New shell-model investigation of the lead-208 mass region: Spectroscopic properties and collectivity

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Within the large-scale shell-model framework, an investigation of even-even nuclei in the vicinity of the ²⁰⁸Pb closed core is reported in the present work. The calculations of the low-lying state energies and the electromagnetic transitions of the Pb, Po, Rn, Ra, and Th nuclei with $126 \le N \le 134$ are performed using a new effective interaction H208 derived from CD-Bonn nucleon-nucleon potential, through the V_{low-k} procedure. A special focus is devoted to the degree of the collectivity, where a transitional region from spherical nuclear shape to weakly collective behavior is revealed.

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I. INTRODUCTION

One of the main challenges in nuclear physics is to accurately describe the structure of atomic nuclei along the whole Segré chart and to develop a comprehensive view of their observed properties.

The neutron-rich nuclei in the neighborhood of ²⁰⁸Pb exhibit particular experimental and theoretical interests. On the experimental side, the access to this region was hampered in the past by the neutron richness and low production cross sections. Currently, substantial progress is achieved with radioactive beams and in advanced setups for decay spectroscopy, which enables to get new nuclear structure information on this relatively unexplored region. Some examples to address are the extended level scheme of ²²⁰Th [1]; the isomer spectroscopy in ²¹⁶Th [2]; the first observation of high-spin states in ²¹⁴Po [3]; the first observation of the isomeric transition strengths in the ^{210–216}Pb isotopes [4]; the first study of the ²¹⁷Bi structure [5]; and the complete spectroscopy of ²¹¹Po below 2 MeV [6].

On the theoretical side, this heavy mass region offers favorable conditions for the formation of isomeric states, due to the presence of high-*j* orbitals such as $0i_{13/2}$ and $0j_{15/2}$, and their position adds up to a useful input in the adjustment of the effective interactions. Additionally, this region marked by a large model space, with the presence of different classes of correlations (pairing, quadrupole, octupole) initiates a real challenge for any theoretical model and represents an ideal system to strengthen its validity.

The standard mean field approach was the only predictive model of these neutron-rich heavy nuclei, because of its ability to reach this region. However, the exclusion of correlations that are extremely present is considered as a deficiency for this class of models. In spite of that, some calculations were performed including approximations, for instance the works of

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Ref. [7], where the authors showed the presence of a subshell closure at Z = 92, using a relativistic mean field approach.

The shell model (SM) has made enormous progress in the investigation of the lead mass region, since the works of Ref. [8], by studying the ²⁰⁶Pb nucleus using a simple neutron-neutron residual interaction. A particular success has been achieved using the residual interaction of Kuo and Herling [9,10], derived from the free nucleon-nucleon potential of Hamada and Johnston [11], using the reaction matrix techniques developed by Kuo and Brown [12]. Some modifications were reported by Warburton and Brown in Ref. [13], to the original version of Kuo-Herling interaction (referred to as KHPE), to improve the agreement with the experimental energy spectra of A = 204-212 nuclei. The same interaction was used in many works: theoretical as the first calculations of binding energies, level schemes, electromagnetic transition rates for N = 126 isotones Po-Pu [14], and experimental as those quoted above [2,4-6]. Some deficiencies in the KHPE interaction are noted in Ref. [14], where the authors suggest modifications in the matrix elements, in order to improve the agreement with the experiment.

In addition, the energy levels of 210 Po, 211 At, 212 Rn [15] and of odd-odd nuclei 210 Bi, 212 Bi, 212 At [16–18] were performed by employing an effective interaction derived from Bonn-A *NN* interaction. These works highlight the close resemblance between proton-neutron multiplets in the 132 Sn and 208 Pb regions.

The state of interest in the current work is mainly to develop a new effective interaction, which is tailored for the model space beyond the ²⁰⁸Pb closed core. In order to prove its reliability, it was applied in the investigation of the spectroscopic properties and collectivity of a large number of even-even nuclei from this region.

In this context, an outline of the model space and the development steps of the new effective interaction H208 are given in Sec. II. Further, the results on low-lying state energies and electromagnetic transitions of several even-even nuclei: ^{210–216}Pb, ^{210–216}Po, ^{212–218}Rn, ^{214–218}Ra, and ^{216–220}Th are discussed in Sec. III, by focusing on the signatures and the degree

of the collectivity in this region. A comparison between the ¹³²Sn and ²⁰⁸Pb mass regions is presented in Sec. IV. Finally, the conclusions and the perspectives are drawn in Sec. V.

II. SHELL-MODEL BACKGROUND

A. $r_5 i - r_6 j$ model space

The present SM calculations are performed in the model space denoted hereafter by $r_5i - r_6j$, including the single particle orbits that are taken above the ²⁰⁸Pb doubly magic core (the notation r_p borrowed from [19] stands for all orbits of the major shell with the principal quantum number p, except the largest one). In this case, the valence space contains the 82–126 particle proton orbits $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $0i_{13/2}$, and the 126–184 particle neutron orbits $1g_{9/2}$, $0i_{11/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $0j_{15/2}$. The proton and neutron single-particle energies employed in present calculations are provided from the experimental energies of the single-particle states in ²⁰⁹Bi and ²⁰⁹Pb [20].

The diagonalization of the matrix elements are carried out using the ANTOINE Strasbourg SM code [21,22], where the full configurational space is included for the nuclei with mass numbers A = 210, 212, 214, and 216. Whereas, the matrices dimension of the ²¹⁸Rn, ²¹⁸Ra, and ^{218–220}Th nuclei is huge (e.g., $\approx 63 \times 10^9$ in ²¹⁸Ra) to be diagonalized. In this case, the introduction of a truncation scheme is necessary, by including the pairs excitations to $0i_{13/2}$ proton orbit and $0j_{15/2}$ neutron orbit with keeping up to 6p-6h excitations. Despite this limitation, the diagonalization remains computationally very challenging (e.g., $\approx 15 \times 10^9$ in ²¹⁸Ra), but possible with the ANTOINE SM code.

B. The H208 effective interaction

The effectiveness of any SM calculations goes first to the used effective interaction V_{eff} , constructed from the free *NN* potentials. It is based on the definition of a restricted valence space, where a suitable Hamiltonian could be diagonalized. New generation of realistic effective interactions was developed, by employing a low-momentum potential approach $V_{\text{low-}k}$ [23,24]. This advanced approach is already proven to be quite successful in describing the spectroscopic properties of nuclei from various regions: sd [25], sdpf [26], $r_4h - r_5i$ [27–30], and $r_5i - r_6j$ [15,31].

In this respect and to explore the ²⁰⁸Pb mass region, a new effective interaction denoted hereafter by H208 is derived from the free nucleon-nucleon interaction, via many-body perturbation theory. It is established following precisely the same procedure as that adopted in our earlier works [28–30], in the development of N3LOP effective interaction for $r_4h - r_5i$ model space around ¹³²Sn closed core.

As a starting point, a realistic interaction V_{NN} is derived from charge-dependant Bonn potential (CD-Bonn) [32], which exhibits a strong short-range repulsive behavior. Making use of the low-momentum approach V_{low-k} [23,24] in the renormalization procedure, a smooth potential is obtained, by integrating V_{NN} down to the cutoff momentum $\Lambda = 2.2$ fm⁻¹. This potential preserves exactly the on-shell properties of the

original V_{NN} , and it is suitable for being used directly in nuclear structure calculations.

Once the renormalized $V_{\text{low-}k}$ interaction is obtained, it is used as an input for the calculation of V_{eff} matrix elements. At this stage, the interaction is adapted to the model space by many body perturbation theory techniques, including all the \hat{Q} -box vertex functions up to the second order of Goldstone diagrams [23]. Then for a given \hat{Q} box, the folded diagram series is summed up to all orders using the Lee-Suzuki iteration method [33]. These diagrams are computed within the harmonic-oscillator basis with the oscillator parameter $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ MeV= 6.88 MeV for the A = 208region. For more detailed description of this approach, the reader is referred to the literature [23].

As pointed out by Zuker in Ref. [34], the two-body monopole corrections of the realistic interaction could imitate the real effect of the three-body forces, that are not considered explicitly in the initial V_{NN} realistic interaction. In this spirit, some monopole adjustments of the effective two-body forces are applied to reproduce the proton and neutron single-particle energies of N = 126 and 127 isotonic chains, respectively.

The resulting interaction named hereafter by H208 is applied in the investigation of the spectroscopic properties of Pb, Po, Rn, Ra, and Th isotopic chains with $126 \le N \le 134$.

III. SHELL-MODEL RESULTS AND DISCUSSIONS

The present work is a complement to my previous study on N = 126 isotones and Z = 82 isotopes [35], providing the opportunity to further examine all the H208 twobody matrix elements (TBME) components: neutron-neutron, proton-proton, and proton-neutron. For this purpose, an extension of the previous calculations [35] to the spectroscopic properties of even-even nuclei with $82 \le Z \le 90$ and $126 \le$ $N \le 134$ is exposed, where a special attention is devoted on the quadrupole collectivity in this region.

A. Energy levels and wave functions

The low-lying state energies of 210,212,214,216 Pb, 210,212,214,216 Po, 212,214,216,218 Rn, 214,216,218 Ra, and 216,218,220 Th isotopes calculated using the H208 effective interaction are displayed, respectively, in Figs. 1, 2, 3, 4, and 5.

At first sight, the calculated energy levels of all considered nuclei indicate an excellent agreement with the experimental data [20]. A small compression is observed in some nuclei with mass numbers A = 218, 220, which could be a consequence of the imposed truncation mentioned in Sec. II A. It is worth noticing that the experimental 6^+ and 8^+ energy levels of ²¹⁴Po, observed in Ref. [3], are well reproduced by the H208 calculations. The observed 10^+ state at 2179 keV in ²¹⁴Po [3] is very close in energy to the calculated 10^+_2 state (2223 keV) with which it could be associated, while the first 10^+ state is predicted at 1829 keV. It is worth noticing that the predicted behavior of 10^+_1 in Po isotopes is the same as



FIG. 1. Experimental and theoretical energy levels of Pb isotopes.

that observed experimentally in Rn and Ra isotopes, where it remains constant in the N = 128 and 130 isotones. This could justify the unlikely increase of 10^+ energy level by about 300 keV from ²¹²Po to ²¹⁴Po.

A deeper insight into the structure of the wave functions of the levels with spins up to $J^{\pi} = 8^+$, indicate that the dominant component 0p0h excitations; corresponds to the configuration $\pi 0h_{9/2}^p \otimes \nu 1g_{9/2}^n$ (*n* and *p* are the neutrons and the protons valence number). This component decreases by increasing the proton and/or the neutron number, where the 2p2h excitations become in competition with the 0p0h excitations, marked by the contribution of the $\pi 1f_{7/2}$ proton orbit $\pi (0h_{9/2}^{(p-2)}1f_{7/2}^2)$ and/or the $\nu 0i_{11/2}$ neutron orbit $\nu (1g_{9/2}^{(n-2)}0i_{11/2}^2)$. Another remark that can be made from the reported energy

Another remark that can be made from the reported energy levels is the decrease of the 10⁺ states from 3000–5000 keV in the N = 126 isotones with Z > 82 to 2000 keV in the N =128 isotones. The analysis of their wave functions indicates that they are characterized by the dominating configuration $\pi 0h_{9/2}^p \otimes \nu(1g_{9/2}^{(n-1)}0i_{11/2})$ with the contribution of $1f_{7/2}$ pro-



FIG. 3. Experimental and theoretical energy levels of Rn isotopes.

ton orbit in some heavy isotopes. In the specific case of the 10^+ state in ²¹⁰Po, which is also calculated at a very high energy (\approx 4800 keV), the same as that already obtained in the works of [13,14] (4674 keV) is arising from $\pi (0i_{13/2})^2$ configuration.

In order to calculate the *E*3 transitions in N = 126 isotonic chain, other positive and negative parity states $(3^-, 11^-, 17^-, 14^+, 8_2^+)$ are calculated and reported in Figs. 2, 3, 4, and 5. As expected, the lowest 3^- octupole phonon energy state is overestimated, and it cannot be reproduced from the present model space, where the 208 Pb core excitations are necessary. However, the other experimentally known states are well reproduced in the present work (see Sec. III C for more details).

Finally, the calculated states using the H208 interaction could be a support for the upcoming experimental measurements, and provide a certitude for the tentatively proposed states (e.g., 2^+ , 4^+ , 6^+ , and 8^+ states in ^{212,214,216}Pb) or unidentified (e.g., 10^+ state in some Pb and Po isotopes).



FIG. 2. Experimental and theoretical energy levels of Po isotopes.



FIG. 4. Experimental and theoretical energy levels of Ra isotopes.

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FIG. 5. Experimental and theoretical energy levels of Th isotopes.

B. E2 electric transitions

The *E*2 electric transition probabilities establish an anchor point in the nuclear structure, where they provide many information. For instance, they constitute an indication of deformation from the quadrupole moment, and they quantify the magnitude of the quadrupole correlations, from the reduced $B(E2; 2^+ \rightarrow 0^+)$ probability.

In this context, the $B(E2; J \rightarrow J - 2)$ transitions are calculated, interconnecting different even-parity states with spins up to J = 8. The results for different transition rates, displayed in Figs. 6, 7, 8, and 9, are calculated using the standard effective charges for neutron $e_v = 0.5e$ and proton $e_{\pi} = 1.5e$ for all the nuclei.

At first glance, a satisfactory agreement with the measurements [20] within errors is reached for most of the experimentally known nuclei. A pronounced discrepancy is noticed for the ²¹⁸Ra nucleus with a visible underestimation of the calculated $B(E2; 4^+ \rightarrow 2^+)$ transition, probably related to the truncation of the model space (see Sec. II A).

Although the experimental $B(E2; 2^+ \rightarrow 0^+) = 0.56(12)$ W.u. [20] value in ²¹⁰Po was revised in [36] [1.8(3)W.u.], but it remains small compared to the calculated value $263 e^2 \text{fm}^4 = 3.54$ W.u. The latter is the same as that obtained by other theoretical calculations (3.55 W.u.) [14,15] using, respectively, KHPE and Bonn-A effective interactions.

Regarding the ²¹²Po nucleus, the weak $B(E2; 2^+ \rightarrow 0^+)$ experimental transition [36] strongly resembles to that already observed in the ¹³⁶Te nucleus [37], which is influenced by the strong neutron dominance in the 2⁺ wave function. To address this point in ²¹²Po, the ratio R(E2) of the neutron $B_{\nu}(E2; 2^+ \rightarrow 0^+)$ and the proton $B_{\pi}(2^+ \rightarrow 0^+)$ transitions is calculated using the charges $(e_{\nu} = 1, e_{\pi} = 0)$ and $(e_{\nu} =$ $0, e_{\pi} = 1)$, respectively. The corresponding R(E2) values, 2.6 in ¹³⁶Te and 2.4 in ²¹²Po, indicate an enhanced neutron amplitude over the proton one. Different experimental values of $B(E2; 6^+ \rightarrow 4^+)$ and $B(E2; 8^+ \rightarrow 6^+)$ transitions in ²¹²Po are reported in Ref. [36] [3.9(11), 2.30(9)] W.u. and [20] [13.2⁺⁴⁹₋₂₉, 4.56(12)] W.u., while my results (4.42, 2.34) W.u. are more coherent with those of [36].

It could be noticed that the experimental isomeric $B(E2; 8^+ \rightarrow 6^+)$ transition in ²¹⁴Po [41(3) e^2 fm⁴] [3] is reasonably well reproduced by the present calculations (69 e^2 fm⁴). Moreover, the calculated quadrupole moments for the 8⁺ states in ²¹⁰Po (-0.58 *e*b) and in ²¹²Rn (-0.26 *e*b) are considerably closer to the experimental data [-0.55(2) *e*b] and [(-)0.18(2) *e*b] [38], respectively.

One observes from Fig. 6, the enhancement of the $B(E2; 2^+ \rightarrow 0^+)$ transitions by increasing the neutron number going from Pb to Th nuclei. The large transition rates reached in ^{216–218}Rn, ²¹⁸Ra and ²²⁰Th nuclei indicate the collective nature of these nuclei. It should be stressed that this behavior is quite similar to that obtained in the investigation of N = 86 and 88 isotones in the ¹³²Sn mass region [28,29] with a maximal value $\approx 2000 \ e^2 \text{fm}^4$ in ¹⁴⁴Ba and ¹⁴⁶Ce nuclei (see Fig. 4(a) of Ref. [29]).

It is worthwhile to mention that the lack of experimental information makes the comparison very sensitive and difficult to fix exactly the effective charges for this mass region.



FIG. 6. The calculated $B(E2; 2^+ \rightarrow 0^+)$ transitions of Pb, Po, Rn, Ra, and Th isotopes compared to the experimental data [20].



FIG. 7. The calculated $B(E2; 4^+ \rightarrow 2^+)$ transitions of Pb, Po, Rn, Ra, and Th isotopes compared to the experimental data [20].



FIG. 8. The calculated $B(E2; 6^+ \rightarrow 4^+)$ transitions of Pb, Po, Rn, Ra, and Th isotopes compared to the experimental data [20].



FIG. 9. The calculated $B(E2; 8^+ \rightarrow 6^+)$ transitions of Pb, Po, Rn, Ra, and Th isotopes compared to the experimental data [20].



FIG. 10. Magnetic dipole moments of several nuclei calculated using the free (green line), the Arima (pink line), and the effective (blue line) spin and orbital *g* factors compared to the data (red symbol) [20] (see text for more details).

C. E3 electric transitions

One of the important E3 transitions in even-even nuclei is the decay of the 3^- state. This specific transition cannot be reproduced within the SM, where the energy of this octupole phonon state is usually overestimated, not only in the present work (see Sec. III A), but also in other SM calculations [15].

Regarding the *E*3 transitions of high-spin states, a great deal of information was obtained in the translead region, characterized by their isomeric decay [39]. It is interesting to calculate these transitions in N = 126 isotones, especially as the excitation energies of the involved states are well reproduced in the present calculations (see Figs. 2, 3, 4, and 5). The *E*3 transition strengths of the N = 126 isotonic chain reported in Table I are calculated using the same effective charge $e_{\pi} = 1.4e$ of Ref. [40], and compared to the available experimental data [20]. One can notice that the present calculations largely underestimate the experimental values, in particular for the strong $11^- \rightarrow 8^+_2$ and $17^- \rightarrow 14^+$ transitions. While, they are very close to those obtained in [15] using the effective operators.

This deficiency in the calculation of E3 transitions within the SM is due to the narrowing of the model space, particularly the omission of particle-hole excitations across the two magic gaps Z = 82 and N = 126. In order to examine this point the development of a new effective interaction will be necessary.

TABLE I. The calculated B(E3) transition strengths in W.u. for ²¹⁰Po, ²¹²Rn, ²¹⁴Ra, and ²¹⁶Th nuclei compared to the experimental data [20] and other SM calculations [15].

Nucleus	$J^{\pi}_i ightarrow J^{\pi}_f$	Exp.	H208	Lit. ^a
²¹⁰ Po	$11^{-} \rightarrow 8^{+}_{1}$	3.71 (10)	0.38	0.55
	$11^- \rightarrow 8^+_2$	19.7 (11)	6.61	6.1
212 Rn	$11^- \rightarrow 8^{\tilde{+}}_1$	1.8^{+6}_{-4}	0.29	
	$11^- \rightarrow 8^+_2$	27^{+10}_{-7}	6.09	
	$17^- \rightarrow 14^+$	21.3^{+17}_{-15}	6.36	6
²¹⁴ Ra	$11^- \rightarrow 8^+_1$	3.09 (18)	0.41	
	$11^{-} \rightarrow 8^{+}_{2}$	21.8(12)	5.35	
	$17^- \rightarrow 14^+$	25.8 (9)	5.67	
²¹⁶ Th	$11^- \rightarrow 8^+_1$	21(2)	3.34	
	$11^- \rightarrow (\overset{1}{8}{}^+_2)$	5.0(15)	1.65	

^aReference [15].

It should be tailored for a larger model space (see Ref. [41] as example), and such treatment is beyond the scope of the present work.

As it was detailed in [39], the different characteristics of E3 transitions identified in ²¹⁰Po, ²¹²Rn, and ²¹⁴Ra nuclei are

- (i) A weak $11^- \rightarrow 8^+_1$ transition dominated by single proton transitions of the type $\pi 0i_{13/2} \rightarrow \pi 0h_{9/2}$, where the spin-flip slows down the *E*3 transition.
- (ii) A strong $11^- \rightarrow 8^+_2$ and $17^- \rightarrow 14^+$ transitions characterized by single particle $\pi 0i_{13/2} \rightarrow \pi 1f_{7/2}$ transitions, where they are considered as a signature of mixing of the 3⁻ core excitation into the involved states. The deficiency in the calculation of the 3⁻ octupole excitations could explain the small values obtained in these transitions.

In the case of the ²¹⁶Th nucleus, the wave functions of the 8_1^+ and 8_2^+ states are marked by the contribution of $\pi 1 f_{7/2}$ in the 8_1^+ state, and a mixing of $\pi (0h_{9/2}^7 1 f_{7/2})$ and $\pi 0h_{9/2}^8$ for the 8_2^+ state. This leads to a fast decay of the 11^- state via an E3 transition to the 8_1^+ state, and a weak decay to the 8_2^+ state.

D. M1 magnetic moments

Another stringent test for the H208 effective interaction is the calculation of the magnetic dipole moment μ . As it could be seen from Fig. 10, the free spin and orbital g-factor values $(g_{\pi}^s, g_{\pi}^l) = (5.5857, 1.0)$ for protons and $(g_{\nu}^s, g_{\nu}^l) = (-3.8263, 0.0)$ for neutrons are inadequate for the lead mass region. This can be expected because the present model space is incomplete with respect to the spin-orbit partners, which significantly affects the magnetic moment. Even larger divergence is observed using the effective values of Arima *et al.* [42] $(g_{\pi}^s, g_{\pi}^l) = (3.536, 1.130)$ and $(g_{\nu}^s, g_{\nu}^l) =$ (-2.026, -0.080), usually used in the KHPE calculations [13].

Nonetheless, a remarkable consistency with the experimental data is reproduced by the H208 effective interaction, using the effective protons $(g_{\pi}^{s}, g_{\pi}^{l}) = (5.4263, 0.8513)$ and neutrons $(g_{\nu}^{s}, g_{\nu}^{l}) = (-1.0244, 0.0039) g$ factors. The values of the last set are fixed by minimization to reproduce accurately with the experimental magnetic dipole moments [20] of different states for a large number of nuclei. It should be



FIG. 11. Magnetic dipole moments of 2^+ , 4^+ , 6^+ , and 8^+ states calculated using the effective g factors and compared to the experimental data shown by the same symbol of different states with red color.

stressed that this method is sensitive to the number of the input data and to their reliability.

Using the new values of the effective spin and orbital g factors, the calculated magnetic dipole moment of 2^+ , 4^+ , 6^+ , and 8^+ states for the nuclei of interest are displayed in Fig. 11, and compared to the data procured from Ref. [20]. As it can be seen, the main feature in this plot is a distinctive staggering for Z > 82 nuclei with an increase of the amplitude going from 2^+ to 8^+ states, which takes maximums in the N = 126 isotones. Whereas, the absence of the protons contribution in the Pb isotopes can be traced as flat lines. Also, a perfect accord is obtained for the $B(M1; 2^+_2 \rightarrow 2^+_1)$ transition rate in 212 Po between theory $(0.102 \, \mu_N^2)$ and experiment $[0.126(16) \mu_N^2]$ [43] using these effective g-factors values.

E. Quadrupole correlations and collectivity

The evolution of the collectivity remains one of the major tasks of the present work. It is strongly influenced by the quadrupole correlations, where the calculations of E2 properties provide a considerable insight into the collective motion. Therefore, they contribute to the evaluation of the degree of deformation in the nuclei.

From the results exposed above, it is worth reminding of the decrease of 2^+ energy levels with increasing the neutron number in the nuclei with Z > 82 (see Sec. III A). For instance, the $E(2^+)$ varies from 1500 keV in ²¹⁴Ra to 400 keV in ²¹⁸Ra. Also, the enhancement of the reduced transition probabilities (see Sec. III B), which is usually caused by the collective nuclear effects, suggests the presence of large quadrupole correlations in the lead region.

Other traditional indicators are distinguished in the studied nuclei, such as

(i) The decrease of 2_2^+ energy levels with increasing the neutron number in the isotopic chains (Z > 82), indicates a significant collective behavior. Albeit, at this step of calculations, the 2_2^+ states energetically are not below the 4⁺ states, so the γ -rigid possibility is rejected, according to the rigid triaxial rotor model [44].

(ii) The $B_{42} = \frac{B(E2;4^+ \rightarrow 2^+)}{B(E2;2^+ \rightarrow 0^+)}$ transitions ratio, reported in Table II, is larger than 1, for most of the nuclei (N > 126), precisely in the vibrational area. It is not the case for the N = 126 isotonic chain (212 Rn, 214 Ra, 216 Th) marked by a spherical character.

Additional sensitive features of triaxiality are the quadrupole moments $Q(2^+_{1,2})$, $Q(3^+)$ and the $B(E2; 3^+ \rightarrow 2^+_2)$ transitions calculated in the laboratory frame. The survey of these quadrupole properties reported in Table II supports, in some cases, the empirical criteria of deformed nuclei, where

- (i) $Q_s(2^+_{\gamma}, K = 2)$ has a similar magnitude with an opposite sign to that of $Q_s(2^+, K = 0)$.
- (ii) $Q_s(3^+, K = 2)$ is close to zero and the low-lying 3^+ state is connected by a strong transition to the second 2^+ excited state.

TABLE II. The quadrupole moments $Q(2^+_{1,2})$, $Q(3^+)$, $B(E2; 3^+ \rightarrow 2^+_2)$, and the $B_{42} = \frac{B(E2; 4^+ \rightarrow 2^+)}{B(E2; 2^+ \rightarrow 0^+)}$ transitions ratio of different nuclei.

Nucleus	$Q_s(2_1^+)$ (<i>e</i> fm ²)	$Q_s(2_2^+)$ (efm ²)	$Q(3^+)$ (<i>e</i> fm ²)	$B(E2; 3^+ \rightarrow 2^+_2)$ $(e^2 \text{fm}^4)$	<i>B</i> ₄₂
210 Do	22.24	12.41	16 59	128.72	1 10
²¹² Po	3.19	22.38	10.38	27.09	1.18
²¹⁴ Po	-18.49	12.26	-5.27	156.62	1.24
²¹⁶ Po	-29.51	10.97	6.14	91.91	1.40
212 Rn	5.54	-3.10	22.22	0.035	0.17
214 Rn	-1.06	12.90	19.72	11.29	1.20
²¹⁶ Rn	-44.16	37.54	-7.82	1705.77	1.52
²¹⁸ Rn	-50.22	48.92	-1.10	2621.32	1.52
²¹⁴ Ra	-7.57	18.68	24.44	1.34	0.00
²¹⁶ Ra	-7.14	0.96	34.31	3.15	1.24
²¹⁸ Ra	-71.22	49.27	-10.39	1073.23	1.53
²¹⁶ Th	-17.25	10.76	21.75	0.47	0.04
²¹⁸ Th	-18.22	-3.10	-5.86	1347.57	1.29
²²⁰ Th	-67.40	35.15	-82.60	0.25	1.55



FIG. 12. Ground state E2 invariants: the expectation values of quadrupole deformation $\langle Q^2 \rangle$ (a) and triaxiality $\cos(3\gamma)$ (b).

All these clues clearly apparent in the ^{216–218}Rn nuclei, support the picture for a γ -soft band. Whereas, in the specific cases of ²¹⁸Ra and ²²⁰Th nuclei distinct by a strong $B(E2; 2^+ \rightarrow 0^+)$ transition strength ($\approx 2000 \ e^2 \text{fm}^4$), some triaxiality criteria are not fulfilled, such as the weak $B(E2; 3^+ \rightarrow 2_2^+)$ transitions. It should be mentioned that the authors of Refs. [1,45] have suggested that the ²¹⁸Ra and ²²⁰Th nuclei exhibit a considerable octupole deformation. As it was discussed in Sec. III C, this class of correlations could be accounted correctly within the SM, only by including a large configuration space constructed by particle-hole excitations across the ²⁰⁸Pb closed core.

To get a further insight into the deformation of the studied nuclei, the deformation parameters are calculated, which give us a complete vision on the intrinsic shape and degree of the collectivity. However, the challenging issue to address is the fact that the shape parameters (β, γ) are defined in the intrinsic frame of the nucleus, whereas the calculations and the observations are made in the laboratory frame. For this, the Kumar-Cline sum rules [46,47] are used, where charge deformation parameters could be obtained using rotationally invariant scalar products of the quadrupole operators. They relate the reduced E2 matrix elements with the quadrupole deformation $\langle Q^2 \rangle$ and triaxiality $\cos(3\gamma)$ parameters. The application of a sum rule technique sets up a challenge for theoretical models, where some are inappropriate for using this method (e.g., the IBM model). However, the SM is well adapted to employ it, where the expectation values could be calculated using the projected Lanczos strength function, already incorporated in the ANTOINE SM code [22,48].

Making use of the quadrupole sum rules method [46,47], the calculated expectation values of quadrupole deformation $\langle Q^2 \rangle$ and triaxiality $\cos(3\gamma)$ for the ground state are illustrated in Fig. 12.

Taking a look at the upper part of Fig. 12, the quadrupole deformation increases with increasing the mass number,

where the larger magnitudes of $\langle Q^2 \rangle$ are achieved in the A = 118, 220 nuclei. This trend reveals a modest deformation of their ground state, corresponding to deformation parameter $\beta = 0.1$. While smaller magnitudes are observed for N = 126 isotones indicating a spherical character.

From the lower part of Fig. 12, illustrating the expectation values of $cos(3\gamma)$, it could be seen that the case of oblate shape is completely excluded for all the nuclei, whereas most ground states are identified with a weak prolate shape.

In summing up this section, the exposed features demonstrate that the studied nuclei possess relatively weak quadrupole correlations, while only ²¹⁶Rn and ²¹⁸Rn nuclei have a small degree of softness in the triaxial direction. One may also notice that these nuclei from this region belong to a transitional region from the spherical regime for the N =126 isotones to slightly weak quadrupole deformation shape for the N = 130,132 isotones. The nuclei beyond N = 132 isotones could be highly deformed, where stronger correlations are supposed to be present.

IV. SIMILARITIES AND DIFFERENCES BETWEEN ¹³²Sn AND ²⁰⁸Pb REGIONS

A close resemblance between the two mass regions ¹³²Sn and ²⁰⁸Pb was already discussed in [16–18], by revealing the same behavior between $\pi 0g_{7/2}\nu 1f_{7/2}$ multiplets in the ¹³⁴Sb and the $\pi 0h_{9/2}\nu 1g_{9/2}$ in the ²¹⁰Bi.

From the results discussed above (see Secs. III A and III B), and those reported in our investigations [28,29,49–51] of the ¹³²Sn region, some similarities in the spectroscopic properties between the two mass regions could be also noticed, such as,

(i) A strong resemblance in the variation of the 2^+ energy level, where it decreases from 1500 keV in N = 126 to 400 keV for the N = 130,132 isotones, the same

as that obtained in the N = 82,86, and 88 isotonic chains.

- (ii) The $B(E2; 2^+ \rightarrow 0^+)$ transition strength grows very slowly in the Po isotopes, similarly to that observed in the Te isotopes.
- (iii) The larger B(E2) strengths are reached in the N = 130, 132 isotones $(2000 e^2 \text{fm}^4)$ with $Q_s(2^+)$ of order 60–80 efm², the same strengths as for the N = 86, 88 isotones in the ¹³²Sn mass region.
- (iv) The different isomeric 8⁺ states observed in ^{210,212,214,216}Pb [4,52] and in ²¹⁰Po [53], have their counterpart 6⁺ isomer decays identified in the ^{134,136,138}Sn [54,55] and ¹³⁴Te [56] nuclei.

Despite all these similarities in the spectroscopic properties between the both systems, but the degree of the quadrupole collectivity in the N = 130, 132 isotones is quite weak $\beta =$ 0.1 compared to the N = 86,88 isotones $\beta = 0.2$. It could be justified by the presence of the octupole deformation in the heavier nuclei from ²⁰⁸Pb mass region, exhibiting the reflection-asymmetric like behavior [1].

V. CONCLUSION

In the present work, a new investigation of even-even nuclei in the vicinity of ²⁰⁸Pb closed core is performed within the SM framework. The calculations include energy levels and electromagnetic transition rates of nuclei with $82 \le Z \le 90$ and $126 \le N \le 134$ are well compared with the experimental data, demonstrating the reliability of the H208 effective interaction.

The analysis of the quadrupole correlations in this mass region indicates the presence of a transitional region from spherical nuclear shape in the N = 126 isotones, to weak quadrupole deformation in the N = 130,132 isotones. Going beyond N = 132 isotones could open up to a highly deformed region, where strong correlations should be present.

The theoretical calculations achieved using the H208 effective interaction provide a large amount of information on energy levels, E2 and M1 transition rates, which compare well with the available experimental data. The predicted spectroscopic properties pave the way to the future measurements and provide a support for different theoretical models.

Finally, this part of work establishes the first step in the development of the ²⁰⁸Pb mass region. In order to explore deeply this region, several subjects will be investigated, such as,

- (i) The spectroscopic study of odd mass nuclei, which provides an important source of information about the evolution of single-particle states, half-lives $(T_{1/2})$, and isomeric states.
- (ii) The investigation of nuclei beyond N = 132 isotones and/or Z = 90 isotopes, where stronger correlations should be present, despite that such a step is extremely challenging computationally.
- (iii) The calculation of the effective E2 and M1 transition operators consistently with the derivation of the effective Hamiltonian, instead the adjustment of the effective charges and gyromagnetic factors to reproduce the experimental observables.

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