Screening of nucleon electric dipole moments in atomic systems

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The electric dipole moments (EDMs) of diamagnetic atoms are expected to be sensitive to charge-parity violation, particularly in nuclei through the nuclear Schiff moment. I explicitly demonstrate that the well-known form of the Schiff moment operator originating from the nucleon EDM is obtained by considering the screening of the nucleon EDMs in a neutral atom. Consequently, an additional contribution to the Schiff moment arises from the screening of the nuclear EDM induced by the interaction of the nucleon EDMs with the protons. This correction to the Schiff moment of ^{199}Hg is evaluated in the independent particle model.

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I. SCHIFF MOMENT

The observation of a permanent electric dipole moment (EDM) of an atom implies the existence of parity (P) and time-reversal (T) violating interactions between constituent particles. The atomic EDM is defined by

$$\boldsymbol{d}_{\text{atom}} = -\sum_{i=1}^{Z} e\boldsymbol{r}'_i, \tag{1}$$

where e is the elementary charge and $\boldsymbol{r}'_i$ indicates the coordinates of the atomic electrons. The interaction of the atomic EDM with an external electric field causes an energy shift to be measured [1,2].

Although the nuclear EDM and the intrinsic EDMs of electrons and nucleons are independently coupled to the external electric field $\boldsymbol{E}_{\text{ext}}$ as

$$V_{\text{ext}} = -\left[\boldsymbol{d}_{\text{atom}} + \boldsymbol{d}_{\text{nucl}} + \sum_{i=1}^{Z}\boldsymbol{d}_i^{(e)} + \sum_{a=1}^{A}\boldsymbol{d}_a\right]\cdot\boldsymbol{E}_{\text{ext}}, \tag{2}$$

they are obscured by the internal interactions with the electrons. Here $\boldsymbol{d}_i^{(e)}$ and $\boldsymbol{d}_a$ are the intrinsic EDMs of electrons and nucleons, respectively, and the nuclear EDM is defined by

$$\boldsymbol{d}_{\text{nucl}} = \sum_{a=1}^{Z} e\boldsymbol{r}_a, \tag{3}$$

where $\boldsymbol{r}_a$ indicates the proton coordinates. In particular, the energy shift due to the EDM of a pointlike nucleus is canceled by the contribution from the atomic EDM induced by the internal interaction of the nuclear EDM with the electrons. However, P, T-odd nucleon-nucleon (NN) interactions allow a finite-size nucleus to have the nuclear Schiff moment as well as the nuclear EDM. Since the atomic EDM induced by the interaction of the Schiff moment with the electrons survives the screening, the atomic EDMs particularly of diamagnetic atoms are sensitive to P, T-odd NN interactions [3–6]. The screening mechanism of the nuclear EDM induced by the P, T-odd meson-exchange NN (πNN) interaction is reviewed in this section.

The Hamiltonian of an atomic system that conserves P and T symmetries is written as

$$H_{\text{atom}} = H_{\text{nucl}} + H_e, \tag{4}$$

$$H_e = T_e + V^{(ee)} + V^{(eN)}_{\text{even}-l}, \tag{5}$$

where H_{nucl} denotes P, T-even NN interactions. The electron kinetic term T_e and the electron-electron interactions $V^{(ee)}$ are not relevant to the nuclear P, T violation of interest. The electrostatic interaction between the electrons and the protons is

$$V^{(eN)} = -e^2\sum_{i=1}^{Z}\sum_{a=1}^{Z}\frac{1}{|\boldsymbol{r}'_i - \boldsymbol{r}_a|}. \tag{6}$$

If $r'_i > r_a$, then each term can be expanded as

$$\frac{1}{|\boldsymbol{r}'_i - \boldsymbol{r}_a|} = \sum_{l=0}^{\infty}\frac{(-1)^l}{l!}(\boldsymbol{r}_a\cdot\boldsymbol{\nabla}'_i)^l\frac{1}{r'_i}. \tag{7}$$

The atomic Hamiltonian H_{atom} does not contain the odd-l electron-nucleon (eN) interactions denoted by $V^{(eN)}_{\text{odd}-l}$, which vanish unless P and T symmetries are both violated in the nucleus.

The nuclear ground state in the existence of the P, T-odd πNN interaction $\widetilde{V}_{\pi NN}$ is given by

$$\big|\widetilde{\psi}^{(N)}_{\text{g.s.}}\big\rangle = \big|\psi^{(N)}_{\text{g.s.}}\big\rangle + \sum_n \frac{1}{E^{(N)}_{\text{g.s.}} - E^{(N)}_n} \times\big|\psi^{(N)}_n\big\rangle\big\langle\psi^{(N)}_n\big|\widetilde{V}_{\pi NN}\big|\psi^{(N)}_{\text{g.s.}}\big\rangle, \tag{8}$$

where $E^{(N)}_{\text{g.s.}}$ and $E^{(N)}_n$ denote the energies of the ground state $|\psi^{(N)}_{\text{g.s.}}\rangle$ and excited states $|\psi^{(N)}_n\rangle$ of the nuclear Hamiltonian H_{nucl}, respectively. As well as the atomic EDM is generated by P, T-violations in the electron system, the P, T-odd πNN interaction can induce the nuclear EDM. The external interaction of the nuclear EDM represented in Fig. 1(a) causes the

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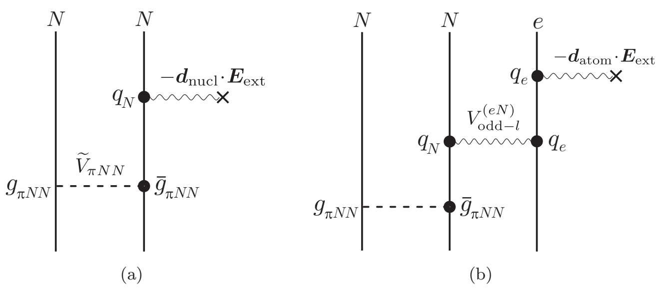

FIG. 1. (a) The second-order and (b) the third-order contributions of the P, T-odd πNN interaction $\widetilde{V}_{\pi NN}$ to the energy shift of an atom immersed in an external electric field. The black circles represent the P, T-odd vertices. The electric charges of proton and electron are denoted by q_N and q_e, respectively. The P, T-odd πNN coupling constant is denoted by $\bar{g}_{\pi NN}$.

energy shift

$$\Delta E_2(\bar{g}_{\pi NN}, q_N) = \langle \widetilde{\psi}^{(N)}_{\rm g.s.} | -\boldsymbol{d}_{\rm nucl}\cdot \boldsymbol{E}_{\rm ext} | \widetilde{\psi}^{(N)}_{\rm g.s.}\rangle, \tag{9}$$

where the coupling constants $\bar{g}_{\pi NN}$ and q_N specify the perturbative interactions.

The P, T-odd πNN interaction also induces the odd-l eN interactions $V^{(eN)}_{{\rm odd}-l}$, which violate P and T symmetries in the electron system. Consequently, the atomic EDM contributes to the energy shift in third-order perturbation as

$$\begin{aligned}\Delta E_3(\bar{g}_{\pi NN}, q_N, q_e) &= \sum_m \frac{1}{E^{(e)}_{\rm g.s.} - E^{(e)}_m} \\ &\times \langle \widetilde{\psi}^{(A)}_{\rm g.s.} | -\boldsymbol{d}_{\rm atom}\cdot \boldsymbol{E}_{\rm ext} | \widetilde{\psi}^{(A)}_m\rangle \langle \widetilde{\psi}^{(A)}_m | V^{(eN)}_{{\rm odd}-l} | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &+ \text{c.c.}\end{aligned} \tag{10}$$

This process is represented in Fig. 1(b). The eigenstates of the atomic system in the existence of the P, T-odd πNN interaction are expressed except for the Clebsch-Gordan coefficients as

$$|\widetilde{\psi}^{(A)}_{\rm g.s.}\rangle = |\widetilde{\psi}^{(N)}_{\rm g.s.}\rangle \otimes |\psi^{(e)}_{\rm g.s.}\rangle, \tag{11}$$

$$|\widetilde{\psi}^{(A)}_m\rangle = |\widetilde{\psi}^{(N)}_{\rm g.s.}\rangle \otimes |\psi^{(e)}_m\rangle, \tag{12}$$

where $|\psi^{(e)}_{\rm g.s.}\rangle$ and $|\psi^{(e)}_m\rangle$ denote the ground state and excited states of the electron system described by H_e with the energies $E^{(e)}_{\rm g.s.}$ and $E^{(e)}_m$, respectively.

Here I summarize the notations used in this paper. The superscripts (A), (N), and (e) represent the atomic, nuclear, and electron systems, respectively. P, T-odd interactions and P, T-violated wave functions are denoted by $\widetilde{V}$ and $|\widetilde{\psi}\rangle$, respectively.

The screening of the nuclear EDM is demonstrated by using a Hermitian operator [7]

$$U_{\rm nucl} = i\frac{1}{Ze}\langle \boldsymbol{d}_{\rm nucl}\rangle \cdot \sum_{i=1}^{Z} \boldsymbol{\nabla}'_i, \tag{13}$$

where

$$\langle \boldsymbol{d}_{\rm nucl}\rangle = \langle \widetilde{\psi}^{(N)}_{\rm g.s.} | \boldsymbol{d}_{\rm nucl} | \widetilde{\psi}^{(N)}_{\rm g.s.}\rangle. \tag{14}$$

The nuclear EDM interactions in Eqs. (9) and (10) are transformed as

$$\begin{aligned}&\langle \widetilde{\psi}^{(A)}_{\rm g.s.} | -\boldsymbol{d}_{\rm nucl}\cdot \boldsymbol{E}_{\rm ext} | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &\quad = i\langle \widetilde{\psi}^{(A)}_{\rm g.s.} | [U_{\rm nucl}, -\boldsymbol{d}_{\rm atom}\cdot \boldsymbol{E}_{\rm ext}] | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle,\end{aligned} \tag{15}$$

and

$$\begin{aligned}&\langle \widetilde{\psi}^{(A)}_m | V^{(eN)}_{l=1} | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &\quad = i\langle \widetilde{\psi}^{(A)}_m | [U_{\rm nucl}, V^{(eN)}_{l=0}] | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &\quad = i\langle \widetilde{\psi}^{(A)}_m | [U_{\rm nucl}, H_e] | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &\qquad - i\langle \widetilde{\psi}^{(A)}_m | [U_{\rm nucl}, V^{(eN)}_{l=2} + V^{(eN)}_{l=4} + \cdots] | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle.\end{aligned} \tag{16}$$

The eN interactions of a pointlike nucleus consist of the $l = 0$, 1 components, which are explicitly given by

$$V^{(eN)}_{l=0} = -Ze^2 \sum_{i=1}^{Z} \frac{1}{r'_i}, \tag{17}$$

$$V^{(eN)}_{l=1} = e\boldsymbol{d}_{\rm nucl}\cdot \sum_{i=1}^{Z} \boldsymbol{\nabla}'_i \frac{1}{r'_i}. \tag{18}$$

The last equality in Eq. (16) follows from the fact that the operator $U_{\rm nucl}$ commutes with the electron kinetic term T_e and the interactions between electrons $V^{(ee)}$.

Although the same transformations are realized even if one adopts

$$U'_{\rm nucl} = i\frac{1}{Ze}\boldsymbol{d}_{\rm nucl}\cdot \sum_{i=1}^{Z} \boldsymbol{\nabla}'_i \tag{19}$$

instead of $U_{\rm nucl}$, the resulting nuclear moment is a more complicated two-body operator than the Schiff moment (24).

Using the transformations (15) and (16), the third-order effect (10) is transformed as

$$\begin{aligned}&\Delta E_3(\bar{g}_{\pi NN}, q_N, q_e) \\ &\quad = -\Delta E_2(\bar{g}_{\pi NN}, q_N) + \sum_m \frac{1}{E^{(e)}_{\rm g.s.} - E^{(e)}_m} \\ &\qquad \times [\langle \psi^{(e)}_{\rm g.s.} | -\boldsymbol{d}_{\rm atom}\cdot \boldsymbol{E}_{\rm ext} | \psi^{(e)}_m\rangle \langle \psi^{(e)}_m | V_{\text{NSM-1}} | \psi^{(e)}_{\rm g.s.}\rangle + \text{c.c.}],\end{aligned} \tag{20}$$

where the first term implies the screening of the nuclear EDM. The remaining terms caused by the finite-size effect can be nonzero in the "pointlike nucleus limit," where

$$\nabla'^2_i \frac{1}{r'_i}\bigg|_{R\to 0} = -4\pi\delta(\boldsymbol{r}'_i). \tag{21}$$

Here R is the nuclear radius.

Considering $l \leqslant 3$, one obtains

$$\begin{aligned}\langle \psi^{(e)}_m | V_{\text{NSM-1}} | \psi^{(e)}_{\rm g.s.}\rangle &= \langle \widetilde{\psi}^{(A)}_m | V^{(eN)}_{l=3} | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &\quad - i\langle \widetilde{\psi}^{(A)}_m | [U_{\rm nucl}, V^{(eN)}_{l=2}] | \widetilde{\psi}^{(A)}_{\rm g.s.}\rangle \\ &= \langle \psi^{(e)}_m | -4\pi e \sum_{i=1}^{Z} \langle \boldsymbol{S}_1\rangle \cdot \boldsymbol{\nabla}'_i \delta(\boldsymbol{r}'_i) | \psi^{(e)}_{\rm g.s.}\rangle,\end{aligned} \tag{22}$$

where the nuclear part is separated from the electron part as explained in Appendix A. The expectation value of the nuclear Schiff moment is given by

$$\langle \boldsymbol{S}_1 \rangle = \sum_n \frac{1}{E_{\text{g.s.}}^{(N)} - E_n^{(N)}} \langle \psi_{\text{g.s.}}^{(N)} | \boldsymbol{S}_1 | \psi_n^{(N)} \rangle \langle \psi_n^{(N)} | \widetilde{V}_{\pi NN} | \psi_{\text{g.s.}}^{(N)} \rangle + \text{c.c.} \tag{23}$$

The explicit form of the Schiff moment operator is

$$S_{1,k} = \frac{e}{10} \sum_{a=1}^{Z} \left[r_a^2 r_{a,k} - \frac{5}{3} r_{a,k} \langle r^2 \rangle_{\text{ch}} - \frac{4}{3} r_{a,j} \langle Q_{jk} \rangle_{\text{ch}} \right], \tag{24}$$

where the charge mean values are defined by

$$\langle r^2 \rangle_{\text{ch}} = \frac{1}{Z} \sum_{a=1}^{Z} \langle \psi_{\text{g.s.}}^{(N)} | r_a^2 | \psi_{\text{g.s.}}^{(N)} \rangle, \tag{25}$$

$$\langle Q_{jk} \rangle_{\text{ch}} = \frac{1}{Z} \sum_{a=1}^{Z} \langle \psi_{\text{g.s.}}^{(N)} | Q_{a,jk} | \psi_{\text{g.s.}}^{(N)} \rangle, \tag{26}$$

and

$$Q_a^{(2)} = \sqrt{\frac{3}{2}} [\boldsymbol{r}_a \otimes \boldsymbol{r}_a]^{(2)} \tag{27}$$

is the quadrupole moment of proton. Since the P, T-odd πNN interaction is scalar, only the z-component S_z can have nonzero values. The third term of the Schiff moment operator (24) must vanish in spin $\frac{1}{2}$ nuclei including ^{199}Hg.

In conclusion of this section, the leading-order contribution from the P, T-odd πNN interaction is given by

$$\begin{aligned} &\Delta E_2(\bar{g}_{\pi NN}, q_N) + \Delta E_3(\bar{g}_{\pi NN}, q_N, q_e) \\ &= \sum_m \frac{1}{E_{\text{g.s.}}^{(e)} - E_m^{(e)}} \\ &\times \langle \psi_{\text{g.s.}}^{(e)} | -\boldsymbol{d}_{\text{atom}} \cdot \boldsymbol{E}_{\text{ext}} | \psi_m^{(e)} \rangle \langle \psi_m^{(e)} | V_{\text{NSM-1}} | \psi_{\text{g.s.}}^{(e)} \rangle \\ &+ \text{c.c.} \end{aligned} \tag{28}$$

This result implies that the interaction of the Schiff moment with the electrons denoted by $V_{\text{NSM-1}}$ induces the atomic EDM that survives the screening. The third-order process is illustrated in Fig. 2.

The Schiff moments S_1 of actinide nuclei would be enhanced thanks to octupole correlations and the parity doubling of the ground states [8]. It is expected from recent nuclear many-body calculations [9–12] that the Schiff moment of ^{225}Ra is greater than that of ^{199}Hg by orders of magnitude, although the uncertainty is still large.

II. NUCLEON EDM

There are several attempts to identify the leading-order contribution from the intrinsic EDMs of nucleons to the atomic EDM. In particular, the Schiff moment of ^{199}Hg that originates from the nucleon EDM was computed in the random-phase approximation [13]. Using their result, an upper bound on the neutron EDM was evaluated from the experimental limit on the atomic EDM as $d_n < 1.6 \times 10^{-26}\ e\,$cm [14]. This constraint is competitive with a recent direct measurement $d_n < 1.8 \times 10^{-26}\ e\,$cm [15]. On the other hand, it was claimed that the nucleon EDMs in a neutral atom are completely screened [16,17]. In this section, I demonstrate that the screening of the nucleon EDMs is incomplete in the pointlike nucleus limit.

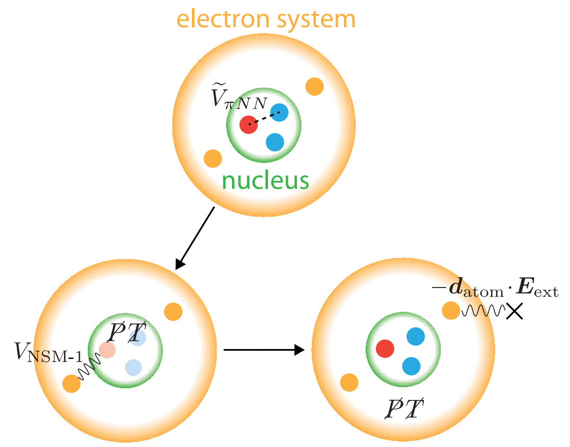

FIG. 2. Schematic illustration of how the P, T-odd πNN interaction $\widetilde{V}_{\pi NN}$ induces the atomic EDM $\boldsymbol{d}_{\text{atom}}$. The P, T-odd πNN interaction induces the nuclear Schiff moment as well as the nuclear EDM. The interaction of the nuclear Schiff moment with the electrons $V_{\text{NSM-1}}$ violates P and T symmetries both in the nucleus and in the electron system. Finally, the P, T violation in the electron system generates the atomic EDM.

Figure 3(a) represents the direct coupling of the nucleon EDMs to the external electric field. This first-order contribution is given by

$$\Delta E_1(d_N) = \sum_{a=1}^{A} \langle \psi_{\text{g.s.}}^{(N)} | -\boldsymbol{d}_a \cdot \boldsymbol{E}_{\text{ext}} | \psi_{\text{g.s.}}^{(N)} \rangle, \tag{29}$$

where $\boldsymbol{d}_a$ denotes the nucleon EDMs.

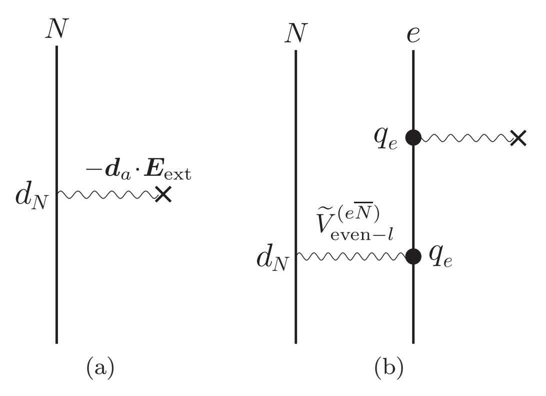

FIG. 3. The external interaction of (a) the nucleon EDMs and (b) the atomic EDM induced by the interaction of the nucleon EDMs with the electrons. The vertices with d_N indicate the interactions of the nucleon EDM.

The internal interaction of the nucleon EDMs with the electrons

$$\widetilde{V}^{(e\bar{N})} = e\sum_{i=1}^{Z}\sum_{a=1}^{A} \boldsymbol{d}_a\cdot\nabla'_i \frac{1}{|\boldsymbol{r}'_i - \boldsymbol{r}_a|} \tag{30}$$

violate P and T symmetries in the electron system. Thus, the induced atomic EDM contributes to the energy shift in second-order perturbation as

$$\begin{aligned}&\Delta E_2(d_N, q_e)\\ &= \sum_m \frac{1}{E^{(e)}_{\text{g.s.}} - E^{(e)}_m}\\ &\times \langle\psi^{(A)}_{\text{g.s.}}| - \boldsymbol{d}_{\text{atom}}\cdot\boldsymbol{E}_{\text{ext}}|\psi^{(A)}_m\rangle\langle\psi^{(A)}_m|\widetilde{V}^{(e\bar{N})}_{\text{even}-l}|\psi^{(A)}_{\text{g.s.}}\rangle\\ &+ \text{c.c.}\end{aligned} \tag{31}$$

This process is represented in Fig. 3(b). The internal interaction (30), which is expanded for $r'_i > r_a$ as

$$\boldsymbol{d}_a\cdot\nabla'_i \frac{1}{|\boldsymbol{r}'_i - \boldsymbol{r}_a|} = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!}(\boldsymbol{r}_a\cdot\nabla'_i)^l \boldsymbol{d}_a\cdot\nabla'_i \frac{1}{r'_i}, \tag{32}$$

is restricted to the even-l components because P and T symmetries are not violated in the nuclear system.

The ground state and excited states of the atomic Hamiltonian H_{atom} without P, T-odd interactions are expressed as

$$|\psi^{(A)}_{\text{g.s.}}\rangle = |\psi^{(N)}_{\text{g.s.}}\rangle \otimes |\psi^{(e)}_{\text{g.s.}}\rangle, \tag{33}$$

$$|\psi^{(A)}_m\rangle = |\psi^{(N)}_{\text{g.s.}}\rangle \otimes |\psi^{(e)}_m\rangle, \tag{34}$$

respectively.

I introduce a Hermitian operator

$$U_N = i\frac{1}{Ze}\sum_{i=1}^{Z}\sum_{a=1}^{A}\langle\boldsymbol{d}_a\rangle\cdot\nabla'_i, \tag{35}$$

where in contrast to $\langle\boldsymbol{d}_{\text{nucl}}\rangle$ in Eq. (13),

$$\langle\boldsymbol{d}_a\rangle = \langle\psi^{(N)}_{\text{g.s.}}|\boldsymbol{d}_a|\psi^{(N)}_{\text{g.s.}}\rangle \tag{36}$$

is the expectation value in the ground state of H_{nucl} conserving P and T symmetries. The external interaction of the nucleon EDMs (29) is transformed as

$$\begin{aligned}&\sum_{a=1}^{A}\langle\psi^{(A)}_{\text{g.s.}}| - \boldsymbol{d}_a\cdot\boldsymbol{E}_{\text{ext}}|\psi^{(A)}_{\text{g.s.}}\rangle\\ &\quad = i\langle\psi^{(A)}_{\text{g.s.}}|[U_N, -\boldsymbol{d}_{\text{atom}}\cdot\boldsymbol{E}_{\text{ext}}]|\psi^{(A)}_{\text{g.s.}}\rangle.\end{aligned} \tag{37}$$

The $l = 0$ component of the internal interaction (30), which is explicitly given by

$$\widetilde{V}^{(e\bar{N})}_{l=0} = e\sum_{i=1}^{Z}\sum_{a=1}^{A}\boldsymbol{d}_a\cdot\nabla'_i\frac{1}{r'_i}, \tag{38}$$

is transformed as

$$\begin{aligned}&\langle\psi^{(A)}_m|\widetilde{V}^{(e\bar{N})}_{l=0}|\psi^{(A)}_{\text{g.s.}}\rangle = i\langle\psi^{(A)}_m|[U_N, V^{(eN)}_{l=0}]|\psi^{(A)}_{\text{g.s.}}\rangle\\ &= i\langle\psi^{(A)}_m|[U_N, H_e]|\psi^{(A)}_{\text{g.s.}}\rangle\\ &\quad - i\langle\psi^{(A)}_m|[U_N, V^{(eN)}_{l=2} + V^{(eN)}_{l=4} + \cdots]|\psi^{(A)}_{\text{g.s.}}\rangle,\end{aligned} \tag{39}$$

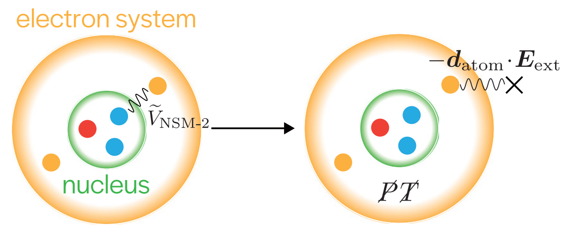

FIG. 4. The leading-order contribution of the nucleon EDMs to the atomic EDM $\boldsymbol{d}_{\text{atom}}$. The P, T-odd eN interaction due to the finite-size effect $\widetilde{V}_{\text{NSM-2}}$ appears in the pointlike nucleus limit.

where $[U_N, T_e] = 0$ and $[U_N, V^{(ee)}] = 0$ are used. Substituting (39) into (31), one can find

$$\begin{aligned}&\Delta E_1(d_N) + \Delta E_2(d_N, q_e)\\ &= \sum_m \frac{1}{E^{(e)}_{\text{g.s.}} - E^{(e)}_m}\\ &\times \langle\psi^{(e)}_{\text{g.s.}}| - \boldsymbol{d}_{\text{atom}}\cdot\boldsymbol{E}_{\text{ext}}|\psi^{(e)}_m\rangle\langle\psi^{(e)}_m|\widetilde{V}_{\text{NSM-2}}|\psi^{(e)}_{\text{g.s.}}\rangle\\ &+ \text{c.c.}\end{aligned} \tag{40}$$

The right-hand side vanishes for a pointlike nucleus, where the eN interactions in Eqs. (6) and (30) are restricted to $l \leqslant 1$. The complete screening of a pointlike nucleus is valid even if the nucleons are relativistic [18].

The remaining second-order process in Eq. (40) is illustrated in Fig. 4. In the pointlike nucleus limit, the finite-size effect is given up to $l = 2$ by

$$\begin{aligned}&\langle\psi^{(e)}_m|\widetilde{V}_{\text{NSM-2}}|\psi^{(e)}_{\text{g.s.}}\rangle\\ &= \langle\psi^{(A)}_m|\widetilde{V}^{(e\bar{N})}_{l=2}|\psi^{(A)}_{\text{g.s.}}\rangle - i\langle\psi^{(A)}_m|[U_N, V^{(eN)}_{l=2}]|\psi^{(A)}_{\text{g.s.}}\rangle\\ &= \langle\psi^{(e)}_m| - 4\pi e\sum_{i=1}^{Z}\langle\boldsymbol{S}_2\rangle\cdot\nabla'_i\delta(\boldsymbol{r}'_i)|\psi^{(e)}_{\text{g.s.}}\rangle,\end{aligned} \tag{41}$$

as derived in Appendix B. The nuclear moment $\boldsymbol{S}_2$ is also called the Schiff moment, and given by

$$\begin{aligned}S_{2,k} &= \frac{1}{6}\sum_{a=1}^{A} d_{a,k}\left(r_a^2 - \langle r^2\rangle_{\text{ch}}\right)\\ &\quad + \frac{2}{15}\sum_{a=1}^{A} d_{a,j}(Q_{a,jk} - \langle Q_{jk}\rangle_{\text{ch}}).\end{aligned} \tag{42}$$

Using the independent particle model (IPM) [12], one obtains

$$S_2(^{199}\text{Hg}) = 2.8 d_n\ (\text{fm}^2), \tag{43}$$

which is consistent with the previous evaluation [8] $S_2 \simeq 2.2 d_n$ (fm^2). This quantity was calculated as $S_2 = (1.895 \pm 0.035) d_n$ (fm^2) in the random-phase approximation [13].

III. NEXT-TO-LEADING-ORDER CONTRIBUTION OF NUCLEON EDM

As discussed in Sec. II, the atomic EDM is sensitive to the Schiff moment S_2, which stems from the screening effect

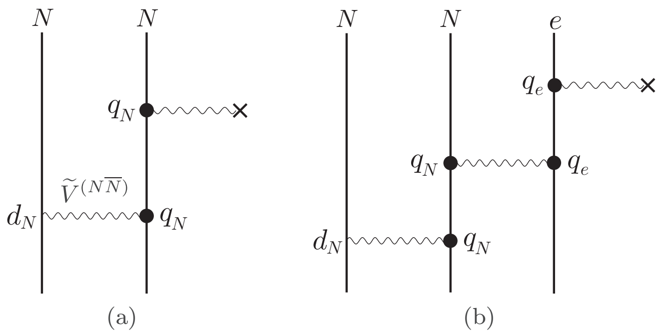

FIG. 5. (a) The second-order and (b) the third-order contributions of the interactions between the nucleon EDMs and the protons.

of the nucleon EDMs themselves. In addition to the nucleon EDMs, the nuclear EDM is independently coupled to the external electric field as shown in Eq. (2). The nuclear EDM is induced not only by the πNN interaction but also by the interaction between the nucleon EDMs and the protons

$$\widetilde{V}^{(N\overline{N})} = ed_p \sum_{a\neq b}^{Z} \frac{\boldsymbol{\sigma}_a\cdot(\boldsymbol{r}_b-\boldsymbol{r}_a)}{|\boldsymbol{r}_b-\boldsymbol{r}_a|^3} + ed_n \sum_{b=1}^{Z}\sum_{a=1}^{N} \frac{\boldsymbol{\sigma}_a\cdot(\boldsymbol{r}_b-\boldsymbol{r}_a)}{|\boldsymbol{r}_b-\boldsymbol{r}_a|^3}. \tag{44}$$

A similar argument as in Sec. I shows that this contribution represented in Fig. 5(a) is screened by the third-order processes represented in Fig. 5(b). The finite-size effect leads to the next-to-leading-order contribution of the nucleon EDM to the Schiff moment

$$\langle \boldsymbol{S}_3\rangle = \sum_n \frac{1}{E^{(N)}_{\text{g.s.}}-E^{(N)}_n} \times \langle\psi^{(N)}_{\text{g.s.}}|\boldsymbol{S}_3|\psi^{(N)}_n\rangle\langle\psi^{(N)}_n|\widetilde{V}^{(N\overline{N})}|\psi^{(N)}_{\text{g.s.}}\rangle + \text{c.c.}, \tag{45}$$

where the operator $\boldsymbol{S}_3$ is the same as $\boldsymbol{S}_1$. This correction is evaluated as

$$S_3(^{199}\text{Hg}) = -0.15 d_n\ (\text{fm}^2) \tag{46}$$

in the IPM.

IV. CONCLUSION

I have examined the screening of the intrinsic EDMs of nucleons and the nuclear EDM in a neutral atom. In the point-like nucleus limit, the Schiff moment of a finite-size nucleus induces the atomic EDM that circumvents the screening. The total Schiff moment is given by

$$S = S_1 + S_2 + S_3, \tag{47}$$

where S_2 and S_3 are due to the nucleon EDM. The nucleon EDM contributions provide constraints on the short-range component, whereas the P, T-odd πNN interaction contributes to the nucleon EDM in the leading-order chiral perturbation theory [2,19,20].

The leading-order contribution S_2 stems from the screening of the nucleon EDMs themselves. The nuclear EDM is induced by the interaction of the nucleon EDMs with the protons as well as the P, T-odd πNN interaction. The screening of the nuclear EDM gives rise to the next-to-leading-order contribution to the Schiff moment S_3. This correction to the Schiff moment of ^{199}Hg is of the order of 5% in the IPM. Here, nuclear octupole correlations would enhance S_3 as well as S_1, which is induced by the πNN interaction, by orders of magnitude. Consequently, the dependence of the Schiff moment on the nucleon EDM can be dominated by S_3 rather than S_2 in octupole deformed nuclei.

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APPENDIX A: SCHIFF MOMENT DUE TO THE *P*, *T*-ODD πNN INTERACTION

The Schiff moment operator $\boldsymbol{S}_1$ is defined by the matrix elements of the remaining eN interaction

$$V^{(eN)}_{l=3} - i\big[U_{\text{nucl}}, V^{(eN)}_{l=2}\big] = \frac{1}{6}e^2\sum_{i=1}^{Z}\sum_{a=1}^{Z}(\boldsymbol{r}_a\cdot\nabla'_i)^3\frac{1}{r'_i} - \frac{e}{2Z}\sum_{i=1}^{Z}\sum_{a=1}^{Z}(\boldsymbol{r}_a\cdot\nabla'_i)^2\langle\boldsymbol{d}_{\text{nucl}}\rangle\cdot\nabla'_i\frac{1}{r'_i}. \tag{A1}$$

The nuclear part can be separated as

$$(\boldsymbol{r}_a\cdot\nabla'_i)^3 = \frac{3}{5}r_a^2(\boldsymbol{r}_a\cdot\nabla'_i)\nabla'^2_i + \frac{2}{5}Q^{(3)}_a\cdot\nabla'^{(3)}_i, \tag{A2}$$

and

$$(\boldsymbol{r}_a\cdot\nabla'_i)^2\langle\boldsymbol{d}_{\text{nucl}}\rangle\cdot\nabla'_i = \frac{1}{3}r_a^2\langle\boldsymbol{d}_{\text{nucl}}\rangle\cdot\nabla'_i\nabla'^2_i - \frac{2}{3}\sqrt{\frac{2}{5}}\big[\langle\boldsymbol{d}_{\text{nucl}}\rangle\otimes Q^{(2)}_a\big]^{(1)}\cdot\nabla'_i\nabla'^2_i - \frac{2}{\sqrt{15}}\big[\langle\boldsymbol{d}_{\text{nucl}}\rangle\otimes Q^{(2)}_a\big]^{(3)}\cdot\nabla'^{(3)}_i, \tag{A3}$$

where

$$Q^{(3)}_a = \sqrt{\frac{5}{2}}[[\boldsymbol{r}_a\otimes\boldsymbol{r}_a]^{(2)}\otimes\boldsymbol{r}_a]^{(3)} \tag{A4}$$

is the nuclear octupole moment and

$$\nabla'^{(3)}_i = \sqrt{\frac{5}{2}}[[\nabla'_i\otimes\nabla'_i]^{(2)}\otimes\nabla'_i]^{(3)} \tag{A5}$$

is a rank 3 operator of electron. Since the last terms in Eqs. (A2) and (A3) can be omitted [8], Eq. (A1) is rewritten as

$$V_{l=3}^{(eN)} - i\left[U_{\rm nucl}, V_{l=2}^{(eN)}\right] = \frac{1}{10}e\sum_{i=1}^{Z}\sum_{a=1}^{Z}\left[er_a^2\boldsymbol{r}_a - \frac{5}{3Z}r_a^2\langle\boldsymbol{d}_{\rm nucl}\rangle + \frac{2}{3Z}\sqrt{10}\left[\langle\boldsymbol{d}_{\rm nucl}\rangle\otimes Q_a^{(2)}\right]^{(1)}\right]\cdot\nabla_i'\nabla_i'^2\frac{1}{r_i'}. \quad \text{(A6)}$$

In the pointlike nucleus limit, $R \to 0$, one then obtain the Schiff moment interaction $V_{\text{NSM-1}}$ in Eq. (22).

APPENDIX B: LEADING-ORDER CONTRIBUTION OF NUCLEON EDM

The P, T-odd interactions between the nucleon EDMs and the electrons in Eq. (40) are written as

$$\widetilde{V}_{l=2}^{(e\overline{N})} - i\left[U_N, V_{l=2}^{(eN)}\right] = \frac{1}{2}e\sum_{i=1}^{Z}\sum_{a=1}^{A}(\boldsymbol{r}_a\cdot\nabla_i')^2\boldsymbol{d}_a\cdot\nabla_i'\frac{1}{r_i'} - \frac{e}{2}\sum_{i=1}^{Z}\sum_{a=1}^{Z}(\boldsymbol{r}_a\cdot\nabla_i')^2\langle\boldsymbol{d}_N\rangle_{\rm ch}\cdot\nabla'\frac{1}{r_i'}, \quad \text{(B1)}$$

where

$$\langle\boldsymbol{d}_N\rangle_{\rm ch} = \frac{1}{Z}\sum_{a=1}^{A}\langle\boldsymbol{d}_a\rangle. \quad \text{(B2)}$$

The nuclear part can be separated by using Eq. (A3). In the pointlike nucleus limit, one obtains the Schiff moment interaction $\widetilde{V}_{\text{NSM-2}}$ in Eq. (41).

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