Low energy kaon-Ξ interaction in an effective chiral model

M. G. L. Nogueira-Santos [*] and C. C. Barros, Jr. [†]
Departamento de Física, CFM, Universidade Federal de Santa Catarina Florianópolis, Santa Catarina CEP 88010-900, Brazil

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We develop a study of the low energy kaon-Ξ interaction taking into account nonlinear chiral invariant Lagragians. We extend a model proposed to describe the πN interaction to the $K\Xi$ and $\overline{K}\Xi$ interactions, considering Λ, Σ, and Ω hyperons and the $\Omega(2012)$ resonance in the intermediate states. We calculate their total cross sections, phase shifts, angular distributions, and polarizations in the center-of-mass frame of reference, accordingly to the proposed formalism.

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I. INTRODUCTION

Nowadays, even with the progressive development of QCD methods, the hyperon interactions at low energies is a subject that is still not properly understood. At the experimental level few data are available, as for example the HyperCP ones [1,2], but we are very far from a scenario similar to the one of the pion-nucleon or nucleon-nucleon interactions, which counts with a large amount of accurate experimental data. At the theoretical level, because it is not straightforward to describe these interactions with QCD, an interesting strategy is to study them with effective Lagrangians that take into account the available proprieties of the considered particles, as it has been done in Refs. [3–6], because this procedure has provided very good results in the study of low energy pion-nucleon interactions.

Low energy hyperon physics is very important in many physical systems of interest. In the study of the hypernuclei structure [7–10], for example, the nucleon-hyperon interaction is a fundamental ingredient, and this interaction depends on meson exchanges, so the pion-hyperon (πY), kaon-nucleon (KN), and kaon-hyperon (KY) interactions are essential subprocesses of the nucleon-hyperon (NY) potential.

In the study of hyperon stars [11–14], the same problem occurs, because the understanding of the NY interactions (and even of the hyperon-hyperon ones) is essential in order to determine a reliable equation of state and then the observables that may be obtained, such as the mass or the radius of the considered star.

In high energy physics these interactions are also very important. When analyzing a Large Hadron Collider process, for example, a large amount of particles is produced in each collision, and after this production these particles may interact. When considering a hydrodynamical picture [15,16], the relative energy of such particles is low, and then the low energy meson-baryon and baryon-baryon interactions become important and have effects in the final experimental results. Hyperon polarization has been studied in this way [15,16] and the obtained results were consistent with the experimental data. Consequently, it is a mechanism that must be considered in addition to other medium properties, such as vorticity [17,18].

As we can see, this kind of study is motivated for many reasons. In Refs. [19–21], low energy πY interactions have been studied, and in Refs. [22,23], the $K\Lambda$, $\overline{K}\Lambda$, $K\Sigma$, and $\overline{K}\Sigma$ interactions have been studied. Therefore, in order to provide a general picture of hyperon interactions, this paper is devoted to the study of the $K\Xi$ and $\overline{K}\Xi$ interactions. With this purpose we follow the same procedure that has been shown in these papers, by considering a model that describes successfully the πN interactions [3–6], based on chiral effective nonlinear Lagrangians, which take into account baryons, baryonic resonances, with spins $1/2$ and $3/2$, and mesons. The scattering amplitudes in terms of the possible isospin states are calculated, and then the phase shifts, cross sections, and polarizations may be determined. Because no experimental data are available for the reactions considered in this paper, this kind of comparison with our results will not be possible.

This paper has the following content. In Sec. II we review the basic formalism to be considered, and in Sec. III the scattering amplitudes are calculated and the results are shown. In Sec. IV the conclusions are presented.

II. THE METHOD

In order to study $K\Xi$ and $\overline{K}\Xi$ interactions, and observing the fact that no experimental data are available, a reasonable strategy is to make an analogy with the πN interaction, which is very well studied [3–6], for which many models and a large amount of experimental data are available. In fact, this procedure has been followed aiming to understand other hyperon systems [19–23] and even Λ_b interactions [24]. Here we use an effective chiral model based on these works, extending it to the kaon-Ξ case.

[*]magwwo@gmail.com
[†]barros.celso@ufsc.br

The chiral Lagrangians of Ref. [3], to describe the $N\pi$ interaction with spin-1/2 and spin-3/2 baryons, are given by

$$\mathcal{L}_{\pi NN} = \frac{g}{2m}(\overline{N}\gamma_\mu\gamma_5\vec{\tau}N)\cdot\partial^\mu\vec{\phi}, \tag{1}$$

$$\mathcal{L}_{\pi N\Delta} = g_\Delta\{\overline{\Delta}^\mu[g_{\mu\nu} - (Z+1/2)\gamma_\mu\gamma_\nu]\vec{M}N\}\cdot\partial^\nu\vec{\phi}, \tag{2}$$

where N, Δ, and $\vec{\phi}$ are the nucleon, Delta, and pion fields with masses m, m_Δ, and m_π, respectively; $\vec{M}$ and $\vec{\tau}$ are isospin matrices; and Z is a parameter representing the possibility of the off-shell-Δ having spin 1/2. The parameters g and g_Δ are coupling constants that depend on the interacting particles. These Lagrangians are adapted to each process of interest.

The Feynman amplitudes will have the general form

$$T^I_{K\Xi} = \overline{u}(\vec{p}\,')\big[A^I + \tfrac{1}{2}(\not{k} + \not{k}')B^I\big]u(\vec{p}), \tag{3}$$

where the subscript $(K\Xi)$ represents the initial particles, a spin-0 kaon, and a spin-1/2 Ξ hyperon (or $\overline{K}\Xi$ for the antikaon-Ξ interaction); $u(\vec{p})$ is the spinor relative to the initial Ξ with momentum $\vec{p}$; and k (k') is the K incoming (outgoing) meson four-momentum. The superscript I represents the isospin channel of the process.

Through the relation (3) and comparing with the appropriate Feynman diagrams, we can calculate the A^I and B^I amplitudes for each total isospin channel I for the $K\Xi$ and $\overline{K}\Xi$ scattering.

The scattering matrix for a given isospin state may be written as

$$M^I_{K\Xi} = \frac{T^I_{K\Xi}}{8\pi\sqrt{s}} = f^I(k,x) + g^I(k,x)i\vec{\sigma}\cdot\hat{n}, \tag{4}$$

which may be decomposed into the spin-nonflip and spin-flip amplitudes, $f^I(k,x)$ and $g^I(k,x)$, and then expanded in terms of partial-wave amplitudes:

$$f^I(k,x) = \sum_{l=0}^{\infty}\big[(l+1)a^I_{l+}(k) + l a^I_{l-}(k)\big]P_l(x), \tag{5}$$

$$g^I(k,x) = \sum_{l=1}^{\infty}\big[a^I_{l-}(k) - a^I_{l+}(k)\big]P^{(1)}_l(x). \tag{6}$$

Here $k = |\vec{k}|$ and $x = \cos\theta$, where θ is the scattering angle, are defined in the center-of-mass frame (c.m.).

Defining new amplitudes as functions of A^I and B^I for each isospin I as

$$f^I_1(k,\theta) = \frac{(E+m)}{8\pi\sqrt{s}}[A^I + (\sqrt{s}-m)B^I], \tag{7}$$

$$f^I_2(k,\theta) = \frac{(E-m)}{8\pi\sqrt{s}}[-A^I + (\sqrt{s}+m)B^I], \tag{8}$$

and considering the Legendre polynomial's orthogonality relations in the partial-wave decomposition, we have that

$$a^I_{l\pm} = \frac{1}{2}\int_{-1}^{1}\big[P_l(x)f^I_1(k,x) + P_{l\pm1}(x)f^I_2(k,x)\big]dx. \tag{9}$$

In the expressions above, considering that E is the Ξ energy and k_0 is the meson energy, we may write $\sqrt{s}$ in terms of the c.m. variables in a usual way (see the Appendix for details).

TABLE I. Intermediate baryons.

Baryon	J^π	I	Mass
Λ	$1/2^+$	0	1116 MeV
Σ	$1/2^+$	1/2	1190 MeV
Ω	$3/2^+$	0	1672 MeV
$\Omega(2012)$	$3/2^+$	0	2012 MeV

As we are interested in the scattering at low energies, just three terms, the S, P_1, and P_3 waves, are considered in this work and it may be verified numerically that this procedure provides a good approximation (obviously, with the formalism presented in this section it is possible to do the calculations for any value of l).

Observables

As the partial-wave amplitudes are real, the unitarity of the S matrix [19,20] is violated. This problem may be overcome by the unitarization of the amplitudes with

$$a^U_{l\pm} = \frac{a_{l\pm}}{1 - i|\vec{k}|a_{l\pm}}. \tag{10}$$

This procedure is followed for all the $a_{l\pm}$ defined above and then the observables may be calculated.

For each isospin channel of Eqs. (5) and (6), the differential cross section is defined in the c.m. system as

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2, \tag{11}$$

and integrating this expression over the solid angle we obtain the total cross section

$$\sigma_T = 4\pi\sum_l\big[(l+1)|a^U_{l+}|^2 + l|a^U_{l-}|^2\big]. \tag{12}$$

The polarization of the final Ξ is defined by

$$\vec{P} = -2\frac{\mathrm{Im}(f^*g)}{|f|^2+|g|^2}\hat{n}, \tag{13}$$

where $\hat{n}$ is a unitary vector normal to the scattering plane, and finally, the phase shifts are given by

$$\delta_{l\pm} = \tan^{-1}(|\vec{k}|a_{l\pm}). \tag{14}$$

The observables are calculated according to these expressions for the reactions of interest in the next section.

III. INTERACTIONS AND RESULTS

This section is devoted to the study of the $K\Xi$ and $\overline{K}\Xi$ interactions. One can observe that both interactions have the same intermediate states, but in different (s and u) channels, as can be seen in Figs. 2 and 7.

The intermediary states for these interactions are the baryons Λ, Σ, Ω, and $\Omega(2012)$, the properties of which are shown in Table I, where J^π is the spin with parity and I is the total isospin. The $\Omega(2012)$ accordingly with Ref. [25] has no established spin yet, so we considered the spin $3/2^+$ following the work [26]. We also take into account a parametrized σ-meson exchange.

The interactions that appear in the diagrams shown in Figs. 2 and 7 are determined by adapting the Lagrangians (1) and (2) that are relative to the πN interaction, to describe the $K\Xi$ interaction with a Λ hyperon coupling,

$$\mathcal{L}_{\Xi K\Lambda} = \frac{g_{\Xi K\Lambda}}{2m_{\Xi}}(\overline{\Lambda}\gamma_{\mu}\gamma_5\Xi)\partial^{\mu}\phi, \tag{15}$$

with a Σ coupling,

$$\mathcal{L}_{\Xi K\Sigma} = \frac{g_{\Xi K\Sigma}}{2m_{\Xi}}(\vec{\overline{\Sigma}}\cdot\vec{\tau}\gamma_{\mu}\gamma_5\Xi)\partial^{\mu}\phi, \tag{16}$$

and with a coupling with the spin-3/2 Ω,

$$\mathcal{L}_{\Xi K\Omega} = g_{\Xi K\Omega}\{\overline{\Omega}^{\mu}[g_{\mu\nu} - (Z+1/2)\gamma_{\mu}\gamma_{\nu}]\Xi\}\partial^{\nu}\phi. \tag{17}$$

For the $\overline{K}\Xi$ case we change the kaon meson fields (ϕ) as follows: $\phi \to \phi'$, where ϕ' represents the $\overline{K}$ field.

A. Isospin states

In order to study the possible reactions for the $K\Xi$ scattering we consider the isospin formalism. The Ξ hyperon has isospin 1/2, with Ξ^0 and Ξ^- states, and the K meson has also isospin 1/2, with states K^+ and K^0 ($\overline{K}$ has also isospin 1/2). By combining two particles of isospin 1/2 we may produce states with isospin $I=0$ or $I=1$. So, in order to calculate the observables of interest we decompose the scattering amplitudes into isospin states considering projection operators. A generic projection operator of an operator O into a state i with an eigenvalue ω_i, P_i, is given by

$$P_i = \prod_{j\neq i}\frac{O-\omega_j}{\omega_i-\omega_j}. \tag{18}$$

For the $K\Xi$ interactions the isospin operator is represented by

$$\vec{I} = \frac{\vec{\tau}_1}{2} + \frac{\vec{\tau}_2}{2}, \tag{19}$$

where $\vec{\tau}_1$ and $\vec{\tau}_2$ are given in terms of the Pauli matrices where the subscripts 1 and 2 are relative to the particles 1 and 2. Squaring Eq. (19) we have

$$(\vec{I})^2 = \left(\frac{\vec{\tau}_1}{2}\right)^2 + \frac{\vec{\tau}_1\cdot\vec{\tau}_2}{2} + \left(\frac{\vec{\tau}_2}{2}\right)^2, \tag{20}$$

and then we may obtain $\omega_0 = -3$ and $\omega_1 = 1$ for isospins $I=0$ and $I=1$, respectively. Then the projection operator (18) for isospin $I=0$ becomes

$$P_0 = \frac{1-\vec{\tau}_1\cdot\vec{\tau}_2}{4}, \tag{21}$$

and for isospin $I=1$ we have

$$P_1 = \frac{3+\vec{\tau}_1\cdot\vec{\tau}_2}{4}. \tag{22}$$

Writing the Feynman amplitudes decomposed into the isospin states $I=0$ and $I=1$ we have

$$T = T_0P_0 + T_1P_1, \tag{23}$$

and using Eqs. (21) and (22), we have

$$T = \frac{T_0+3T_1}{4} + \frac{T_1-T_0}{4}\vec{\tau}_1\cdot\vec{\tau}_2. \tag{24}$$

Now defining

$$T_\delta = \frac{T_0+3T_1}{4}, \quad T_\tau = \frac{T_1-T_0}{4}, \tag{25}$$

where

$$T_\delta = \overline{u}(\vec{p}')\left[A_\delta + \frac{\not{k}+\not{k}'}{2}B_\delta\right]u(\vec{p}), \tag{26}$$

$$T_\tau = \overline{u}(\vec{p}')\left[A_\tau + \frac{\not{k}+\not{k}'}{2}B_\tau\right]u(\vec{p}), \tag{27}$$

the scattering amplitude has the structure

$$T = \overline{u}(\vec{p}')\left[A_\delta + \frac{\not{k}+\not{k}'}{2}B_\delta + \left(A_\tau + \frac{\not{k}+\not{k}'}{2}B_\tau\right)\vec{\tau}_1\cdot\vec{\tau}_2\right]u(\vec{p}), \tag{28}$$

so we achieve that

$$T_0 = \overline{u}(\vec{p}')\left[A_\delta - 3A_\tau + \frac{\not{k}+\not{k}'}{2}(B_\delta - 3B_\tau)\right]u(\vec{p}), \tag{29}$$

$$T_1 = \overline{u}(\vec{p}')\left[A_\delta + A_\tau + \frac{\not{k}+\not{k}'}{2}(B_\delta + B_\tau)\right]u(\vec{p}). \tag{30}$$

Finally we may write the A_I and B_I amplitudes in terms of the expressions defined above:

$$A_0 = A_\delta - 3A_\tau, \quad B_0 = B_\delta - 3B_\tau, \tag{31}$$

$$A_1 = A_\delta + A_\tau, \quad B_1 = B_\delta + B_\tau, \tag{32}$$

where the δ and τ amplitudes are determined in the diagrams to be considered.

B. Coupling constants

In this subsection we define the coupling constant for each interaction of interest.

To determine the coupling constants with the Λ and Σ baryons of Eqs. (15) and (16), we consider the octet SU(3) symmetry in quark space. In Refs. [27,28], we find the relations for the $\Xi K\Lambda$ and $\Xi K\Sigma$ coupling constants, which are respectively

$$g_{\Xi K\Lambda} = \frac{f}{\sqrt{3}}(1-4\alpha), \tag{33}$$

$$g_{\Xi K\Sigma} = f, \tag{34}$$

where we use the constant defined as $f = g_{\pi NN} = 13.4$ [29], and then the relation $g_{\pi\Lambda\Sigma} = 2f(1-\alpha)/\sqrt{3}$. Taking $g_{\pi\Lambda\Sigma} = 11.7$ [30], we calculate the parameter $\alpha = 0.244$, which is in accord with Ref. [28].

For the coupling constant of Eq. (17) with the spin-3/2 Ω member of the decuplet symmetry, we use the Goldberger-Treiman relation [31] that relates a pseudoscalar meson, with an octet and a decuplet baryon ($P_8B_8B_{10}$) as done in [32]

$$g_{P_8B_8B_{10}} = \frac{M_8+M_{10}}{\sqrt{2}f_8}C_5^A(0), \tag{35}$$

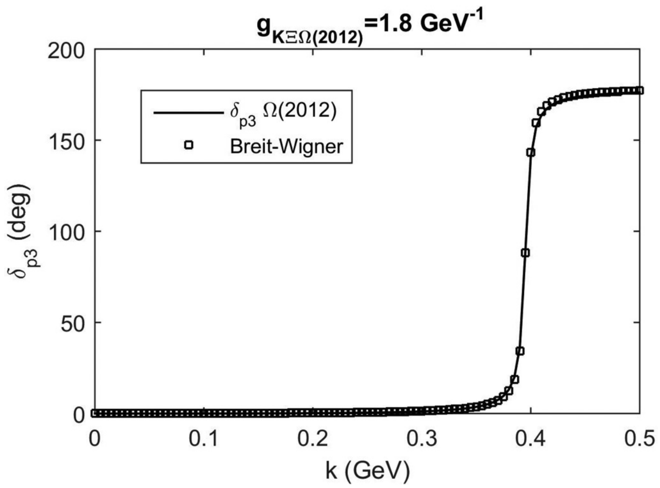

FIG. 1. Fitting of the Ω(2012) coupling constant by the comparison of the Breit-Wigner and δ_{p_3} results.

where M_8 and M_{10} are the Ξ and Ω masses, respectively, $f_8 = 113$ MeV is the decay constant of the kaon, and $C_5^A(0) = 1.612 \pm 0.007$ was calculated in Ref. [33] for the $\Xi K \Omega$ case in the symmetry breaking, which as shown in Ref. [32] is in accord with the experimental constraints. In accordance with the Lagrangian (17) we adjust the dimensional unit of the coupling constant by the convention $g_{\Xi K \Omega} = \frac{g_{P_8 B_8 B_{10}}}{2m_\Xi}$.

The coupling constant with the Ω(2012) resonance is calculated with the aid of the relativistic Breit-Wigner expression (see the Appendix, Sec. 2), the resonance phase shift is fitted with the relation (A13) by adjusting the coupling constant, as it is shown in Fig. 1, in the same way it has been done in Refs. [19,22].

The final results of the coupling constants to be considered in this work are shown in Table II.

C. The $K\Xi$ interaction

Now we turn our attention to the calculation of the scattering amplitudes and the corresponding observables. The diagrams considered in the study of the $K\Xi$ interaction are shown in Fig. 2.

According to the Feynman rules, the Lagrangian (15) for the spin-1/2 Λ coupling [Fig. 2(a)], and the projection relations (31) and (32), we find that

$$A_\Lambda^0 = A_\delta^\Lambda, \quad A_\tau^\Lambda = 0, \tag{36}$$

$$B_\Lambda^0 = B_\delta^\Lambda, \quad B_\tau^\Lambda = 0, \tag{37}$$

TABLE II. Coupling constants for the $K\Xi$ and $\overline{K}\Xi$ interactions.

$g_{\Xi K\Lambda}$	0.2
$g_{\Xi K\Sigma}$	13.4
$g_{\Xi K\Omega}$	8.3 GeV^{-1}
$g_{\Xi K\Omega(2012)}$	1.8 GeV^{-1}

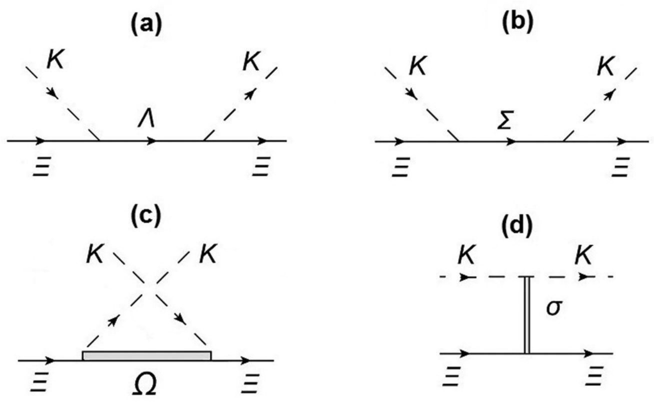

FIG. 2. Diagrams of the $K\Xi$ interaction.

with the amplitudes given by

$$A_\Lambda^0 = \frac{g_{\Xi K\Lambda}^2}{4m_\Xi^2}(m_\Lambda + m_\Xi)\left(\frac{s - m_\Xi^2}{s - m_\Lambda^2}\right), \tag{38}$$

$$B_\Lambda^0 = -\frac{g_{\Xi K\Lambda}^2}{4m_\Xi^2}\left[\frac{2m_\Xi(m_\Xi + m_\Lambda) + s - m_\Xi^2}{s - m_\Lambda^2}\right], \tag{39}$$

where m_Λ and m_Ξ are the Λ and Ξ hyperon masses.

In the same fashion, for a Σ (spin-1/2) pole in the s channel [Fig. 2(b)], the amplitudes for the isospin $I = 0$ state are

$$A_\Sigma^0 = -3A_\tau^\Sigma, \quad A_\delta^\Sigma = 0, \tag{40}$$

$$B_\Sigma^0 = -3B_\tau^\Sigma, \quad B_\delta^\Sigma = 0, \tag{41}$$

where

$$A_\Sigma^0 = -\frac{3g_{\Xi K\Sigma}^2}{4m_\Xi^2}(m_\Sigma + m_\Xi)\left(\frac{s - m_\Xi^2}{s - m_\Sigma^2}\right), \tag{42}$$

$$B_\Sigma^0 = \frac{3g_{\Xi K\Sigma}^2}{4m_\Xi^2}\left[\frac{2m_\Xi(m_\Xi + m_\Sigma) + s - m_\Xi^2}{s - m_\Sigma^2}\right]. \tag{43}$$

For the isospin $I = 1$ channel we have

$$A_\Sigma^1 = A_\tau^\Sigma, \quad A_\delta^\Sigma = 0, \tag{44}$$

$$B_\Sigma^1 = B_\tau^\Sigma, \quad B_\delta^\Sigma = 0, \tag{45}$$

with the amplitudes

$$A_\Sigma^1 = \frac{g_{\Xi K\Sigma}^2}{4m_\Xi^2}(m_\Sigma + m_\Xi)\left(\frac{s - m_\Xi^2}{s - m_\Sigma^2}\right), \tag{46}$$

$$B_\Sigma^1 = -\frac{g_{\Xi K\Sigma}^2}{4m_\Xi^2}\left[\frac{2m_\Xi(m_\Xi + m_\Sigma) + s - m_\Xi^2}{s - m_\Sigma^2}\right]. \tag{47}$$

For the diagram with a spin-3/2 Ω hyperon in the intermediate state [Fig. 2(c)] we use the Lagrangian (17) and the relations (31) for the $I = 0$ channel, and then

$$A_\Omega^0 = A_\delta^\Omega, \quad A_\tau^\Omega = 0, \tag{48}$$

$$B_\Omega^0 = B_\delta^\Omega, \quad B_\tau^\Omega = 0, \tag{49}$$

which results in the amplitudes

$$A_\Omega = \frac{g^2_{\Xi K\Omega}}{6}\left[\frac{2\hat{A}' + 3(m_\Xi + m_\Omega)t}{m_\Omega^2 - u} + c_0 + c_z\big(u - m_\Xi^2\big)\right], \tag{50}$$

$$B_\Omega = \frac{g^2_{\Xi K\Omega}}{6}\left[-\frac{2\hat{B}' + 3t}{m_\Omega^2 - u} + d_0 + d_z\big(u - m_\Xi^2\big)\right], \tag{51}$$

where we have

$$\hat{A}' = 3(m_\Xi + m_\Omega)(q_\Omega)^2 + (m_\Omega - m_\Xi)(E_\Omega + m_\Xi)^2, \tag{52}$$

$$\hat{B}' = 3(q_\Omega)^2 - (E_\Omega + m_\Xi)^2, \tag{53}$$

and

$$c_0 = -\frac{(m_\Xi + m_\Omega)}{m_\Omega^2}\big(2m_\Omega^2 + m_\Xi m_\Omega - m_\Xi^2 + 2m_K^2\big), \tag{54}$$

$$c_z = \frac{4}{m_\Omega^2}[(m_\Omega + m_\Xi)Z + (2m_\Omega + m_\Xi)Z^2], \tag{55}$$

$$d_0 = \frac{8}{m_\Omega^2}\big[\big(m_\Xi^2 + m_\Xi m_\Omega - m_K^2\big)Z + \big(2m_\Xi m_\Omega + m_\Xi^2\big)Z^2\big] + \frac{(m_\Xi + m_\Omega)^2}{m_\Omega^2}, \tag{56}$$

$$d_z = \frac{4Z^2}{m_\Omega^2}, \tag{57}$$

where t, q_Ω, and E_Ω are defined in the Appendix, and m_K and m_Ω are the kaon and the Ω masses, respectively. The $g_{\Xi K\Omega}$ coupling constant is given in Table II. The $\Omega(2012)$ resonance is considered and the same procedure may be followed.

For the scalar σ-meson exchange represented in Fig. 2(d), we used a parametrization of the amplitude in the same way it was done in Refs. [19,20],

$$A_\sigma = a + bt, \tag{58}$$

$$B_\sigma = 0, \tag{59}$$

with $a = 1,05m_\pi^{-1}$ and $b = -0,8m_\pi^{-3}$, where m_π is the pion mass. Some discussions about this term may be found in Refs. [21,34–37].

With the knowledge of the scattering amplitudes for the diagrams of Fig. 2 and their projections into isospin states, we can compute the amplitudes for the elastic and charge exchange reactions, which for the $K\Xi$ interactions are labeled as R_1, R_2, and R_3:

$$R_1 \equiv \langle K^+\Xi^0|T|K^+\Xi^0\rangle = \langle K^0\Xi^-|T|K^0\Xi^-\rangle = T_1, \tag{60}$$

$$R_2 \equiv \langle K^+\Xi^-|T|K^+\Xi^-\rangle = \langle K^0\Xi^0|T|K^0\Xi^0\rangle = \tfrac{1}{2}(T_1 + T_0), \tag{61}$$

TABLE III. Parameters for the $K\Xi$ and $\overline{K}\Xi$ interactions.

m_π	140 MeV
m_K	496 MeV
m_Ξ	1320 MeV
Z	−0.5

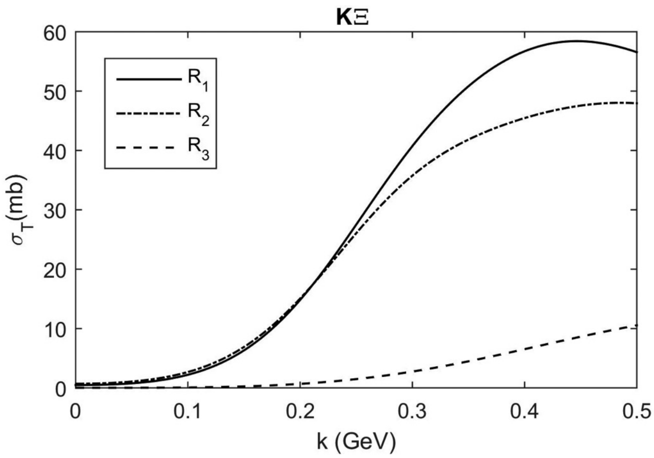

FIG. 3. The $K\Xi$ total cross section.

$$R_3 \equiv \langle K^0\Xi^0|T|K^+\Xi^-\rangle = \langle K^+\Xi^-|T|K^0\Xi^0\rangle = \tfrac{1}{2}(T_1 - T_0). \tag{62}$$

Finally, considering the parameters shown in Tables I, II, and III, the amplitudes A and B for each diagram (Fig. 2) may be calculated, and then applying the relations (5)–(8) and (10)–(14), we may compute the resulting observables.

In Fig. 3 the total cross sections for the reactions R_1, R_2, and R_3 are shown, and in Fig. 4 we display the phase shifts

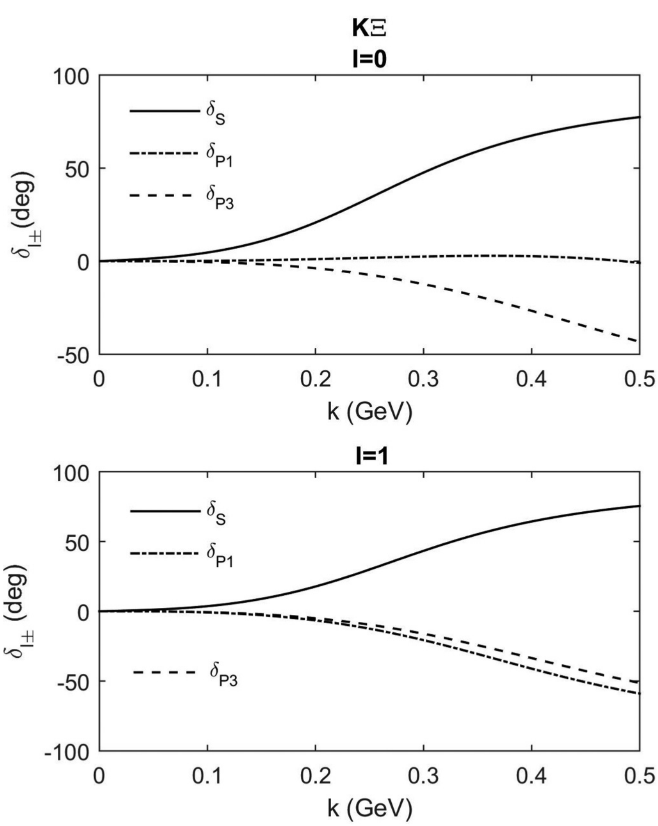

FIG. 4. Phase shifts for the S, P_1, and P_3 partial-wave amplitudes in the $K\Xi$ interaction.

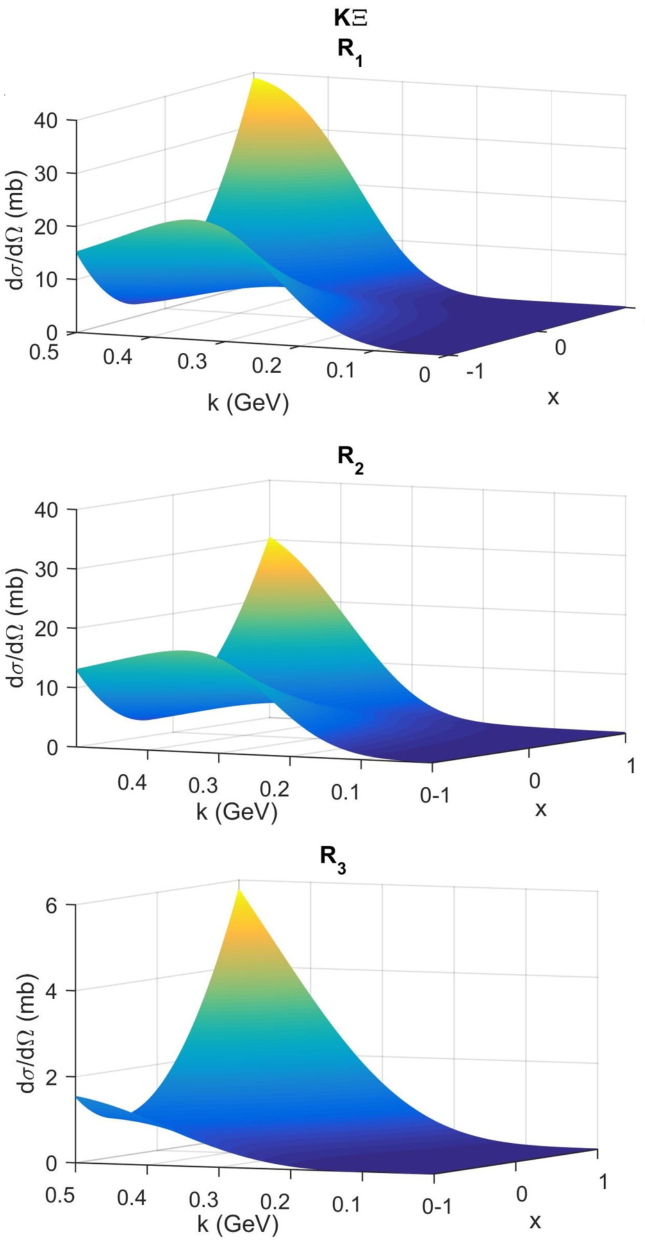

FIG. 5. Differential cross sections for the $K\Xi$ scattering.

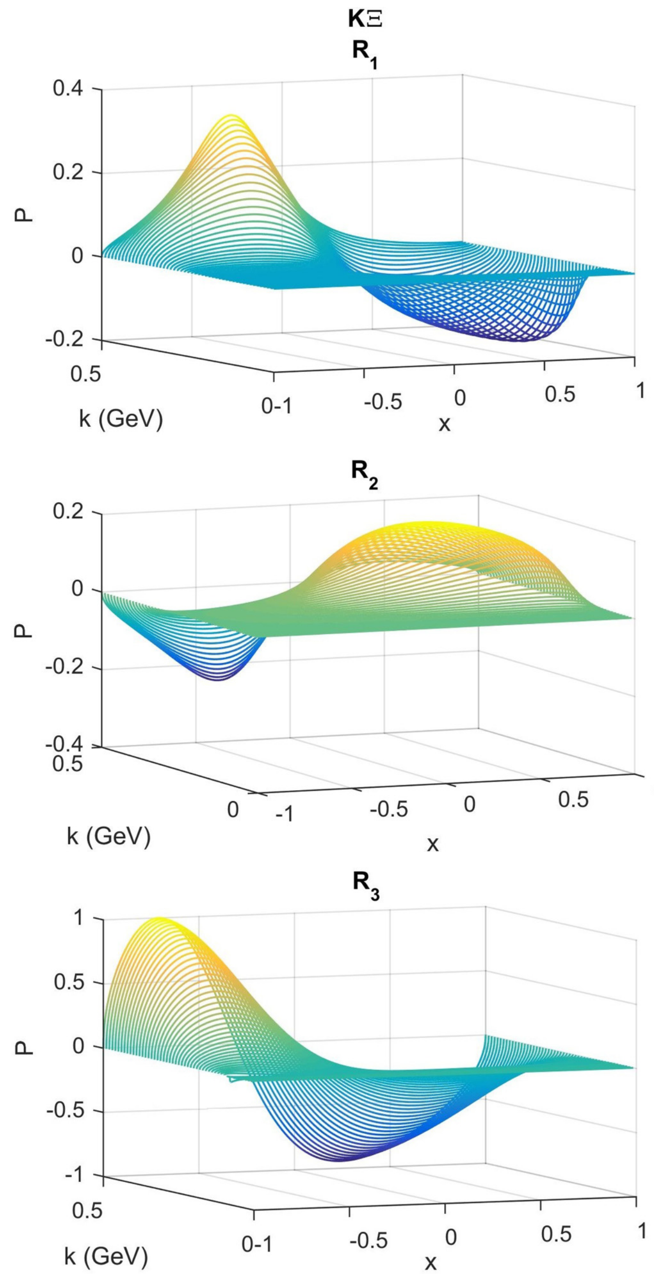

FIG. 6. $K\Xi$ polarization.

for the partial waves S, P_1, and P_3 in the isospin channels 0 and 1 as functions of the momentum k, in the center-of-mass system. The angular distributions are shown in Figs. 5, and in Fig. 6 the polarizations are shown, with a momentum gap of 10 MeV, as functions of $k = |\vec{k}|$ and $x = \cos\theta$ considering the S, P_1, and P_3 waves.

D. The $\overline{K}\Xi$ interaction

In this subsection we study the $\overline{K}\Xi$ scattering with the same procedure that has been adopted before. Basically, when considering the quark composition of the system one observes that is possible to make the change $K \to \overline{K}$ that corresponds to the change $s \to u$ in the Λ and Σ diagrams and $u \to s$ in the Ω one, as it may be seen in Fig. 7. So, for a Λ particle in the u channel [Fig. 7(b)], we have

$$A_{\Lambda}^{0} = \frac{g_{\Xi K\Lambda}^{2}}{4m_{\Xi}^{2}}(m_{\Lambda} + m_{\Xi})\left(\frac{u - m_{\Xi}^{2}}{u - m_{\Lambda}^{2}}\right), \tag{63}$$

$$B_{\Lambda}^{0} = \frac{g_{\Xi K\Lambda}^{2}}{4m_{\Xi}^{2}}\left[\frac{2m_{\Xi}(m_{\Xi} + m_{\Lambda}) + u - m_{\Xi}^{2}}{u - m_{\Lambda}^{2}}\right]. \tag{64}$$

For a Σ hyperon in the intermediate state, as show in Fig. 7(c), in the isospin 0 channel we have

$$A_{\Sigma}^{0} = -\frac{3g_{\Xi K\Sigma}^{2}}{4m_{\Xi}^{2}}(m_{\Sigma} + m_{\Xi})\left(\frac{u - m_{\Xi}^{2}}{u - m_{\Sigma}^{2}}\right), \tag{65}$$

$$B_{\Sigma}^{0} = -\frac{3g_{\Xi K\Sigma}^{2}}{4m_{\Xi}^{2}}\left[\frac{2m_{\Xi}(m_{\Xi} + m_{\Sigma}) + u - m_{\Xi}^{2}}{u - m_{\Sigma}^{2}}\right], \tag{66}$$

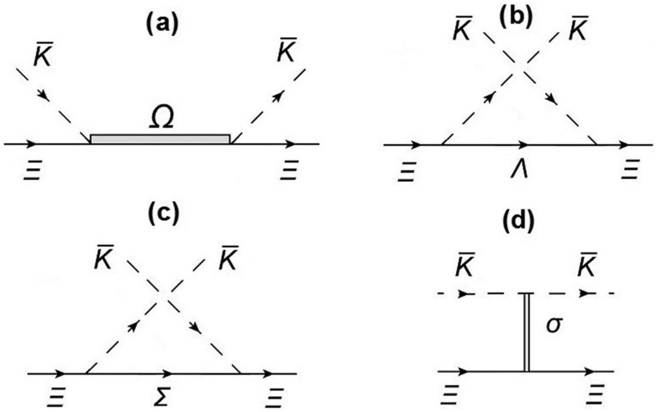

FIG. 7. The $\overline{K}\Xi$ scattering diagrams.

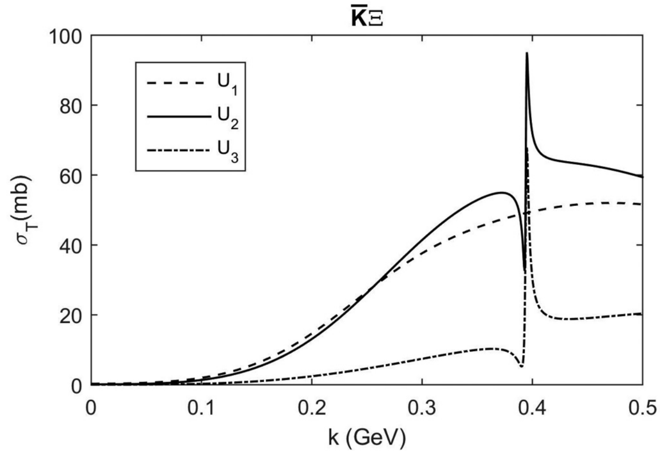

FIG. 8. Total cross section for the $\overline{K}\Xi$ interaction.

and for the isospin 1 channel the amplitudes are

$$A_{\Sigma}^{1} = \frac{g_{\Xi K\Sigma}^{2}}{4m_{\Xi}^{2}}(m_{\Sigma} + m_{\Xi})\left(\frac{u - m_{\Xi}^{2}}{u - m_{\Sigma}^{2}}\right), \tag{67}$$

$$B_{\Sigma}^{1} = \frac{g_{\Xi K\Sigma}^{2}}{4m_{\Xi}^{2}}\left[\frac{2m_{\Xi}(m_{\Xi} + m_{\Sigma}) + u - m_{\Xi}^{2}}{u - m_{\Sigma}^{2}}\right]. \tag{68}$$

Now, considering the Ω (spin 3/2) in the intermediate state shown in Fig. 7(a), we have

$$A_{\Omega} = \frac{g_{\Xi K\Omega}^{2}}{6}\left[\frac{2\hat{A} + 3(m_{\Xi} + m_{\Omega})t}{m_{\Omega}^{2} - s} + a_0\right], \tag{69}$$

$$B_{\Omega} = \frac{g_{\Xi K\Omega}^{2}}{6}\left[\frac{2\hat{B} + 3t}{m_{\Omega}^{2} - s} - b_0\right], \tag{70}$$

with

$$\hat{A} = 3(m_{\Xi} + m_{\Omega})(q_{\Omega})^2 + (m_{\Omega} - m_{\Xi})(E_{\Omega} + m_{\Xi})^2, \tag{71}$$

$$\hat{B} = 3(q_{\Omega})^2 - (E_{\Omega} + m_{\Xi})^2, \tag{72}$$

and

$$a_0 = -\frac{(m_{\Xi} + m_{\Omega})}{m_{\Omega}^{2}}\left(2m_{\Omega}^{2} + m_{\Xi}m_{\Omega} - m_{\Xi}^{2} + 2m_{\overline{K}}^{2}\right) + \frac{4}{m_{\Omega}^{2}}[(m_{\Omega} + m_{\Xi})Z + (2m_{\Omega} + m_{\Xi})Z^2]\left[s - m_{\Xi}^{2}\right], \tag{73}$$

$$b_0 = \frac{8}{m_{\Omega}^{2}}\left[\left(m_{\Xi}^{2} + m_{\Xi}m_{\Omega} - m_{\overline{K}}^{2}\right)Z + \left(2m_{\Xi}m_{\Omega} + m_{\Xi}^{2}\right)Z^2\right] + \frac{(m_{\Xi} + m_{\Omega})^2}{m_{\Omega}^{2}} + \frac{4Z^2}{m_{\Omega}^{2}}\left[s - m_{\Xi}^{2}\right]. \tag{74}$$

To evaluate the Ω(2012) contribution, the same procedure is adopted.

For Fig. 7(d), the σ-meson exchange, we use the same parametrization shown in the last subsection.

The reactions studied for the $\overline{K}\Xi$ interaction are labeled as U_1, U_2, and U_3:

$$U_1 \equiv \langle \overline{K}^0\Xi^0|T|\overline{K}^0\Xi^0\rangle = \langle K^-\Xi^-|T|K^-\Xi^-\rangle = T_1, \tag{75}$$

$$U_2 \equiv \langle \overline{K}^0\Xi^-|T|\overline{K}^0\Xi^-\rangle = \langle K^-\Xi^0|T|K^-\Xi^0\rangle = \tfrac{1}{2}(T_1 + T_0), \tag{76}$$

$$U_3 \equiv \langle K^-\Xi^0|T|\overline{K}^0\Xi^-\rangle = \langle \overline{K}^0\Xi^-|T|K^-\Xi^0\rangle = \tfrac{1}{2}(T_1 - T_0). \tag{77}$$

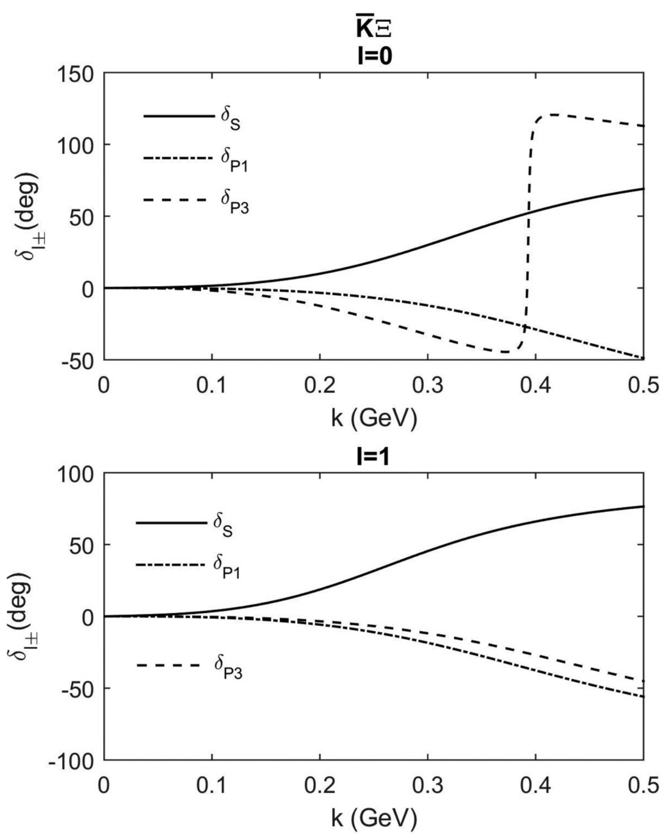

FIG. 9. Phase shifts for the partial-wave amplitudes S, P_1, and P_3 in the $\overline{K}\Xi$ scattering.

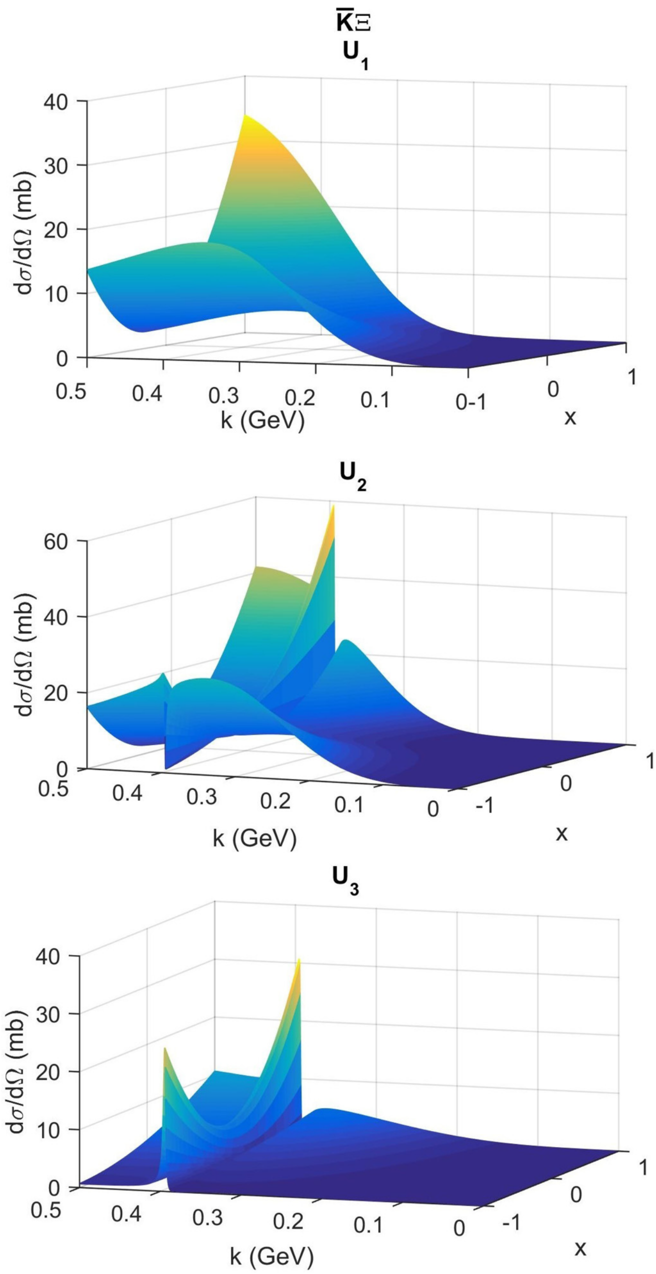

FIG. 10. Differential cross sections for the U_1, U_2, and U_3 reactions.

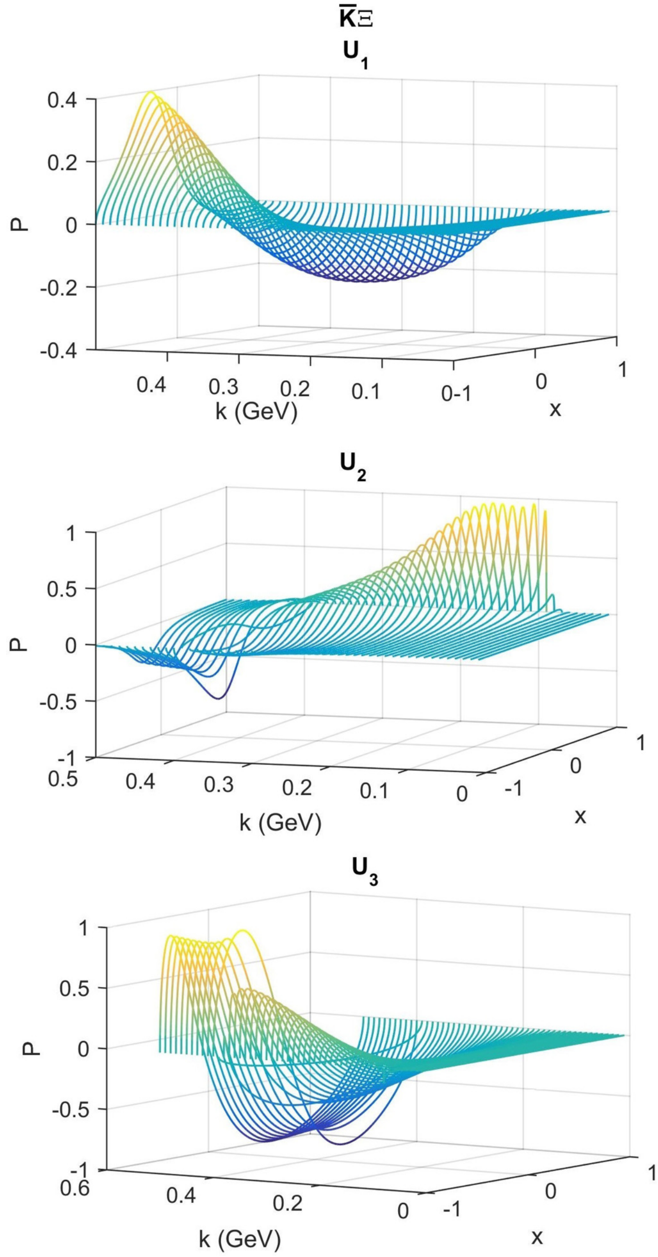

FIG. 11. Polarization in the U_1, U_2, and U_3 reactions.

The observables for the $\overline{K}\Xi$ scattering may be calculated in the same way it was done in the last section, as functions of the $\overline{K}$ momentum $k = |\vec{k}|$ and the scattering angle θ in the c.m. system. These results are shown in Figs. 8, 9, 10, and 11 for the total cross sections, phase shifts, differential cross sections, and polarizations, respectively. We considered $m_{\overline{K}} = m_K$.

IV. CONCLUSIONS

In this work we have studied the $K\Xi$ and $\overline{K}\Xi$ interactions. We have considered the effective Lagrangians shown in the preceding sections and calculated the amplitudes at the tree level. The phase shifts have been calculated and then the total cross sections, angular distributions, and polarizations, in an approximation that takes into account the S and P waves ($l = 0$ and $l = 1$). The higher values of l, at the considered energies, provide only small corrections and for this reason have been neglected for the sake of the simplicity of the model. However, it is very simple to include these waves with the formalism presented in this paper and no new expression is needed.

As we can see in Figs. 6 and 11, the polarizations may be large for some conditions. So, if one considers this effect as a final-state interaction in the study of the hyperon production in high energy collisions, some polarization may remain

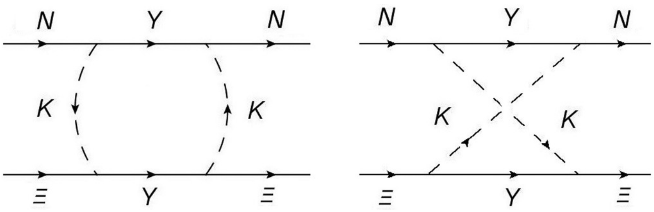

FIG. 12. Two kaons exchanged in the $N\Xi$ interaction.

when the average polarization of the process is calculated. In Refs. [15,16] only the pion-hyperon interaction has been considered and so an interesting study is to investigate this kind of process. This effect shall be calculated in a future work.

As it has been pointed out before, there are no experimental data for the reactions considered in this paper, so it is not possible to confirm the precision of our results. Although, some comments can be made about this question. When the model for the pion-hyperon interaction was proposed in Ref. [19], no experimental data were available, but our prediction for the $\pi\Lambda$ phase shifts, $\delta_P - \delta_S = 4.3^\circ$, at the Ξ mass, based on the same model were confirmed a few years later in the HyperCP experiment [1,2] where the value $\delta_P - \delta_S = (4.6 \pm 1.4 \pm 1.2)^\circ$ was obtained. We may also compare our results obtained in Ref. [22] for the $K\Lambda$ and $K\Sigma$ interactions with the ones presented in [38] by the ANL-Osaka Collaboration for the πp and γp collisions that produce $K\Lambda$ and $K\Sigma$. Observing the results, it is possible to identify a very similar behavior of the cross sections that shows that these results are consistent.

If one considers hypernuclei or hyperon stars, the $N\Xi$ interaction is a fundamental element. In the formulation of the interaction, if an accurate result is needed, processes of the types shown in Fig. 12 for the $K\Xi$ interactions may provide important contributions to the potential. We must emphasize that, even considering all these systems where the $K\Xi$ and $\overline{K}\Xi$ interactions play important roles, this study provides important information about the proprieties of the strong interactions, and we think that this fact justifies the exploration of this subject and indicates the need for future experimental results.

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APPENDIX

1. Kinematics relations

Considering a process where p and p' are the initial and final hyperon four-momenta, and k and k' are the initial and final meson four-momenta, the Mandelstam variables are given by

$$s = (p+k)^2 = (p'+k')^2 = m^2 + m_K^2 + 2Ek_0 - 2\vec{k}\cdot\vec{p}, \tag{A1}$$

$$u = (p'-k)^2 = (p-k')^2 = m^2 + m_K^2 - 2Ek_0 - 2\vec{k}'\cdot\vec{p}, \tag{A2}$$

$$t = (p-p')^2 = (k-k')^2 = 2|\vec{k}|^2 x - 2|\vec{k}|^2\,. \tag{A3}$$

In the center-of-mass frame, the energies are defined as

$$k_0 = k_0' = \sqrt{|\vec{k}|^2 + m_K^2}, \tag{A4}$$

$$E = E' = \sqrt{|\vec{k}|^2 + m^2}, \tag{A5}$$

and the total momentum is null:

$$\vec{p} + \vec{k} = \vec{p}' + \vec{k}' = 0. \tag{A6}$$

We also define the variable

$$x = \cos\theta, \tag{A7}$$

where θ is the scattering angle. Other variables of interest are

$$\nu_r = \frac{m_r^2 - m^2 - k\cdot k'}{2m}, \tag{A8}$$

$$\nu = \frac{s-u}{4m} = \frac{2Ek_0 + |\vec{k}|^2 + |\vec{k}|^2 x}{2m}, \tag{A9}$$

$$k\cdot k' = m_K^2 + |\vec{k}|^2 - |\vec{k}|^2 x = k_0^2 - |\vec{k}|^2 x, \tag{A10}$$

where m, m_r, and m_K are the hyperon mass, the resonance mass, and the kaon mass, respectively.

For the energy and the three-momentum of the intermediary particles we also have the following relations:

$$(E_{B^*} \pm m_\Xi) = \frac{(m_{B^*} \pm m_\Xi)^2 - m_K^2}{2m_{B^*}}, \tag{A11}$$

$$(q_{B^*})^2 = |\vec{q}_{B^*}|^2 = E_{B^*}^2 - m_\Xi^2 = (E_{B^*} + m_\Xi)(E_{B^*} - m_\Xi), \tag{A12}$$

where E_{B^*} and $\vec{q}_{B^*}$ are the energy and the momentum of the intermediate baryon B^* in the center-of-mass frame, respectively.

2. Breit-Wigner expression

The relativistic Breit-Wigner expression is determined in terms of experimental quantities:

$$\delta_{l\pm} = \tan^{-1}\left[\frac{\Gamma\left(\frac{|\vec{k}|}{|\vec{k}_0|}\right)^{2J+1}}{2(m_r - \sqrt{s})}\right], \tag{A13}$$

where Γ is the width branch, $|\vec{k}_0|$ is the momentum at the peak of the resonance in the center-of-mass system, m_r is its mass, and J is the total angular momentum (spin) of the resonance.

[1] A. Chakravorty *et al.*, Phys. Rev. Lett. **91**, 031601 (2003).
[2] M. Huang *et al.*, Phys. Rev. Lett. **93**, 011802 (2004).
[3] H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C **28**, 1812 (1983).
[4] H. Pilkuhn, *The Interaction of Hadrons* (North-Holland, Amsterdam, 1967).
[5] E. T. Osypowski, Nucl. Phys. B **21**, 615 (1970).
[6] M. G. Olsson and E. T. Osypowski, Nucl. Phys. B **101**, 136 (1975).
[7] R. S. Hayano *et al.*, Phys. Lett. B **231**, 355 (1989).
[8] T. Nagae *et al.*, Phys. Rev. Lett. **80**, 1605 (1998).
[9] S. Bart *et al.*, Phys. Rev. Lett. **83**, 5238 (1999).
[10] J. Schaffner, C. Greiner, and H. Stöcker, Phys. Rev. C **46**, 322 (1992).
[11] N. K. Glendenning, Astrophys. J. **293**, 470 (1985).
[12] J. Schaffner-Bielich, Nucl. Phys. A **804**, 309 (2008).
[13] M. Baldo, G. F. Burgio, and H. J. Schulze, Phys. Rev. C **61**, 055801 (2000).
[14] I. Vidana, A. Polls, A. Ramos, L. Engvik, and M. Hjorth-Jensen, Phys. Rev. C **62**, 035801 (2000).
[15] C. C. Barros, Jr. and Y. Hama, Int. J. Mod. Phys. E **17**, 371 (2008).
[16] C. C. Barros, Jr. and Y. Hama, Phys. Lett. B **699**, 74 (2011).
[17] F. Becattini and M. A. Lisa, Ann. Rev. Nucl. Part. Sci. **70**, 395 (2020).
[18] F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, Phys. Rev. C **95**, 054902 (2017).
[19] C. C. Barros and Y. Hama, Phys. Rev. C **63**, 065203 (2001).
[20] C. C. Barros, Jr., Phys. Rev. D **68**, 034006 (2003).
[21] C. C. Barros, Jr. and M. R. Robilotta, Eur. Phys. J. C **45**, 445 (2006).
[22] M. G. L. N. Santos and C. C. Barros, Jr., Phys. Rev. C **99**, 025206 (2019).
[23] M. G. L. Nogueira-Santos and C. C. Barros, Jr., Int. J. Mod. Phys. E **29**, 2050013 (2020).
[24] M. G. L. Nogueira-Santos and C. C. Barros, Jr., Eur. Phys. J. C **80**, 578 (2020).
[25] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[26] T. M. Aliev, K. Azizi, Y. Sarac, and H. Sundu, Eur. Phys. J. C **78**, 894 (2018).
[27] V. Stoks and T. A. Rijken, Nucl. Phys. A **613**, 311 (1997).
[28] J. J. Swart, Rev. Mod. Phys. APS, **35**, 916 (1963).
[29] T. Ericson and W. Weise, *Pions and Nuclei* (Oxford University Press, London, 1988).
[30] H. Pilkuhn *et al.*, Nucl. Phys. B **65**, 460 (1973).
[31] I. J. General and S. R. Cotanch, Phys. Rev. C **69**, 035202 (2004).
[32] G. S. Yang and H.-Ch. Kim, Phys. Lett. B **785**, 434 (2018).
[33] G. S. Yang and H.-Ch. Kim, Phys. Rev. C **92**, 035206 (2015).
[34] J. Gasser, M. E. Sainio, and A. Švarc, Nucl. Phys. B **307**, 779 (1988); T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999); J. High Energy Phys. 06 (2001) 017.
[35] J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. B **253**, 252 (1991); **253**, 260 (1991).
[36] A. I. L'vov, S. Scherer, B. Pasquini, C. Unkmeir, and D. Drechsel, Phys. Rev. C **64**, 015203 (2001).
[37] M. R. Robilotta, Phys. Rev. C **63**, 044004 (2001).
[38] H. Kamano, S. X. Nakamura, T.-S. H. Lee, and T. Sato, Phys. Rev. C **88**, 035209 (2013).