

## Nuclear matrix elements for Majoron-emitting double- $\beta$ decay

J. Kotila<sup>1,2,\*</sup> and F. Iachello<sup>2,†</sup>

<sup>1</sup>*Finnish Institute for Educational Research, University of Jyväskylä, P.O. Box 35, 40014 Jyväskylä, Finland*

<sup>2</sup>*Center for Theoretical Physics, Sloane Physics Laboratory Yale University, New Haven, Connecticut 06520-8120, USA*



(Received 3 November 2020; accepted 18 March 2021; published 1 April 2021)

A complete calculation of the nuclear matrix elements (NMEs) for Majoron emitting neutrinoless double beta decay within the framework of IBM-2 for spectral indices  $n = 1, 3, 7$  is presented. By combining the results of this calculation with previously calculated phase-space factors (PSFs) we give predictions for expected half-lives. By comparing with experimental limits on the half-lives we set limits on the coupling constants  $\langle g_{ee}^M \rangle$  of all proposed Majoron-emitting models.

DOI: [10.1103/PhysRevC.103.044302](https://doi.org/10.1103/PhysRevC.103.044302)

### I. INTRODUCTION

In recent years, increased accuracy has been achieved in the measurement of double- $\beta$  decay (DBD) with the emission of two neutrinos,  $2\nu\beta\beta$  decay, especially in the measurement of the summed electron spectra. High-statistics experiments have been reported by GERDA ( $^{76}\text{Ge}$ ) [1], NEMO3 ( $^{100}\text{Mo}$ ) [2], CUORE ( $^{130}\text{Te}$ ) [3], EXO ( $^{136}\text{Xe}$ ) [4], and KamLAND-Zen ( $^{136}\text{Xe}$ ) [5]. High-statistics experiments have provided information on the mechanism of DBD in  $2\nu\beta\beta$  decay, in particular on the question of single-state dominance (SSD) versus high-state dominance (HSD), CUPID-0 ( $^{82}\text{Se}$ ) [6] and CUPID-Mo ( $^{100}\text{Mo}$ ) [7]. With the degree of accuracy reached in the latest experiments, one can also test nonstandard mechanisms of DBD and set stringent limits on them [8].

One of the nonstandard mechanisms is that occurring with the emission of additional bosons called Majorons. Majorons were introduced years ago [10,11] as massless Nambu-Goldstone bosons arising from global  $B - L$  (baryon number minus lepton number symmetry) broken spontaneously in the low-energy regime. These bosons couple to the Majorana neutrinos and give rise to neutrinoless double- $\beta$  decay, accompanied by Majoron emission  $0\nu\beta\beta M$  [12], as schematically shown in Fig. 1(a). Although these older models are disfavored by precise measurements of the width of the  $Z$  boson decay to invisible channels [13], several other models of  $0\nu\beta\beta M$  decay have been proposed in which one or two Majorons, denoted by  $\chi_0$ , are emitted (see Fig. 1):

$$\begin{aligned} (A, Z) &\rightarrow (A, Z + 2) + 2e^- + \chi_0, \\ (A, Z) &\rightarrow (A, Z + 2) + 2e^- + 2\chi_0. \end{aligned} \quad (1)$$

Table I lists some of the models proposed to describe these decays [14–17]. The different models are distinguished by the nature of the emitted Majoron(s), i.e., whether it is a Nambu-

Goldstone boson (NG), the leptonic charge of the emitted Majoron ( $L$ ), and the spectral index of the model,  $n$ .

The half-life for all these models can be written as

$$[\tau_{1/2}^{0\nu M}]^{-1} = G_{m\chi_0 n}^{(0)} \langle g_{\chi_{ee}^M} \rangle^{2m} |M_{0\nu M}^{(m,n)}|^2, \quad (2)$$

where  $G_{m\chi_0 n}^{(0)}$  is a phase-space factor (PSF),  $\langle g_{\chi_{ee}^M} \rangle$  is the effective coupling constant of the Majoron to the neutrino,  $m = 1, 2$  for the emission of one or two Majorons, respectively, and  $M_{0\nu M}^{(m,n)}$  the nuclear matrix element (NME).

### II. PHASE-SPACE FACTORS

In a previous article [9] we calculated the PSF and from these the single-electron spectrum, the summed electron spectrum, and the angular correlation between the two electrons. Particularly interesting are the summed electron spectra whose shape depends crucially on the spectral index  $n$ . In Fig. 2, the summed electron spectra for  $n = 1$ ,  $n = 3$  and  $n = 7$ , obtained from Ref. [9] by normalizing the spectra so that the area covered by each of them is the same, are plotted as a function of  $\varepsilon_1 + \varepsilon_2 - 2m_e c^2$ . In this figure, also the summed electron spectrum for  $2\nu\beta\beta$  decay [18] is shown again with area normalized to 1. This spectrum has a spectral index  $n = 5$ . The summed electron spectrum of the “bulk” model  $n = 2$  is also shown in Fig. 2. Exact Dirac wave functions, nuclear finite size, and electron screening are included in this calculation, as discussed in Ref. [18]. Previous calculations [19–22] make use of Fermi functions, which are an approximation to the relativistic Dirac wave functions. For comparison between the values reported in Ref. [22] and our values [9] we note that our PSFs are divided by a factor of  $g_A^4 = 2.593$  since we include this factor in the NME. We estimated the error in using the old calculation of the PSFs [19–22] instead of the new [9],  $(G_{m\chi_0 n}^{(0)old} - G_{m\chi_0 n}^{(0)new})/G_{m\chi_0 n}^{(0)new}$ , to be 6% in  $^{76}\text{Ge}$  and 28% in  $^{136}\text{Xe}$ . The reason why the error is larger in  $^{136}\text{Xe}$  ( $Z = 54$ ) than in  $^{76}\text{Ge}$  ( $Z = 32$ ) is the neglect in the old calculation of relativistic effects and electron screening which increase as a large power of  $Z$ . While in  $^{76}\text{Ge}$  and  $^{82}\text{Se}$  the use of the old

\*jenni.kotila@jyu.fi

†francesco.iachello@yale.edu

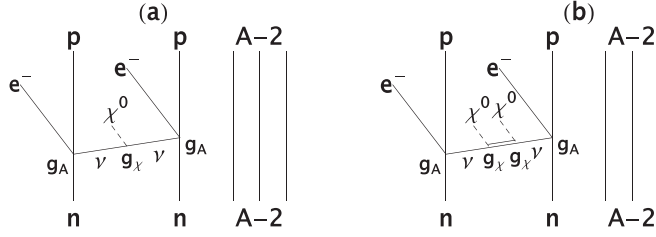


FIG. 1. Schematic representation of neutrinoless double- $\beta$  decay accompanied by the emission of one or two Majorons. Adapted from Ref. [9].

calculation may still be reasonable, it is definitely not so in the other nuclei of current interest:  $^{100}\text{Mo}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ , and  $^{150}\text{Nd}$ . Although experimentally not easily accessible, we also plot in Fig. 3 the single electron spectra with area normalized to 1, and in Fig. 4 the angular correlation between the two electrons, for  $n = 1$ ,  $n = 2$ ,  $n = 3$ ,  $n = 7$ , and  $2\nu\beta\beta$  ( $n = 5$ ) as a function of  $\epsilon_1 - m_e c^2$ , both of which have been measured by the NEMO3 collaboration in  $^{130}\text{Te}$  [23]. In this article we present a calculation of the nuclear matrix elements  $M_{0\nu M}^{(m,n)}$ .

### III. NUCLEAR MATRIX ELEMENTS

Nuclear matrix elements for Majoron-emitting DBD were derived in a seminal paper by Hirsch *et al.* [22]. These authors derived an explicit form for the nuclear matrix elements of all the models of Table I, except for the “bulk” model. We have converted the form of Ref. [22] to our notation, added some higher-order terms not included in the original form and calculated the corresponding matrix elements within the framework of the microscopic interacting boson model IBM-2 [24,25] with isospin restoration [26]. Explicitly, we introduce the matrix elements

$$\begin{aligned} \mathcal{M}_F &= \langle f || v_m || i \rangle, \\ \mathcal{M}_{GT} &= \langle f || v_m \sigma_1 \cdot \sigma_2 || i \rangle, \\ \mathcal{M}_T &= \langle f || v_m S_{12} || i \rangle, \end{aligned}$$

TABLE I. Different Majoron-emitting models [14–17]. The third, fourth, and fifth columns indicate whether the Majoron is a Nambu-Goldstone boson, its leptonic charge  $L$ , and the model’s spectral index,  $n$ . The sixth column indicates the nuclear matrix elements of Sec. II appropriate for each model.

Model	Decay mode	NG boson	$L$	$n$	NME
IB	$0\nu\beta\beta\chi_0$	No	0	1	$M_1$
IC	$0\nu\beta\beta\chi_0$	Yes	0	1	$M_1$
ID	$0\nu\beta\beta\chi_0\chi_0$	No	0	3	$M_3$
IE	$0\nu\beta\beta\chi_0\chi_0$	Yes	0	3	$M_3$
IIB	$0\nu\beta\beta\chi_0$	No	-2	1	$M_1$
IIC	$0\nu\beta\beta\chi_0$	Yes	-2	3	$M_2$
IID	$0\nu\beta\beta\chi_0\chi_0$	No	-1	3	$M_3$
IIE	$0\nu\beta\beta\chi_0\chi_0$	Yes	-1	7	$M_3$
IIF	$0\nu\beta\beta\chi_0$	Gauge boson	-2	3	$M_2$
“Bulk”	$0\nu\beta\beta\chi_0$	Bulk field	0	2	

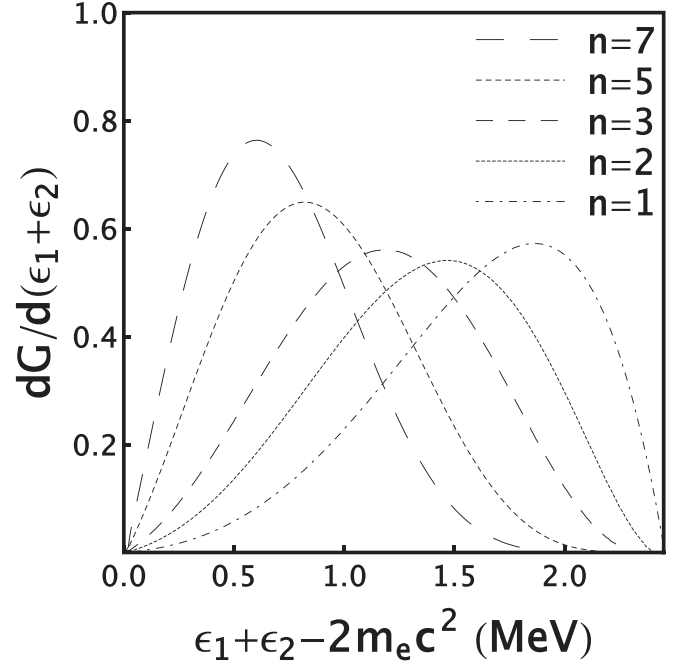


FIG. 2. Summed electron spectra for the  $n = 1, 2, 3$ , and  $7$ , as well as for the  $2\nu\beta\beta$  ( $n = 5$ ) decays of  $^{136}\text{Xe}$ .

$$\begin{aligned} \mathcal{M}_{GTR} &= \langle f || v_R \sigma_1 \cdot \sigma_2 || i \rangle, \\ \mathcal{M}_{TR} &= \langle f || v_R S_{12} || i \rangle, \\ \mathcal{M}_{F\omega^2} &= \langle f || v_{\omega^2} || i \rangle, \\ \mathcal{M}_{GT\omega^2} &= \langle f || v_{\omega^2} \sigma_1 \cdot \sigma_2 || i \rangle, \\ \mathcal{M}_{T\omega^2} &= \langle f || v_{\omega^2} S_{12} || i \rangle, \end{aligned} \quad (3)$$

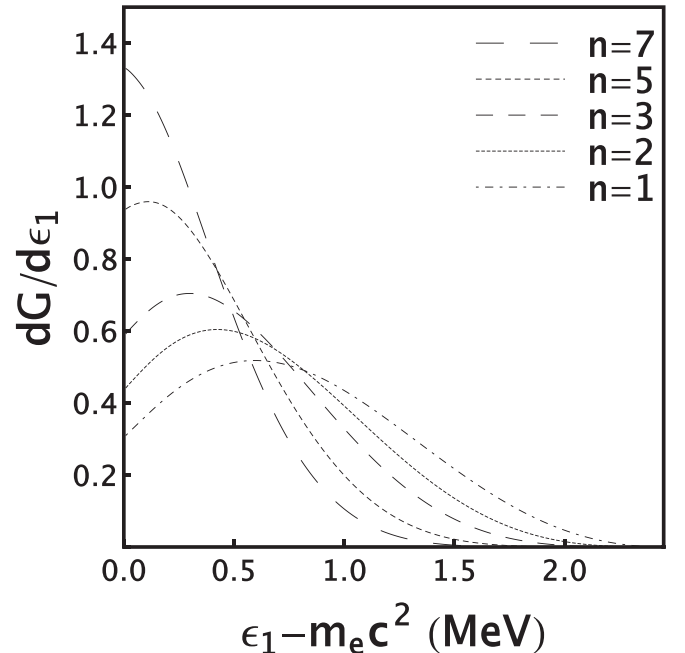


FIG. 3. Single electron spectra for the  $n = 1, 2, 3$ , and  $7$ , as well as for the  $2\nu\beta\beta$  ( $n = 5$ ) decays of  $^{136}\text{Xe}$ .

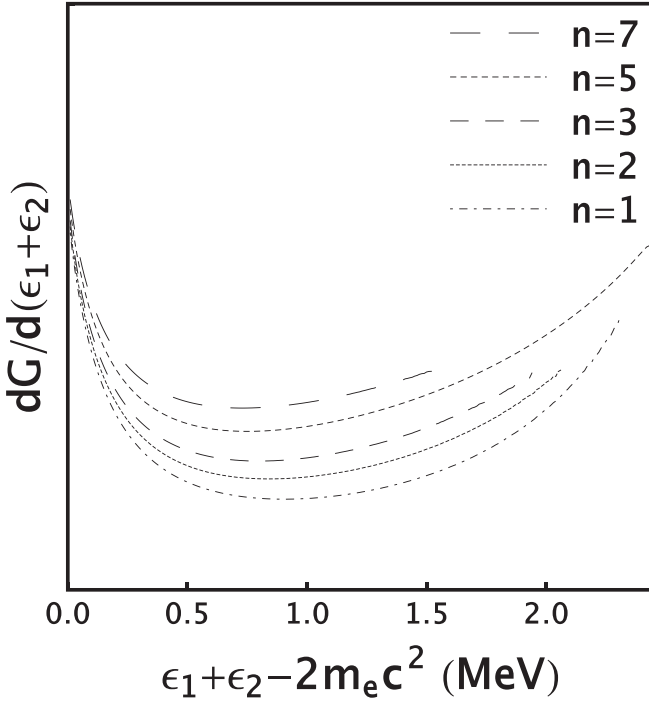


FIG. 4. Angular correlation between the two emitted electrons for the  $n = 1, 2, 3,$  and  $7$ , as well as for the  $2\nu\beta\beta$  ( $n = 5$ ) decays of  $^{136}\text{Xe}$ . The calculation for  $n = 1, 2, 3, 7$  stops at the point where the single electron spectrum goes to zero. Beyond that point it becomes unstable as  $G^{(0)}$  goes to zero faster than  $G^{(1)}$ . For  $2\nu\beta\beta$  ( $n = 5$ ) this is avoided by taking into account the individual energies of the neutrinos,  $\omega_1$  and  $\omega_2$ , as in Eq. (22) of Ref. [18], instead of the Majoron energy  $q$  as in Eq. (5) of Ref. [9].

where the isospin operators  $\tau_1^+\tau_2^+$  have been dropped for simplicity. These matrix elements are the same as in Ref. [22] with the addition of the tensor matrix elements.

The neutrino potentials needed for the calculation of these matrix elements, when converted to the notation used in IBM-2 [24–26] are

$$\begin{aligned} v_m &= \frac{2}{\pi} \frac{1}{q(q+\tilde{A})}, & v_R &= \frac{2}{\pi} \frac{1}{Rm_p} \frac{q + \frac{\tilde{A}}{2}}{q(q+\tilde{A})^2}, \\ v_{\omega^2} &= \frac{2}{\pi} m_e^2 \frac{q^2 + \frac{9}{8}q\tilde{A} + \frac{3}{8}\tilde{A}^2}{q^3(q+\tilde{A})^3}, \end{aligned} \quad (4)$$

with  $R = 1.2A^{1/3}$  fm,  $m_p = 938$  MeV =  $4.76$  fm $^{-1}$ ,  $m_e = 0.511$  MeV =  $0.00259$  fm $^{-1}$ .  $\tilde{A}$  is the closure energy that we take as in Refs. [24–26],  $\tilde{A} = 1.12A^{1/2}$  MeV, where  $A$  denotes the mass number. We note that the last term in  $v_{\omega^2}$  diverges at the origin as  $q^{-3}$ . We regularize this term by multiplying it by  $q/(q+\tilde{A})$ , that is

$$v_{\omega^2} = \frac{2}{\pi} m_e^2 \frac{q^2 + \frac{9}{8}q\tilde{A} + \frac{3}{8}\tilde{A}^2 \frac{q}{q+\tilde{A}}}{q^3(q+\tilde{A})^3}. \quad (5)$$

From the neutrino potentials we construct the quantities

$$h(q) = v(q)\tilde{h}(q), \quad (6)$$

where  $\tilde{h}_{F,GT,T}(q) = \tilde{h}_{F\omega^2,GT\omega^2,T\omega^2}$  are given in Table II of Ref. [25] which includes the form factors and higher-order corrections and

$$\tilde{h}_R(q) = \frac{1}{(1+q^2/m_V^2)^2} \frac{1}{(1+q^2/m_A^2)^2}, \quad (7)$$

which includes the form factors with  $m_V = 0.84$  GeV and  $m_A = 1.09$  GeV, as in Refs. [25,26].

The matrix elements for the three classes of Majoron models are

$$\begin{aligned} M_1 &= g_A^2 \mathcal{M}_1 = g_A^2 \left[ -\left(\frac{g_V^2}{g_A^2}\right) \mathcal{M}_F + \mathcal{M}_{GT} - \mathcal{M}_T \right], \\ M_2 &= g_A^2 \mathcal{M}_2 = g_A^2 \left[ \left(\frac{g_V}{g_A}\right) \frac{f_W}{3} \mathcal{M}_{GTR} - \left(\frac{g_V}{g_A}\right) \frac{f_W}{6} \mathcal{M}_{TR} \right], \\ M_3 &= g_A^2 \mathcal{M}_3 = g_A^2 \left[ -\left(\frac{g_V^2}{g_A^2}\right) \mathcal{M}_{F\omega^2} + \mathcal{M}_{GT\omega^2} - \mathcal{M}_{T\omega^2} \right], \end{aligned} \quad (8)$$

where we have used the overall sign convention as in Ref. [27] and in our previous papers [24–26]. In Eq. (8),  $f_W = 1 + \kappa_\beta = 4.70$ , where  $\kappa_\beta$  is the isovector magnetic moment of the nucleon. In the calculation of the matrix elements in Eq. (8) also short-range correlations are included as in Refs. [24–26]. Our results are shown in Table II. The nuclear matrix elements  $M_1, M_2, M_3$  are associated with Majoron-emitting models of  $0\nu\beta\beta M$  decays as in the last column of Table I.

#### Sensitivity to parameter changes, model assumptions, and operator assumptions

The matrix element  $\mathcal{M}_1$  for index  $n = 1$  is identical to the matrix element of ordinary  $0\nu\beta\beta$  decay without Majoron emission. The sensitivity of IBM-2 calculations to parameter changes, model assumptions, and operator assumption for this NME was discussed in great detail in Refs. [25,26]. Our error estimate for  $M_1$  is therefore 16% for all nuclei, as in Ref. [26].

For the matrix element  $\mathcal{M}_2$  we have an additional error coming from the neglect of higher-order terms of the type

$$\begin{aligned} \frac{(\mathbf{Q} \cdot \boldsymbol{\sigma}_1)(\mathbf{q} \cdot \boldsymbol{\sigma}_2)}{4m_p^2} &\simeq \frac{(\mathbf{q} \cdot \boldsymbol{\sigma}_1)(\mathbf{q} \cdot \boldsymbol{\sigma}_2)}{4m_p^2} \\ &= \frac{q^2}{4m_p^2} \left[ \frac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{1}{3} S_{12} \right], \end{aligned} \quad (9)$$

where  $\mathbf{Q}$  is the total momentum and  $\mathbf{q}$  the relative momentum of the nucleons and we have assumed  $\mathbf{Q} \simeq \mathbf{q}$  [20]. We estimate the neglected contribution of these higher-order terms to be about 4%, giving a total estimated error of 20% for  $\mathcal{M}_2$ .

The matrix element  $\mathcal{M}_3$  depends strongly on the closure energy  $\tilde{A}$  as given in Eq. (4). In the present calculation we have assumed the standard choice  $\tilde{A} = 1.12A^{1/2}$  MeV. We have investigated variations of  $\tilde{A}$  around the standard values and estimate an additional error in the calculation of  $\mathcal{M}_3$  of  $\approx 10\%$ , bringing the total estimated error to 30%. An estimate of the sensitivity of  $\mathcal{M}_3$  to the closure energy was also given in Ref. [22]. In this reference also a discussion of the sensitivity to model assumptions of Majoron-emitting DBD was given.

TABLE II. Majoron-emitting DBD NMEs  $\mathcal{M}_i$  ( $i = 1, 2, 3$ ) calculated in this work using the quenched value  $g_A = 1.0$  and the convention  $\mathcal{M}_i > 0$ .

Isotope	$\mathcal{M}_F$	$\mathcal{M}_{GT}$	$\mathcal{M}_T$	$\mathcal{M}_1$	$\mathcal{M}_{GTR}$	$\mathcal{M}_{TR}$	$\mathcal{M}_2$	$\mathcal{M}_{F\omega^2}$ $\times 10^3$	$\mathcal{M}_{GT\omega^2}$ $\times 10^3$	$\mathcal{M}_{T\omega^2}$ $\times 10^3$	$\mathcal{M}_3$ $\times 10^3$
<sup>76</sup> Ge	-0.780	5.582	-0.281	6.642	0.225	-0.037	0.381	-0.017	2.530	-0.009	2.556
<sup>82</sup> Se	-0.667	4.521	-0.270	5.458	0.178	-0.034	0.305	-0.014	1.967	-0.009	1.993
<sup>96</sup> Zr	-0.361	3.954	0.250	4.065	0.147	0.031	0.205	-0.006	1.672	0.009	1.668
<sup>100</sup> Mo	-0.511	5.075	0.318	5.268	0.187	0.038	0.263	-0.008	1.904	0.011	1.901
<sup>110</sup> Pd	-0.425	4.024	0.243	4.206	0.144	0.030	0.203	-0.006	1.411	0.009	1.409
<sup>116</sup> Cd	-0.335	2.888	0.118	3.105	0.102	0.019	0.144	-0.005	0.945	0.006	0.945
<sup>124</sup> Sn	-0.572	3.099	-0.118	3.789	0.104	-0.017	0.177	-0.013	1.161	-0.005	1.179
<sup>128</sup> Te	-0.718	3.965	-0.115	4.798	0.132	-0.020	0.223	-0.016	1.505	-0.006	1.527
<sup>130</sup> Te	-0.651	3.586	-0.159	4.396	0.118	-0.018	0.199	-0.014	1.291	-0.005	1.311
<sup>134</sup> Xe	-0.686	3.862	-0.121	4.669	0.126	-0.018	0.212	-0.015	1.456	-0.006	1.477
<sup>136</sup> Xe	-0.522	2.958	-0.123	3.603	0.096	-0.013	0.160	-0.012	1.161	-0.004	1.112
<sup>148</sup> Nd	-0.362	2.283	0.125	2.521	0.074	0.012	0.107	-0.006	0.648	0.004	0.650
<sup>150</sup> Nd	-0.507	3.371	0.119	3.759	0.110	0.017	0.159	-0.008	0.836	0.005	0.839
<sup>154</sup> Sm	-0.340	2.710	0.122	2.928	0.086	0.015	0.122	-0.006	0.858	0.005	0.859
<sup>160</sup> Gd	-0.415	3.838	0.250	4.002	0.120	0.023	0.170	-0.006	1.261	0.008	1.260
<sup>198</sup> Pt	-0.329	2.021	0.119	2.230	0.061	0.009	0.089	-0.005	0.393	0.003	0.395
<sup>232</sup> Th	-0.444	3.757	0.251	3.950	0.104	0.019	0.148	-0.006	0.930	0.007	0.930
<sup>238</sup> U	-0.525	4.470	0.244	4.751	0.122	0.022	0.174	-0.007	1.118	0.008	1.118

IV. LIMITS ON THE COUPLING CONSTANTS

From the PSF of Ref. [9], the NME of this article, and experimental limits on half-lives for each type of Majoron

model, one can derive limits on the coupling constants  $g_{\chi_{ee}^M}$ . These limits depend on the value of the coupling constant  $g_A$ . This coupling constant is renormalized in nuclei by many-body effects. Three possible values are [28] (i) the free

TABLE III. Limits on the Majoron-neutrino coupling constants  $\langle g_{\chi_{ee}^M} \rangle$  for  $g_A = 1$ . PSF from Ref. [9]. NME from this paper.

Decay mode	Spectral index	Model type	$\mathcal{M}$	$G_{m\chi_0 n}^{(0)} [10^{-18} \text{ yr}]$	$\tau_{1/2} [\text{yr}]$	$ \langle g_{\chi_{ee}^M} \rangle $
<sup>76</sup> Ge [32]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	6.64	44.2	$>4.2 \times 10^{23}$	$<3.5 \times 10^{-5}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0026	0.22	$>0.8 \times 10^{23}$	$<1.7$
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.381	0.073	$>0.8 \times 10^{23}$	$<0.34 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0026	0.420	$>0.3 \times 10^{23}$	$<1.9$
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.8 \times 10^{23}$	
<sup>130</sup> Te [29]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>2.2 \times 10^{21}$	$<2.4 \times 10^{-4}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0013	3.21	$>0.9 \times 10^{21}$	$<3.8$
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.199	1.51	$>2.2 \times 10^{21}$	$<0.87 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0013	14.4	$>0.9 \times 10^{21}$	$<2.6$
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.2 \times 10^{21}$	
<sup>130</sup> Te [23]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>1.6 \times 10^{22}$	$<8.8 \times 10^{-5}$
<sup>136</sup> Xe [31]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>1.2 \times 10^{24}$	$<1.3 \times 10^{-5}$
$0\nu\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>2.7 \times 10^{22}$	$<1.8$
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>2.7 \times 10^{22}$	$<0.31 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>6.1 \times 10^{21}$	$<1.8$
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.5 \times 10^{23}$	
<sup>136</sup> Xe [30]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>2.6 \times 10^{24}$	$<8.5 \times 10^{-6}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>4.5 \times 10^{24}$	$<0.49$
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>4.5 \times 10^{24}$	$<0.24 \times 10^{-2}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>1.1 \times 10^{22}$	$<1.6$
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.0 \times 10^{24}$	

value,  $g_A = 1.269$ , (ii) the quark value,  $g_A = 1.0$ , and (iii) the value extracted from  $2\nu\beta\beta$  decay, which, in IBM-2 can be parametrized as  $g_{A,eff}^{IBM-2} = 1.269A^{-0.18}$ . To allow for different values of  $g_A$ , we rewrite Eq. (2) as

$$[\tau_{1/2}^{0\nu M}]^{-1} = G_{m\chi_0^n}^{(0)} |g_{\chi_{ee}^M}|^{2m} g_A^4 |\mathcal{M}_{0\nu M}^{(m,n)}|^2, \quad (10)$$

where  $\mathcal{M}_{0\nu M}^{(m,n)}$  are the NME given in Table II. In extracting limits on  $g_{\chi_{ee}^M}$  we use in this article  $g_A = 1$ . From Eq. (10) it is straightforward to obtain limits for other values of  $g_A$ .

Limits on half-lives for Majoron-emitting models have been reported by several groups [23,29–32]. In Table III we provide our limits on the coupling constants  $g_{\chi_{ee}^M}$ .

The most stringent limits come from the KamLAND-Zen collaboration [30] and from the EXO collaboration [31]. The reason why one obtains such small limits for Majoron-emitting models with index  $n = 1$  was discussed in Ref. [22]. The larger limits of  $g_{\chi_{ee}^M}$  for Majoron-emitting models with

index  $n = 3$  and  $n = 7$  are due to the smaller values of the PSF for these indices.

## V. CONCLUSIONS

We have presented here a complete calculation of NME for Majoron-emitting neutrinoless double- $\beta$  decay within the framework of the Interacting Boson Model IBM-2. Our results when combined with the phase space factors of Ref. [9] provide up-to-date predictions for lifetimes, single-electron spectra, summed electron spectra, and angular distributions for Majoron-emitting neutrinoless double- $\beta$  decay which can be used in the analysis of recent high-statistics experiments [1–7].

## ACKNOWLEDGMENTS

This work was supported in part by the Academy of Finland Grants No. 314733 and No. 320062.

- 
- [1] M. Agostini *et al.* (GERDA Collaboration), *Nature (London)* **544**, 47 (2017).
- [2] R. Arnold *et al.* (NEMO3 Collaboration), *Phys. Rev. D* **92**, 072011 (2015).
- [3] K. Alfonso *et al.* (CUORE Collaboration), *Phys. Rev. Lett.* **115**, 102502 (2015).
- [4] N. Ackerman *et al.* (EXO Collaboration), *Phys. Rev. Lett.* **107**, 212501 (2011).
- [5] A. Gando *et al.* (KamLAND-Zen Collaboration), *Phys. Rev. C* **85**, 045504 (2012).
- [6] A. Azzolini *et al.* (CUPID-0 Collaboration), *Phys. Rev. Lett.* **123**, 262501 (2019).
- [7] E. Armengaud *et al.*, *Eur. Phys. J. C* **80**, 674 (2020).
- [8] F. F. Deppisch, L. Graf, and F. Šimkovic, *Phys. Rev. Lett.* **125**, 171801 (2020).
- [9] J. Kotila, J. Barea, and F. Iachello, *Phys. Rev. C* **91**, 064310 (2015).
- [10] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, *Phys. Rev. Lett.* **45**, 1926 (1980).
- [11] G. B. Gelmini and M. Roncadelli, *Phys. Lett. B* **99**, 411 (1981).
- [12] H. M. Georgi, S. L. Glashow, and S. Nussinov, *Nucl. Phys. B* **193**, 297 (1981).
- [13] The ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, SLD Collaboration, LEP Electroweak Working Group, and Electroweak and Heavy Flavor Group, *Phys. Rep.* **427**, 257 (2006).
- [14] P. Bamert, C. Burgess, and R. Mohapatra, *Nucl. Phys. B* **449**, 25 (1995).
- [15] C. D. Carone, *Phys. Lett. B* **308**, 85 (1993).
- [16] C. Burgess and J. Cline, in *Proceedings of the First International Conference on Nonaccelerator Physics, Bangalore, India, 1994*, edited by R. Cowsik (World Scientific, Singapore, 1995).
- [17] R. Mohapatra, A. Perez-Lorenzana, and C. D. S. Pires, *Phys. Lett. B* **491**, 143 (2000).
- [18] J. Kotila and F. Iachello, *Phys. Rev. C* **85**, 034316 (2012).
- [19] M. Doi, T. Kotani, and E. Takasugi, *Prog. Theor. Phys. Suppl.* **83**, 1 (1985).
- [20] T. Tomoda, *Rep. Prog. Phys.* **54**, 53 (1991).
- [21] J. Suhonen and O. Civitarese, *Phys. Rep.* **300**, 123 (1998).
- [22] M. Hirsch, H. V. Klapdor-Kleingrothaus, S. G. Kovalenko, and H. Päs, *Phys. Lett. B* **372**, 8 (1996).
- [23] R. Arnold *et al.* (NEMO3 Collaboration), *Phys. Rev. Lett.* **107**, 062504 (2011).
- [24] J. Barea and F. Iachello, *Phys. Rev. C* **79**, 044301 (2009).
- [25] J. Barea, J. Kotila, and F. Iachello, *Phys. Rev. C* **87**, 014315 (2013).
- [26] J. Barea, J. Kotila, and F. Iachello, *Phys. Rev. C* **91**, 034304 (2015).
- [27] F. Šimkovic, G. Pantis, J. D. Vergados, and A. Faessler, *Phys. Rev. C* **60**, 055502 (1999).
- [28] S. Dell’Oro, S. Marcocci, and F. Vissani, *Phys. Rev. D* **90**, 033005 (2014).
- [29] C. Arnaboldi *et al.* (CUORE Collaboration), *Phys. Lett. B* **557**, 167 (2003).
- [30] A. Gando, Y. Gando, H. Hanakago, H. Ikeda, K. Inoue, R. Kato, M. Koga, S. Matsuda, T. Mitsui, T. Nakada, K. Nakamura, A. Obata, A. Oki, Y. Ono, I. Shimizu, J. Shirai, A. Suzuki, Y. Takemoto, K. Tamae, K. Ueshima *et al.* (KamLAND-Zen Collaboration), *Phys. Rev. C* **86**, 021601(R) (2012).
- [31] J. B. Albert *et al.* (EXO-200 Collaboration), *Phys. Rev. D* **90**, 092004 (2014).
- [32] S. Hemmer (GERDA Collaboration), *Eur. Phys. J. Plus* **130**, 139 (2015).