bigg\}\\
&\quad+\Bigg(A_0\vec{\xi}_n+a_0\frac{\vec{k}_{\bar\nu}}{E_{\bar\nu}}\Bigg)\cdot\bigg\{\bigg[\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)\frac{\vec{k}_e}{E_e}+\Big(\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)+\frac{1}{2}(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{\varepsilon}_{\lambda'})(k_e\cdot q)\Big)\vec{n}\frac{\omega}{E_e}\\
&\quad+\frac{1}{2}\Big(\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\vec{\varepsilon}_{\lambda'}+\vec{\varepsilon}^{\,*}_\lambda\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)\Big)\frac{k_e\cdot q}{E_e}\bigg]-\frac{m_e}{E_e}\Big[\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)\vec{\zeta}_e+\frac{1}{2}\Big(\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{\zeta}_e\big)\\
&\quad+\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{\zeta}_e\big)\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)+(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{\varepsilon}_{\lambda'})(\zeta_e\cdot q)\Big)\omega\vec{n}+\frac{1}{2}\Big(\big(\vec{\varepsilon}^{\,*}_\lambda\cdot\vec{k}_e\big)\vec{\varepsilon}_{\lambda'}+\vec{\varepsilon}^{\,*}_\lambda\big(\vec{\varepsilon}_{\lambda'}\cdot\vec{k}_e\big)\Big)(\zeta_e\cdot q)\Big]\bigg\}\Bigg\}. \qquad \text{(B10)}
\end{aligned}$$

Summing up over polarizations of the photon we get

$$\begin{aligned}
&\frac{d^8\lambda_{\beta^-_c\gamma}\big(E_e,\omega,\vec{k}_e,\vec{k}_{\bar\nu},\vec{q},\vec{\xi}_n,\vec{\xi}_e\big)}{d\omega dE_e d\Omega_e d\Omega_{\bar\nu}\Omega_\gamma}\\
&= \big(1+3g_A^2\big)\frac{\alpha}{\pi}\frac{|G_V|^2}{(2\pi)^6}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)(E_0-E_e-\omega)^2\frac{1}{\omega}\\
&\quad\times\Bigg\{\Bigg(1+B_0\frac{\vec{\xi}_n\cdot\vec{k}_{\bar\nu}}{E_{\bar\nu}}\Bigg)\Bigg\{\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Big(1+\frac{\omega}{E_e}\Big)+\frac{\omega^2}{E_e^2}\frac{1}{1-\vec{n}\cdot\vec{\beta}}\Bigg]\\
&\quad-\frac{m_e}{E_e}\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\zeta_e^0+\frac{\omega}{E_e}\frac{\zeta_e^0-\big(\vec{n}\cdot\vec{\beta}\big)\big(\vec{n}\cdot\vec{\zeta}_e\big)}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}+\frac{\omega^2}{E_e^2}\frac{\zeta_e^0-\vec{n}\cdot\vec{\zeta}_e}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Bigg]\Bigg\}\\
&\quad+\Bigg(A_0\vec{\xi}_n+a_0\frac{\vec{k}_{\bar\nu}}{E_{\bar\nu}}\Bigg)\cdot\Bigg\{\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Bigg(\frac{\vec{k}_e}{E_e}+\vec{n}\frac{\omega}{E_e}\Bigg)+\frac{\omega}{E_e}\frac{\vec{\beta}-\vec{n}\big(\vec{n}\cdot\vec{\beta}\big)}{1-\vec{n}\cdot\vec{\beta}}+\frac{\omega^2}{E_e^2}\frac{\vec{n}}{1-\vec{n}\cdot\vec{\beta}}\Bigg]\\
&\quad-\frac{m_e}{E_e}\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\vec{\zeta}_e+\frac{\omega}{E_e}\frac{\zeta_e^0-\vec{n}\cdot\vec{\zeta}_e}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\vec{\beta}+\frac{\omega}{E_e}\frac{\vec{n}}{1-\vec{n}\cdot\vec{\beta}}\zeta_e^0+\frac{\omega^2}{E_e^2}\frac{\vec{n}\big(\zeta_e^0-\vec{n}\cdot\vec{\zeta}_e\big)}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Bigg]\Bigg\}\Bigg\}, \qquad \text{(B11)}
\end{aligned}$$

where we have used that $\vec{\beta}\cdot\vec{\zeta}_e=\zeta_e^0$. The next step is to average over directions of the three-momentum $\vec{q}=\omega\vec{n}$ of the real photon. This gives

$$\begin{aligned}
&\frac{d^6\lambda_{\beta^-_c\gamma}\big(E_e,\omega,\vec{k}_e,\vec{k}_{\bar\nu},\vec{\xi}_n,\vec{\xi}_e\big)}{d\omega dE_e d\Omega_e d\Omega_{\bar\nu}}\\
&= \big(1+3g_A^2\big)\frac{\alpha}{\pi}\frac{|G_V|^2}{16\pi^5}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)(E_0-E_e-\omega)^2\\
&\quad\times\frac{1}{\omega}\int\frac{d\Omega_\gamma}{4\pi}\Bigg\{\Bigg(1+B_0\frac{\vec{\xi}_n\cdot\vec{k}_{\bar\nu}}{E_{\bar\nu}}\Bigg)\Bigg\{\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Big(1+\frac{\omega}{E_e}\Big)+\frac{\omega^2}{E_e^2}\frac{1}{1-\vec{n}\cdot\vec{\beta}}\Bigg]\\
&\quad-\frac{m_e}{E_e}\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\zeta_e^0+\frac{\omega}{E_e}\frac{\zeta_e^0-\big(\vec{n}\cdot\vec{\beta}\big)\big(\vec{n}\cdot\vec{\zeta}_e\big)}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}+\frac{\omega^2}{E_e^2}\frac{\zeta_e^0-\vec{n}\cdot\vec{\zeta}_e}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Bigg]\Bigg\}
\end{aligned}$$

$$+\Big(A_0\vec{\xi}_n + a_0\frac{\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)\cdot\Bigg\{\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Big(\frac{\vec{k}_e}{E_e}+\vec{n}\frac{\omega}{E_e}\Big)+\frac{\omega}{E_e}\frac{\vec{\beta}-\vec{n}\big(\vec{n}\cdot\vec{\beta}\big)}{1-\vec{n}\cdot\vec{\beta}}+\frac{\omega^2}{E_e^2}\frac{\vec{n}}{1-\vec{n}\cdot\vec{\beta}}\Bigg]$$
$$-\frac{m_e}{E_e}\Bigg[\frac{\beta^2-\big(\vec{n}\cdot\vec{\beta}\big)^2}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\vec{\zeta}_e+\frac{\omega}{E_e}\frac{\zeta_e^0-\vec{n}\cdot\vec{\zeta}_e}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\vec{\beta}+\frac{\omega}{E_e}\frac{\vec{n}}{1-\vec{n}\cdot\vec{\beta}}\zeta_e^0+\frac{\omega^2}{E_e^2}\frac{\vec{n}\big(\zeta_e^0-\vec{n}\cdot\vec{\zeta}_e\big)}{\big(1-\vec{n}\cdot\vec{\beta}\big)^2}\Bigg]\Bigg\}\Bigg\}. \quad \text{(B12)}$$

The integration over the directions of the vector $\vec{n}$ we carry out by using the results obtained in Ref. [22]. We get

$$\frac{d^6\lambda_{\beta^-_c\gamma}\big(E_e,\omega,\vec{k}_e,\vec{k}_{\bar{\nu}},\vec{\xi}_n,\vec{\xi}_e\big)}{d\omega dE_e d\Omega_e d\Omega_{\bar{\nu}}}$$
$$=\big(1+3g_A^2\big)\frac{\alpha}{\pi}\frac{|G_V|^2}{16\pi^5}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)(E_0-E_e-\omega)^2\frac{1}{\omega}\Bigg\{\Big(1+B_0\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)$$
$$\times\Bigg\{\Bigg[\Big(1+\frac{\omega}{E_e}+\frac{1}{2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{\omega^2}{E_e^2}\Bigg]-\frac{m_e}{E_e}\zeta_e^0\Big[1+\frac{1}{\beta^2}\frac{\omega}{E_e}\Big(1+\frac{1}{2}\frac{\omega}{E_e}\Big)\Big]\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]\Bigg\}$$
$$+\Big(A_0\vec{\xi}_n+a_0\frac{\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\Big)\cdot\Bigg\{\frac{\vec{k}_e}{E_e}\Big[1+\frac{1}{\beta^2}\frac{\omega}{E_e}\Big(1+\frac{1}{2}\frac{\omega}{E_e}\Big)\Big]\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]-\vec{\zeta}_e\frac{m_e}{E_e}\Big(1-\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]$$
$$-\vec{\beta}\frac{m_e}{E_e}\zeta_e^0\Big[\frac{1}{\beta^2}\frac{\omega}{E_e}\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big(\frac{3-\beta^2}{\beta^2}\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]-2\Big)\Big]\Bigg\}\Bigg\}. \quad \text{(B13)}$$

The r.h.s. of Eq. (B13), rewritten in terms of irreducible correlation structures [see Eq. (1)], takes the form

$$\frac{d^6\lambda_{\beta^-_c\gamma}\big(E_e,\omega,\vec{k}_e,\vec{k}_{\bar{\nu}},\vec{\xi}_n,\vec{\xi}_e\big)}{d\omega dE_e d\Omega_e d\Omega_{\bar{\nu}}}$$
$$=\big(1+3g_A^2\big)\frac{\alpha}{\pi}\frac{|G_V|^2}{16\pi^5}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)(E_0-E_e-\omega)^2$$
$$\times\Bigg\{\frac{1}{\omega}\Big[\Big(1+\frac{\omega}{E_e}+\frac{1}{2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{\omega^2}{E_e^2}\Big]+a_0\frac{\vec{k}_e\cdot\vec{k}_{\bar{\nu}}}{E_eE_{\bar{\nu}}}\frac{1}{\omega}\Big(1+\frac{1}{\beta^2}\frac{\omega}{E_e}+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)$$
$$\times\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+A_0\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e}\frac{1}{\omega}\Big(1+\frac{1}{\beta^2}\frac{\omega}{E_e}+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+B_0\frac{\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}$$
$$\times\frac{1}{\omega}\Big[\Big(1+\frac{\omega}{E_e}+\frac{1}{2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{\omega^2}{E_e^2}\Big]+(-1)\frac{\vec{\xi}_e\cdot\vec{k}_e}{E_e}\frac{1}{\omega}\Big(1+\frac{1}{\beta^2}\frac{\omega}{E_e}+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)$$
$$\times\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+(-1)\frac{m_e}{E_e}a_0\frac{\vec{\xi}_e\cdot\vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}}\frac{1}{\omega}\Big(1-\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+(-1)\frac{m_e}{E_e}A_0$$
$$\times\vec{\xi}_n\cdot\vec{\xi}_e\frac{1}{\omega}\Big(1-\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+(-1)A_0\frac{\big(\vec{\xi}_n\cdot\vec{k}_e\big)\big(\vec{\xi}_e\cdot\vec{k}_e\big)}{(E_e+m_e)E_e}\Bigg\{\frac{1}{\omega}\Big(1-\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)$$
$$\times\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\big(1+\sqrt{1-\beta^2}\big)\Bigg[\frac{1}{\beta^2}\frac{\omega}{E_e}\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Bigg(\frac{3-\beta^2}{\beta^2}$$
$$\times\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]-2\Bigg)\Bigg]\Bigg\}+(-1)a_0\frac{\big(\vec{\xi}_e\cdot\vec{k}_e\big)\big(\vec{k}_e\cdot\vec{k}_{\bar{\nu}}\big)}{(E_e+m_e)E_eE_{\bar{\nu}}}\Bigg\{\frac{1}{\omega}\Big(1-\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]$$
$$+\big(1+\sqrt{1-\beta^2}\big)\Bigg[\frac{1}{\beta^2}\frac{\omega}{E_e}\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big(\frac{3-\beta^2}{\beta^2}\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]-2\Big)\Bigg]\Bigg\}$$
$$-B_0\frac{\big(\vec{\xi}_n\cdot\vec{k}_{\bar{\nu}}\big)\big(\vec{\xi}_e\cdot\vec{k}_e\big)}{E_eE_{\bar{\nu}}}\frac{1}{\omega}\Big(1+\frac{1}{\beta^2}\frac{\omega}{E_e}+\frac{1}{2\beta^2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]\Bigg\}. \quad \text{(B14)}$$

It is seen that the terms with the correlation structures $(\vec{\xi}_n \cdot \vec{\xi}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})$ and $(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})$, inducing the correlation coefficients $S(E_e)$ and $U(E_e)$, respectively, do not appear in the electron-energy and angular distribution of the neutron radiative β decay for polarized neutrons, polarized electrons, unpolarized protons, and unpolarized photons. This confirms the results, obtained in Appendix A, that there are no contributions of the radiative corrections of order $O(\alpha/\pi)$ caused by one-virtual-photon exchanges to the correlation coefficients $S(E_e)$ and $U(E_e)$, respectively.

[1] J. D. Jackson, S. B. Treiman, and H. W. Wyld Jr., Possible tests of time reversal invariance in beta decay, Phys. Rev. **106**, 517 (1957).

[2] M. E. Ebel and G. Feldman, Further remarks on Coulomb corrections in allowed beta transitions, Nucl. Phys. **4**, 213 (1957).

[3] T. D. Lee and C. N. Yang, Question of parity conservation in weak interactions, Phys. Rev. **104**, 254 (1956).

[4] R. P. Feynman and M. Gell–Mann, Theory of Fermi interaction, Phys. Rev. **109**, 193 (1958).

[5] R. E. Marshak, Riazuddin, and C. P. Ryan, in *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, A Division of John Wiley & Sons, Inc. New York, 1969), p. 41.

[6] H. Abele, The neutron. Its properties and basic interactions, Prog. Part. Nucl. Phys. **60**, 1 (2008).

[7] J. S. Nico, Neutron beta decay, J. Phys. G **36**, 104001 (2009).

[8] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, On the correlation coefficient $T(E_e)$ of the neutron beta decay, caused by the correlation structure invariant under discrete P, C and T symmetries, Phys. Lett. B **816**, 136263 (2021).

[9] B. Märkisch, H. Mest, H. Saul, X. Wang, H. Abele, D. Dubbers, M. Klopf, A. Petoukhov, C. Roick, T. Soldner, and D. Werder, Measurement of the Weak Axial-Vector Coupling Constant in the Decay of Free Neutrons Using a Pulsed Cold Neutron Beam, Phys. Rev. Lett. **122**, 242501 (2019).

[10] A. Czarnecki, W. J. Marciano, and A. Sirlin, Neutron Lifetime and Axial Coupling Constant, Phys. Rev. Lett. **120**, 202002 (2018).

[11] P. A. Zyla *et al.*, Review of particle physics (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).

[12] V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, in *Currents in Hadronic Physics* (Noth-Holland Publishing Company, Amsterdam, London, American Elsevier Publishing Company, Inc., New York, 1973).

[13] S. M. Bilen'kii, R. M. Ryndin, Ya. A. Smorodinskii, and Ho Tso-Hsiu, On the theory of the neutron beta decay, JETP **37**, 1759 (1959); [Sov. Phys. JETP **37**, 1241 (1960)].

[14] A. Sirlin, General properties of the electromagnetic corrections to the beta decay of a physical nucleon, Phys. Rev. **164**, 1767 (1967).

[15] D. H. Wilkinson, Analysis of neutron beta decay, Nucl. Phys. A **377**, 474 (1982).

[16] V. Gudkov, G. I. Greene, and J. R. Calarco, General classification and analysis of neutron beta-decay experiments, Phys. Rev. C **73**, 035501 (2006).

[17] A. N. Ivanov, M. Pitschmann, and N. I. Troitskaya, Neutron β-decay as a laboratory for testing the standard model, Phys. Rev. D **88**, 073002 (2013).

[18] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Theoretical description of the neutron beta decay in the standard model at the level of 10^{-5}, arXiv:2104.11080.

[19] R. T. Shann, Electromagnetic effects in the decay of polarized neutrons, Nuovo Cimento A **5**, 591 (1971).

[20] S. Ando, H. W. Fearing, V. Gudkov, K. Kubodera, F. Myhrer, S. Nakamura, and T. Sato, Neutron beta-decay in effective field theory, Phys. Lett. B **595**, 250 (2004).

[21] D. H. Wilkinson and B. E. F. Macfield, The numerical evaluation of radiative corrections of order α to allowed nuclear β-decay, Nucl. Phys. A **158**, 110 (1970).

[22] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Precision analysis of electron energy spectrum and angular distribution of neutron beta decay with polarized neutron and electron, Phys. Rev. C **95**, 055502 (2017).

[23] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Test of the standard model in neutron beta decay with polarized electrons and unpolarized neutrons and protons, Phys. Rev. D **99**, 053004 (2019).

[24] T. D. Lee and C. N. Yang, Remarks on possible noninvariance under time reversal and charge conjugation, Phys. Rev. **106**, 340 (1957).

[25] C. Itzykson and J.–B. Zuber, in *Quantum Field Theory* (McGraw–Hill Inc., New York, 1980).

[26] C. G. Callan and S. B. Treiman, Electromagnetic simulation of T violation in beta decay, Phys. Rev. **162**, 1494 (1967).

[27] S. I. Ando, J. A. McGovern, and T. Sato, The *D* coefficient in neutron beta decay in effective field theory, Phys. Lett. B **677**, 109 (2009).

[28] J. D. Jackson, S. B. Treiman, and H. W. Wyld Jr., Coulomb corrections in allowed beta transitions, Nucl. Phys. **4**, 206 (1957).

[29] J. D. Jackson, S. B. Treiman, and H. W. Wyld, Jr., Note on relativistic Coulomb wave functions, Z. Phys. **150**, 640 (1958).

[30] M. Fierz, Zur fermischen theorie des β-zerfalls, Eur. Phys. J. A **104**, 553 (1937).

[31] J. C. Hardy and I. S. Towner, Superallowed $0^+ \to 0^+$ nuclear beta decays: 2020 critical survey, with implications for V_{ud} and CKM unitarity, Phys. Rev. C **102**, 045501 (2020).

[32] M. González–Alonso, O. Naviliat–Cuncic, and N. Severijns, New physics searches in nuclear and neutron beta decay, Prog. Part. Nucl. Phys. **104**, 165 (2019).

[33] H. Saul, Ch. Roick, H. Abele, H. Mest, M. Klopf, A. Petukhov, T. Soldner, X. Wang, D. Werder, and B. Märkisch, Limit on the Fierz Interference Term *b* From a Measurement of the Beta Asymmetry in Neutron Decay, Phys. Rev. Lett. **125**, 112501 (2020).

[34] X. Sun *et al.* (UCNA Collaboration), Improved limits on Fierz interference using asymmetry measurements from the ultracold neutron asymmetry (UCNA) experiment, Phys. Rev. C **101**, 035503 (2020).

[35] V. Cirigliano, A. Garcia, D. Gazit, O. Naviliat-Cuncic, G. Savard, and A. Young, Precision beta decay as a probe of new physics, arXiv:1907.02164.

[36] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Neutron dark matter decays and correlation coefficients of neutron beta decays, Nucl. Phys. B **938**, 114 (2019).

[37] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Precision analysis of pseudoscalar interactions in neutron beta decays, Nucl. Phys. B **951**, 114891 (2020).

[38] S. Weinberg, Charge symmetry of weak interactions, Phys. Rev. **112**, 1375 (1958).

[39] K. Bodek, Beta-decay correlations in the LHC era, Acta Phys. Pol., B **47**, 349 (2016).

[40] S. M. Berman, Radiative corrections to muon and neutron decay, Phys. Rev. **112**, 267 (1958).

[41] T. Kinoshita and A. Sirlin, Radiative corrections to Fermi interactions, Phys. Rev. **113**, 1652 (1959).

[42] S. M. Berman and A. Sirlin, Some considerations on the radiative corrections to muon and neutron decay, Ann. Phys. (NY) **20**, 20 (1962).

[43] G. Källén, Radiative corrections to beta decay and nucleon form factors, Nucl. Phys. B **1**, 225 (1967).

[44] E. S. Abers, D. A. Dicus, R. E. Norton, and H. R. Queen, Radiative corrections to the Fermi part of strangeness-conserving beta decay, Phys. Rev. **167**, 1461 (1968).

[45] T. D. Lee and C. N. Yang, Charge conjugation, a new quantum number G, and selection rules concerning a nucleon antinucleon system, Nuovo Cimento **10**, 749 (1956).

[46] B. R. Holstein, Recoil effects in allowed beta decay: The elementary particle approach, Rev. Mod. Phys. **46**, 789 (1974).

[47] S. Gardner and C. Zhang, Sharpening Low-Energy, Standard-Model Tests via Correlation Coefficients in Neutron Beta Decay, Phys. Rev. Lett. **86**, 5666 (2001).

[48] S. Gardner and B. Plaster, Framework for maximum likelihood analysis of neutron beta decay observables to resolve the limits of the $V-A$ law, Phys. Rev. C **87**, 065504 (2013).

[49] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Tests of the standard model in neutron β decay with a polarized neutron and electron and an unpolarized proton, Phys. Rev. C **98**, 035503 (2018).

[50] P. Herczeg, Beta decay beyond the standard model, Prog. Part. Nucl. Phys. **46**, 413 (2001).

[51] N. Severijns, M. Beck, and O. Naviliat-Cuncic, Tests of the standard electroweak model in nuclear beta decay, Rev. Mod. Phys. **78**, 991 (2006).

[52] T. Bhattacharya, V. Cirigliano, S. D. Cohen, A. Filipuzzi, M. González-Alonso, M. L. Graesser, R. Gupta, and Huey-Wen Lin, Probing novel scalar and tensor interactions from (ultra)cold neutrons to the LHC, Phys. Rev. D **85**, 054512 (2012).

[53] S. Gardner, V. Cirigliano, P. Fierlinger, C. Fischer, K. Jansen, S. Paul, F. J. Llanes-Estrada, H. W. Lin, and W. M. Snow, Round table: Resolving Physics bSM at low energies, PoS **171**, 024 (2012).

[54] V. Cirigliano, M. González-Alonso, M. L. Graesser, Non-standard charged current interactions: Beta decays versus the LHC, J. High Energy Phys. 02 (2013) 046.

[55] V. Cirigliano and M. J. Ramsey-Musolf, Low energy probes of physics beyond the standard model, Prog. Part. Nucl. Phys. **71**, 2 (2013).

[56] V. Cirigliano, S. Gardner, and B. Holstein, Beta decays and non-standard interactions in the LHC era, Prog. Part. Nucl. Phys. **71**, 93 (2013).

[57] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wily & Sons, New York, 1952).

[58] A. Antognini *et al.*, Proton structure from the measurement of $2S$-$2P$ transition frequencies of muonic hydrogen, Science **339**, 417 (2013).

[59] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Corrections of order $O(E_e^2/m_N^2)$, caused by weak magnetism and proton recoil, to the neutron lifetime and correlation coefficients of the neutron beta decay, Results Phys. **21**, 103806 (2021).

[60] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Precision theoretical analysis of neutron radiative beta decay to order $O(\alpha^2/\pi^2)$, Phys. Rev. D **95**, 113006 (2017).