Spherical-deformed mixing in ^{94}Zr

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I have applied a simple two-state mixing model to $E2$ strengths from a recent ^{94}Zr$(n, n'\gamma)$ experiment in order to obtain the mixing amplitudes between spherical and deformed basis states in ^{94}Zr. The fits also provide the transition matrix elements connecting the basis states.

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I. INTRODUCTION

Recently, Chakraborty *et al.* [1] used the ^{94}Zr$(n, n'\gamma)$ reaction to measure lifetimes, and hence $E2$ strengths, for low-lying levels in ^{94}Zr. They discussed their results in terms of subshell effects in shape coexistence. A recent review discussed coexistence in many different nuclei [2]. Shape coexistence has been suggested in ^{96}Zr [3,4], ^{98}Zr [2], and ^{100}Zr [5]. I examined mixing in ^{96}Zr [6]. Chakraborty *et al.* stressed the importance of *direct* evidence in terms of $E2$ transition strengths, rather than *indirect* evidence involving energy patterns and/or $E0$ transition strengths. With their work, all four of the $0 \leftrightarrow 2$ strengths are now known in ^{94}Zr [1,7], and can be used in a simple mixing model. Earlier reports that $B(E2; 2_2 \rightarrow 0_1)$ is significantly larger than $B(E2; 2_1 \rightarrow 0_1)$ [8,9] turned out to be erroneous [1,10–13]. All four strengths are listed in Table I. These are suggested to involve mixing between spherical and deformed basis states. In the present work, I examine that idea quantitatively with a simple two-state mixing model [14,15] that has been successful in other nuclei, including ^{96}Zr [6].

II. CALCULATIONS AND RESULTS

For the first two 0^+ and 2^+ states of ^{94}Zr, I write

$$\Psi(0_1) = a\Phi(0_g) + b\Phi(0_e), \quad \Psi(0_2) = -b\Phi(0_g) + a\Phi(0_e),$$

$$\Psi(2_1) = A\Phi(2_g) + B\Phi(2_e), \quad \Psi(2_2) = -B\Phi(2_g) + A\Phi(2_e);$$

and

$$M_g = \langle 0_g \parallel M(E2) \parallel 2_g \rangle, \quad M_e = \langle 0_e \parallel M(E2) \parallel 2_e \rangle.$$

It is convenient to think of basis states g and e as spherical and deformed, respectively, but nothing about their structure is assumed at the outset, except that I assume that the $E2$ operator does not connect g and e. Other properties of the basis states emerge from the mixing.

It is then a simple matter to construct equations for the $E2$ transition matrix elements, for example, $M_0 = aAM_g + bBM_e$, and likewise for the others. Some sign ambiguities can arise when taking square roots of $B(E2)$ to get $M(E2)$. In my phase convention, M_0 and M_3 are positive, whereas M_1 and M_2 can have either sign, because they involve destructive interference. However, if M_e is significantly larger than M_g, all four matrix elements will be positive.

Whenever all four of the relevant matrix elements are available, the four parameters (0^+ mixing, 2^+ mixing, M_g, and M_e) can be uniquely determined. Results of the procedure for ^{94}Zr are listed in Table II.

As expected, the $E2$ strength is much larger in the excited basis band than in the ground basis band. I note that the ratio $M_g/M_e = 0.249(34)$ is remarkably close to the value of $M_{\rm sph}/M_{\rm def} = 0.231$ that was successful earlier in ^{96}Zr [6]. The absolute values indicate that g and e are both slightly less collective in ^{94}Zr than in ^{96}Zr.

The analysis of $E2$ strengths in ^{96}Zr obtained a 0^+ mixing amplitude of 0.128(18) [6]. With a generalized coexistence model, analysis of $2n$ transfer strengths determined 0^+ mixing amplitudes for four even Zr nuclei in terms of a dimensionless parameter R [16]. From that analysis, an amplitude of 0.128(18) in ^{96}Zr corresponds (Fig. 5 of [16]) to an amplitude of 0.477(14) in ^{94}Zr, only a 1.4σ difference from the value of 0.519(26) obtained above.

I turn now to the $2 \leftrightarrow 4$ transitions. As above, I write

$$\Psi(4_1) = C\Phi(4_g) + D\Phi(4_e), \quad \Psi(4_2) = -D\Phi(4_g) + C\Phi(4_e),$$

and

$$M'_g = \langle 4_g \parallel M(E2) \parallel 2_g \rangle, \quad M'_e = \langle 4_e \parallel M(E2) \parallel 2_e \rangle.$$

Here, only three of the four relevant matrix elements are known (Table III), but with the 2^+ mixing already established, there are only three parameters (4^+ mixing, M'_g, and M'_e) to be determined. It turns out that the uncertainties in M'_2 and M'_3 are so large that a range of solutions exists, covering the range $0 < M'_g/M'_e < 0.16$, with D in the range $0.33 > D > 0$. Of special interest is the solution with $D \sim 0$, because it has $M'_e = 1.6M_e$, which is the condition expected for a $K = 0$ rotational band. Coincidently, this solution corresponds to $M'_g \sim M_g$. This solution is listed in Table IV. Calculated values of the relevant matrix elements are listed in the last column of Table III. Note that the prediction from this solution is that

TABLE I. $E2$ strengths of $0 \leftrightarrow 2$ transitions in ^{94}Zr.

Label	Initial	Final	$B(E2)$ (W.u.)[a]	$M(E2)$(W.u.)$^{1/2}$[b]
M_0	2_1	0_1	4.9(11)	4.95(56)
M_1	0_2	2_1	9.3(4)	±3.05(7)
M_2	2_2	0_1	3.9(3)	±4.42(17)
M_3	2_2	0_2	19(2)	9.75(51)

[a]References [1,7].
[b]$M^2(E2) = (2J_i + 1)B(E2; i \rightarrow f)$.

TABLE II. Results of mixing for $0 \leftrightarrow 2$ transitions in ^{94}Zr.

Quantity	Value
b	0.519(26)
B	0.438(7)
M_g	2.94(38)(W.u.)$^{1/2}$
M_e	11.8(5)(W.u.)$^{1/2}$

TABLE III. $E2$ strengths of $4 \leftrightarrow 2$ transitions in ^{94}Zr.

				$M(E2)$(W.u.)$^{1/2}$	
Label	Initial	Final	Expt. $B(E2)$ (W.u.)[a]	Expt.	Fit
M'_0	4_1^+	2_1^+	0.880(23)	2.81(4)	2.81
M'_1	2_2^+	4_1^+	Unknown		−0.87
M'_2	4_2^+	2_1^+	13^{+4}_{-7}	$10.8^{+1.7}_{-2.9}$	8.4
M'_3	4_2^+	2_2^+	34^{+10}_{-17}	$17.5^{+2.6}_{-4.4}$	17.5

[a]Reference [1].

TABLE IV. Results of one solution for $4 \leftrightarrow 2$ transitions in ^{94}Zr.

Quantity	Value
D	0.023[a]
B	0.438
M'_g	2.91(W.u.)$^{1/2}$
M_e	19.4(W.u.)$^{1/2}$

[a]See text.

TABLE V. Mixing matrix elements in ^{94,96}Zr.

	V (keV)	
J	^{94}Zr	^{96}Zr
0	576(52)	199(28)
2	296(10)	85(14)
4	0–270[a]	

[a]Large uncertainties in two of the $2 \leftrightarrow 4$ strengths cause large uncertainty in 4^+ mixing. Any value of D in the range $0 < D < 0.33$ reproduces the data.

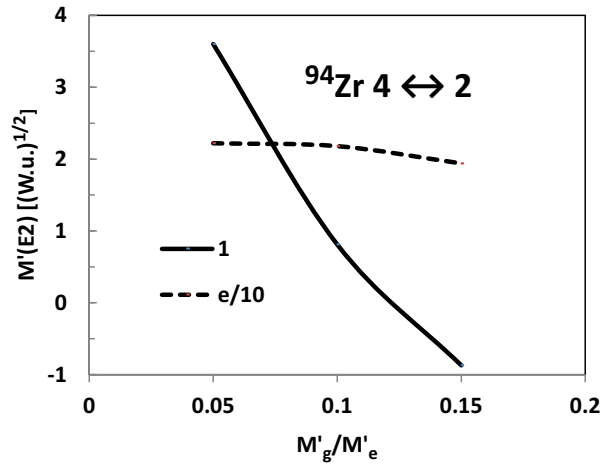

FIG. 1. Plotted vs M'_g/M'_e are values of M'_1 (solid curve) and $M'_e/10$ (dashed curve) over most of the allowed range of solutions.

the unknown M'_1 is small and negative. Plotted in Fig. 1 are values of M'_1 (solid) and M'_e (dashed) over most of the continuous range of solutions. For this entire range, the computed values of M'_2 and M'_3 agree with experimental results within their uncertainties. Note that M'_1 changes considerably over this range, but M'_e changes very little. If M'_1 could ever be measured, the range of solutions would be considerably narrowed.

With a knowledge of the mixing amplitudes and the energies of the various states, the potential matrix elements responsible for the mixing can be determined, as, for example $V_0 = ab\Delta E_0$. These are listed in Table V. They are seen to decrease rapidly as J increases. Combining the 0^+ mixing in ^{96}Zr [6] with the $2n$ transfer analysis for four Zr nuclei [16] produces $V_0 = 545(25)$ keV in ^{94}Zr, consistent with the present value. By comparison, V was found to be about 560 keV for all of $J = 0$, 2, and 4 in ^{76}Se [17]. In a simple model, Heyde *et al.* concluded that the matrix element that is most important for mixing the first two 0^+ states in $^{90-96}$Zr, namely $\langle (p_{1/2})^2 \parallel V \parallel (g_{9/2})^2 \rangle$ in the proton space, is approximately constant in those nuclei at about 800 keV [18]. Gloeckner and Serduke [19] found this matrix element to be 853 keV. Of course, these two 0^+ states contain other components, and that configuration mixing will tend to reduce the value of V.

Previous results for ^{96}Zr [16] are also listed in Table V. There is no expectation that V will be the same for different J when the mixing is between spherical and deformed basis states. Both the $E2$ and $2n$-transfer analyses demonstrated that the mixing in ^{94}Zr is significantly larger than in ^{96}Zr. However, the change from $J = 0$ to 2 is about the same in the two nuclei: the ratio V_2/V_0 is 0.43(9) in ^{96}Zr and 0.51(5) in ^{94}Zr.

Whenever the mixing is between members of two deformed rotational bands, it is more likely that V will be approximately independent of J. That is the case, for example, for ^{76}Se mentioned above and for ^{152}Sm—where V is about 310 [20] or 325 [21] keV for $J = 0$, 2, and 4.

III. SUMMARY

Using $E2$ strengths from a recent ^{94}Zr$(n, n'\gamma)$ experiment, I have applied a simple two-state mixing model in order to obtain the mixing amplitudes between spherical and deformed basis states. The fits also provide the transition matrix elements connecting the basis states. The 0^+ mixing obtained here is remarkably close to that obtained by combining an $E2$ analysis in ^{96}Zr with an analysis of $2n$ transfer among the even Zr nuclei.

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