# $2n$ transfer and $E2$ strengths in $^{154}$Sm

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Two separate analyses of $E2$ strengths among the first two bands in $^{154}$Sm are consistent. They indicate that mixing is small and decreases with increasing $J$.

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## I. INTRODUCTION

A generalized coexistence model [1,2] was developed for use in an analysis of $2n$ transfer data in a series of isotopes in which an intruder $0^+$ state mixed with the normal ground state (g.s.). Experimental cross-section ratios for $2n$ stripping and pickup leading to the two experimental $0^+$ states provided mixing amplitudes for all the isotopes in terms of a single dimensionless parameter, which was of order unity. An alternative view of the analysis was the derivation of mixing amplitudes for all nuclei in terms of that for any one. The model turned out to be useful also for proton [3] and $\alpha$ [4] transfer and $E2$ strengths [5,6] in such coexistence nuclei. The model was initially applied to Ge [7] and Zn [6] nuclei and more recently to Zr [8] and Mo [9].

Earlier, I examined results for $2n$ transfer and $E2$ strengths in $^{150,152}$Sm [10]. That $2n$-transfer analysis provided $0^+$ mixing amplitudes for $^{150,154}$Sm in terms of the mixing in $^{152}$Sm. A separate band-mixing analysis [11] of the $E2$ strengths in $^{152}$Sm [12] selected one value of this mixing for that nucleus ($0^+$ mixing intensity of 0.341 [11]). It turned out that the $^{150}$Sm $0^+$ mixing that resulted from the $2n$ analysis with that value of $^{152}$Sm mixing was in agreement with the $0^+$ mixing that emerged from a band-mixing analysis of $E2$ strengths in $^{150}$Sm [10]. I did not include the $E2$ strengths in $^{154}$Sm in that analysis because only three of the four strengths needed were available. I address the $^{154}$Sm case here.

## II. ANALYSIS

A great deal of information is available concerning the structure of $^{154}$Sm. Here, I am interested only in the first two rotational bands. Values of $E2$ transition strengths for $2 \leftrightarrow 0$ and $4 \leftrightarrow 2$ transitions in $^{154}$Sm are listed in Table I [13,14]. Energies of the first two bands are plotted in Fig. 1. Strengths of $J \to J{-}2$ transitions within the ground-state band are plotted in Fig. 2, compared with predictions for a deformed rotor. Agreement is good. Long ago, Fraser *et al.* [15] measured transition matrix elements between states in the ground-state bands of $^{152,154}$Sm with $J$ up to 10. They observed large deviations from rotational-model predictions in $^{152}$Sm but not in $^{154}$Sm. Their conclusion for $^{154}$Sm agrees with Fig. 2.

Takemasa *et al.* [16] analyzed $2n$ transfer data in the Sm isotopes with a two-state mixing model. They assumed the ground states of $^{148}$Sm and $^{154}$Sm were spherical and deformed, respectively, and that both $^{150}$Sm and $^{152}$Sm were mixtures of the two structures.

Kumar [17] found $^{154}$Sm to be "a well-deformed nucleus," with weak mixing between low-energy rotations and vibrations. Bhardwaj *et al.* [18] considered mixing of ground, $\beta$, and $\gamma$ bands in several so-called transitional nuclei and selected $^{154}$Sm as "a representative of well deformed nuclei."

The conventional picture of the band head of the excited $0^+$ band is that it is a so-called $\beta$ vibration [13,19]. Krücken *et al.* [19] measured lifetimes of the $0_2$ and $2_\gamma$ states in $^{154}$Sm following Coulomb excitation. They stated that "$^{154}$Sm was identified as one of the few deformed nuclei where the first excited $0^+$ state is the $\beta$ vibration of the ground state." Quite recently, Otsuka *et al.* [20] have performed Monte Carlo shell-model calculations for $^{154}$Sm and concluded that "the present calculation indicates a coexistence between prolate and triaxial shapes in a stark contrast to the conventional picture of the $\beta$ and $\gamma$ vibrations." General and specific details of coexistence are discussed in an excellent review [21]. Perhaps surprisingly, this review does not mention $^{154}$Sm.

Here, I undertake a two-state mixing analysis of members of the first two bands without needing to specify anything about the structure of the underlying basis states.

As elsewhere, I write for $^{154}$Sm,

$$0_1 = a\,0_g + b\,0_e, \quad 0_2 = -b\,0_g + a\,0_e,$$
$$2_1 = A\,2_g + B\,2_e, \quad 2_2 = -B\,2_g + A\,2_e,$$

and

$$4_1 = C\,4_g + D\,4_e, \quad 4_2 = -D\,4_g + C\,4_e$$

I define $M_g = \langle 0_g \| E2 \| 2_g \rangle$, $M_e = \langle 0_e \| E2 \| 2_e \rangle$, $M_g' = \langle 2_g \| E2 \| 4_g \rangle$, $M_e' = \langle 2_e \| E2 \| 4_e \rangle$.

Furthermore, I assume the $g$ states are not connected to the $e$ states by the $E2$ operator. No other assumptions are necessary. Whenever all four of the relevant $E2$ transition matrix elements are available, the solution of a mixing fit is unique. Central values of the fit parameters reproduce the

 

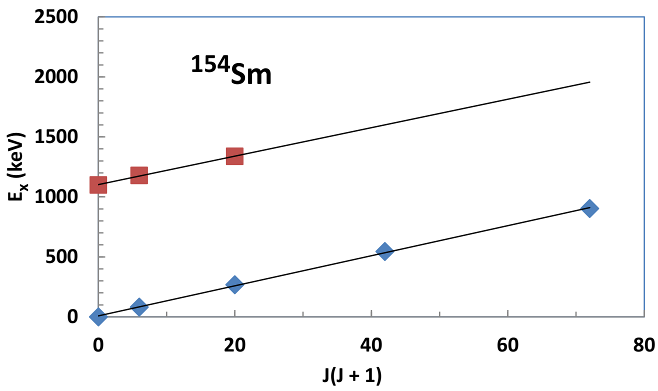


FIG. 1. Energies of members of the first two bands in $^{154}$Sm are plotted vs $J(J+1)$.

TABLE I. $E2$ Transition strengths $B$ [in Weisskopf units (W.u.)] and matrix elements $M[(\mathrm{W.u.})^{1/2}]$ for $2 \leftrightarrow 0$ and $4 \leftrightarrow 2$ transitions in $^{154}$Sm.[a]

| Label | Init. | Fin. | $B$ [b] | unc | $M$ | unc | Fit 1 | Fit2 |
|---|---|---|---|---|---|---|---|---|
| $M_0$ | $2_1$ | $0_1$ | 176 [c] | (1) | 29.66 | (0.08) | 29.66 | 29.66 |
| $M_1$ | $0_2$ | $2_1$ | 11.2 | (2.1) | ± 3.35 | (0.31) | 3.35 | 3.11 |
| $M_2$ | $2_2$ | $0_1$ | 0.32 | (0.04) | ± 1.26 | (0.08) | −1.26 | −1.47 |
| $M_3$ | $2_2$ | $0_2$ | Unknown | | | | 24.8 | 25.2 |
| $M_0'$ | $4_1$ | $2_1$ | 245 [c] | (6) | 46.96 | (0.57) | 46.96 | 46.96 |
| $M_1'$ | $2_2$ | $4_1$ | 1.32 | (0.15) | ± 2.57 | (0.15) | 3.19 | 3.16 |
| $M_2'$ | $4_2$ | $2_1$ | 0.32 | (0.11) | ± 1.68 | 0.29 | −0.93 | −1.56 |
| $M_3'$ | $4_2$ | $2_2$ | Unknown | | | | 39.0 | 39.7 |

[a]Init. stands for initial, Fin. for final, and unc for uncertainty.
[b]From Ref. [13] unless noted otherwise.
[c]From Ref. [14].

central values of the experimental $M$'s, and uncertainties in the fit parameters are computed from uncertainties in the experimental numbers. If only $B(E2)$'s and not the $M$'s are known, a sign ambiguity can exist. Because of destructive interference, $M_1$ and $M_2$ can have either sign; $M_0$ and $M_3$ are positive by definition. In the present case, preferred signs emerge from the mixing analysis.

In the $2n$ transfer analysis mentioned above, the combination of $2n$ and $E2$ strengths in $^{152}$Sm selected a $0^+$ mixing intensity of 0.341 in that nucleus. From $2n$ transfer data alone, this mixing in $^{154}$Sm corresponds to a $0^+$ mixing *amplitude* of 0.245 in $^{154}$Sm. I have first attempted to reproduce the $E2$ matrix elements of Table I with this value of $0^+$ mixing. Results of the fit for the transition matrix elements are listed in Table I (Fit 1). The fitted parameters are listed in Table II. It can be noted that mixing is small for each $J$ and appears to decrease as $J$ increases. (see also Fig. 3.)

Without the input from $2n$ transfer, there is not enough experimental information to determine the seven unknown parameters: mixing in $0^+$, $2^+$, and $4^+$ states, together with the four basis-state transition matrix elements. With only six

TABLE II. Fit parameters in $^{154}$Sm.

| $J$ | Fit 1[a] $g$ | Fit 1[a] $e$ | Fit 2[b] $g$ | Fit 2[b] $e$ |
|---|---|---|---|---|
| 0 | 0.245(15) | 0.970 | 0.216(13) | 0.976 |
| 2 | 0.162(13) | 0.987 | 0.134(11) | 0.991 |
| 4 | 0.115(30) | 0.993 | 0.0801(20) | 0.997 |
| $M[(\mathrm{W.u.})^{1/2}]$ | 24.7(3) | 30.0(5) | 25.2(3) | 29.9 |
| $M'\,[(\mathrm{W.u.})^{1/2}]$ | 38.9(13) | 47.2(15) | 39.6(13) | 47.1(15) |

[a]$0^+$ mixing solely from $2n$ transfer analysis [10], others from $M(E2)$ from Table I.
[b]Used only $E2$ strengths from Table I, plus the assumption $M_g/M_e = M_g'/M_e'$.

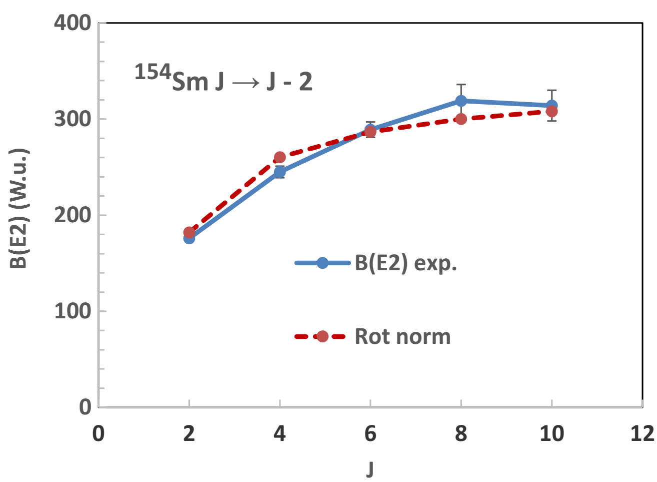


FIG. 2. Plotted vs $J$ are $B(E2)$ values for $J \to J-2$ transitions in $^{154}$Sm, compared with predictions for a deformed rotor.

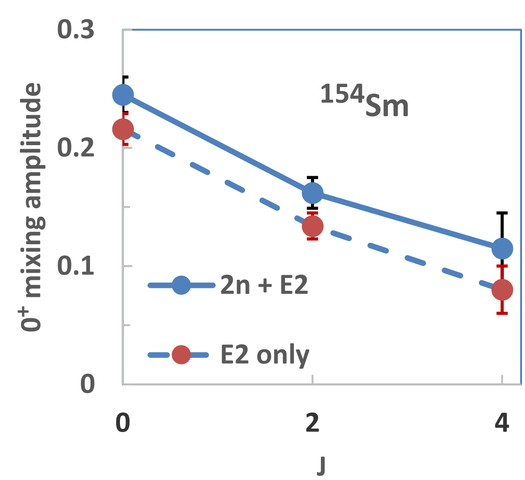


FIG. 3. Mixing amplitudes for $J = 0, 2,$ and 4 from the present analysis.

TABLE III. Mixing matrix elements (keV) for Fit 2 of Table II.

| $J$ | $V$ |
|---|---|
| 0 | 232(14) |
| 2 | 146(12) |
| 4 | 86(22) |

experimental quantities known, one additional constraint is needed to enable a fit. I have chosen to assume the relationship $M_g/M_e = M'_g/M'_e$. If either of the two missing $E2$ strengths ever becomes available, the analysis can be repeated with this constraint removed. The results of this fit are also listed in Table I, and the fitted parameters are given in Table II (Fit 2). Note that $M'_g/M_g = 1.57$, very close to the ratio of 1.60 expected for a $0^+$ rotational band. The similarity of the results of the two fits is apparent. Both fits allow predictions for the two missing matrix elements as given also in Table I. Thus, the mixing in the g.s. of $^{154}$Sm, which is frequently assumed to be zero, is indeed small, but definitely not zero.

The potential matrix elements responsible for the mixing in Fit 2 are listed in Table III for each $J$. The difference for $J = 0$ and $J = 2$ is a 4.7$\sigma$ effect; for $J = 2$ and 4, the difference is 2.4$\sigma$.

As mentioned above, the pattern of $E2$ strengths in $^{154}$Sm is considerably different from that in $^{152}$Sm. And yet, the basis-state matrix elements that emerge from the mixing are very similar in the two nuclei as noted in Table IV. Thus, the properties of the basis states are about the same in the two nuclei. The differences in experimental quantities are due to

TABLE IV. Basis-state $E2$ matrix elements [(W.u.)$^{1/2}$] in $^{152}$Sm [a] and $^{154}$Sm.

| $A$ | $M_g$ | $M_e$ | $M'_g$ | $M'_e$ |
|---|---|---|---|---|
| 152 | 29.5(13) | 21.0(9) | 49.4(22) | 37.1(17) |
| 154 | 30.0(5) | 24.7(3) | 47.2(15) | 38.9(13) |

[a]Reference [11].

the difference in mixing intensities: 0.341 in $^{152}$Sm [11] and 0.060 in $^{154}$Sm (present) for the $0^+$ states. Concerning the $E0$ strength, because $\rho^2(E0)$ scales as $a^2b^2$, and given the value of $56(8) \times 10^{-3}$ in $^{152}$Sm [22], I expect $\rho^2(E0)$ to be about $14 \times 10^{-3}$ in $^{154}$Sm.

## III. SUMMARY

I have performed two separate band-mixing analyses of strengths for $2 \leftrightarrow 0$ and $4 \leftrightarrow 2$ transitions in $^{154}$Sm for which a full data set is incomplete. The first analysis used $0^+$ mixing from an earlier analysis of $2n$ transfer data, whereas the second used only $E2$ information (plus the assumption of one additional constraint). Results of the two fits are consistent at approximately the 1$\sigma$ level and indicate small mixing, with a small decrease with increasing $J$. It is significant that the $E2$ and $2n$ transfer analyses are in agreement. Although the mixing is small, it is distinctively different from zero. The smallness of the mixing agrees with several earlier works but disagrees with those that assumed zero mixing. The present analysis confirms that $^{154}$Sm is an excellent example of a deformed collective nucleus.


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