Decay modes of superheavy nuclei using a modified generalized liquid drop model and a mass-inertia-dependent approach for spontaneous fission

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A modified generalized liquid drop model with the proximity 77 parametrization is used for studying the alpha decay of superheavy nuclei in the range $104 \leqslant Z \leqslant 118$. The spontaneous fission (SF) half-lives are computed using a new formula which depends on the shells effect (E_{shell}) and the mass inertia (I_{rigid}). The modes of decay are predicted by comparing $T^{\alpha}_{1/2}$ with $T^{\text{SF}}_{1/2}$ and are compared with the experimental observations. These studies may induce further investigations in the superheavy region.

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I. INTRODUCTION

Studies on superheavy nuclei have received much attention since the prediction of island of stability [1–5] in the 1960s. The two fusion evaporation techniques which are widely used for the production of superheavy elements are the cold fusion reaction [6] and the hot fusion reaction [7]. Using cold fusion reaction superheavy elements up to $Z = 113$ have been synthesized [8–10], whereas hot fusion reactions are successful in synthesizing elements up to to $Z = 118$ [11].

One of the reliable tools to identify the newly synthesized superheavy nuclei is to observe their decay modes. Main decay modes observed in superheavy nuclei are alpha decay and spontaneous fission (SF). The investigations on alpha decay and SF gives a large variety of information about the nuclear structure, including shell effects and stability, nuclear spins and parities, fission barrier, rotational properties, etc. The theoretical studies on the superheavy nuclei [12–25] which includes the various decay mechanisms are very important in the current nuclear research.

Theoretically, the process involved in alpha decay and SF is the quantum tunneling effect. Various formulations like the cluster model [26], the generalized liquid drop model (GLDM) [27], and the fission model [28] are used for studying the alpha decay from superheavy nuclei. Similarly, different theoretical approaches [29–32] are used to successfully describe the phenomenon of spontaneous fission.

Among the various theoretical approaches, the GLDM, where the already existing liquid drop model is modified by adding the nuclear proximity energy and the quasimolecular shape, has been taken as an effective tool to study the fusion, fission, and alpha emission processes [33–35].

In the present work, we use the modified generalized liquid drop model (MGLDM), with the inclusion of proximity potential proposed by Blocki *et al.* [36,37] for studying alpha emission from superheavy nuclei. The predictions on the modes of decay of these nuclei are done by analyzing the alpha half-lives using the present approach with the SF half-lives using a new formula that depends on the shell effect (E_{shell}) and the mass inertia (I_{rigid}).

The paper is arranged as follows: Section II describes the MGLDM and the SF formula that depends on the shell effect and mass inertia used for predicting the decay modes. The results and discussions are given in Sec. III. Section IV summarizes the complete work.

II. THEORETICAL MODELS

A. Modified generalized liquid drop model (MGLDM)

In MGLDM, the macroscopic energy for a deformed nucleus is defined as

$$E = E_V + E_S + E_C + E_R + E_P. \tag{1}$$

Here E_V, E_S, E_C, E_R, and E_P represents the volume, surface, Coulomb, rotational and proximity energy respectively.

For the prescission region the volume, surface and Coulomb energies in MeV are given by Royer *et al.* [33],

$$E_V = -15.494(1 - 1.8I^2)A, \tag{2}$$

$$E_S = 17.9439(1 - 2.6I^2)A^{2/3}\left(S/4\pi R_0^2\right), \tag{3}$$

$$E_C = 0.6e^2(Z^2/R_0) \times 0.5 \int [V(\theta)/V_0][R(\theta)/R_0]^3 \sin\theta\, d\theta. \tag{4}$$

Here I is the relative neutron excess and S is the surface of the deformed nucleus, $V(\theta)$ is the electrostatic potential at the surface, and V_0 the surface potential of the sphere.

For the postscission region [33],

$$E_V = -15.494\left[\left(1 - 1.8I_1^2\right)A_1 + \left(1 - 1.8I_2^2\right)A_2\right], \tag{5}$$

$$E_S = 17.9439\left[\left(1 - 2.6I_1^2\right)A_1^{2/3} + \left(1 - 2.6I_2^2\right)A_2^{2/3}\right], \tag{6}$$

$$E_C = \frac{0.6e^2Z_1^2}{R_1} + \frac{0.6e^2Z_2^2}{R_2} + \frac{e^2Z_1Z_2}{r}. \tag{7}$$

Here A_i, Z_i, R_i, and I_i represent the masses, charges, radii, and relative neutron excess of the fragments, r is the distance

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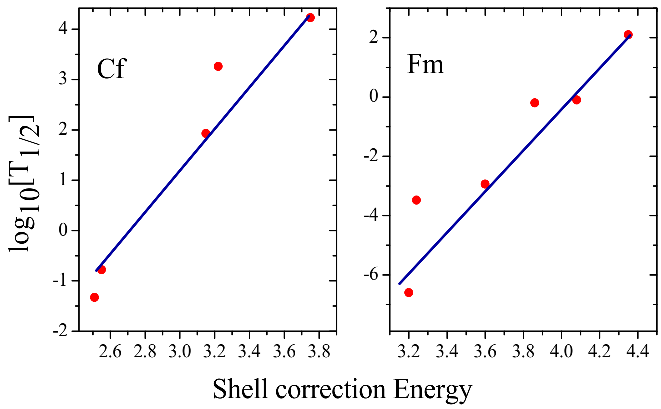

FIG. 1. Graph showing the variation of logarithm of experimental SF half-life of Cf and Fm with respect to shell correction energy. Half-lives are in years.

between the fragment centers,

$$E_p(z) = 4\pi\gamma b\left[\frac{C_1C_2}{(C_1+C_2)}\right]\Phi\left(\frac{z}{b}\right), \tag{8}$$

is the nuclear proximity potential of Blocki *et al.* [36] with γ the nuclear surface tension coefficient and Φ the universal proximity potential [37].

The barrier penetrability P is calculated using the following integral [33]:

$$P = \exp\left\{-\frac{2}{\hbar}\int_{R_{\rm in}}^{R_{\rm out}}\sqrt{2B(r)[E(r)-E(\text{sphere})]}dr\right\}, \tag{9}$$

where $R_{\rm in} = R_1 + R_2$, $B(r) = \mu$ and $R_{\rm out} = e^2Z_1Z_2/Q$. R_1, R_2 are the radius of the daughter nuclei and emitted alpha particle respectively, and μ is the reduced mass and Q is the released energy.

The partial half-life is related to the decay constant λ by

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P}\right). \tag{10}$$

The assault frequency ν has been taken as 10^{20} s^{-1}.

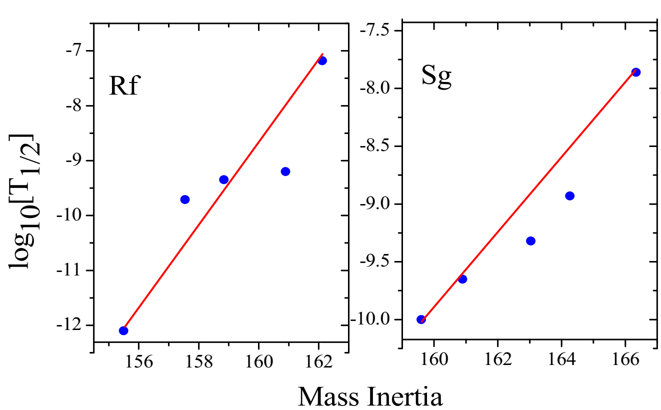

FIG. 2. Graph showing the variation of logarithm of experimental SF half-life of Rf and Sg with respect to mass inertia. Half-lives are in years.

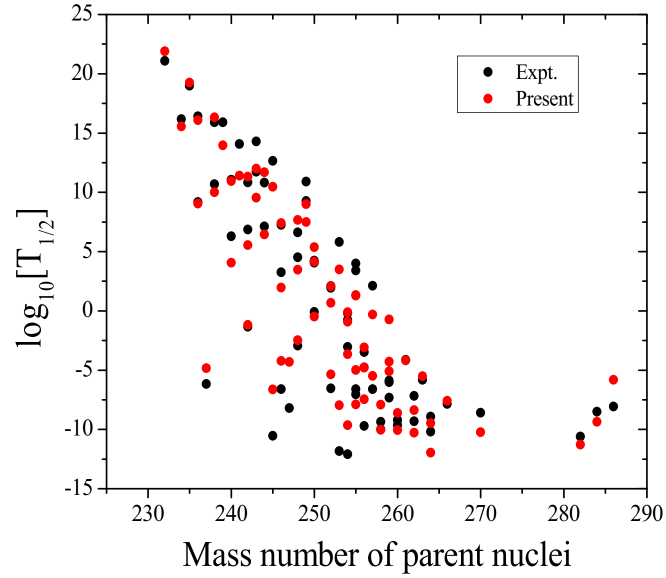

FIG. 3. Comparison of predicted logarithm of SF half-live values with the experimental data. Half-lives are in years.

B. Shell effect and mass-inertia-dependent formula for spontaneous fission half-lives

Spontaneous fission is probably the nuclear process, less accurately described with current models due to its complex nature. It is very hard to find models, deep rooted in physics, that could reproduce fission observables precisely. The early description of spontaneous fission half-lives was done within the framework of the charged liquid drop model [38]. In 1955, Swiatecki *et al.* [29] proposed a semiempirical formula for the first time to calculate SF half-lives. Swiatecki pointed out the importance of the shell structure in SF by proving that the ir-

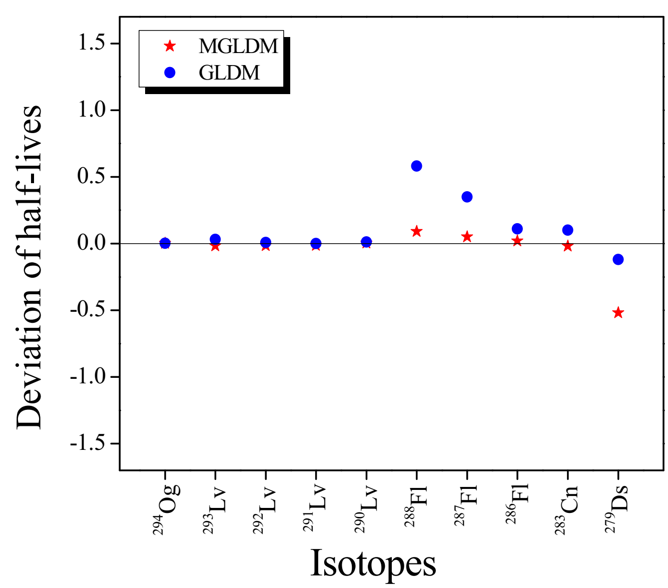

FIG. 4. Deviation of the calculated alpha half-lives, $[T_{1/2}^{\rm Expt.} - T_{1/2}^{\rm Theory}]$ with the corresponding experimental data for recently synthesized SHN. Half-lives are in seconds.

regularities in the original plot of SF life time against Z^2/A occur due to the irregularities in the ground-state masses, associated with shell structure. By checking how SF depends on the fissionability parameter and the isospin effect, we developed a formula for finding the half-lives of spontaneous fission [39].

Bao *et al.* [40] included a shell correction energy term to calculate SF half-life in the modified Swiatecki formula. In order to verify the dependence of SF life time on shell correction energy, we studied the variation of SF half-life time with shell correction energy. In Fig. 1, we plotted a graph with the logarithm of experimental SF half-life of Cf and Fm on the Y axis and shell correction energy taken from Moller *et al.* [41] on the X axis. Straight lines are drawn in these figures to guide the eye. As expected, we got a pattern that shows a

TABLE I. Comparison of alpha half-lives with SF half-lives for even Z nuclei with $Z = 104$–118.

					Decay modes	
Parent nuclei	Q value (MeV)	$T_{1/2}^{\mathrm{Expt.}}$ (s) [57]	$T_{1/2}^{\alpha}$ (s)	$T_{1/2}^{\mathrm{SF}}$ (s)	Theor.	Expt. [57]
^{253}Rf	9.355		1.01×10^{-01}	3.53×10^{-01}	α/SF	
^{254}Rf	9.215		2.51×10^{-01}	7.14×10^{-03}	SF	
^{255}Rf	9.055		7.38×10^{-01}	$3.17 \times 10^{+02}$	α	
^{256}Rf	8.926		$1.79 \times 10^{+00}$	$1.09 \times 10^{+00}$	α/SF	
^{257}Rf	9.082		5.61×10^{-01}	$1.25 \times 10^{+03}$	α	
^{258}Rf	9.192		2.49×10^{-01}	4.13×10^{-01}	α/SF	
^{259}Rf	9.128		3.74×10^{-01}	$2.57 \times 10^{+02}$	α	
^{260}Rf	8.903		$1.79 \times 10^{+00}$	7.46×10^{-02}	SF	
^{261}Rf	8.648		$1.15 \times 10^{+01}$	$2.08 \times 10^{+02}$	α	
^{262}Rf	8.485		$3.89 \times 10^{+01}$	1.31×10^{-01}	SF	
^{263}Rf	8.256		$2.35 \times 10^{+02}$	$1.79 \times 10^{+03}$	α	
^{264}Rf	8.045		$1.31 \times 10^{+03}$	$4.01 \times 10^{+00}$	SF	
^{265}Rf	7.805	$1.20 \times 10^{+02}$	$1.03 \times 10^{+04}$	$3.76 \times 10^{+04}$	α	SF
^{266}Rf	7.555		$9.73 \times 10^{+04}$	$9.48 \times 10^{+01}$	SF	
^{267}Rf	7.885	$4.68 \times 10^{+00}$	$4.65 \times 10^{+03}$	$1.02 \times 10^{+05}$	α	SF
^{268}Rf	8.045		$1.12 \times 10^{+03}$	$2.90 \times 10^{+00}$	SF	
^{258}Sg	9.615		7.74×10^{-02}	2.92×10^{-03}	SF	
^{259}Sg	9.765		2.80×10^{-02}	$5.58 \times 10^{+00}$	α	
^{260}Sg	9.901		1.13×10^{-02}	2.82×10^{-03}	SF	
^{261}Sg	9.714		3.58×10^{-02}	$4.03 \times 10^{+00}$	α	
^{262}Sg	9.605		7.01×10^{-02}	1.68×10^{-03}	SF	
^{263}Sg	9.405		2.57×10^{-01}	$7.26 \times 10^{+00}$	α	
^{264}Sg	9.205		9.89×10^{-01}	1.08×10^{-02}	SF	
^{265}Sg	9.045		$2.98 \times 10^{+00}$	$2.53 \times 10^{+02}$	α	
^{266}Sg	8.805		$1.69 \times 10^{+01}$	8.19×10^{-01}	SF	
^{267}Sg	8.625		$6.48 \times 10^{+01}$	$1.57 \times 10^{+04}$	α	
^{268}Sg	8.295		$8.85 \times 10^{+02}$	$5.32 \times 10^{+01}$	SF	
^{269}Sg	8.645	$1.20 \times 10^{+02}$	$5.11 \times 10^{+01}$	$6.08 \times 10^{+04}$	α	α
^{270}Sg	8.985		$3.78 \times 10^{+00}$	$2.37 \times 10^{+00}$	SF	
^{271}Sg	8.895	$9.60 \times 10^{+01}$	$7.05 \times 10^{+00}$	$9.86 \times 10^{+01}$	α	SF : 58 ± 23
^{272}Sg	8.675		$3.60 \times 10^{+01}$	9.16×10^{-04}	SF	
^{263}Hs	10.735		3.35×10^{-04}	3.09×10^{-02}	α	
^{264}Hs	10.590		7.36×10^{-04}	3.42×10^{-05}	SF	
^{265}Hs	10.470		1.42×10^{-03}	3.38×10^{-01}	α	
^{266}Hs	10.345		2.87×10^{-03}	8.89×10^{-04}	SF	
^{267}Hs	10.035		1.84×10^{-02}	$2.48 \times 10^{+01}$	α	
^{268}Hs	9.625		2.54×10^{-01}	2.79×10^{-01}	α/SF	
^{269}Hs	9.345		$1.66 \times 10^{+00}$	$3.94 \times 10^{+03}$	α	
^{270}Hs	9.065		$1.20 \times 10^{+01}$	$1.50 \times 10^{+01}$	α/SF	
^{271}Hs	9.505		5.06×10^{-01}	$5.51 \times 10^{+04}$	α	
^{272}Hs	9.785		7.49×10^{-02}	$3.11 \times 10^{+00}$	α	
^{273}Hs	9.705	2.00×10^{-01}	1.22×10^{-01}	$1.35 \times 10^{+02}$	α	α
^{274}Hs	9.575		2.79×10^{-01}	9.45×10^{-04}	SF	
^{275}Hs	9.435	2.00×10^{-01}	6.99×10^{-01}	5.99×10^{-02}	SF	α
^{276}Hs	9.285		$1.92 \times 10^{+00}$	6.17×10^{-07}	SF	
^{277}Hs	9.045	3.00×10^{-03}	$1.05 \times 10^{+01}$	6.20×10^{-04}	SF	SF

TABLE I. (*Continued.*)

Parent nuclei	Q value (MeV)	$T_{1/2}^{\mathrm{Expt.}}$ (s) [57]	$T_{1/2}^{\alpha}$ (s)	$T_{1/2}^{\mathrm{SF}}$ (s)	Decay modes Theor.	Expt. [57]
^{267}Ds	11.775		5.02×10^{-06}	5.46×10^{-04}	α	
^{268}Ds	11.662		8.50×10^{-06}	4.10×10^{-06}	α/SF	
^{269}Ds	11.505		1.82×10^{-05}	1.83×10^{-01}	α	
^{270}Ds	11.115		1.39×10^{-04}	1.86×10^{-03}	α	
^{271}Ds	10.875		5.06×10^{-04}	$8.87 \times 10^{+01}$	α	
^{272}Ds	10.765		9.09×10^{-04}	9.68×10^{-01}	α	
^{273}Ds	11.375		3.06×10^{-05}	$4.24 \times 10^{+03}$	α	
^{274}Ds	11.665		6.61×10^{-06}	5.07×10^{-01}	α	
^{275}Ds	11.405		2.42×10^{-05}	$2.23 \times 10^{+01}$	α	
^{276}Ds	11.105		1.16×10^{-04}	3.31×10^{-04}	α	
^{277}Ds	10.825	6.00×10^{-03}	5.30×10^{-04}	2.51×10^{-02}	α	α
^{278}Ds	10.465		4.16×10^{-03}	4.69×10^{-07}	SF	
^{279}Ds	10.085	2.10×10^{-01}	4.17×10^{-02}	2.95×10^{-03}	SF	SF : 89^{+4}_{-6}
^{280}Ds	9.805		2.47×10^{-01}	5.86×10^{-06}	SF	
^{281}Ds	9.515	$1.27 \times 10^{+01}$	$1.70 \times 10^{+00}$	9.67×10^{-02}	SF	SF : 93^{+5}_{-9}
^{276}Cn	11.905		7.65×10^{-06}	7.09×10^{-04}	α	
^{277}Cn	11.615		3.21×10^{-05}	1.43×10^{-01}	α	
^{278}Cn	11.305		1.60×10^{-04}	3.76×10^{-06}	SF	
^{279}Cn	11.035		6.79×10^{-04}	3.30×10^{-03}	α	
^{280}Cn	10.735		3.66×10^{-03}	1.51×10^{-06}	SF	
^{281}Cn	10.455	1.00×10^{-01}	1.88×10^{-02}	2.63×10^{-02}	α	α
^{282}Cn	10.175	9.10×10^{-04}	1.04×10^{-01}	1.66×10^{-04}	SF	SF
^{283}Cn	9.935	$4.20 \times 10^{+00}$	4.76×10^{-01}	$4.90 \times 10^{+00}$	α	α: $\geqslant$93
^{284}Cn	9.605	9.80×10^{-02}	$4.31 \times 10^{+00}$	1.35×10^{-02}	SF	SF
^{285}Cn	9.315	$2.80 \times 10^{+01}$	$3.30 \times 10^{+01}$	$3.10 \times 10^{+02}$	α	α
^{284}Fl	10.795		1.02×10^{-02}	$1.74 \times 10^{+00}$	α	
^{285}Fl	10.555	1.30×10^{-01}	4.17×10^{-02}	$2.91 \times 10^{+04}$	α	α
^{286}Fl	10.365	1.20×10^{-01}	1.31×10^{-01}	$4.83 \times 10^{+01}$	α	α : 60^{+10}_{-11}
^{287}Fl	10.155	4.80×10^{-01}	4.83×10^{-01}	$7.91 \times 10^{+05}$	α	α
^{288}Fl	10.065	6.60×10^{-01}	8.39×10^{-01}	$3.03 \times 10^{+03}$	α	α
^{289}Fl	9.965	$1.90 \times 10^{+00}$	$1.57 \times 10^{+00}$	$5.39 \times 10^{+07}$	α	α
^{289}Lv	11.105		6.27×10^{-03}	$3.02 \times 10^{+07}$	α	
^{290}Lv	11.005	8.30×10^{-03}	1.08×10^{-02}	$2.17 \times 10^{+04}$	α	α
^{291}Lv	10.895	1.90×10^{-02}	1.98×10^{-02}	$1.81 \times 10^{+08}$	α	α
^{292}Lv	10.775	1.30×10^{-02}	3.91×10^{-02}	$9.32 \times 10^{+05}$	α	α
^{293}Lv	10.685	5.70×10^{-02}	6.51×10^{-02}	$8.17 \times 10^{+09}$	α	α
^{293}Og	11.915		2.70×10^{-04}	$8.41 \times 10^{+08}$	α	
^{294}Og	11.835	6.90×10^{-04}	3.96×10^{-04}	$1.44 \times 10^{+05}$	α	α
^{295}Og	11.695		8.07×10^{-04}	$7.84 \times 10^{+09}$	α	

linear dependence of SF life time on shell correction energy, i.e., SF half-life time increases with shell correction energy.

The fissioning nucleus undergoes deformation and can be considered as a cluster configuration of more than one nucleus that touch each other, keeping their individuality. Nuclear inertia decreases from a relatively larger value for a sphere to the reduced mass of the separated fragments. A crucial physical quantity in calculating angular anisotropy of fission fragments after fission is the effective moment of inertia or mass inertia of a parent nucleus in fission dynamics. Also, deformation in the shape of the nucleus occurs in fission. Hence mass inertia that depends on deformation is a useful tool to study SF. The rigid body mass inertia of a nucleus [42,43] is given by $I_{\mathrm{rigid}} = B_{\mathrm{rigid}}[1 + 0.31\beta_2 + 0.44\beta_2^2 + \ldots]$ where the mass parameter $B_{\mathrm{rigid}} = \frac{2}{5}MR^2 = 0.0138A^{5/3}(\hbar^2/\mathrm{MeV})$. Here M is the mass of the nucleus, β_2 is the quadrupole deformation, and $R = 1.2A^{1/3}(\mathrm{fm})$. To investigate the dependence of SF half-life on the nuclear mass inertia parameter, we draw a graph connecting the logarithm of experimentally available SF half-life values of Sg and Rf on the Y axis and mass inertia on the X axis, as shown in Fig. 2. Interestingly we got a linear relationship for mass inertia similar to that of the shell correction energy. Since the role of shell structure is something which cannot be neglected, and another important

TABLE II. Comparison of alpha half-lives with SF half-lives for odd Z nuclei with $Z = 105$–117.

Parent nuclei	Q value (MeV)	$T_{1/2}^{\text{Expt.}}$(s) [57]	$T_{1/2}^{\alpha}$ (s)	$T_{1/2}^{\text{SF}}$ (s)	Decay modes Theor.	Decay modes Expt. [57]
^{255}Db	9.435		1.26×10^{-01}	4.06×10^{-01}	α/SF	
^{256}Db	9.335		2.38×10^{-01}	$1.66 \times 10^{+04}$	α	
^{257}Db	9.205		5.60×10^{-01}	$1.02 \times 10^{+02}$	α	
^{258}Db	9.505		6.95×10^{-02}	$1.28 \times 10^{+05}$	α	
^{259}Db	9.618		3.17×10^{-02}	$6.52 \times 10^{+01}$	α	
^{260}Db	9.495		6.84×10^{-02}	$7.86 \times 10^{+04}$	α	
^{261}Db	9.215		4.42×10^{-01}	$2.32 \times 10^{+01}$	α	
^{262}Db	9.045		$1.41 \times 10^{+00}$	$7.19 \times 10^{+04}$	α	
^{263}Db	8.835		$6.30 \times 10^{+00}$	$9.64 \times 10^{+01}$	α	
^{264}Db	8.655		$2.36 \times 10^{+01}$	$1.37 \times 10^{+06}$	α	
^{265}Db	8.495		$7.90 \times 10^{+01}$	$4.34 \times 10^{+03}$	α	
^{266}Db	8.215	$1.32 \times 10^{+03}$	$7.37 \times 10^{+02}$	$4.54 \times 10^{+07}$	α	SF
^{267}Db	7.915	$4.68 \times 10^{+03}$	$9.29 \times 10^{+03}$	$1.64 \times 10^{+05}$	α	SF
^{268}Db	8.255	$9.36 \times 10^{+04}$	$4.88 \times 10^{+02}$	$1.82 \times 10^{+08}$	α	SF
^{269}Db	8.495		$6.73 \times 10^{+01}$	$5.41 \times 10^{+03}$	α	
^{270}Db	8.265	$5.40 \times 10^{+04}$	$4.15 \times 10^{+02}$	$1.69 \times 10^{+05}$	α	SF
^{260}Bh	10.395		1.24×10^{-03}	$6.13 \times 10^{+02}$	α	
^{261}Bh	10.495		6.63×10^{-04}	6.49×10^{-01}	α	
^{262}Bh	10.315		1.83×10^{-03}	$1.28 \times 10^{+03}$	α	
^{263}Bh	10.085		7.11×10^{-03}	6.22×10^{-01}	α	
^{264}Bh	9.965		1.44×10^{-02}	$5.72 \times 10^{+03}$	α	
^{265}Bh	9.685		8.39×10^{-02}	$1.09 \times 10^{+01}$	α	
^{266}Bh	9.425		4.62×10^{-01}	$2.37 \times 10^{+05}$	α	
^{267}Bh	9.235		$1.67 \times 10^{+00}$	$1.62 \times 10^{+03}$	α	
^{268}Bh	9.025		$7.29 \times 10^{+00}$	$3.72 \times 10^{+07}$	α	
^{269}Bh	8.575		$2.18 \times 10^{+02}$	$8.53 \times 10^{+04}$	α	
^{270}Bh	9.065	$6.10 \times 10^{+01}$	$5.02 \times 10^{+00}$	$1.27 \times 10^{+08}$	α	
^{271}Bh	9.425	$1.50 \times 10^{+00}$	3.78×10^{-01}	$6.48 \times 10^{+03}$	α	α
^{272}Bh	9.305	$1.09 \times 10^{+01}$	8.35×10^{-01}	$2.76 \times 10^{+05}$	α	α
^{273}Bh	9.055		$4.79 \times 10^{+00}$	$2.08 \times 10^{+00}$	SF	
^{274}Bh	8.945	$4.40 \times 10^{+01}$	$1.04 \times 10^{+01}$	$1.51 \times 10^{+02}$	α	SF
^{265}Mt	11.125		7.63×10^{-05}	4.09×10^{-03}	α	
^{266}Mt	10.995		1.49×10^{-04}	$5.03 \times 10^{+01}$	α	
^{267}Mt	10.865		2.94×10^{-04}	2.00×10^{-01}	α	
^{268}Mt	10.665		8.79×10^{-04}	$9.04 \times 10^{+03}$	α	
^{269}Mt	10.525		1.91×10^{-03}	$7.93 \times 10^{+01}$	α	
^{270}Mt	10.185		1.43×10^{-02}	$1.83 \times 10^{+06}$	α	
^{271}Mt	9.905		8.09×10^{-02}	$1.56 \times 10^{+04}$	α	
^{272}Mt	10.345		4.96×10^{-03}	$6.87 \times 10^{+07}$	α	
^{273}Mt	10.805		3.24×10^{-04}	$4.24 \times 10^{+03}$	α	
^{274}Mt	10.595	4.40×10^{-01}	1.04×10^{-03}	$3.06 \times 10^{+05}$	α	α
^{275}Mt	10.485	2.00×10^{-02}	1.91×10^{-03}	$1.98 \times 10^{+00}$	α	α
^{276}Mt	10.105	4.50×10^{-01}	1.86×10^{-02}	$1.17 \times 10^{+02}$	α	α
^{277}Mt	9.915	5.00×10^{-03}	5.98×10^{-02}	1.04×10^{-03}	SF	SF
^{278}Mt	9.635	$4.50 \times 10^{+00}$	3.65×10^{-01}	$1.61 \times 10^{+00}$	α	α
^{279}Mt	9.385		$1.97 \times 10^{+00}$	1.78×10^{-03}	SF	
^{272}Rg	11.195		1.75×10^{-04}	$3.51 \times 10^{+03}$	α	
^{273}Rg	10.905		8.43×10^{-04}	$3.78 \times 10^{+01}$	α	
^{274}Rg	11.475		3.62×10^{-05}	$3.17 \times 10^{+05}$	α	
^{275}Rg	11.775		7.47×10^{-06}	$2.29 \times 10^{+01}$	α	
^{276}Rg	11.485		3.18×10^{-05}	$2.82 \times 10^{+03}$	α	
^{277}Rg	11.205		1.36×10^{-04}	1.04×10^{-01}	α	
^{278}Rg	10.845	4.20×10^{-03}	9.73×10^{-04}	$1.40 \times 10^{+01}$	α	α
^{279}Rg	10.525	9.00×10^{-02}	6.09×10^{-03}	1.48×10^{-03}	α/SF	α
^{280}Rg	10.145	$4.60 \times 10^{+00}$	6.11×10^{-02}	$1.66 \times 10^{+01}$	α	α
^{281}Rg	9.905	$1.70 \times 10^{+01}$	2.78×10^{-01}	3.32×10^{-02}	SF	SF : 88^{+7}_{-9}

TABLE II. (*Continued.*)

Parent nuclei	Q value (MeV)	$T_{1/2}^{\mathrm{Expt.}}$(s) [57]	$T_{1/2}^{\alpha}$ (s)	$T_{1/2}^{\mathrm{SF}}$ (s)	Decay modes Theor.	Decay modes Expt. [57]
^{282}Rg	9.635	$1.00 \times 10^{+02}$	$1.65 \times 10^{+00}$	$5.60 \times 10^{+02}$	α	α
^{283}Rg	9.355		$1.14 \times 10^{+01}$	$1.21 \times 10^{+00}$	SF	
^{278}Nh	11.855		1.85×10^{-05}	$3.32 \times 10^{+01}$	α	
^{279}Nh	11.515		1.04×10^{-04}	4.39×10^{-02}	α	
^{280}Nh	11.225		4.79×10^{-04}	$9.23 \times 10^{+01}$	α	
^{281}Nh	11.045		1.26×10^{-03}	3.46×10^{-01}	α	
^{282}Nh	10.785	7.30×10^{-02}	5.43×10^{-03}	$7.95 \times 10^{+03}$	α	α
^{283}Nh	10.505	7.50×10^{-02}	2.80×10^{-02}	$2.99 \times 10^{+01}$	α	α
^{284}Nh	10.275	9.10×10^{-01}	1.13×10^{-01}	$7.45 \times 10^{+05}$	α	α
^{285}Nh	10.005	$4.20 \times 10^{+00}$	6.24×10^{-01}	$1.60 \times 10^{+03}$	α	α
^{286}Nh	9.785	$9.50 \times 10^{+00}$	$2.64 \times 10^{+00}$	$3.38 \times 10^{+07}$	α	α
^{287}Nh	9.545		$1.35 \times 10^{+01}$	$4.86 \times 10^{+04}$	α	
^{287}Mc	10.765	3.70×10^{-02}	2.34×10^{-02}	$1.59 \times 10^{+06}$	α	α
^{288}Mc	10.755	1.64×10^{-01}	2.39×10^{-02}	$2.18 \times 10^{+10}$	α	α
^{289}Mc	10.515	3.30×10^{-01}	9.97×10^{-02}	$2.39 \times 10^{+07}$	α	α
^{290}Mc	10.455	6.50×10^{-01}	1.40×10^{-01}	$2.41 \times 10^{+11}$	α	α
^{291}Mc	10.315		3.27×10^{-01}	$9.54 \times 10^{+08}$	α	
^{291}Ts	11.475		1.51×10^{-03}	$2.02 \times 10^{+08}$	α	
^{292}Ts	11.385		2.40×10^{-03}	$1.39 \times 10^{+12}$	α	
^{293}Ts	11.295	2.20×10^{-02}	3.83×10^{-03}	$2.87 \times 10^{+09}$	α	α
^{294}Ts	11.205	5.10×10^{-02}	6.15×10^{-03}	$2.56 \times 10^{+13}$	α	α

factor that should be considered while calculating SF half-lives is mass inertia, a new formula is introduced by including the shell correction term (E_{shell}) and mass-inertia parameter (I_{rigid}), given below:

$$\log_{10}[T_{1/2}(yr)] = c_1 + c_2\left(\frac{Z^2}{(1-kI^2)A}\right) + c_3\left(\frac{Z^2}{(1-kI^2)A}\right)^2 + c_4E_{\mathrm{shell}} + c_5I_{\mathrm{rigid}} + h_i. \tag{11}$$

The surface asymmetry coefficient $k = 2.6$ is taken from Royer *et al.* [33]. Here $I = (N-Z)/A$ is the isospin effect and E_{shell} is the shell correction energy taken from Moller *et al.* [41].

The constants for the equation are $c_1 = 1208.763\,104$, $c_2 = -49.264\,392\,88$, $c_3 = 0.486\,222\,575$, $c_4 = 3.557\,962\,857$, $c_5 = 0.042\,925\,714\,94$, and h_i is the blocking effect for the unpaired nucleon. For even-even heavy and superheavy nuclei $h_i = 0$, odd N nuclei, $h_{eo} = 2.749\,814$, and odd Z nuclei, $h_{oe} = 2.490\,760$. For odd-odd heavy and superheavy nuclei h_i is the sum of h_{eo} and h_{oe}. The above formula is used to determine SF half-lives of 44 even-even, 12 odd N, and 12 odd Z heavy and superheavy nuclei and the obtained values are in agreement with the experimental SF half-lives with a standard deviation 1.772. Figure 3 shows the comparison of the predicted logarithm of SF half-lives with experimental values.

III. RESULTS AND DISCUSSION

The GLDM of Royer used the proximity parametrization of Feldmeier *et al.* [44], and it has been improved by using the proximity potential of Blocki and co-workers [36,37]. The potential, proximity 77 (PP77), was used for the first time by Shi and Swiatecki [45] and after that a proximity potential with several modifications [46,47] has been used by several authors. Yao *et al.* [48] and Ghodsi *et al.* [49] studied α emission from various isotopes using different nuclear potentials and found that PP77 will give results close to experiments. Our studies on alpha decay of Po isotopes [50], Hg isotopes [51], and cluster decay of various nuclei [52] in the translead region using different versions of proximity potential have also shown that PP77 is the apt potential with least standard deviation.

The half-lives of even-even superheavy nuclei [53,54] which are synthesized experimentally are calculated through MGLDM [55]. The obtained results are then cross checked with the experimental values and with the values using GLDM [56]. It was found that the MGLDM can be used effectively to replicate the experimental half-lives in the superheavy regions. Figure 4 shows the deviation of alpha half-lives $[T_{1/2}^{\mathrm{Expt.}} - T_{1/2}^{\mathrm{Theory}}]$ calculated using MGLDM [55] and GLDM [56] with the experimental results [53,54]. The figure shows that the MGLDM is appropriate in reproducing the experimental half-lives of most of the superheavy nuclei [53,54]. It is to be noted that the deviation of MGLDM and GLDM predictions is very small, for, e.g., in the case of ^{288}Fl, $T_{\alpha}^{\mathrm{MGLDM}} = 0.71$ s, $T_{\alpha}^{\mathrm{GLDM}} = 0.22$ s, and $T_{\alpha}^{\mathrm{Expt}} = 0.8$ s. The calculations of the proximity energy within the MGLDM are easier than those using the GLDM and the improvement in accuracy is fair. We would like to mention that we are also able to reproduce the alpha half-lives of most of the experimentally synthesized superheavy nuclei [57] using the MGLDM [55].

In this work, the decay modes of superheavy nuclei with $Z = 104$–118 are studied by comparing $T^{\alpha}_{1/2}$ calculated using MGLDM with $T^{\mathrm{SF}}_{1/2}$ using a formula that depends on the shell effect and mass inertia.

The alpha decay energies of all the isotopes are computed using the equation

$$Q = \Delta M_p - (\Delta M_\alpha + \Delta M_d). \tag{12}$$

The mass excess values ΔM_p, ΔM_d, and ΔM_α of all the isotopes are taken from the recent mass table of Wang *et al.* [58].

The comparison of $T^{\alpha}_{1/2}$ with $T^{SF}_{1/2}$ for even Z isotopes in the range $104 \leqslant Z \leqslant 118$ are given in Table I. The first and second columns represent the parent nuclei and Q values for the decay respectively. The experimental half-lives taken from Oganessian *et al.* [57] are given in the third column. The fourth column represents the alpha half-lives computed using MGLDM and the fifth column in the table represents the SF half-lives using the formula which depends on shell effect and mass inertia. Column 6 shows predicted decay modes, and the experimental decay modes [57] are depicted in column 7. Table II gives similar studies performed for odd Z superheavy nuclei within the range $105 \leqslant Z \leqslant 117$. The tables clearly shows that the experimental half-lives and decay modes can be reproduced moderately well using the present formalism.

From Tables I and II, it is clear that, among 53 experimentally synthesized superheavy nuclei, the decay modes of 44 nuclei are exactly reproduced using the present formalism, thereby proving the applicability of the present models, the MGLDM, and the formula which depends on shell effect and mass inertia, for studying the decay modes of the isotopes in the superheavy regions.

IV. SUMMARY

The decay modes of the isotopes of superheavy nuclei with $Z = 104-118$ are studied by comparing $T^{\alpha}_{1/2}$ calculated using MGLDM with $T^{\mathrm{SF}}_{1/2}$ using the formula which depends on shell effect and mass inertia. The predicted results are compared with the experimental observations. The decay modes of most of the experimentally synthesized nuclei are reproduced well with the present formalism. The half-lives predicted are also compared with the experimental results. We hope that the studies may induce further investigation in the superheavy region.

[1] A. Sobiczewski, F. A. Gareev, and B. N. Kalinkin, Phys. Lett. B **22**, 500 (1966).
[2] H. Meldner, Ark. Fys. **36**, 593 (1967).
[3] W. D. Myers and W. J. Swiatecki, Ark. Fys. **36**, 343 (1967).
[4] U. Mosel and W. Greiner, Z. Phys. **222**, 261 (1969).
[5] S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafson, I.-L. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. A **131**, 1 (1969).
[6] S. Hofmann and G. Münzenberg, Rev. Mod. Phys. **72**, 733 (2000).
[7] Yu. Ts. Oganessian, J. Phys. G: Nucl. Part. Phys. **34**, R165 (2007).
[8] D. Ackermann, Nucl. Phys. A **787**, 353 (2007).
[9] K. Morita, Nucl. Phys. A **944**, 30 (2015).
[10] S. Hofmann, F. P. Heßberger, D. Ackermann, S. Antalic, P. Cagarda, B. Kindler, P. Kuusiniemi, M. Leino, B. Lommel, O. N. Malyshev, R. Mann, G. Münzenberg, A. G. Popeko, S. S'aro, B. Streicher, and A. V. Yeremin, Nucl. Phys. A **734**, 93 (2004).
[11] Yu. Ts. Oganessian and S. N. Dmitriev, Russ. Chem. Rev. **78**, 1077 (2009).
[12] Z. Ren, Y. Mao, Q. Zhi, C. Xu, and T. Dong, J. Radioanal. Nucl. Chem. **272**, 209 (2007).
[13] D. N. Poenaru, R. A. Gherghescu, and N. Carjan, Europhys. Lett. **77**, 62001(2007).
[14] K. P. Santhosh and R. K. Biju, J. Phys. G: Nucl. Part. Phys. **36**, 015107 (2009).
[15] K. P. Santhosh and C. Nithya, Int. J. Mod. Phys. E **25**, 1650079 (2016).
[16] K. P. Santhosh, A. Augustine, C. Nithya, and B. Priyanka, Nucl. Phys. A **951**, 116 (2016).
[17] K. P. Santhosh and C. Nithya, Eur. Phys. J. A **52**, 371 (2016).
[18] K. P. Santhosh and C. Nithya, Eur. Phys. J. A **53**, 189 (2017).
[19] C. Tang, X. Jin, N. Wang, and E. G. Zhao, Front. Phys. **10**, 102401 (2015).
[20] K. P. Santhosh and C. Nithya, Phys. Rev. C **97**, 044615 (2018).
[21] K. P. Santhosh and C. Nithya, Phys. Rev. C **96**, 044613 (2017).
[22] Z. Y. Wang, Z. M. Niu, Q. Liu, and J. Y. Guo, J. Phys. G: Nucl. Part. Phys. **42**, 055112 (2015).
[23] Z. H. Fei, L. J. Qing, Z. Wei, Z. X. Hong, and G. Z. Guo, Commun. Theor. Phys. **48**, 545 (2007).
[24] D. N. Poenaru, H. Stöcker, and R. A. Gherghescu, Eur. Phys. J. A **54**, 14 (2018).
[25] M. Ikram, A. A. Rather, A. A. Usmani, B. Kumar, and S. K. Patra, arXiv:1709.07311v2.
[26] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. C **45**, 2247 (1992).
[27] G. Royer and R. Moustabchir, Nucl. Phys. A **683**, 182 (2001).
[28] D. N. Poenaru, M. Ivaşcu, and A. Săndulescu, J. Phys. G: Nucl. Phys. **5**, L169 (1979).
[29] W. J. Swiatecki, Phys. Rev. **100**, 937 (1955).
[30] C. Xu and Z. Ren, Phys. Rev. C **71**, 014309 (2005).
[31] M. Warda and J. L. Egido, Phys. Rev. C **86**, 014322 (2012).
[32] R. Smolánczuk, J. Skalski, and A. Sobiczewski, Phys. Rev. C **52**, 1871 (1995).
[33] G. Royer and B. Remaud, Nucl. Phys. A **444**, 477 (1985).
[34] G. Royer and R. A. Gherghescu, Nucl. Phys. A **699**, 479 (2002).
[35] G. Royer, J. Phys. G: Nucl. Part. Phys. **26**, 1149 (2000).
[36] J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. (N.Y.) **105**, 427 (1977).
[37] J. Blocki and W. J. Swiatecki, Ann. Phys. (N.Y.) **132**, 53 (1981).
[38] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

[39] K. P. Santhosh, R. K. Biju, and S. Sabina, Nucl. Phys. A **832**, 220 (2010).
[40] X. J. Bao, S. Q. Guo, H. F. Zhang, Y. Z. Xing, J. M. Dong, and J. Q. Li, J. Phys. G: Nucl. Part. Phys. **42**, 085101 (2015).
[41] P. Möller, A. J. Sierk, T. Ichikawa, and H. Sagawa, At. Data Nucl. Data Tables **109-110**, 1 (2016).
[42] A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd. **30**, 1 (1955).
[43] J. M. Allmond, and J. L. Wood, Phys. Lett. B **767**, 226 (2017).
[44] H. Feldmeier, in *12th Summer School on Nuclear Physics* (Mikolajski, Poland 1979).
[45] Y. J. Shi and W. J. Swiatecki, Nucl. Phys. A **438**, 450 (1985).
[46] W. Reisdorf, J. Phys. G: Nucl. Part. Phys. **20**, 1297 (1994).
[47] I. Dutt and R. K. Puri, Phys. Rev. C **81**, 064609 (2010).
[48] Y. J. Yao, G. L. Zhang, W. W. Qu, and J. Q. Qian, Eur. Phys. J. A **51**, 122 (2015).
[49] O. N. Ghodsi and A. Daei-Ataollah, Phys. Rev. C **93**, 024612 (2016).
[50] K. P. Santhosh and Indu Sukumaran, Eur. Phys. J. A **53**, 246 (2017).
[51] K. P. Santhosh and Indu Sukumaran, CR Phys. **19**, 347 (2018).
[52] K. P. Santhosh and Indu Sukumaran, Eur. Phys. J. A **53**, 136 (2017).
[53] Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, I. V. Shirokovsky, Yu. S. Tsyganov, G. G. Gulbekian, S. L. Bogomolov, B. N. Gikal, A. N. Mezentsev, S. Iliev, V. G. Subbotin, A. M. Sukhov, A. A. Voinov, G. V. Buklanov, K. Subotic, V. I. Zagrebaev, M. G. Itkis, J. B. Patin *et al.*, Phys. Rev. C **70**, 064609 (2004).
[54] Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, I. V. Shirokovsky, Yu. S. Tsyganov, G. G. Gulbekian, S. L. Bogomolov, B. N. Gikal, A. N. Mezentsev, S. Iliev, V. G. Subbotin, A. M. Sukhov, A. A. Voinov, G. V. Buklanov, K. Subotic, V. I. Zagrebaev, M. G. Itkis, J. B. Patin *et al.*, Phys. Rev. C **71**, 029902(E) (2005).
[55] K. P. Santhosh, C. Nithya, H. Hassanabadi, and D. T. Akrawy, Phys. Rev. C **98**, 024625 (2018).
[56] H. Zhang, W. Zuo, J. Li, and G. Royer, Phys. Rev. C **74**, 017304 (2006).
[57] Yu. Ts. Oganessian, A. Sobiczewski, and G. M. Ter-Akopian, Phys. Scr. **92**, 023003 (2017).
[58] M. Wang, G. Audi, F. G. Kondev, W. J. Huang, S. Naimi, and X. Xu, Chin. Phys. C **41**, 030003 (2017).