Level structures of the transfermium odd-odd nucleus ^{252}Md

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Low-energy level scheme of doubly odd transfermium nucleus $^{252}_{101}$Md$_{151}$ is constructed using our well-tested two-quasiparticle rotor model with explicit inclusion of residual n-p interaction contribution. Energy levels thus deduced are examined critically to assign spin-parity $J^{\pi}K$ and two-quasiparticle (2qp) configurations to the ^{252}Md ground state and other energy levels with $E_x < 400$ keV. Our low-energy level structures are then examined to predict a high-spin ($J^{\pi} = 8^{+}$) long-lived isomer with $E_x \approx 125$ keV and possibly also a lower-lying short-lived isomeric state. Comparison with ^{256}Lr (α) populated ^{252}Md excited states leads to $J^{\pi}K$ and 2qp assignment to each of these experimentally postulated levels. Further, the five γ's ascribed to ^{252}Md from α-γ coincidence experiments are placed in our evaluated levels corresponding to $\Delta I = 2$ transition in each case. Additionally, exploration of the upward extended radioactive connection of ^{232}Th heading the $A = 4n$ naturally occurring radioactive series to the heaviest known superheavy element with $A = 4n$, namely $^{272}_{111}$Rg$_{161}$, has led us to establish landmark position of ^{252}Md in this ^{272}Rg - ^{232}Th extended decay chain.

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I. INTRODUCTION

As a part of our ongoing investigations of level structures of odd-odd actinides [1,2], we have lately been focusing on heavy ($A > 250$) odd-odd nuclei. In this context we have earlier published results of our investigations on level structures in ^{250}Md [3], ^{252}Es [4], ^{254}Es [5], and ^{254}Md [6]. Within this framework, we present herein our analysis of low-lying structures in the odd-odd transfermium nucleus $^{252}_{101}$Md$_{151}$. This study is primarily motivated by five considerations. First, a number of experiments aimed at identifying and confirming the heaviest $A = 4n$ superheavy element (SHE) $^{272}_{111}$Rg$_{161}$ synthesized through cold fusion (CF) process have been reported from GSI, Germany [7,8], RIKEN, Japan [9], and Berkeley, USA [10]. In each of these studies, the SHE with $A = 4n$ ^{272}Rg decays through five genetically correlated α's to the transfermium odd-odd $A = 4n$ nucleus ^{252}Md, whereafter this α chain ends. By extending this decay chain downwards right up to the naturally occurring radioactive series (NORS) headed by $A = 4n$ ^{232}Th, we unravel the distinctive position of ^{252}Md in this process.

The other four questions addressed herein concern the identification and characterization of individual energy levels in ^{252}Md spectrum through our model calculations, taken together with the currently available experimental information. First therein, even though ^{252}Md was first identified in 1973 [11,12] and inferred a couple of years earlier in 1971 [13], no characterization of any of its levels has been reported so far [14,15]. Even the spin parity (J^{π}) of its ground state (gs) remains undetermined to date. We use our well-tested two-quasiparticle rotor model (TQRM) [1–6] to deduce the level energies and two-quasiparticle (2qp) configurations of low-lying spectra of this nucleus. Another point of interest in this context is the possible occurrence of a speculated [16,17] long-lived ^{252}Md isomer. Also ^{256}Lr α decay [14] is reported to populate six ^{252}Md levels with excitation energies $E_x <$ 370 keV. However, no characterization (J^{π} or 2qp configuration) for any of these six ^{252}Md levels has been suggested to this date [15]. Further, recent α-γ coincidence spectroscopy of n-deficient $_{103}$Lr isotopes [18] has ascribed five γ transitions with E_{γ} ranging from 84 keV to 190 keV in the ^{252}Md daughter nucleus. However, due to very large spread ΔE_{α} (up to 370 keV) in a specific α branch, placement of these γ's in the ^{252}Md level scheme is still an open question. We address each of these questions in the present report. Preliminary results of these investigations have earlier been published in a symposium proceeding [19].

Essentially we follow the procedure adopted in our recent investigation of ^{254}Md [6]. However, it turns out that the low-energy 2qp configuration space in the two cases is totally different. Whereas $N = 151$ isotope ^{252}Md involves only the 1qp neutron levels below the $N = 152$ deformed shell gap, the $N = 153$ isotope ^{254}Md includes neutron orbitals above the $N = 152$ shell gap. Hence no overlap in the 2qp domain occurs in the two cases.

In Sec. II, we examine the specific features of the extended decay path of the heaviest CF synthesized SHE with $A = 4n$, namely $^{272}_{111}$Rg, to establish the “landmark” role of ^{252}Md in this decay process. In Sec. III, we briefly outline our three-step TQRM. Therein successively we scan the available single-particle (1qp) configuration space for $N = 151$ isotones and $Z = 101$ isotopes, respectively, thence enumerate the physically admissible 2qp configurations in ^{252}Md and finally evaluate the bandhead energies of the 2qp bands within the specified energy range. In Sec. IV we critically examine our calculated level energies with a view to characterize ^{252}Md low-lying levels and to physically interpret results of ^{256}Lr (α) decay and also α-γ coincidence experiments. Summary and conclusions of our study are presented in Sec. V.

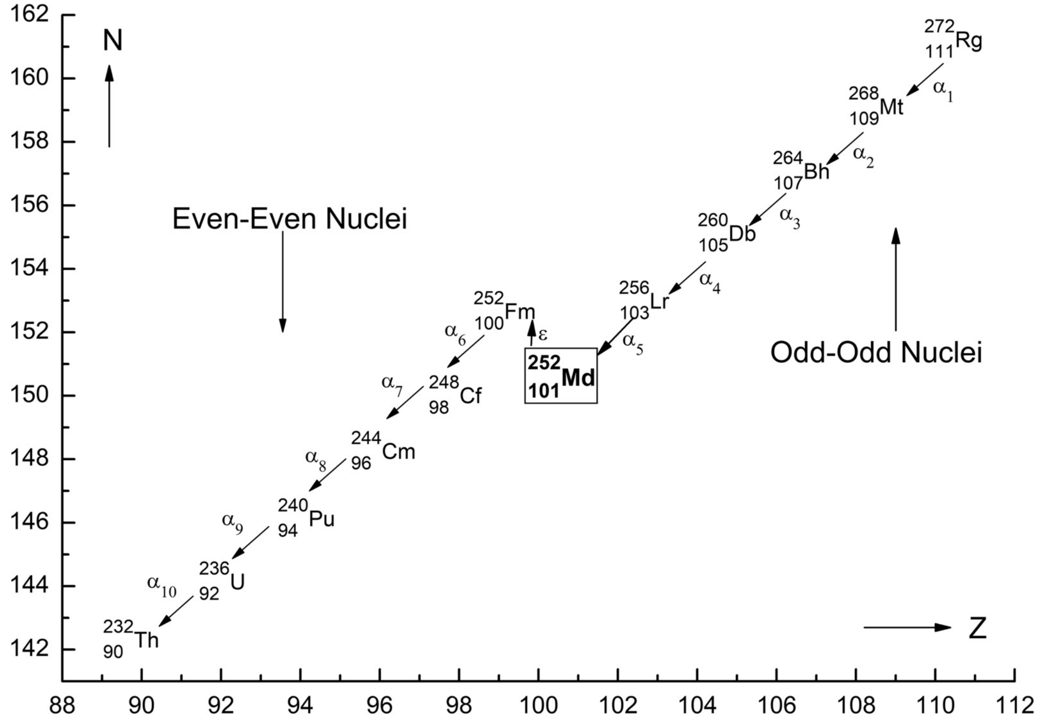

FIG. 1. Schematic plot of radioactive connection between head of $A = 4n$ NORS ^{232}Th and the heaviest CF synthesized SHE with $A = 4n$ ^{272}Rg. The plot establishes the role of ^{252}Md as the transition point between the odd-odd nuclei for $Z \geqslant 101$ and even-even nuclei for $Z < 101$.

II. ^{252}Md: A LANDMARK IN DECAY PATH OF 4n SHE

The heaviest CF synthesised SHE with $A = 4n$ presently known is $^{272}_{111}\mathrm{Rg}_{161}$. It was first identified in 1995 in ^{209}Bi(^{64}Ni, 1n) reaction in GSI, Germany [7] on the basis of three events attributed to genetically correlated sequential 5α decay chain originating from ^{272}Rg and ending in earlier known [12] ^{256}Lr(α) ^{252}Md reaction, as shown in the top right part of Fig. 1. The same experiment was repeated in GSI in 2002 [8], wherein three additional ^{272}Rg(5α) ^{252}Md decay chains were observed. An independent confirmation of this $Z = 111$ isotope production was provided by a long-duration (50 d) study of same CF reaction in RIKEN, Japan in 2004 [9]. Therein eight events corresponding to ^{272}Rg (5α) decay chain were observed. Final confirmation of ^{272}Rg identification in CF process came in 2004 from Berkeley, USA [10] in a different projectile-target (^{65}Cu - ^{208}Pb) combination experiment.

A common feature of all these four investigations from three different continents is that, in each of the 15 reported events, the ^{272}Rg decay chain of genetically correlated five α's ended at predominantly ϵ ($\leqslant$100%) decaying ^{252}Md and its long-lived (23.4 h) daughter ^{252}Fm [15]. Thus it can be confidently stated that ^{252}Md is an important milestone in the decay process of SHE with $A = 4n$.

Another significant facet of the specific location of ^{252}Md in SHE with $A = 4n$ decay chain is revealed when we seek to establish a connection of this decay chain with the $A = 4n$ NORS. The result of this exercise, based on information from current nuclear data sets [15], is plotted in the bottom left segment of Fig. 1. As seen therein, the end product of SHE with $A = 4n$ radioactive decay chain, namely ^{252}Fm, connects to the top nucleus of $A = 4n$ NORS, namely ^{232}Th, via a 5α decay chain. One significant difference between these two successive 5α chains is that the SHE (5α) chain consists of only odd-odd nuclei whereas the ^{252}Fm (5α) actinide chain has only even-even members. Considering all these features, as evident in Fig. 1 also, we conclude that ^{252}Md can be said to occupy landmark position in the ^{272}Rg - ^{232}Th extended radioactive decay path on the following counts:

(i) It marks the endpoint of genetically correlated α chain of SHE with $A = 4n$.
(ii) It is the only non-α decaying member of this extended decay chain.
(iii) It marks the boundary between the odd-odd nuclei for $Z \geqslant 101$ members and the even-even nuclei for $Z < 101$ members of this chain.

III. MODEL FORMULATION AND CALCULATION

In the independent particle formalism including pairing forces, specific nuclear spectrum originates from the valence (unpaired) nucleon(s). In case of deformed nuclei, particle states are defined by Nilsson orbitals with characteristic quantum number Ω, while the 2qp states of odd-odd nuclei are labeled by the band quantum number K. Thus the bandhead energies in latter case are, in principle, given by the expression [2,22]

$$E(K : \Omega_p, \Omega_n) = E_0 + E_p(\Omega_p) + E_n(\Omega_n) + E_{\mathrm{rot}} + \langle V_{pn} \rangle, \tag{1}$$

wherein E_p/E_n are the observed [15] excitation energies of the respective orbitals in neighboring odd mass isotope/isotone, E_{rot} is the correction for the zero point rotational energy, and $\langle V_{pn}\rangle$ is the contribution from the residual neutron-proton interaction. In the Bohr-Mottelson-Nilsson formalism for a single valence nucleon coupled to quadrupole deformed core (Rotor), the total angular momentum I is the sum of the rotational angular momentum R and the particle angular momentum J

$$\vec{I} = \vec{R} + \vec{J}. \tag{2}$$

For axially symmetric rotor, the leading term in the rotational Hamiltonian then is [23]

$$H_{\rm rot} = \frac{\hbar^2}{2\mathcal{I}}\left[\left(I^2 - I_3^2\right) + \cdots + \cdots\right]. \tag{3}$$

The corresponding eigenvalues of rotational levels in the $K = I_3$ rotational band are,

$$E_{\rm rot}(I, K) = \frac{\hbar^2}{2\mathcal{I}}[I(I+1) - K^2]. \tag{4}$$

The term $\hbar^2/2\mathcal{I}$ is the usual rotational band inertial parameter A. Accordingly, the bandhead (zero point) with $I = K$, level energy,

$$E_{\rm rot}(I, I) = \frac{\hbar^2}{2\mathcal{I}}K \tag{5}$$

relative to which the energy of individual rotational levels follow $I(I+1)$ law. Extending the above formalism to the case of odd-odd nuclei the correction for the bandhead (zero point energy) in the 2qp system is given by [2,22]

$$E_{\rm rot} = \frac{\hbar^2}{2\mathcal{I}}[K - (\Omega_p + \Omega_n)] = -\frac{\hbar^2}{2\mathcal{I}}(2\Omega_<)\delta_{K,K^-}. \tag{6}$$

In this formalism, each 2qp (Ω_p, Ω_n) structure couples to give rise to two bands with quantum numbers $K^{\pm} = |\Omega_p \pm \Omega_n|$. Relative energy ordering of these two bands is governed by the Gallagher-Moszkowski (GM) rule [24], which places the spins-parallel triplet ($\Sigma = 1$) K_T band lower in energy than its GM doublet partner spins-antiparallel singlet ($\Sigma = 0$) K_S band.

For over 35 years, we have been engaged in investigating level structures of odd-odd deformed nuclei, both of the actinide [1–6] and of the rare-earth regions [22,25–27]. These investigations have used our three-step TQRM with variants of $\langle V_{pn}\rangle$ as briefly outlined below.

In the first step, we map the relevant 1qp configuration spaces using the current data bases [15,20]. Since the presently postulated ^{252}Md levels [14,15] have $E_x < 370$ keV, we limit our searches for 1qp, and also 2qp, levels to $E_x < 400$ keV. The experimental 1qp levels within the specified energy range for $N = 151$ isotones are shown in Fig. 2 and those for $Z = 101$ isotopes in Fig. 3.

In the second step, we enumerate in Table I the physically admissible K_T and K_S GM doublet bands from coupling of the low-lying ($E_x < 400$ keV) 1qp orbitals of $N = 151(A-1)$ isotones [15,20,21] as shown in Fig. 2 and Table I and of $Z = 101$ (A-1) isotopes [15,20,28] as shown in Fig. 3.

Finally the 2qp bandhead energies are evaluated using the following expression along with Eq. (6) in Eq. (1) [2,22]:

$$\langle V_{pn}\rangle = -\left(\tfrac{1}{2} - \delta_{\Sigma,0}\right)E_{\rm GM} + (-)^I E_N \delta_{K,0}. \tag{7}$$

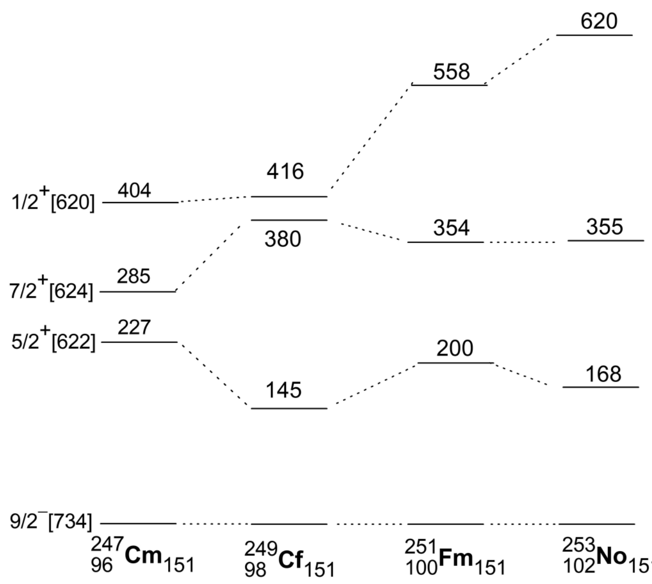

FIG. 2. Systematics of the gs and excited energy levels in $N = 151$ isotones taken from Refs. [15,20,21].

The terms $E_{\rm GM}$ and E_N in Eq. (3) denote, respectively, the GM doublet splitting energy and the Newby [29] odd-even shift for only the $K = 0$ bands arising from the residual n-p interaction V_{pn} for the specified configuration.

The model parameters $E_{\rm GM}$ and E_N can be evaluated theoretically, as described in our earlier papers [1,30,31]. Over the last two decades, we have been using a semiempirical approach [2,22] on the assumption that these parameters are only configuration specific and not nucleus dependent. However, in the present transfermium region, no corresponding experimental value of $E_{\rm GM}$ for any of the GM bands listed in our Table I is available. Further, even the postulated level energies on the experimental side are only approximately indicated [14]. Under these constraints, numerical agreement between experiment and theory is neither feasible, nor sought. Hence our bandhead energies are calculated using an average $E_{\rm GM} = 80$ keV and rotational parameter $A = 6$ keV. The results thus obtained, which are intended to serve as location guides, are plotted in Fig. 4 for the lowest four GM doublets (eight 2qp bands) from Table I. It is to be remembered that each bandhead has rotational levels above it following the

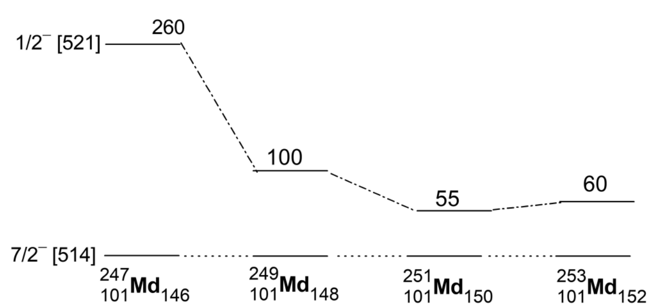

FIG. 3. Systematics of the gs and excited energy levels in $Z = 101$ Md isotopes taken from Refs. [15,20,28].

TABLE I. Physically admissible 2qp GM doublet bands in $^{252}_{101}Md_{151}$ arising from coupling of the 1qp p orbitals (top row) and the n orbitals (first column); numbers beside p_i/n_j are E_x(keV) and those within parentheses are the summed $[E(p_i) + E(n_j)]$ energies in keV.

p_i →	p_0:0 $7/2^-[514\downarrow]$		p_1:55 $1/2^-[521\downarrow]$	
n_j ↓	K_T	K_S	K_T	K_S
n_0:0 $9/2^-[734\uparrow]$	1^+	8^+	4^+	5^+
	(0)		(55)	
n_1:200 $5/2^+[622\uparrow]$	1^-	6^-	2^-	3^-
	(200)		(255)	
n_2:354 $7/2^+[624\downarrow]$	7^-	0^-	4^-	3^-
	(354)		(409)	

$AI(I+1)$ spacing rule. A detailed analysis and physical insights therefrom in respect of ^{252}Md spectrum are presented in the following section.

IV. ANALYSIS AND DISCUSSION

A. ^{252}Md ground state

The nuclide $^{252}_{101}Md_{151}$ was first discovered [11] in 1973 at Berkeley in the (HI, xn) reaction

$$^{243}_{95}\mathrm{Am}(^{13}_{6}\mathrm{C}, 4n)^{252}_{101}\mathrm{Md}(\epsilon)^{252}_{100}\mathrm{Fm}(\alpha) \quad (8)$$

by observation of characteristic α's from decay of long-lived ($t_{1/2} = 25.4$ h) ^{252}Fm produced in ϵ decay of short-lived ($t_{1/2} = 2.3$ m) and almost 100% ϵ-decaying ^{252}Md. Except for the deduced $t_{1/2}$ and ϵ-decay mode, no other characteristic of ^{252}Md gs is known even after 47 years of its discovery [14]. However, NDS2005 evaluators [14] had surmised its "J^π: possibly {n:9/2[734] - p:7/2[514]} orbitals coupled to 1^+." Our detailed TQRM evaluation of ^{252}Md energy levels, as described in the preceding section, places the K_T member with $J^\pi = 1^+$ of (p_0n_0) GM doublet as the lowest-energy level in this level scheme. In view of these considerations, we confirm the following assignment:

$$^{252}Md(gs) : J^\pi = 1^+\{p_0 : 7/2[514\downarrow] \otimes n_0 : 9/2[734\uparrow]\}. \quad (9)$$

B. First excited state: A short-lived isomer?

The lowest ^{252}Md level postulated from the α decay of ^{256}Lr is placed around 56 keV [14]. In our TQRM evaluation the closest thereto is $J^\pi K = 3^+1$ rotational level of gsb with (p_0n_0) configuration and $E_x \approx 60$ keV. However, as discussed later in Sec. IV D, α decay from (p_1n_j) gs of ^{256}Lr to an (p_0n_0) level involves change of both the orbitals from the parent to the daughter state, which, according to α decay transition rules, should correspond to hindrance factor (HF) $\gg 10^3$; the experimental HF for transition to the 56 keV level is just 267 [14]. Accordingly, identification of ≈56 keV level with the $J^\pi K = 3^+1$ level at $E_x \approx 60$ keV is inconsistent with known experimental information. Our TQRM places the (p_1n_0) 2qp state $K_T^\pi = 4^+\{p_1{:}1/2^-[521\downarrow] \otimes n_0{:}9/2^-[734\uparrow]\}$ at ~ 90 keV as the first (lowest) excited state in ^{252}Md spectrum. This $J^\pi K = 4^+4$ bandhead is the only intrinsic level (aside from the $I^\pi = 1^+ - 3^+$ rotational levels of gsb) below 100 keV excitation in this level scheme. This $J^\pi K = 4^+4$ level can, in principle, decay by an $E2$ transition to the $J^\pi K = 2^+1$ level of gsb. However, this transition is $\Delta K = 3$

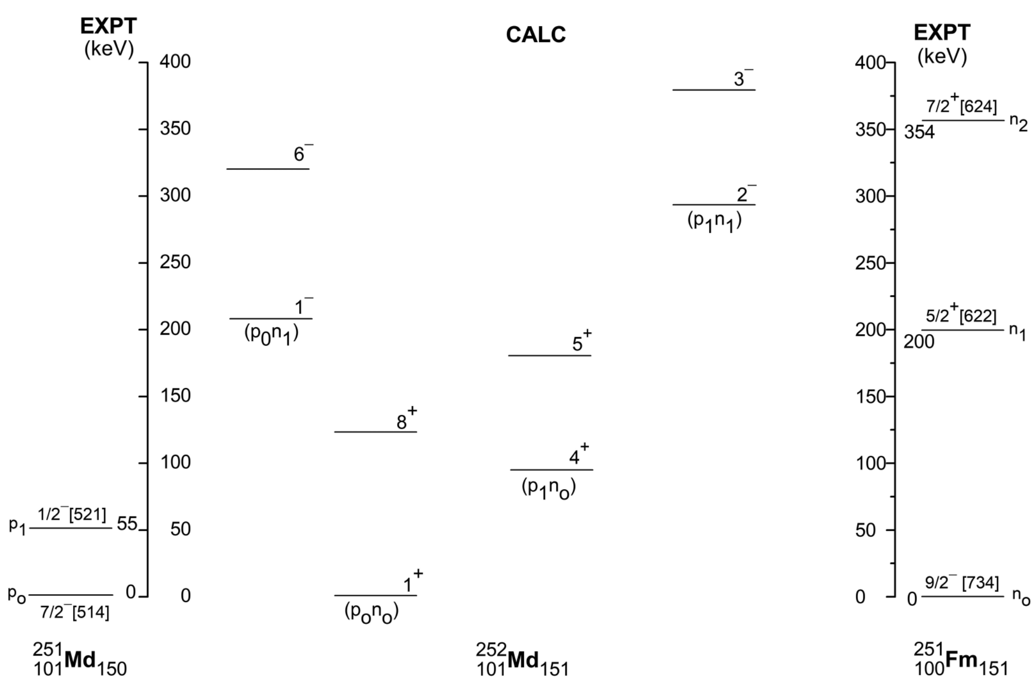

FIG. 4. Plot of model calculated energies of 2qp bandheads up to 400 keV in ^{252}Md (center) constructed from the experimental proton 1qp orbitals (left) and the neutron orbitals (right).

forbidden, and hence the $4^+4(p_1n_0)$ level with $E_x \sim 90$ keV is expected to be an isomeric state with measurable half-life. Also this $4^+4(p_1n_0)$ level can be populated with small HF from ^{256}Lr α decay from the parent state with one orbital, namely p_1, unchanged.

Further, since $E_x \approx 56$ keV is only an approximate value for the lowest level populated in α decay of ^{256}Lr, and our $E_x \approx 90$ keV is also a rough estimate for the lowest intrinsic state in ^{252}Md, we suggest $E_x = 75(25)$ keV for the lowest isomeric (excited) state, which is also the lowest populated level in α decay of ^{256}Lr.

C. $K^\pi = 8^+$: A long-lived high-spin isomer

Following a preliminary examination of long-lived isomers (LLI) in Md ($Z = 101$) isotopes with $A = 248(2)258$ by Sood in 1986 [16], an exhaustive and critical survey of then known, and also expected to occur, LLI in low-energy spectra of deformed nuclei from both the medium-weight (rare-earths) and heavy (actinides and beyond) regions was reported a year later by Sood and Sheline [32]. Therein they had predicted the occurrence of non-γ decaying, high-spin, long-lived, low-lying nuclear species in the heavy actinide and transactinide regions. In particular, they had explicitly pointed out the possible existence of $K^\pi = 8^+$ LLI above the $J^\pi = 1^+$ ground state in ^{252}Md. Our present detailed evaluation of low-lying 2qp intrinsic excitations, as shown in Fig. 4, clearly places a $K^\pi = 8^+(p_0n_0)$ around $E_x \approx 125$ keV as an LLI, which admits of only $\Delta K \geqslant 4$ em decay in this level scheme.

Recently we undertook a survey [33] of spectra of heavy actinides and neighboring transactinides in an attempt to unravel the occurrence and the nature of low-lying $K = 8$ states therein. Among other features, our study included 11 such instances in even-A (including both even-even and odd-odd) nuclei over the $A = 246(2)258$ mass range. These investigations examined the corresponding configuration space with a view to understand the physical basis of this wide-ranging phenomenon. In the present context, it is of interest to take note of the analogous occurrence in ^{256}Es [34] wherein a 7.6 h (8^+) isomer has been identified lying above a 25.4 m $(1^+, 0^-)$ ground state for well over four decades. Recently we have reported [35] our TQRM results on low-lying ^{256}Es levels, with particular focus on the $(1^+, 8^+)$ doublet as the gs GM pair. These investigations were primarily aimed at resolving the ambiguity about the ^{256}Es gs configuration, and also the excitation energy of the 7.6 h high-spin $(K^\pi = 8^+)$ isomer.

On the basis of the above discussion, we conclude that the $K^\pi = 8^+$ singlet member of the gs(p_0n_0) GM doublet lying around $E_x \approx 125$ keV is a low-lying high-spin isomer in the ^{252}Md level scheme.

D. ^{252}Md levels from ^{256}Lr α decay

Two years before the formal discovery of the nuclide ^{252}Md in 1973 [12], ^{256}Lr (α) decay studies at Berkeley [13] had observed six α branches, which led them to infer corresponding six excited levels in daughter nucleus, namely ^{252}Md. The respective excitation energies of these ^{252}Md levels were calculated from the α energies measured in ^{256}Lr α decay and $Q_\alpha(^{256}\text{Lr}) = 8810$ keV obtained from Q_α systematics [14]. As pointed out therein, E_α had uncertainty of 15–25 keV, whereas the recent data tables [34] quote an uncertainty of 100 keV in Q_α. In consideration of these uncertainties, NDS2005 evaluators [14] list only approximate level energies with no stated uncertainty. The numerical values listed therein fit the relation

$$E_x^i(^{252}\text{Md}) = 8682 - E_\alpha^i(^{256}\text{Lr}) \tag{10}$$

for $i = 1$–6. Thus, almost for 50 years now, we have these six excited ^{252}Md levels with imprecise level energies, no identifying spin-parity J^π labels, no interconnecting γ transitions, no connection to the $t_{1/2} = 2.3$ m ^{252}Md gs, or any other configurational assignment. This is the case even though ^{256}Lr (α) ^{252}Md has been subject of several SHE decay studies as described in our Sec. II above and also in certain α-γ coincidence experiments. Against this background we examine the ^{252}Md level structures using our well-tested TQRM formulation and attempt to correlate the results with the α decay data of ^{256}Lr.

In this process, we first seek the 2qp character of the ^{252}Md α parent, namely, $^{256}_{103}\text{Lr}_{153}$. As it happens, even the latest available $A = 256$ NDS2017 [36] just lists its 27.9 s gs with no indication of its structure. However, regarding its 1qp constituents, latest ENSDF [15] lists the ground-state configuration of all odd-mass $N = 153$ isotones from $Z = 96$(Cm) through 104(Rf) to be $1/2^+[620\uparrow]_n$. Further, these data tables also list $1/2^-[521\downarrow]_p$ as the configuration of the 31.1 s ^{255}Lr gs with its $7/2^-[514\downarrow]_p$ isomeric ($t_{1/2} = 1.94$ s) state placed 38 keV above the gs. With these inputs, we can reasonably assign

$$^{256}Lr(gs) : 0^-\{1/2^-[521]_p \otimes 1/2^+[620]_n\}1^- \tag{11}$$

with the triplet $K_T = 0^-$ member of the corresponding 2qp GM doublet lying lower than its $K_S = 1^-$ singlet counterpart.

Next we seek directions for characterizing ^{252}Md levels deduced in ^{256}Lr α decay based on the experimentally determined hindrance factors (HF) for respective α branches populating specific ^{252}Md energy levels. According to the policies adopted by nuclear data sheets (NDS) evaluators, for odd-odd nuclei, HF $\leqslant 4$ identifies favored α transitions, and these connect states having the same spin, parity, and configuration. Further detailed examination of unfavored α transitions established that for $4 <$ HF $< 10^3$, only one of the constituent states of odd-odd nuclei remains unchanged (same J^π and configuration) in such an α transition. In the present instance of ^{256}Lr(α) ^{252}Md, NDS2005 [14] lists six α branches having HF = 7–267, and hence each of them connects states having same J, π and configuration for one of the constituent orbital. Examination of ^{256}Lr gs 2qp configuration [see Eq. (11) above] and ^{252}Md 2qp states (as shown in Fig. 4) reveals the common (to remain unchanged) orbital in each case as p_1:$1/2^-[521]$. Taken together with the level energies listed in NDS2005 and our TQRM evaluated level energies, we conclude the spin, parity, and configuration for these six ^{256}Lr populated states as given in Table II and also shown in Fig. 5.

TABLE II. Suggested spin-parity $J^\pi K$ and 2qp configurations of α populated ^{252}Md excited states.

Sr. No	$\approx E_x$ (keV)	Calc E_x	$J^\pi K$	K^π {2qp config}
1	$\approx$366	375	4^-2	$2^-\{1/2^-[521]_p$ - $5/2^+[622]_n\}$
2	$\approx$294	291	2^-2	
3	$\approx$253	250	6^+5	$5^+\{1/2^-[521]_p + 9/2^-[734]_n\}$
4	$\approx$165	177	5^+5	
5	$\approx$211	223	6^+4	$4^+\{1/2^-[521]_p$ - $9/2^-[734]_n\}$
6	$\approx$56[a]	91[a]	4^+4	

[a]Entries in the last line (6) in this table are the approximate lowest-energy values respectively from α decay experiments and from model calculations; as indicated in text, these values are grouped together with assigned 75(25) keV value.

E. γ transitions from α-γ coincidence experiments

The only other experimental input for studying the level spectrum of ^{252}Md comes from 'observation of several γ transitions in coincidence with ^{256}Lr α decay in the energy range 8300–8700 keV'. These experiments in GSI [18] have deduced five γ transitions with E_γ in the range 85–190 keV. Evidently these γ's should originate from some of the five ^{252}Md levels with E_x ranging 165–366 keV deduced from ^{256}Lr α decay experiments [12] discussed in preceding section. However, the placement of γ's with specific E_x is not straightforward, since E_α in coincidence studies for a given γ is spread over $\Delta E_\alpha \approx$ 170–370 keV, which corresponds to multiple level energies. Under the circumstances, we examine our TQRM specified levels to fit the observed E_γ with acceptable constraints for em transitions. This exercise has led us to the γ placements between ^{252}Md levels as listed in Table III and plotted in Fig. 5. The distinctive feature of these placements is that each of the five instances corresponds to $\Delta I = 2$ transition, with four out of five listed transitions being $E2$ type and two being intraband $E2$s. Thus the specified placements may not be unique, but they fall in the most likely category.

V. SUMMARY AND CONCLUSIONS

Based on the current databases, we sought radioactive decay connection of the heaviest known SHE with $A = 4n$, namely ^{272}Rg, to the top nucleus heading the naturally occurring radioactive series, namely $A = 4n$ ^{232}Th (our Fig. 1). This process led to proclaiming the landmark position of ^{252}Md in the extended ^{272}Rg - ^{232}Th radioactive series on various counts detailed in our Sec. II.

We then constructed a low-lying (<400 keV) level scheme for this odd-odd ^{252}Md using our well-tested two quasiparticle rotor model (TQRM) with respective 1qp orbitals from neighboring odd-A isotope ^{251}Md and odd-A isotone ^{251}Fm and with the inclusion of residual $\langle V_{pn}\rangle$ contribution. This exercise resulted in identification of eight 2qp bandheads (four GM doublets, see Fig. 4) and their respective $I(I+1)$-dependent rotational levels within the specified energy domain. A critical analysis thereof resulted in a J^π and 2qp characteristics of ^{252}Md gs as given in Fig. 5. This process also revealed the existence of a high-spin $K^\pi = 8^+$ long-lived isomer around

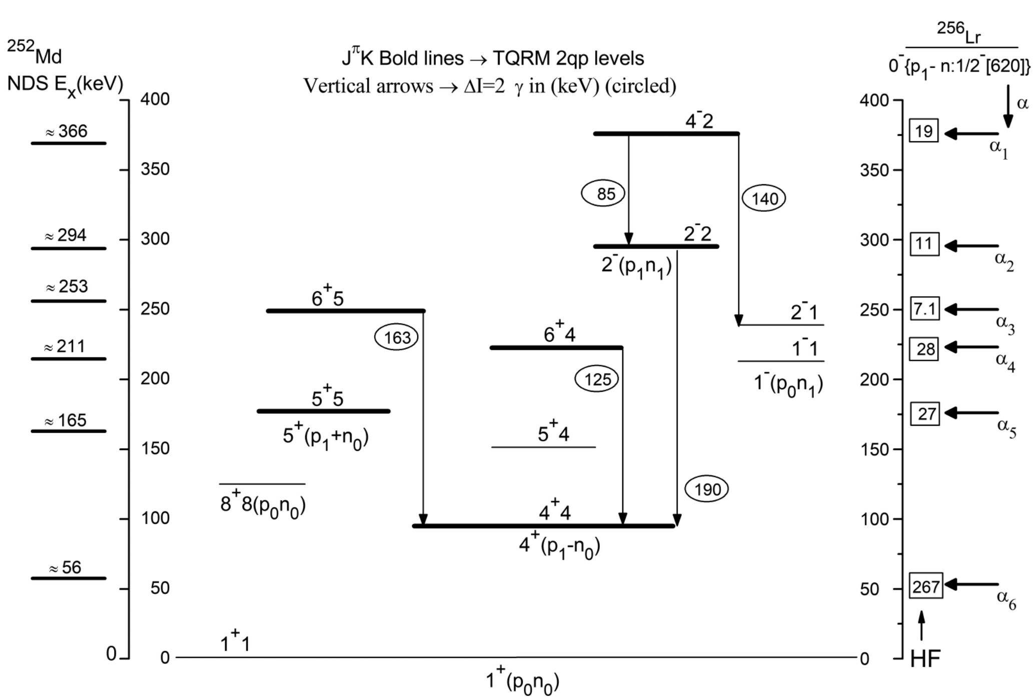

FIG. 5. ^{256}Lr decay observed α branches (arrows on right-hand side) and respective HF (boxed) on the left-hand side with the corresponding TQRM deduced energy levels (bold lines in middle) vide our Table II. The vertical arrows show our placement of γ's observed in α-γ coincidence experiments [18] between TQRM deduced ^{252}Md levels (Table III) for $\Delta I = 2$ transitions. The corresponding γ energies (in keV) are shown as circled numbers.

TABLE III. Placement of γ energies (first column) from α-γ coincidence experiments [18] between TQRM calculated energy levels, as explicitly shown in Fig. 5, which corresponds to $\Delta I = 2$ transitions.

Sr. No	E_γ (keV)	$J^\pi K$(upper)	$J^\pi K$(lower)
1	190	$2^-2(p_1n_1)$	$4^+4(p_1n_0)$
2	163	$6^+5(p_1n_0)$	$4^+4(p_1n_0)$
3	140	$4^-2(p_1n_1)$	$2^-1(p_0n_1)$
4	125	$6^+4(p_1n_0)$	$4^+4(p_1n_0)$
5	85	$4^-2(p_1n_1)$	$2^-2(p_1n_1)$

$E_x \approx 125$ keV and a possible existence of a lower-lying short-lived isomer in ^{252}Md spectrum. A comparison of our evaluated level energies with the NDS2005 indicated approximate experimental values has led us to assign $J^\pi K$ and 2qp configuration to each of these levels as listed in our Table II and shown in Fig. 5. Further, a side-by-side examination of our level spacings and α-γ coincidence deduced γ energies has enabled us to the more likely/preferred $\Delta I = 2$ placement of the experimental γ's in our ^{252}Md level scheme, as shown in Table III and Fig. 5. The agreement between the model results and presently available experimental values in each case is quite satisfactory. However, in view of the rather large uncertainties in the experimental values and also in the model evaluations (with *ad hoc* choice of model parameters in absence of sufficient supporting data) we presently do not emphasize the quantitative aspect here. We are of the considered view that our analysis provides valuable guidelines for further studies to elucidate the physical picture in this unexplored region.

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