# Ground state inversions in hole nuclei near <sup>132</sup>Sn driven by the monopole interaction

H. K. Wang<sup>0</sup>,<sup>1,2,3,\*</sup> Z. Q. Chen,<sup>4,5</sup> H. Jin<sup>0</sup>,<sup>6</sup> Z. H. Li,<sup>4</sup> G. S. Li<sup>0</sup>,<sup>7</sup> Y. M. Feng,<sup>1</sup> and Q. Wang<sup>8</sup>

<sup>1</sup>College of Physics and Telecommunication Engineering, Zhoukou Normal University, Henan 466000, China

<sup>3</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

<sup>4</sup>School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

<sup>5</sup>Department of Physics, University of Surrey, Guildford GU27XH, United Kingdom

<sup>6</sup>School of Arts and Sciences, Shanghai Dianji University, Shanghai 201306, China

<sup>7</sup>Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

<sup>8</sup>School of Network Engineering, Zhoukou Normal University, Henan 466000, China

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The neutron-rich nuclei to the southwest of <sup>132</sup>Sn are studied comprehensively by large-scale, shell-model calculations with the extended pairing plus multipole-multipole force (EPQQM) model. A regular correlation driven by the monopole interaction between the neutron orbits  $h_{11/2}$  and  $d_{3/2}$  is found in this nuclear region for different isotonic chains with N = 79, 80, 81. The ground-state inversions from <sup>130</sup>In (<sup>129</sup>Cd) to <sup>128</sup>In (<sup>127</sup>Cd) seen experimentally are well described for the first time by this regular correlation. The regular correlation in different isotonic chains is also supported by a systematic comparison between the observed spectra of <sup>126</sup>Ag and <sup>128</sup>In, and further confirmed by the investigation of isomeric states of <sup>126</sup>Pd, <sup>128</sup>Cd, and <sup>129</sup>In the in N = 80 isotonic chain. This regular correlation in different isotonic chains should provide useful guidance for further experiments in this region of nuclei.

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# I. INTRODUCTION

Many exotic and intriguing phenomena have been reported in neutron-rich nuclei near the doubly magic <sup>132</sup>Sn. The second abundance peak at  $A \approx 130$  appears through the astrophysical rapid neutron capture process, and the doubly magic nature of <sup>132</sup>Sn was explored in experiment and also theory [1–5]. Recently, the doubly-magic nucleus <sup>132</sup>Sn was reconfirmed again by the first charge-radius measurement of a neutron-rich Sn isotope beyond N = 82 [6] and the investigations for nuclei near  $^{132}$ Sn are important for the *r*-process study around the A = 130 abundance peak [7]. The strong fragmentation of single-hole strength is reported in <sup>131</sup>In from measurements of the spectroscopic factors of protonhole states [8]. From mass measurements of the neutron-rich cadmium isotopes, the N = 82 shell closure was confirmed below Z = 50 and the  $h_{11/2}$  neutron orbital near <sup>132</sup>Sn was reported to be a key for the evolution of the N = 82 shell gap towards Z = 40 [9]. A reduction of the Z = 40 subshell gap was suggested in the Ag isotopes approaching N = 82 and the tensor force was found to play a crucial role in the proton shell evolution [10]. The first spectroscopic information on the excited states in <sup>125,127</sup>Pd suggested competition between proton excitations and neutron excitations of hole nuclei in the vicinity of the doubly-magic <sup>132</sup>Sn [11] nucleus. The isomeric states in <sup>128</sup>Cd have been identified and compared to the results of large-scale shell-model calculations [12]. A microsecond isomer has been identified in <sup>127</sup>Cd, and the experimental data have been investigated by two theoretical shell-model approaches with different model spaces and interactions [13].

Note that a new  $\beta$ -decaying high-spin isomer has been discovered in <sup>128</sup>In at 1797.6(20) keV by using Penningtrap techniques [14]. The isotopes of neutron-rich indium can provide essential data to test the nuclear shell model and to develop the nucleon-nucleon effective interaction. The ground state of  $^{128}$ In was suggested as  $(3)^+$  by Fogelberg and Carl in Ref. [15], and the change in position of the  $1^-$  state from <sup>130</sup>In to <sup>128</sup>In was investigated in Ref. [16]. Such ground-state inversion also exists for <sup>129</sup>Cd to <sup>127</sup>Cd. In <sup>129</sup>Cd, the ground state is assigned as  $11/2^{-}$ , while the excited  $3/2^+$  level changes into the ground state in <sup>127</sup>Cd. It is very interesting to search for the nuclear structure reason for these ground-state inversions. The isomers of neutron-rich exotic nuclei have been investigated comprehensively, and a  $\gamma$  ray at  $E_{\gamma} = 254.8$  keV was observed in <sup>126</sup>Ag [17]. The systematic study assigned  $3^+$  as the ground state of  ${}^{126}Ag$  by comparison with states in  ${}^{128}In$ , while the ground state of  $^{126}$ Ag was assigned as 1<sup>-</sup> using the jj45pna interaction. The two-body effective Hamiltonians have been well established for decades in shell-model calculations, and the monopole interaction is crucial for obtaining agreement with experiment [18]. Different effects of the monopole-driven shell evolutions are discussed for tensor forces [19] and other terms in the nucleon-nucleon interaction [20–23]. In fact, the monopole corrections are necessary for two-body interactions [24], as confirmed by the *ab initio* studies in Ref. [25].

<sup>&</sup>lt;sup>2</sup>School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

<sup>\*</sup>whk2007@163.com



FIG. 1. The EPQQM calculations on ground-state inversions in comparison with experiment and jj45pna results.

In this paper, the neutron-rich hole nuclei of N =79, 80, 81 are investigated using the extended pairing plus multipole-multipole force (EPQQM model [26-29]. This model employs monopole correction (Mc) terms that provide an advantage to study monopole effects. With <sup>78</sup>Ni as the frozen core, the present model space includes six proton orbits  $(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}, 0g_{7/2}, 1d_{5/2})$  and seven neutron orbits  $(0g_{7/2}, 1d_{5/2}, 2s_{1/2}, 0h_{11/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2})$ . The two orbits above Z = 50 (N = 82) added for proton (neutron) core excitations are frozen in this work, to make sure of using a uniform model space to study the nuclei a little further from the doubly magic <sup>132</sup>Sn. The single-particle energies and the two-body force strengths employed in the present work are consistent with our previous paper [30]. There is no truncation in major shell orbits, and the shell-model code NUSHELLX@MSU is used for the calculations [31].

# **II. THE MONOPOLE CORRECTIONS**

In the present interaction, the monopole effects can be investigated using the monopole correction terms

$$Mc = k(ia, i'c) \sum_{JM} A^{\dagger}_{JM}(ia, i'c) A_{JM}(ia, i'c).$$
(1)

Here  $A_{JM}^{\dagger}(ia, i'c)$  and  $A_{JM}$  are the pair operators, and  $k_{\rm mc}$  is the monopole force strength (*pn* representation). Three monopole terms have already been employed, namely Mc1  $\equiv k_{mc}(vh_{11/2}, vf_{7/2}) = 0.52$  MeV, Mc2  $\equiv k_{mc}(\pi g_{9/2}, vh_{11/2}) = -0.4$  MeV, and Mc3  $\equiv k_{mc}(\pi g_{9/2}, vg_{7/2}) = -1.0$  MeV. These play crucial roles in explaining the energy spectrum to the southwest of <sup>132</sup>Sn [30,32,33]. In <sup>128</sup>In, the ground state was suggested as (3)<sup>+</sup> by Fogelberg and Carl [15], while the (3<sup>+</sup>) state becomes an excited level in <sup>130</sup>In with the configuration  $\pi g_{9/2}^{-1}vd_{3/2}^{-1}$ , and the 1<sup>(-)</sup> is suggested as the ground state in <sup>130</sup>In with the configuration  $\pi g_{9/2}^{-1}vh_{11/2}^{-1}$ .

#### **III. GROUND STATE INVERSIONS**

It is very interesting to see why the ground state is reversed experimentally from <sup>130</sup>In to <sup>128</sup>In. Such a ground-state inversion also exists from <sup>127</sup>Cd to <sup>129</sup>Cd (Fig. 1) with increasing neutron number. The ground state of odd Cd isotopes from



FIG. 2. The monopole effects in  $3^+$  and  $1^-$  levels in  $^{128}$ In.

<sup>121</sup>Cd to <sup>127</sup>Cd is (3/2<sup>+</sup>), while the excited level 11/2<sup>-</sup> becomes the ground state in <sup>129</sup>Cd. Note that the (3)<sup>+</sup> level in <sup>128</sup>In has a main configuration of  $\pi g_{9/2}^{-1} \nu d_{3/2}^{-1} h_{11/2}^{-2}$ , and the 3<sup>+</sup> level in <sup>128</sup>In would be influenced by monopole effects involving the  $\pi g_{9/2}$ ,  $\nu d_{3/2}$ , and  $\nu h_{11/2}$  orbits. So, the monopole terms  $Mc(\pi g_{9/2}, \nu d_{3/2})$ ,  $Mc(\pi g_{9/2}, \nu h_{11/2})$ , and  $Mc(\nu h_{11/2}, \nu d_{3/2})$ are studied as the function of the monopole force strength  $k_{mc}$ in Fig. 2. The monopole term  $Mc(\pi g_{9/2}, \nu d_{3/2})$  shifts the 3<sup>+</sup> level down about 40 keV as  $k_{mc}$  varies from 0.5 to 0 MeV, while there is a slight increase from 0 to -0.5 MeV.

For Mc( $\pi g_{9/2}$ ,  $\nu h_{11/2}$ ), the 3<sup>+</sup> level keeps linearly decreasing as  $k_{\rm mc}$  varies from 0.5 to -0.2 MeV, and then is almost stable from -0.3 to -0.5 MeV. As the monopole interaction between neutron orbitals  $h_{11/2}$  and  $d_{3/2}$ , the monopole term Mc( $\nu h_{11/2}$ ,  $\nu d_{3/2}$ ) has been found to provide influence obviously on the 3<sup>+</sup> level and reverses the 3<sup>+</sup> level into the ground state after  $k_{\rm mc} = -0.1$  MeV. With two more neutron holes in <sup>128</sup>In than in <sup>130</sup>In, the additional monopole strength is quantitatively fixed as Mc4( $\nu d_{3/2}$ ,  $\nu h_{11/2}$ ) = -0.4. With this Mc4 monopole correction, the excited 3<sup>+</sup> level in <sup>128</sup>In reverses into the ground state, and the 1<sup>-</sup> level becomes an excited state closer to the experimental datum 0.248 MeV (Fig. 3). The 16<sup>+</sup> state also has a positive change that shifts it down close to the datum 1.798 MeV.

To validate this regular correlation driven by monopole interactions, the isotone N = 79 is studied. In <sup>127</sup>Cd, the reversal of the excited level  $3/2^+$  into the ground state is driven by Mc4 after increasing by 2 the number of neutron holes from <sup>129</sup>Cd. In experiments, the  $11/2^-$  level is the ground state in <sup>129</sup>Cd, and the  $3/2^+$  level is the excited one. Based on this regular correlation, the  $3/2^+$  state is suggested as the ground state of <sup>127</sup>Cd in theory. As a further test, more N = 79 isotones are investigated. In Ref. [34], an isomeric state with a half-life of 27(6)  $\mu$ s is observed in <sup>126</sup>Ag, and (1<sup>-</sup>) and (3<sup>+</sup>) are assigned to the 254 keV and the groundstate levels based on the systematics by comparing with the level scheme of <sup>128</sup>In. In Fig. 1, both the present calculations and the jj45pna results reproduce well these reversed ground states of <sup>128</sup>In and <sup>127</sup>Cd in experiments, while the jj45pna



FIG. 3. The theoretical low-lying levels of  $^{128}$ In in comparison with experimental data. "+Mc4" means considering monopole effects of Mc4 in the calculations (Th.).

interaction gives the 1<sup>-</sup> level as the ground state of <sup>126</sup>Ag. As shown in Fig. 4, the 3<sup>+</sup> level in <sup>126</sup>Ag is reversed well into the ground state driven by this new monopole effect of Mc4. In this work, the ground states keep the same parity and J value, and they are connected by the same regular correlation driven by the monopole interaction. This systematic study provides evidence for extending the regular correlation from <sup>128</sup>In to <sup>126</sup>Ag, which suggests 3<sup>+</sup> as the ground state of <sup>126</sup>Ag. In the next N = 79 isotone <sup>125</sup>Pd, the excited state 3/2<sup>+</sup>

In the next N = 79 isotone  ${}^{125}$ Pd, the excited state  $3/2^+$ is also shifted down as the ground state even in four protonhole configurations such as  $\pi g_{9/2}^{-4}$ . With increasing number of proton holes from  ${}^{128}$ In to  ${}^{125}$ Pd, the *p-n* monopole interaction does not have more impact, while the neutron monopole interaction keeps dominating the ground-state inversions in N = 79 isotones. Figure 2 also shows the greater monopole effects of Mc( $\nu h_{11/2}$ ,  $\nu d_{3/2}$ ) between neutrons. The competition between 1<sup>-</sup> (11/2<sup>-</sup>) and 3<sup>+</sup> (3/2<sup>+</sup>) will be determined by their configurations. As shown in Fig. 5, the 3<sup>+</sup> (or 3/2<sup>+</sup>) states of these N = 79 isotones have the main neutron configuration  $\nu d_{3/2}^{-1} h_{11/2}^{-2}$ , and the 1<sup>-</sup> (11/2<sup>-</sup>) states have the main neutron configuration  $h_{11/2}^{-1} h_{11/2}^{-2}$ , as well as  $h_{11/2}^{-1} d_{3/2}^{-2}$ .



FIG. 4. The comparison of  $1^-$  and  $3^+$  states in  ${}^{126}$ Ag and  ${}^{128}$ In with monopole effects from Mc4.



FIG. 5. The variation of configurations in the ground states of N = 79 isotones with the monopole effects of Mc4.

Considering the attractive monopole correction between  $vh_{11/2}$  and  $vd_{3/2}$ , the components of  $vd_{3/2}^{-1}h_{11/2}^{-2}$  and  $vd_{3/2}^{-2}h_{11/2}^{-1}$  increase in these N = 79 isotones, while the part of  $vh_{11/2}^{-1}h_{11/2}^{-2}$  distinctly shrinks, as in the 1<sup>-</sup> and 11/2<sup>-</sup> levels (Fig. 5). The configurational competition is the structural reason for the ground-state inversions in N = 79 isotones, which are driven by the monopole interaction between  $vh_{11/2}$  and  $vd_{3/2}$ .

The same regular correlation is also found in N = 80 isotones. Three isomeric states, in <sup>129</sup>In, <sup>128</sup>Cd, and <sup>126</sup>Pd especially, shown in Fig. 6, exhibit this regular correlation driven by the monopole interaction. The calculations can reproduce well the experimental data with Mc4( $\nu d_{3/2}$ ,  $\nu h_{11/2}$ ) = -0.4 MeV, and the jj45pna results are listed for comparison. In the present work, the 7<sup>-</sup> level in <sup>126</sup>Pd has 59% of  $\pi g_{9/2}^{-4} \nu h_{11/2}^{-1} \nu d_{3/2}^{-1}$  (43% without Mc4), and jj45pna has about 18% of this configuration as the biggest component. For the 5<sup>-</sup> level in <sup>128</sup>Cd, the main configuration is 74% of  $\pi g_{9/2}^{-2} \nu h_{11/2}^{-1} \nu d_{3/2}^{-1}$  (58% without Mc4), while jj45pna has about 11% of  $\pi g_{9/2}^{-2} \nu h_{11/2}^{-1} \nu d_{3/2}^{-1}$  and 16% of  $\pi g_{9/2}^{-2} \nu h_{11/2}^{-1} \nu s_{1/2}^{-1}$ .



FIG. 6. The Mc4 monopole effects in negative-parity levels of the N = 80 isotones. The present results are compared to experimental data and jj45pna calculations.

If we sum up the just the proton configurations in this 5<sup>-</sup> level, the jj45pna model predicts about 33% of  $\pi g_{9/2}^{-2}$  and 25% of  $\pi g_{9/2}^{-1} p_{1/2}^{-1}$ , while 70% of  $\pi g_{9/2}^{-1} p_{1/2}^{-1}$  is reported in Ref. [12]. As a further transitional test, the  $B(E2, 7^- \rightarrow 5^-)$ in <sup>126</sup>Pd is 2.76 W.u. with Mc4 (almost zero before) compared with 2.13(14) W.u. experimentally [35], while jj45pna provides 6.27 W.u. for this transition. In <sup>128</sup>Cd, the value of  $B(E2, 7^- \rightarrow 5^-)$  is 0.89 W.u. with Mc4 (1.08 W.u. before) compared with the datum 1.5(3) W.u. [12]. The value is 2.87 W.u. for the jj45pna interaction. These isomeric states with B(E2) transitions provide more evidence for the regular correlation in this region of hole nuclei.

# **IV. THEORETICAL PREDICTIONS**

In <sup>128</sup>In, the reversed ground state is driven by the monopole correction Mc4 between  $\nu h_{11/2}$  and  $d_{3/2}$ . This regular correlation dominates the ground-state inversions in these N = 79 isotones and supports the systematic analogy between <sup>128</sup>In and <sup>126</sup>Ag that assigned the isomeric state (1<sup>-</sup>) and the ground state (3<sup>+</sup>) in <sup>126</sup>Ag experimentally. As a theoretical prediction, the reversed ground state of <sup>126</sup>Ag will change back to (1<sup>-</sup>) in <sup>128</sup>Ag. For odd mass nuclei, with two more neutron holes in <sup>129</sup>Cd (<sup>127</sup>Pd), the regular correlation reverses the excited  $3/2^+$  level into the ground state in <sup>127</sup>Cd(<sup>125</sup>Pd). The ground state of <sup>127</sup>Cd has already been suggested as  $3/2^+$  in Ref. [36], and then the theoretical prediction in <sup>125</sup>Pd will become more reasonable.

#### V. SUMMARY

The neutron-rich isotones of N = 79, 80, 81 are studied comprehensively with large-scale shell model calculations to the southwest of <sup>132</sup>Sn. A regular correlation driven by the monopole interaction between the neutron orbits  $h_{11/2}$  and  $d_{3/2}$  is found and quantified with the extended pairing plus multipole-multipole force model. The data on the ground-state inversion from <sup>130</sup>In to <sup>128</sup>In can be well described by this regular correlation, as well as the inversion from <sup>129</sup>Cd to <sup>127</sup>Cd. The configurational competition between 1<sup>-</sup> (11/2<sup>-</sup>) and 3<sup>+</sup> (3/2<sup>+</sup>) provides the structural reason for these ground-state inversions in the N = 79 isotones. Furthermore, this regular correlation agrees with the fact that <sup>126</sup>Ag and <sup>128</sup>In have the same parity and J value in their ground states. The study of isomeric states with B(E2) transitions in the N = 80 isotones provides more evidence, and such a regular correlation in different isotonic chains should provide useful guidance for further experiments in this region of nuclei.

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