Exploring the Λ-deuteron interaction via correlations in heavy-ion collisions

J. Haidenbauer

Institute for Advanced Simulation, Institut für Kernphysik (Theorie), and Jülich Center of Hadron Physics,
Forschungszentrum Jülich, D-52425 Jülich, Germany

Λ-deuteron two-particle momentum correlation functions, to be measured in high-energy heavy-ion collisions, are investigated. In particular, the question is addressed whether such correlations can serve as an additional and alternative source of information on the elementary ΛN interaction. The study is performed within the Lednicky-Lyuboshits formalism, utilizing an effective range expansion for the two relevant S-wave Λd amplitudes with parameters taken from the literature. It is found that in collisions characterized by a large emitting source the Λd correlation function is predominantly sensitive to the quartet state ($^4S_{3/2}$). In contrast, for small source sizes the contribution from the doublet partial wave ($^2S_{1/2}$) could be significant. Though the latter is constrained by the hypertriton binding energy, its present experimental uncertainty impedes an accurate determination of the doublet amplitude and, in turn, complicates conclusions on the quartet state.

I. INTRODUCTION

Contrary to the nucleon-nucleon (NN) interaction, the forces between strange baryons (Λ, Σ, Ξ) and nucleons are still poorly understood, not least because of the limited number and accuracy of available scattering data [1–4]. At least with regard to the ΛN interaction the main features such as the overall strength are roughly known due to the aforementioned scattering data but also from measurements and studies of hypernuclei [5–7]. Indeed, the spectroscopic study of Λ hypernuclei, especially the γ-ray hypernuclear spectroscopy, allows also conclusions on the spin dependence of the ΛN effective interaction [5,6]. However, so far there is no direct empirical constraint on the spin dependence of the elementary ΛN interaction, necessary to resolve the relative strength of the forces in the two possible spin configurations, $S = 0, 1$, notably in the $^1S_0$ and $^3S_1$ states. With respect to that, few-body systems constitute a valuable complementary source of information [8]. In particular, this concerns the bound systems $^3_\Lambda$H (hypertriton) and the four-body states $^4_\Lambda$H and $^4_\Lambda$He, where empirical values for the binding energies have been available for a long time already [9]. Light systems are amenable to a microscopic treatment within, e.g., the Faddeev/Yakubovsky approach [10–13] or via *ab initio* calculations based on the no-core shell model [14–16], allowing for a rigorous inclusion of the underlying ΛN interaction and of the important coupling to the ΣN system. Clearly, there should be also three-body forces (3BF) [17], which complicate conclusions on the elementary ΛN interaction from few-body studies. However, since the systems are very light and only loosely bound, effects from 3BFs are expected to be small. Indeed, this notion has been adopted by the Jülich-Bonn-Munich group in their studies of the hyperon-nucleon (YN) interaction within chiral effective field theory (EFT) by considering not only the Λp (and ΣN) data but also

the hypertriton binding energy to fix the interaction strength in the spin-singlet ($^1S_0$) and spin-triplet ($^3S_1$) channels [18–21].

In the present paper I want to explore the potential of an additional and independent source of information, namely the Λ-deuteron (Λd) system. Certainly, empirical information on direct Λd scattering is even harder to get than on, say, Λp scattering, and as far as I know has never been considered. However, there is another possibility to access such information, namely by means of two-particle momentum correlation functions [22–29], measured in heavy-ion collisions and/or high-energetic pp collisions. Such correlations were initially considered as a tool to learn more about the emission process and/or the properties of the emitting source. But they provide likewise a doorway to information on hadron-hadron forces at low energies, specifically on those that are inaccessible by other means. Experiments with that aim in mind have been suggested and (in part) already successfully performed for multistrange systems such as $\Lambda\Lambda$ [26,27,30–32], $\Xi^- p$ [29,33–35], $p\Omega$ [35–38], or $\Omega\Omega$ [38], and also for charmed baryons [39]. Extending the measurements to Λd correlations could be feasible too, judging from the available production yields [40]. In fact, in the past, experimental studies of correlations for pd, dd and even for light nuclei have been already performed [41–45], and a measurement of $K^- d$ correlation functions is in progress [46]. Actually, even Λd has been on the agenda [47,48].

Whereas calculations of the hypertriton abound in the literature there is little to be found about Λd scattering. This is not too surprising, since, as said before, the prospects of pertinent scattering experiments are practically nonexistent. Nonetheless, there is a series of Faddeev-type studies and also variational calculations starting with the pioneering work of Schick and collaborators in the 1960s [49–52] and followed by others [53–59]. Recently, Λd scattering at energies close to

the threshold has been studied within pionless effective field theory ($\not\pi$EFT) [60–62].

The present work is intended to serve as illustration for what can be expected from measuring Λd correlations. It is an exploratory study and, therefore, it is done on a simple technical level. For the interpretation of actual data on Λd correlation functions it is certainly advisable to perform solid and full-fledged calculations of the Λd system. It goes without saying that such calculations are challenging and technically demanding. Since the hypertriton is weakly bound and the binding energy is known [9] (see, however, Refs. [63,64]), effective range theory can be used to pin down the Λd S-wave amplitude in the spin-doublet state ($^2S_{1/2}$) at low energies in an essentially model-independent way [60,65]. The situation is much less satisfactory for the spin-quartet ($^4S_{3/2}$) amplitude as demonstrated by the results reported in Ref. [62]. Thus, the essential question to be addressed is in how far a measurement of the Λd correlation function could help to pin down the latter amplitude.

The paper is structured in the following way. In Sec. II the formalism for two-particle momentum correlation functions is briefly reviewed. Specifically, the simple and compact expression for the correlation function due to Lednicky and Lyuboshits [23] is provided, which is used for the present investigation. Results for the Λd correlation functions are presented in Sec. III. A variety of amplitudes for the $^2S_{1/2}$ and $^4S_{3/2}$ partial waves is considered, all taken from the literature, and their influence on the resulting correlation functions is discussed. In addition the role of the size of the emitting source (parameterized in terms of a Gaussian source function) on the results is explored. The paper ends with concluding remarks.

II. CORRELATION FUNCTION

The formalism for calculating the two-particle correlation function has been described in detail in various publications [22–28]. I summarize it here very briefly and provide only an overview of the essential formulas. The two-particle momentum correlation function is defined by

$$C(\mathbf{p_1}, \mathbf{p_2}) = \frac{\int d^4 x_1 d^4 x_2 S_1(x_1, \mathbf{p_1}) S_2(x_2, \mathbf{p_2}) |\Psi^{(-)}(\mathbf{r}, \mathbf{k})|^2}{\int d^4 x_1 d^4 x_2 S_1(x_1, \mathbf{p_1}) S_2(x_2, \mathbf{p_2})}$$
$$\simeq \int d\mathbf{r} S_{12}(\mathbf{r}) |\Psi^{(-)}(\mathbf{r}, \mathbf{k})|^2. \quad (1)$$

Here the quantity $S_i(x_i, \mathbf{p}_i)$ $(i = 1, 2)$ is the single-particle source function of particle i with momentum $\mathbf{p}_i$. As already indicated by Eq. (1), I evaluate the quantity in question in the center-of-mass (c.m.) frame where the wave function $\Psi^{(-)}$ is then a function of the relative coordinate $\mathbf{r}$ and the c.m. momentum, $\mathbf{k} = (m_2 \mathbf{p_1} - m_1 \mathbf{p_2})/(m_1 + m_2)$, and $S_{12}(\mathbf{r})$ is the normalized pair source function that depends likewise only on the relative coordinate. Furthermore, I consider only interactions in the S wave.

Assuming a static and spherical Gaussian source with radius R, $S(x, \mathbf{p}) \propto \exp(-\mathbf{x}^2/2R^2)\delta(t - t_0)$, a partial wave expansion can be performed straightforwardly and the correlation function can be written in a compact form [27]. In

particular, for systems with two nonidentical particles such as Λp or Λd the correlation function amounts to

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2 dr \, S_{12}(\mathbf{r})[|\psi(k, r)|^2 - |j_0(kr)|^2], \quad (2)$$

where the properly normalized source function is given by $S_{12}(\mathbf{r}) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$ and $j_l(kr)$ is the spherical Bessel function for $l = 0$. $\psi(k, r)$ is the scattering wave function. For two-body systems it can be obtained easily by solving the Schrödinger equation for a given potential, but also from the Lippmann-Schwinger (LS) equation [29]. In case of Λd, in principle, the wave functions can be deduced from the solution of the configuration-space Faddeev equations or from variational calculations [66–68]. As a less ambitious alternative one could construct effective Λd two-body potentials [69] following corresponding studies for the Nd case [70,71], and use them for generating wave functions. In general, S-wave states of two particles can be formed with different spins. For example, ΛN can be in the partial waves $^1S_0$ and $^3S_1$, respectively, and Λd in the $^2S_{1/2}$ and $^4S_{3/2}$ states. Accordingly, an averaging over the spin has to be performed in Eq. (2). It is usually assumed that the weight is the same as for free scattering. For the Λd system the weights are $1/3$ and $2/3$, respectively, i.e., $|\psi(r, k)|^2 \to \frac{1}{3}|\psi_{1/2}(r, k)|^2 + \frac{2}{3}|\psi_{3/2}(r, k)|^2$.

A much simpler expression for the correlation function can be derived if one assumes that the wave function entering Eq. (2) can be approximated by its asymptotic form, $\psi(k, r) \to j_0(kr) + f(k) \exp(ikr)/r$. Then one arrives at a formula often called the Lednicky-Lyuboshits (LL) approach or model [23]:

$$\int_0^\infty 4\pi r^2 dr \, S_{12}(r)[|\psi(k, r)|^2 - |j_0(kr)|^2]$$
$$\approx \frac{|f(k)|^2}{2R^2} F(r_0) + \frac{2\text{Re}f(k)}{\sqrt{\pi}R} F_1(x) - \frac{\text{Im}f(k)}{R} F_2(x). \quad (3)$$

Here $f(k)$ is the scattering amplitude which is related to the S matrix by $f(k) = (S - 1)/2ik$, and in practical applications is often replaced by the effective range expansion (ERE), i.e., $f(k) \approx 1/(-1/a_0 + r_0 k^2/2 - ik)$ with a_0 and r_0 being the scattering length and the effective range, respectively. Furthermore, $F_1(x) = \int_0^x dt \, e^{t^2 - x^2}/x$ and $F_2(x) = (1 - e^{-x^2})/x$, with $x = 2kR$. The factor $F(r_0) = 1 - r_0/(2\sqrt{\pi}R)$ is a correction that accounts for the deviation of the true wave function from the asymptotic form [23,27]. The approximation (3) works reasonably well for source sizes R larger than the range of interaction. For smaller values of R there might be noticeable differences between the results with the LL formula and those based on the full wave function [23,28].

In the present work I show results for three different R values, where the choice is motivated by values suggested by analyses of measurements of the Λp correlation function in pp collisions at 7 TeV by the ALICE Collaboration ($R = 1.2$ fm) [31] and that of $p\Omega$ in peripheral and central Au + Au collisions at 200 GeV by the STAR Collaboration ($R = 2.5, 5$ fm) [36]. For a general discussion of the dependence of correlation functions on the source size in combination with the scattering length see, e.g., Refs. [28,38].

As noted in Ref. [72], Eq. (2) is true only if the deuteron behaves like an elementary particle, in the sense that it is directly emitted from the source. If it is formed afterwards then modifications are required as discussed in detail in that reference. Specifically, in the latter case there will be a modification of the source size R. I perform the calculations for fixed R values and, thus, I will not consider this effect here, which is in the order of 15% or so [72]. In any case, it is an open question which R values to expect for Λd production in different collisions and at different collision energies.

III. Λd SCATTERING AND Λd CORRELATION FUNCTIONS

In this exploratory study I calculate the Λd correlation functions in the LL formalism (3), based on Λd ERE parameters taken from the literature. Values for the parameters in the doublet S wave can be found in a variety of works [56–61]. As mentioned above, there is a bound state in this partial wave, the hypertriton, which provided an important incentive for pertinent calculations. Indeed the $^3_\Lambda$H binding energy is related to the ERE parameters in terms of the Bethe formula [65], which reads

$$\frac{1}{a_{1/2}} = \gamma - \frac{1}{2} r_{1/2} \gamma^2. \tag{4}$$

Here γ is the binding momentum; the binding energy itself is given by $B_\Lambda = \frac{\gamma^2}{2\mu_{\Lambda d}}$ with $\mu_{\Lambda d}$ being the reduced mass of the Λd system. Since the binding energy is experimentally known, $B_\Lambda = 0.13 \pm 0.05$ MeV [9], and very small, it provides substantial constraints on the ERE parameters and, in turn, on the contribution of this partial wave to the Λd correlation function.

For the present study I consider ERE parameters from three-body calculations, which predict a $^3_\Lambda$H binding energy close to the aforementioned value of 0.13 MeV. This is fulfilled for some of the phenomenological potential sets considered in Ref. [56] and for the calculation of Hammer [60] based on $\not\pi$EFT. Actually, in the latter work the binding energy is used as input. The found ERE parameters are $a_{1/2} = 16.8^{+4.4}_{-2.4}$ fm, $r_{1/2} = 2.3 \pm 0.3$ fm, where the errors are due to the uncertainty in the $^3_\Lambda$H binding energy. With regard to potential models I take the result for the combination GC1-E5rb from Cobis et al. [56] (cf. Table 6), i.e., $a_{1/2} = 16.3$ fm, $r_{1/2} = 3.2$ fm. Taking into account the uncertainty in B_Λ leads to $a_{1/2} = 16.3^{+4.0}_{-2.1}$ fm. Since the calculations in Ref. [56] suggest that $r_{1/2}$ is largely insensitive to variations of the potentials (the hypertriton binding energy), the effective range is kept fixed. Note that both sets of effective range parameters are reasonably well in line with Eq. (4).

Results for the ERE parameters in the quartet state are much harder to find in the literature. Here the ones from the recent calculations of Schäfer et al. [62] performed in $\not\pi$EFT are used. This work reports values (cf. Table II therein) ranging from $a_{3/2} = -17.3$ fm, $r_{3/2} = 3.6$ fm, based on a ΛN interaction fixed by the scattering lengths of Alexander et al. [2] over $a_{3/2} = -10.8$ fm, $r_{3/2} = 3.8$ fm (ΛN properties adjusted to the Nijmegen YN potential NSC97f [73])

to $a_{3/2} = -7.6$ fm, $r_{3/2} = 3.6$ fm, with ΛN fixed by the YN results from a potential derived within SU(3) chiral EFT up to next-to-leading order (NLO13) [19]. An even larger value is suggested in Ref. [59], namely $a_{3/2} = -31.9$ fm, but the pertinent value for $r_{3/2}$ is not provided. Actually, for some potentials considered in that work the quartet scattering length is positive, in other words the $I = 0$, $J^P = \frac{3}{2}^+$ state is predicted to be bound. Since there is no evidence for that experimentally this possibility is not considered here.

The results for the Λd correlation function are presented in Figs. 1 (source radius $R = 1.2$ fm), 2 ($R = 2.5$ fm), and 3 ($R = 5$ fm), respectively, for different combinations of the doublet and quartet amplitudes. Evidently, the available studies suggest that the scattering lengths for the doublet and quartet S states are both large. Indeed, while the former partial wave is governed by the shallow $^3_\Lambda$H bound state, the latter is characterized by the presence of a near-threshold virtual state, as pointed out in Ref. [62]. Standard experiments allow one only to measure an average over the two states. As mentioned above, here the usual assumption is made that the weights of the spin components in the correlation function is the same as for free scattering (1/3 and 2/3, respectively), which puts a somewhat larger weight on the quartet contribution. However, equally important for the concrete results in the Λd case is the characteristic dependence of the correlation function on the source radius R, cf. the exemplary discussion in Refs. [28,38]. Specifically, for a combination of large scattering length and small R the correlation function $C(k)$ is significantly enhanced at small values of k, independently of the sign of a. Accordingly, one has to expect that in this limit the two Λd partial waves yield similar effects. With increasing R the enhancement of $C(k)$ decreases continuously in case of a moderately attractive interaction (negative a). On the other hand, if a bound state is present (positive a), the $C(k)$ drops rather rapidly and eventually even falls below the nominal value of $C(k) \equiv 1$, i.e., there is a depletion as compared to the case without any two-particle interaction [28,38]. Thus, now the two partial-wave contributions should show a rather different trend.

After these general statements, let me discuss the results in more detail. The predictions for $R = 1.2$ fm displayed in Fig. 1 represent roughly the first scenario. As expected $C(k)$ is strongly enhanced at small momenta; compare this with measurements and calculations for the Λp system [31,74–76] where the scattering lengths are typically in the order of 2 fm. Besides that, one sees a sizable dependence of the correlation functions on the properties in the doublet wave. For the Cobis amplitude the contribution of the $^2S_{1/2}$ itself is relatively small and well separated from the results that include the quartet contributions based on the effective range parameters from Schäfer, irrespective of the uncertainty due to the $^3_\Lambda$H binding energy. In case of the Hammer parameters the doublet contribution is much larger and the uncertainties too. Indeed, now there is an overlap between the various results including the quartet contribution. A closer inspection revealed that the differences in the doublet contribution are primarily caused by the differences in the effective range r. Thus, a more elaborate evaluation of the doublet amplitude within EFT, beyond the present LO level, could presumably allow one to pin down the
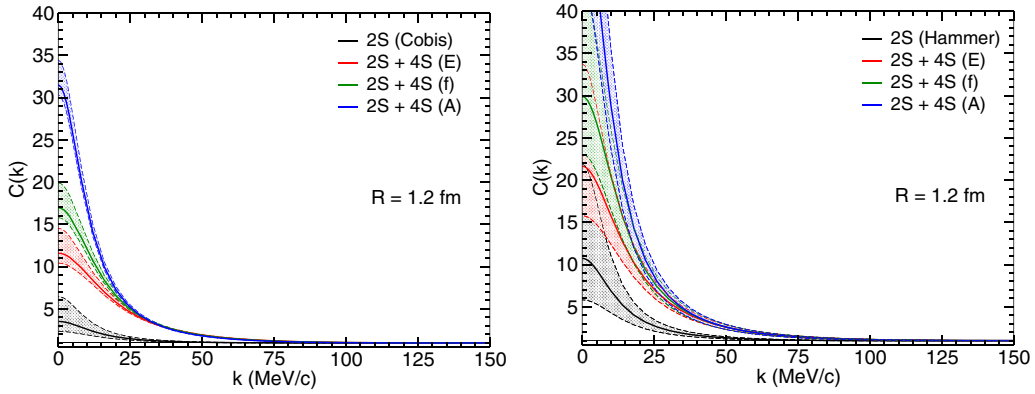
FIG. 1. Λd correlation functions for the source size $R = 1.2$ fm. Spin-averaged results are shown wherein the $^2S_{1/2}$ state either the ERE parameters of Cobis [56] (left) or of Hammer [60] (right) are employed. For the $^4S_{3/2}$ state results from Schäfer [62] are used, building on ΛN scattering lengths from Alexander (A) [2], NSC97f (f) [73], or chiral EFT (E) [19] (from top to bottom), see text. The bands are the error due to the uncertainty in the $^3_\Lambda$H binding energy.

effective range more reliably. As known from studies of the NN and ΛN systems within chiral EFT, LO calculations tend to underestimate the effective range. Of course, a reduction in the uncertainty of the $^3_\Lambda$H binding energy would be also extremely useful. Anyway, under the present circumstances one has to concede that drawing reliable conclusions on the magnitude of the quartet amplitude from measurements in reactions where the source size is small is difficult.

The second scenario discussed above is more or less realized in the results for $R = 5$ fm presented in Fig. 3. Here the signal is clearly dominated by the quartet contribution. Those from the doublet state are small so that the difference between the ERE parameters from Cobis and Hammer and even the uncertainty in the $^3_\Lambda$H binding energy do not play a decisive role. Therefore, a measurement of the correlation function under these conditions would certainly yield valuable constraints on the quartet contribution and, in turn, on the corresponding scattering length.

Disentangling the doublet and quartet amplitudes from investigating the source size dependence itself appears to be difficult. However, such a study certainly provides additional constraints once results for large values of R are available and/or provided that independent information allows one to narrow down the uncertainty in the doublet amplitude.

Note that the result for the quartet channel depends primarily on the ΛN spin-triplet interaction [11,59] and, thus, provides directly constraints on the latter quantity. But of course, there can be also contributions from 3BFs. Indeed, in the $\not\pi$EFT calculation of Schäfer *et al.* their influence appears to be significant [62]. In that work the arising 3BF is fixed by considering the binding energy of the 1^+ state of $^4_\Lambda$H. However, one should not forget that in $\not\pi$EFT 3BFs appear at LO [60,62,77]. I expect the situation to be different in calculations within chiral EFT where pion exchange and, specifically, the important coupling of ΛN to ΣN are taken into account explicitly [20]. In this scheme 3BFs, which appear first at next-to-next-to-leading order (N^{2}LO) [17], should be much weaker. In any case, besides the direct inclusion of the ΛN-ΣN coupling, an evaluation of the Λd amplitudes in chiral EFT and with YN forces beyond the LO level [19,20] would be desirable for other reasons too. In particular, as mentioned, such a calculation should lead to more reliable results for the Λd effective range r.

Finally, it should be mentioned that evidence for a possibly larger hypertriton binding energy, $B_\Lambda = 0.41 \pm 0.12$ MeV, has been reported recently by the STAR Collaboration [63]. Such an energy implies $a_{1/2} = 10.2^{+1.5}_{-0.9}$ fm, assuming the Cobis value for $r_{1/2}$. Clearly, with that the contribution of the
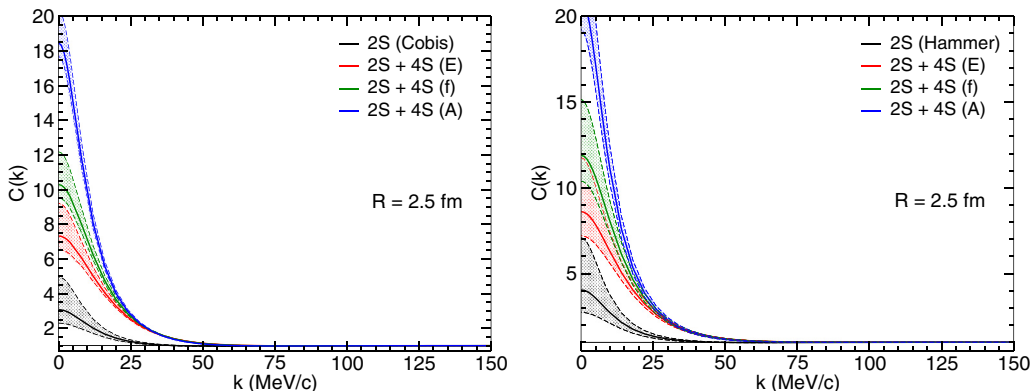

FIG. 2. Λd correlation functions for the source size $R = 2.5$ fm. Same description of curves as in Fig. 1.
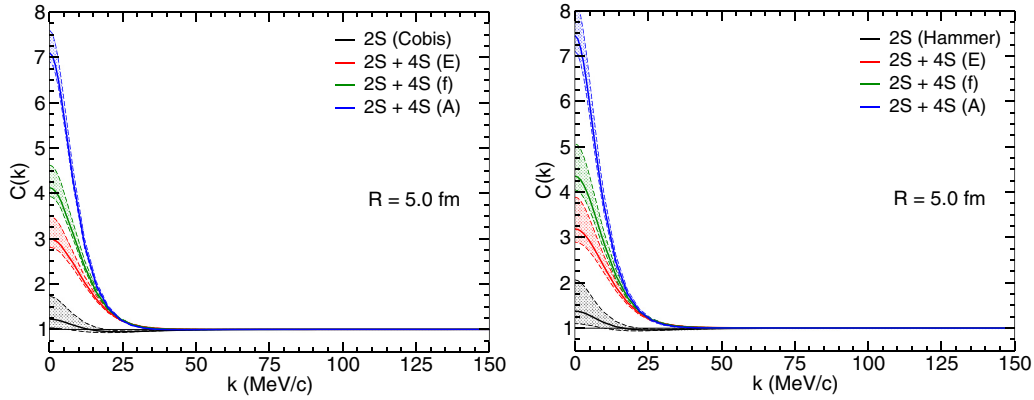
FIG. 3. Λd correlation functions for the source size $R = 5$ fm. Same description of curves as in Fig. 1.

doublet state to $C(k)$ (and its uncertainty) would be drastically reduced. It is below the lower bound of the uncertainty shown in the figures, a situation that would be certainly beneficial for the determination of the quartet amplitude from a measurement of the Λd correlation function. In fact, a recent calculation based on $\pi\!\!\!/$EFT suggests also a distinctly smaller value, namely $a_{1/2} = 13.8^{+3.75}_{-2.03}$ fm [61]. However, the central value here leads to a negative result for $r_{1/2}$, when inserted in the Bethe formula together with the $^3_\Lambda$H binding energy of 0.13 MeV. There is a well-known anomaly in the corresponding doublet state of nd scattering, which requires a modification of the effective range function [78]. But in case of Λd there is no indication for an unusual behavior, judging from the plot of $k \cot \delta$ in Ref. [61]. Incidentally, assuming that $r_{1/2} \equiv 0$ leads to $a_{1/2} = 14.6^{+4.1}_{-2.2}$ fm based on Eq. (4).

Since the correlation functions for small momenta are rather large selected results are shown on a different scale in Fig. 4 so that one can see the behavior in the region of $k = 25$–100 MeV/c in detail. There is a sizable effect from the quartet state in the region of $k = 25$–50 MeV/c. On the other hand, differences between the different strength of the quartet amplitudes are rather difficult to resolve in this momentum region. One should be aware that in the calculation of Schäfer *et al.* all the effective ranges are practically the same and about 3.6–3.8 fm. It remains unclear whether that is a realistic range or rather a consequence of the LO treatment. Noticeably

different values of $r_{3/2}$ could lead to stronger variations in the momentum region of $k = 25$–100 MeV/c. In this context let me mention that the momentum corresponding to the Λd breaking threshold is $k \approx 55$ MeV/c. However, judging from results for the elastic and total Λd cross sections shown in the works of Schick and collaborators [49,52], and by results for the $^4S_{3/2}$ nd phase shift, see, e.g., Ref. [79], drastic changes in the Λd amplitude for energies near to or above the breakup threshold are rather unlikely, at least in the momentum region up to 100 MeV/c, where $C(k)$ is noticeably different from 1.

IV. CONCLUSIONS

In the present paper I have investigated the potential of Λd two-particle momentum correlation functions as an additional and alternative source of information on the elementary ΛN interaction. Since the present work is primarily an exploratory study, intended as an illustration for what can be expected from measuring Λd correlations, it has been done on a simple technical level. Specifically, it has been performed within the Lednicky-Lyuboshits approach [23] and by utilizing an effective range expansion for the relevant Λd amplitudes. The effective range parameters for the two S-wave states that can contribute, the $^2S_{1/2}$ and $^4S_{3/2}$ partial waves, have been taken from results available in the literature [56,60,62].
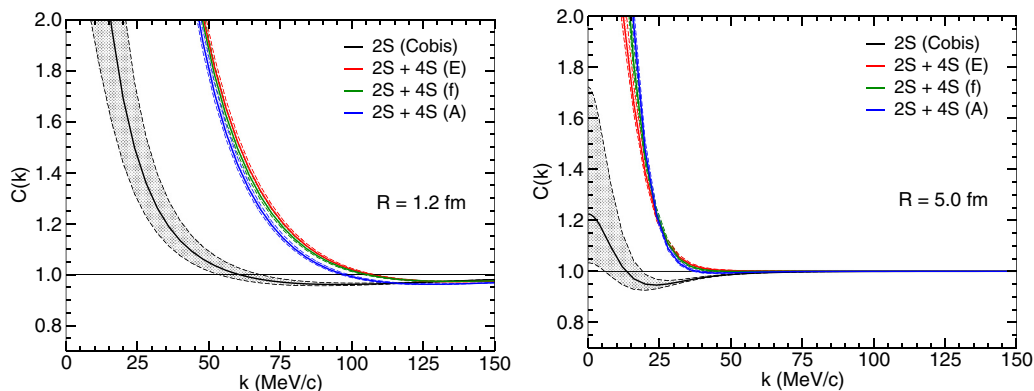

FIG. 4. Λd correlation functions for the source sizes $R = 1.2$ and 5 fm, based on the $^2S_{1/2}$ effective range parameters of Cobis [56]. Same description of curves as in Fig. 1.

Of specific interest is the situation in the $^4S_{3/2}$ partial wave, since presently there is no constraint on its properties. Thus, the main question is whether measurements of the correlation function could allow one to pin it down. The $^4S_{3/2}$ state is strongly linked to the properties of the ΛN interaction in the spin-triplet ($^3S_1$) state and could facilitate conclusions on the pertinent interaction strength. Note that the $^2S_{1/2}$ partial wave is fixed to a large extent by the presence of a weakly bound state, the $^3_\Lambda$H.

The results suggest that measurements of the Λd correlation function in configurations characterized by a large emitting source such as in central heavy-ion collisions [38] look indeed very promising. For large source sizes the contribution from the $^2S_{1/2}$ partial wave is significantly suppressed, as a consequence of the presence of the hypertriton, and one is predominantly sensitive to the quartet state. In contrast, for small sizes both states contribute in a similar way

and with comparable magnitude. Given the present uncertainty in the doublet effective range parameters and the $^3_\Lambda$H binding energy, respectively, conclusions are much more difficult to draw based on the experiment alone. Nonetheless, also in this case a more refined evaluation of the effective range parameters in the $^2S_{1/2}$ state, say based on a Λd calculation utilizing NLO ΛN forces [19,20], and/or more accurate data on the $^3_\Lambda$H binding energy could improve the prospects.

[1] R. Engelmann, H. Filthuth, V. Hepp, and E. Kluge, Phys. Lett. **21**, 587 (1966).

[2] G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, and W. Lughofer, Phys. Rev. **173**, 1452 (1968).

[3] B. Sechi-Zorn, B. Kehoe, J. Twitty, and R. Burnstein, Phys. Rev. **175**, 1735 (1968)

[4] J. A. Kadyk, G. Alexander, J. H. Chan, P. Gaposchkin, and G. H. Trilling, Nucl. Phys. B **27**, 13 (1971).

[5] O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. **57**, 564 (2006).

[6] A. Gal, E. V. Hungerford, and D. J. Millener, Rev. Mod. Phys. **88**, 035004 (2016).

[7] E. Botta, T. Bressani, and A. Feliciello, Nucl. Phys. A **960**, 165 (2017).

[8] B. Gibson, I. Afnan, J. Carlson, and D. Lehman, Prog. Theor. Phys. Suppl. **117**, 339 (1994).

[9] M. Jurič et al., Nucl. Phys. B **52**, 1 (1973).

[10] K. Miyagawa and W. Glöckle, Phys. Rev. C **48**, 2576 (1993).

[11] K. Miyagawa, H. Kamada, W. Glöckle, and V. Stoks, Phys. Rev. C **51**, 2905 (1995).

[12] A. Nogga, H. Kamada, and W. Glöckle, Phys. Rev. Lett. **88**, 172501 (2002).

[13] A. Nogga, Nucl. Phys. A **914**, 140 (2013).

[14] R. Wirth, D. Gazda, P. Navrátil, A. Calci, J. Langhammer, and R. Roth, Phys. Rev. Lett. **113**, 192502 (2014).

[15] D. Gazda and A. Gal, Nucl. Phys. A **954**, 161 (2016).

[16] R. Wirth and R. Roth, Phys. Rev. C **100**, 044313 (2019).

[17] S. Petschauer, N. Kaiser, J. Haidenbauer, Ulf-G. Meißner, and W. Weise, Phys. Rev. C **93**, 014001 (2016).

[18] H. Polinder, J. Haidenbauer, and U.-G. Meißner, Nucl. Phys. A **779**, 244 (2006).

[19] J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, and W. Weise, Nucl. Phys. A **915**, 24 (2013).

[20] J. Haidenbauer, U.-G. Meißner, and A. Nogga, Eur. Phys. J. A **56**, 91 (2020).

[21] S. Petschauer, J. Haidenbauer, N. Kaiser, U.-G. Meißner, and W. Weise, Front. Phys. **8**, 12 (2020).

[22] S. E. Koonin, Phys. Lett. B **70**, 43 (1977).

[23] R. Lednicky and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1982) [Sov. J. Nucl. Phys. **35**, 770 (1982)].

[24] S. Pratt, T. Csorgo, and J. Zimanyi, Phys. Rev. C **42**, 2646 (1990).

[25] W. Bauer, C. K. Gelbke, and S. Pratt, Ann. Rev. Nucl. Part. Sci. **42**, 77 (1992).

[26] K. Morita, T. Furumoto, and A. Ohnishi, Phys. Rev. C **91**, 024916 (2015).

[27] A. Ohnishi, K. Morita, K. Miyahara, and T. Hyodo, Nucl. Phys. A **954**, 294 (2016).

[28] S. Cho et al. (ExHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017).

[29] J. Haidenbauer, Nucl. Phys. A **981**, 1 (2019).

[30] L. Adamczyk et al. (STAR Collaboration), Phys. Rev. Lett. **114**, 022301 (2015).

[31] S. Acharya et al. (ALICE Collaboration), Phys. Rev. C **99**, 024001 (2019).

[32] S. Acharya et al. (ALICE Collaboration), Phys. Lett. B **797**, 134822 (2019).

[33] T. Hatsuda, K. Morita, A. Ohnishi, and K. Sasaki, Nucl. Phys. A **967**, 856 (2017).

[34] S. Acharya et al. (ALICE Collaboration), Phys. Rev. Lett. **123**, 112002 (2019).

[35] S. Acharya et al. (ALICE Collaboration), arXiv:2005.11495 [nucl-ex].

[36] J. Adam et al. (STAR Collaboration), Phys. Lett. B **790**, 490 (2019).

[37] K. Morita, A. Ohnishi, F. Etminan, and T. Hatsuda, Phys. Rev. C **94**, 031901(R) (2016).

[38] K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, and A. Ohnishi, Phys. Rev. C **101**, 015201 (2020).

[39] J. Haidenbauer, G. Krein, and T. Peixoto, Eur. Phys. J. A **56**, 184 (2020).

[40] P. Braun-Munzinger and B. Dönigus, Nucl. Phys. A **987**, 144 (2019).

[41] C. Chitwood et al., Phys. Rev. Lett. **54**, 302 (1985).

[42] J. Pochodzalla et al., Phys. Rev. C **35**, 1695 (1987).

[43] R. Kotte et al. (FOPI Collaboration), Eur. Phys. J. A **6**, 185 (1999).

[44] K. Wosińska et al., Eur. Phys. J. A **32**, 55 (2007).

[45] M. Aggarwal *et al.* (WA98 Collaboration), arXiv:0709.2477 [nucl-ex].

[46] W. Rzęsa, https://twiki.cern.ch/twiki/pub/Main/WutStudents/raport_WR.pdf; https://indico.cern.ch/event/819610/contributions/3425246/attachments/1845730/3028148/hirg1605.pdf

[47] K. Wiśniewksi, presentation at "Meson2012", Kraków, Poland, 31 May–5 June 2012, http://meson.if.uj.edu.pl/meson2012/talks/Wisniewski.pdf

[48] Y. Zhang, Ph.D. thesis, University of Heidelberg, 2012.

[49] J. H. Hetherington and L. H. Schick, Phys. Rev. **139**, B1164 (1965).

[50] L. H. Schick and J. H. Hetherington, Phys. Rev. **156**, 1602 (1967).

[51] L. H. Schick, Phys. Rev. **188**, 2089 (1969).

[52] B. Ghaffary Kashef and L. H. Schick, Phys. Rev. D **3**, 2661 (1971).

[53] N. Kolesnikov and V. Kopylov, Sov. Phys. J. **24**, 890 (1982).

[54] K. Hasegawa, Y. Yamamoto, and H. Bando, Prog. Theor. Phys. **69**, 171 (1983).

[55] I. R. Afnan and B. F. Gibson, Phys. Rev. C **47**, 1000 (1993).

[56] A. Cobis, A. S. Jensen, and D. V. Fedorov, J. Phys. G **23**, 401 (1997).

[57] I. Filikhin and S. Yakovlev, Phys. Atom. Nucl. **63**, 223 (2000).

[58] H. Garcilazo, T. Fernandez-Caramés, and A. Valcarce, Phys. Rev. C **75**, 034002 (2007).

[59] H. Garcilazo, A. Valcarce, and T. Fernandez-Carames, Phys. Rev. C **76**, 034001 (2007).

[60] H. W. Hammer, Nucl. Phys. A **705**, 173 (2002).

[61] F. Hildenbrand and H.-W. Hammer, Phys. Rev. C **100**, 034002 (2019).

[62] M. Schäfer, B. Bazak, N. Barnea, and J. Mareš, Phys. Lett. B **808**, 135614 (2020).

[63] J. Adam *et al.* (STAR Collaboration), Nature Phys. **16**, 409 (2020).

[64] H. Le, J. Haidenbauer, U.-G. Meißner, and A. Nogga, Phys. Lett. B **801**, 135189 (2020).

[65] H. Bethe, Phys. Rev. **76**, 38 (1949).

[66] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **39**, 1261 (1989).

[67] A. Kievsky, M. Viviani, and S. Rosati, Nucl. Phys. A **577**, 511 (1994).

[68] W. Glöckle, H. Witala, D. Huber, H. Kamada, and J. Golak, Phys. Rept. **274**, 107 (1996).

[69] J. Congleton, J. Phys. G **18**, 339 (1992).

[70] L. Tomio, A. Delfino, and S. K. Adhikari, Phys. Rev. C **35**, 441 (1987).

[71] Yu. V. Orlov, Yu. P. Orevkov, and L. I. Nikitina, Phys. Atom. Nuclei **63**, 328 (2000).

[72] S. Mrówczyński and P. Słoń, arXiv:1904.08320 [nucl-th].

[73] T. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).

[74] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **74**, 064906 (2006).

[75] V. M. Shapoval, B. Erazmus, R. Lednicky, and Y. M. Sinyukov, Phys. Rev. C **92**, 034910 (2015).

[76] S. Acharya *et al.* (ALICE Collaboration), arXiv:2004.08018 [nucl-ex].

[77] L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. **121**, 102502 (2018).

[78] W. van Oers and J. Seagrave, Phys. Lett. B **24**, 562 (1967).

[79] P. F. Bedaque and H. W. Grießhammer, Nucl. Phys. A **671**, 357 (2000).