New estimator for symmetry plane correlations in anisotropic flow analyses

Ante Bilandzic,[1,2,*] Marcel Lesch,[1] and Seyed Farid Taghavi[1]
[1]*Physik Department, Technische Universität München, 80333 Munich, Germany*
[2]*Excellence Cluster Universe, Technische Universität München, 80333 Munich, Germany*

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Correlations of symmetry planes are important observables used to quantify anisotropic flow phenomena and constrain independently the properties of strongly interacting nuclear matter produced in the collisions of heavy ions at the highest energies. In this paper, we point out current problems in measuring correlations between symmetry planes and elaborate on why the available analysis techniques have a large systematic bias. To overcome this problem, we develop the first estimator for true symmetry plane correlations, and we introduce a new approach to approximate multiharmonic flow fluctuations via a two-dimensional Gaussian distribution. Employing this approximation, we introduce a new estimator, dubbed the Gaussian estimator (GE), to extract pure correlations between symmetry planes. We validate the GE by using the realistic event generator iEBE-VISHNU and demonstrate that it outperforms all existing estimators. Based on event-shape engineering, we propose an experimental strategy to improve the GE accuracy even further.

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I. INTRODUCTION

The past years have witnessed the advent of large-statistics heavy-ion data sets at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) facilities, comprising events with very large multiplicities. It is therefore becoming feasible to study the details of strongly interacting nuclear matter produced in heavy-ion collisions with unprecedented precision by employing multiparticle correlation techniques. When two heavy ions collide at ultrarelativistic energies a very rich and nontrivial sequence of stages emerges in the evolution of the produced fireball. Since each of these stages typically involves different underlying physics, ideally they would be described separately in theoretical models and probed one at a time in an experiment. To date, however, most of the analyzed heavy-ion observables are final-state observables in the momentum space, which pick up cumulatively the contributions from all stages in the heavy-ion evolution, starting all the way down from the details of the initial collision geometry. To leading order, these stages can be divided into the following categories: initial conditions, deconfined quark-gluon plasma stage, hadronization, chemical freezeout, rescatterings, kinetic freezeout, and, finally, free streaming. An important program in the field is the development of new observables which would be sensitive to only one particular stage at a time in the heavy-ion evolution [1–3].

For an idealized description of the heavy-ion collision geometry the initial volume containing interacting nucleons is ellipsoidal in noncentral collisions. In this case, anisotropic flow develops the shapes in the final-state momentum distribution which can be captured solely with the even Fourier amplitudes v_{2n} and only one symmetry plane $\Psi_{\rm RP}$ (the reaction plane, spanned by the impact parameter vector and the beam axis) [4–6]. However, in a more realistic description of the collision geometry, the initial energy density profiles fluctuate both in magnitude and in shape from one heavy-ion collision to another. Such initial-state fluctuations are also transferred into the final-state momentum fluctuations via anisotropic pressure gradients which develop in the fireball. Therefore, the full Fourier series expansion needs to be employed to quantify the anisotropies in the azimuthal distribution of emitted particles in the plane transverse to the beam axis,

$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]\right], \tag{1}$$

where v_n's are anisotropic flow amplitudes, and Ψ_n's the corresponding symmetry planes [7]. In the past, anisotropic flow studies have focused mostly on the flow amplitudes v_n. These results helped a great deal in establishing the perfect fluid paradigm about quark-gluon plasma properties [8,9]. From the above Fourier series expansion it can be seen immediately that, due to event-by-event flow fluctuations, v_n's and Ψ_n's are independent and equally important degrees of freedom to quantify anisotropic flow phenomenon, and therefore both sets of observables need to be studied and measured.

Before discussing the physics of v_n and Ψ_n observables, we summarize the most important formal mathematical properties, which are used later in the derivation of our main results (additional details can be found in Appendix A). Solely from the definition of Fourier series one can prove that $v_{-n} = v_n$ and $\Psi_{-n} = \Psi_n$, therefore in this paper we use them interchangeably. Bhalerao *et al.* have derived in Ref. [10] the most general relation between flow degrees of freedom v_n and Ψ_n

*ante.bilandzic@tum.de

and multiparticle azimuthal correlators, which is valid for any number of azimuthal angles $\varphi_1, \varphi_2, \ldots, \varphi_k$ and for any choice of harmonics $n_1, n_2, \ldots, n_k$:

$$v_{n_1}^{a_1} \ldots v_{n_k}^{a_k} \, e^{i(a_1 n_1 \Psi_{n_1} + \ldots + a_k n_k \Psi_{n_k})} = \langle e^{i(a_1 n_1 \varphi_1 + \ldots + a_k n_k \varphi_k)} \rangle. \quad (2)$$

The average on the right-hand side in the above expression goes over all distinct tuples of k azimuthal angles in an event. Compared to the original result in Ref. [10], we have used a slightly different notation in the above expression by introducing a_i coefficients, which are by definition positive integers. The precise meaning of the a_i coefficient is the following: a_i is the number of appearances of harmonic n_i associated with different azimuthal angles in the azimuthal correlator on the right-hand side of Eq. (2) (positive and negative harmonics are counted separately). The advantage of this more general notation is that harmonics n_i in Eq. (2) are now all unique by definition. In addition, n_i and a_i naturally split off when associated with flow amplitudes on the left-hand side of Eq. (2), which makes their physical interpretation straightforward. It is easy to choose harmonics $n_1, n_2, \ldots, n_k$ in this general result in order to cancel the contribution from symmetry planes Ψ_n and estimate solely the flow amplitudes v_n (e.g., the choice $n_1 = n,\ n_2 = -n,\ n_k = 0$ for $k > 2$, $a_1 = a_2 = 1$, yields the standard formula for two-particle azimuthal correlation $\langle \cos[n(\varphi_1 - \varphi_2)] \rangle = v_n^2$). However, it is much more of a challenge to derive an analogous expression that would express multiparticle azimuthal correlators only in terms of symmetry planes, i.e., the expression from which the prefactors $v_{n_1}^{a_1} \ldots v_{n_k}^{a_k}$ in Eq. (2) would cancel out exactly.

The fundamental difference between v_n and Ψ_n flow degrees of freedom lies in the fact that only v_n's are invariant with respect to the arbitrary rotations of a laboratory coordinate system in which azimuthal angles φ are measured. We also remark that due to periodicity the symmetry plane angle Ψ_n is uniquely determined only in the range $0 \leqslant \Psi_n < 2\pi/n$ [11]. Therefore, in order to eliminate trivial periodicity of each symmetry plane, and to ensure invariance of our observables with respect to random event-by-event fluctuations of the impact parameter vector, we arrive at the conclusion that the fundamental nontrivial observables involving symmetry planes are the following correlators and constraints [10,12,13]:

$$\left\langle e^{i(a_1 n_1 \Psi_{n_1} + \ldots + a_k n_k \Psi_{n_k})} \right\rangle, \qquad \sum_i a_i n_i = 0. \quad (3)$$

The meaning of a_i and n_i is clarified in the text following Eq. (2). In the rest of the paper, we call observables in Eq. (3) symmetry plane correlations (SPCs). They can be estimated precisely only in theoretical models in which it is possible to compute each symmetry plane Ψ_n for each heavy-ion collision. The main purpose of this paper is to establish a reliable experimental way to estimate SPCs indirectly by using only the azimuthal angles of reconstructed particles, since only they can be measured reliably in experiments.

The importance of SPCs in anisotropic flow measurements has been fully acknowledged only in the LHC era, even though the first results were obtained 20 years ago in the E877 experiment [14]. The first results at the RHIC for SPCs involving two symmetry planes were published by the PHENIX Collaboration in Refs. [15,16], using the standard event plane method with the subevent technique [11]. Both the NA49 and the STAR collaborations have analyzed three-particle azimuthal correlators in mixed harmonics, which by definition do have contributions from symmetry planes, but their contribution was neglected in these early analyses [17,18]. The first SPC studies at the LHC provided only binary statements on whether or not certain symmetry planes are correlated, without providing quantitative details; in Ref. [19] the ALICE Collaboration, using the carefully designed five-particle azimuthal correlator (the technical details can be found in Appendix H of [20]), has demonstrated that the fluctuations of symmetry planes Ψ_2 and Ψ_3 are independent in all considered centralities. Finally, the most thorough experimental analysis to date, which includes also the first measurements of correlations among three symmetry planes, has been published by the ATLAS Collaboration in Refs. [12,21,22], using the analysis technique discussed in Refs. [13,23].

Theoretical studies have investigated SPCs separately in coordinate (typically using the Monte Carlo Glauber model [24] in combination with the event plane method) and momentum [12,13,23,25–30] space. In these studies the values of symmetry planes are typically the direct output of the model in each heavy-ion collision, and therefore they do not need to be estimated indirectly by utilizing the azimuthal angles of produced particles. A notable independent approach to SPCs in terms of conditional probabilities has been established in Ref. [31]. Other types of studies involving symmetry planes which we do not discuss in our paper have been performed in Refs. [32–36]. Finally, for the previous attempts to use azimuthal correlators to estimate SPCs indirectly, we refer the reader to [10,37–40].

This paper is organized as follows. After this introduction, in Sec. II we present the ideal form of SPC measurements and point out the inherent systematic biases that plagued the previous approaches. Section III discusses the concrete realization of our new estimator, dubbed the Gaussian estimator (GE). In Sec. IV we present a comparison with the theoretical models, for both new and old SPC estimators, and indicate in which regime our estimators outperform the existing ones. Finally, in Sec. V we summarize our results and outline the next steps. The technical details are provided in Appendixes A and B.

II. COMPARISON TO PREVIOUS METHODS

In this section we summarize the systematic biases of previous analyses that used azimuthal correlators to estimate SPCs and introduce our new approach, which improves on those biases. SPCs were estimated previously using the scalar product (SP) method [5,41] or event plane method [11], both of which yield the theoretical results for SPCs only in the absence of correlated fluctuations of different flow magnitudes. Our new method, which we illustrate in the next paragraphs and elaborate in detail in Sec. IV, provides a further step forward in the sense that it yields the theoretical result for SPCs also when such correlated fluctuations of flow magnitudes are present in the data. In fact, at RHIC and LHC energies correlations of event-by-event fluctuations of v_2 and v_3, and of v_2 and v_4, are large, and if they are not taken into account and corrected for, the final results for SPCs can exhibit large systematic biases, as we demonstrate in Monte Carlo studies presented in Sec. IV.

As indicated in Sec. I, correlations between k symmetry planes in unique harmonics $n_1, \dots, n_k$ (i.e., correlations between $\Psi_{n_1}, \dots, \Psi_{n_k}$) can be investigated by the measurement of the correlator $\langle \cos{(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k})} \rangle$, where the coefficients a_i have to be fixed in such a way that this expression is invariant with respect to the randomness of the reaction plane. In theory, such correlators can be built from an event-by-event ratio of two multiparticle azimuthal correlators. As a concrete example, by using the analytic formula in Eq. (2), one can derive the following result:

$$\frac{\langle \cos(2\varphi_1 + 2\varphi_2 - \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6) \rangle}{\langle \cos(2\varphi_1 - 2\varphi_2 + \varphi_3 - \varphi_4 + \varphi_5 - \varphi_6) \rangle} = \frac{v_2^2 v_1^4 \cos 4(\Psi_2 - \Psi_1)}{v_2^2 v_1^4} = \cos 4(\Psi_2 - \Psi_1). \quad (4)$$

This idea, which works only for correlators involving six or more azimuthal angles, demonstrates that for general ratios of this kind, the numerator consists of both flow amplitudes and symmetry planes, while the denominator consists only of the respective flow amplitudes, without any contribution of symmetry planes. The correlators in the numerator and denominator were carefully chosen so that the final expression depends only on the symmetry planes, even in the case of large correlated fluctuations of flow amplitudes v_1 and v_2.

We now generalize this starting example and write, in the most general case,

$$\left\langle \cos{\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)} \right\rangle_{\mathrm{EbE}} = \left\langle \frac{v_{n_1}^{a_1} \ \dots \ v_{n_k}^{a_k} \cos{\left(a_1 n_1 \Psi_{n_1} + \dots + a_k n_k \Psi_{n_k}\right)} + \delta}{v_{n_1}^{a_1} \dots v_{n_k}^{a_k} + \delta'} \right\rangle. \quad (5)$$

As such, this event-by-event ratio exhibits only the symmetry planes. This remains true by definition also if the event-by-event (EbE) fluctuations of flow amplitudes are correlated, and it is precisely this point which is not satisfied for the currently used estimators. Since both the numerator and the denominator in Eq. (5) have to be estimated with different k-particle azimuthal correlators ($k \geqslant 6$), they will have different statistical errors, which we denote δ and δ', respectively. Such a direct event-by-event approach is at the moment experimentally not feasible due to large statistical uncertainties which prevent such a per-event ratio.

We overcome the limitation of the event-by-event estimator in Eq. (5) by introducing a new approximate method to estimate the SPC, which we refer to as the *Gaussian estimator*, in the next section. By using the same notation we now clarify the currently used approximation methods, hereby focusing on the SP method. The explicit form of SP estimation is given by [40]

$$\left\langle \cos{\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)} \right\rangle_{\mathrm{SP}} = \frac{\left\langle v_{n_1}^{a_1} \ \dots \ v_{n_k}^{a_k} \cos{\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)} \right\rangle}{\sqrt{\left\langle v_{n_1}^{2a_1} \right\rangle \dots \left\langle v_{n_k}^{2a_k} \right\rangle}}. \quad (6)$$

The powers a_i are chosen in such a way that the numerator and the denominator are valid multiparticle correlators. We demonstrate later that within the GE approximation the same kind of powers a_i appear, and we provide a set of constraints which a_i must satisfy in Appendix B. We see further that in most cases, the SP method is not an accurate estimator of the true SPC, since the denominator in Eq. (6) cannot be written in the factorized form. This statement is supported by experimental evidence of large correlations between the flow amplitudes [42–45], which therefore leads in general to the nonnegligible bias in the SP method. On the other hand, estimators for SPCs from the event plane method are plagued by finite resolutions in estimating each symmetry plane directly event by event [12].

III. GAUSSIAN ESTIMATOR

Before introducing our new estimators, it could be illuminating to briefly point out the distribution of monochromatic flow $v_n e^{in\Psi_n}$. It is known that, due to the central limit theorem, the quantities $v_{n,x} = v_n \cos n\Psi_n$ and $v_{n,y} = v_n \sin n\Psi_n$ obey a two-dimensional (2D) Gaussian distribution approximately. The randomness of the reaction plane angle, however, forces us to average out the angular part of the 2D Gaussian to obtain the Bessel-Gaussian distribution, only for the flow amplitude [7,46]. It is shown that considering the Bessel-Gaussian as an approximate distribution for the flow harmonic, the cumulants of the flow harmonic fluctuations, $v_n\{2k\}$ ($k > 1$), are estimators of the genuine average of the flow harmonic originated from the initial geometry [47]. Here, we follow a rather similar concept to estimate SPCs.

Instead of focusing on only one harmonic, we study the distribution of a product of flow harmonics. Let us define the quantities

$$\mathcal{R} = v_{n_1}^{a_1} \ \dots \ v_{n_k}^{a_k}, \qquad \Theta = a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}, \qquad \sum a_i n_i = 0 \quad (7)$$

or equivalent quantities in the Cartesian coordinate system,

$$\mathcal{R}_x = \mathcal{R} \cos \Theta, \quad \mathcal{R}_y = \mathcal{R} \sin \Theta. \quad (8)$$

One notes that only averages of quantities $\mathcal{R}_x$ and $\mathcal{R}_y$ are experimentally accessible where we can write them in terms of Eq. (2). In general, the nonvanishing $\langle \mathcal{R}_x^p \mathcal{R}_y^q \rangle$ (the angular brackets indicate the average over events) can be expanded in the basis spanned by the moments

$$\left\langle \langle m \rangle_{(a_1,n_1),\dots,(a_k,n_k)} \right\rangle = \left\langle v_{n_1}^{a_1} \dots v_{n_k}^{a_k} \ e^{i(a_1 n_1 \Psi_{n_1} + \dots + a_k n_k \Psi_{n_k})} \right\rangle, \qquad m = \sum a_i, \quad (9)$$

and estimated experimentally by employing multiparticle correlation techniques [48–52] (the above notation is introduced in the paragraph below Eq. (2)). For positive integers p and positive even q, the moment $\langle \mathcal{R}_x^p \mathcal{R}_y^q \rangle = \langle \mathcal{R}^{p+q} \cos^p \Theta \sin^q \Theta \rangle$ is nonvanishing. The moments with odd q are 0 because of the presence of the sin term with odd power. A simple example of a nonvanishing moment $\langle \mathcal{R}^{p+q} \cos^p \Theta \sin^q \Theta \rangle$ is the case $k = 2$, $n_1 = -n_2 = n$, and $a_1 = a_2 = \ell$, leading to $\mathcal{R} = v_n^{2\ell}$ and

$\Theta = 0$. In this case, the moments $\langle v_n^{2\ell} \rangle$ have been extensively studied over the past years. In the present study, the following specific moments are employed:

$$\langle \mathcal{R}_x \rangle = \mathrm{Re}\big\langle \langle m \rangle_{(a_1,n_1),\ldots,(a_k,n_k)} \big\rangle,$$
$$\big\langle \mathcal{R}_x^2 \big\rangle + \big\langle \mathcal{R}_y^2 \big\rangle = \big| \big\langle \langle 2m \rangle_{(2a_1,n_1),\ldots,(2a_k,n_k)} \big\rangle \big|. \tag{10}$$

The quantity $\langle \mathcal{R}_x^p \mathcal{R}_y^q \rangle$ is a moment of the p.d.f. $P(\mathcal{R}_x, \mathcal{R}_y)$. If we had been able to measure $P(\mathcal{R}_x, \mathcal{R}_y)$, we could have computed the moment $\langle \cos \Theta \rangle$ immediately. Although $P(\mathcal{R}_x, \mathcal{R}_y)$ is not experimentally accessible, we are still able to approximately estimate the distribution as a 2D Gaussian (normal) distribution, $P(\mathcal{R}_x, \mathcal{R}_y) \simeq \mathcal{N}(\mathcal{R}_x, \mathcal{R}_y)$, where

$$\mathcal{N}(\mathcal{R}_x, \mathcal{R}_y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp\left[-\frac{(\mathcal{R}_x - \mu_x)^2}{2\sigma_x^2} - \frac{\mathcal{R}_y^2}{2\sigma_y^2} \right], \tag{11}$$

and $\mu_x = \langle \mathcal{R}_x \rangle$, $\sigma_x^2 = \langle \mathcal{R}_x^2 \rangle - \langle \mathcal{R}_x \rangle^2$, and $\sigma_y^2 = \langle \mathcal{R}_y^2 \rangle$. Similarly to the 2D Gaussian distribution for one harmonic flow, this estimation is based on the central limit theorem. Since we are only interested in the angular part of the normal distribution in Eq. (11), we integrate out the radial part. After some algebra, we find

$$\mathcal{N}_\theta(\Theta) = \int \mathcal{R}\, d\mathcal{R}\, \mathcal{N}(\mathcal{R}, \Theta)$$
$$= \frac{\sigma_x^3 \sigma_y\, e^{-\mu_x^2/2\sigma_x^2}}{\pi \sigma_\theta^2} \left[1 + \frac{\sqrt{\pi}\, \mu_x \sigma_y\, e^{\mu_\theta^2}}{\sigma_\theta} [1 + \mathrm{erf}(\mu_\theta)] \right], \tag{12}$$

where

$$\sigma_\theta(\Theta) = \sigma_x \sqrt{2\sigma_y^2 \cos^2 \Theta + 2\sigma_x^2 \sin^2 \Theta},$$
$$\mu_\theta(\Theta) = \frac{\mu_x \sigma_y \cos \Theta}{\sigma_\Theta}. \tag{13}$$

As a result, one can straightforwardly compute the average $\langle \cos \Theta \rangle$ by computing the integral

$$\langle \cos \Theta \rangle_{\mathrm{GE}} = \int d\Theta\, \mathcal{N}_\Theta(\Theta) \cos \Theta, \tag{14}$$

which is the Gaussian estimator for the true value of $\langle \cos \Theta \rangle$. To find an analytical result for our estimator, we still need some simplifications as we discuss in the following.

The quantity σ_θ has no Θ dependence upon considering $\sigma_x \sim \sigma_y \sim \sigma_r/\sqrt{2}$, where $\sigma_r = \sqrt{\sigma_x^2 + \sigma_y^2}$. This leads to an analytical result for the integral in Eq. (14) written in terms of two first modified Bessel functions. By expanding the result in terms of μ_x/σ_r and keeping only the leading term, we obtain

$$\langle \cos \Theta \rangle_{\mathrm{GE}} \simeq \sqrt{\frac{\pi}{4}} \left(\frac{\mu_x}{\sigma_r} \right). \tag{15}$$

Using Eq. (10), the above approximation can be written explicitly as

$$\big\langle \cos \big(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k} \big) \big\rangle_{\mathrm{GE}}$$
$$\simeq \sqrt{\frac{\pi}{4}} \frac{\big\langle v_{n_1}^{a_1} \ldots\ v_{n_k}^{a_k} \cos \big(a_1 n_1 \Psi_{n_1} + \ldots + a_k n_k \Psi_{n_k} \big) \big\rangle}{\sqrt{\big\langle v_{n_1}^{2a_1} \ldots\ v_{n_k}^{2a_k} \big\rangle}}, \tag{16}$$

where for the denominator we have used the fact that $\sigma_r = \sqrt{\langle \mathcal{R}_x^2 \rangle - \mu_x^2 + \langle \mathcal{R}_y^2 \rangle} \simeq \sqrt{\langle \mathcal{R}_x^2 \rangle + \langle \mathcal{R}_y^2 \rangle}$. The error we have made in the second equality is of the order of $(\mu_x/\sigma_r)^2$. The expression Eq. (16) is our main result and is experimentally measurable. In order to avoid the nonflow effects one can employ the rapidity gap technique in computing particle correlations [37,53,54]. After comparing Eq. (16) with Eq. (6), one finds that apart from a numerical factor, $\sqrt{\pi/4} \simeq 0.886$, we have a joint moment of flow amplitudes in the denominator. After introducing the technical details of the GE approximation for SPCs, in the next section we validate it by using realistic Monte Carlo simulation. We demonstrate that although the above approximation works accurately for most cases, there is still room to improve the accuracy by employing event shape engineering.

IV. VALIDATION OF THE GAUSSIAN ESTIMATOR AND ITS FURTHER IMPROVEMENT

A new estimator of the SPC has been introduced in the previous section by assuming that the $(\mathcal{R}_x, \mathcal{R}_y)$ fluctuation is approximately described by a 2D normal distribution. The accuracy and applicability of the method depend on this assumption. To examine the estimator's accuracy and in order to study possible ways for its improvement, we employ the realistic Monte Carlo event generator iEBE-VISHNU, which is based on hydrodynamic simulation in $2 + 1$ dimensions with longitudinal boost-invariant symmetry [55]. We initiate the events at $\tau = 0.6$ fm/c by the Monte Carlo–Glauber model [24] implemented in the iEBE-VISHNU. For the hydrodynamic evolution DNMR [56,57] the causal hydrodynamic is solved at a fixed shear viscosity over the entropy density $\eta/s = 0.08$, while the Cooper-Frye freezeout [58] prescription has been implemented in the model for the particleization stage. Evolution in the hadronic stage is not considered in our simulation. For each centrality bin, 14 000 events of Pb-Pb collisions ($\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV) have been generated and the flow magnitudes v_n and symmetry planes Ψ_n are computed in each event for $\pi^{\pm}$, $K^{\pm}$, and $p/\bar{p}$ in the final state. One notes that the flow harmonics in the simulation are computed from a continuous single-particle momentum distribution in each event. As a result, there is no concern about the statistical uncertainty of flow harmonics in a single event and consequently we have access to the true value of the SPC at every single event. The SPCs obtained from directly computed event-by-event symmetry planes in the simulation are referred to as the true values of the SPCs in the comparisons which we present next. As has been pointed out, however, we need to perform many event averages and employ suitable techniques for removing nonflow effects in the real experiment.

Our first study in Fig. 1 shows eight different choices for the correlation of two symmetry planes, and it demonstrates that the true value of the SPC can be approximated much better with the GE approach [Eq. (16)] than with the SP estimator [Eq. (6)]. Especially in cases where the two symmetry planes are strongly correlated [Figs. 1(a)–1(d)] due to their geometric correlations that preexisted in the initial state (e.g., between Ψ_2 and Ψ_4), our new method reproduces

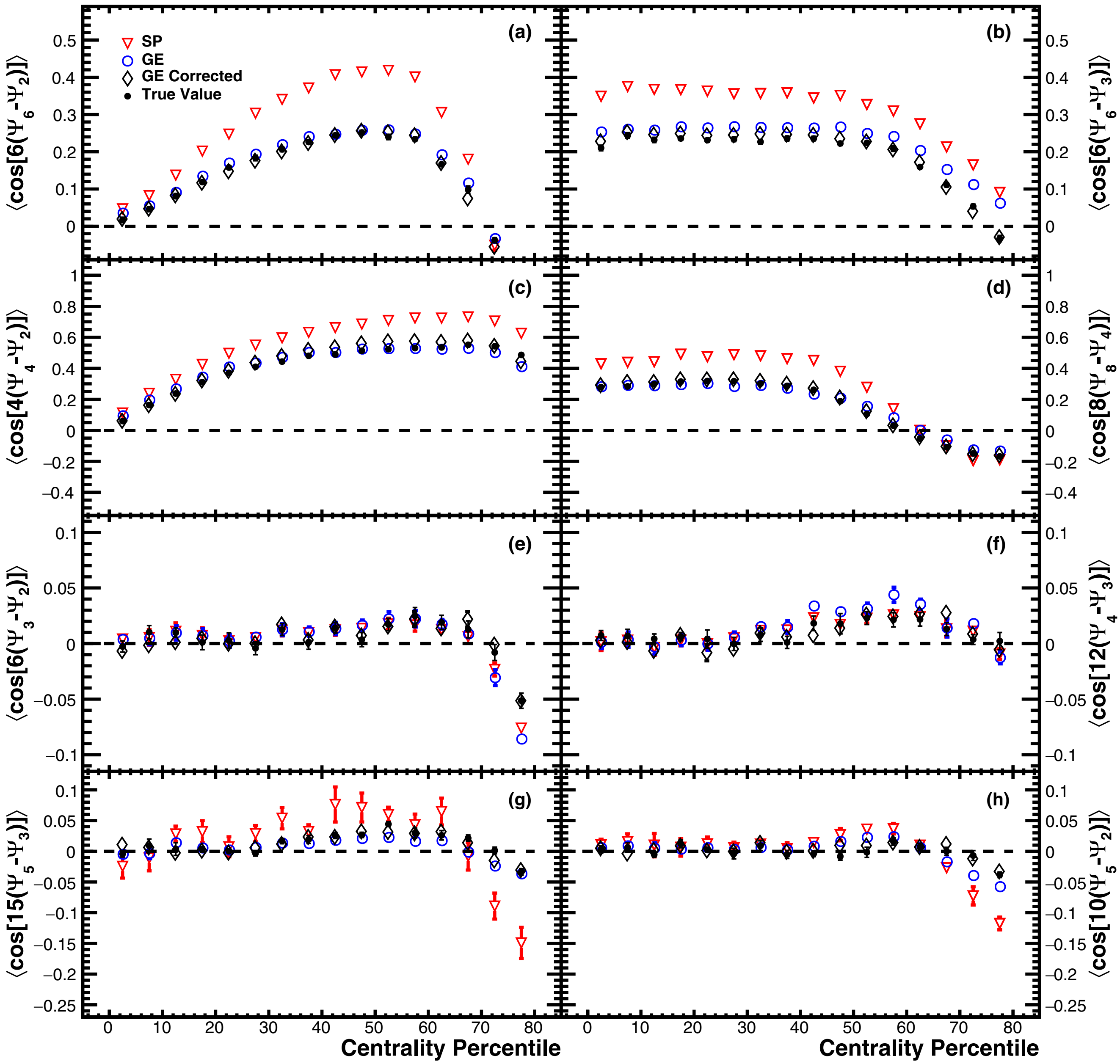

FIG. 1. Comparison of the GE and SP methods to the true value of the SPC between two symmetry planes in iEBE-VISHNU.

the true value very well in all centrality classes of interest. This demonstrates clearly that the systematic bias caused by neglecting correlations between the flow amplitudes in the SP method is large and, therefore, cannot be neglected. Only in a few cases can it be observed that the GE and SP yield comparable results (e.g., for SPCs between Ψ_4 and Ψ_3). We elaborate on this in more detail later and present a way to improve the GE method even further. The centrality dependence of each SPC in Fig. 1 presents strikingly different features and, therefore, provides independent constraints for the system properties. Further, we present results for correlations between three symmetry planes (Fig. 2) as well as between four symmetry planes (Fig. 3). It can be observed clearly that for each SPC the GE approach outperforms the SP estimator in most of the considered centralities, while for the remaining few centralities the accuracy of the methods is comparable.

Although the Gaussian estimator in Eq. (16) works accurately in almost all cases, in contrast to the SP method, which in most cases exhibits large systematic biases, there are still minor discrepancies between our estimator and the true value in a few cases [see, e.g., $\Theta = 2\Psi_2 + 3\Psi_3 - 5\Psi_5$ in Fig. 2(d)]. To investigate the reason more deeply, we focus on an extreme example: $\mathcal{R} = v_2 v_4 v_6$, $\Theta = 2\Psi_2 + 4\Psi_4 - 6\Psi_6$ at 40% centrality [see Fig. 2(c)]. In this case, there is a clear discrepancy between the true value and the Gaussian approximation, Eq. (16). In Fig. 4, the iEBE-VISHNU outcome for $(\mathcal{R}_x, \mathcal{R}_y)$ fluctuations is shown. As shown in the figure,

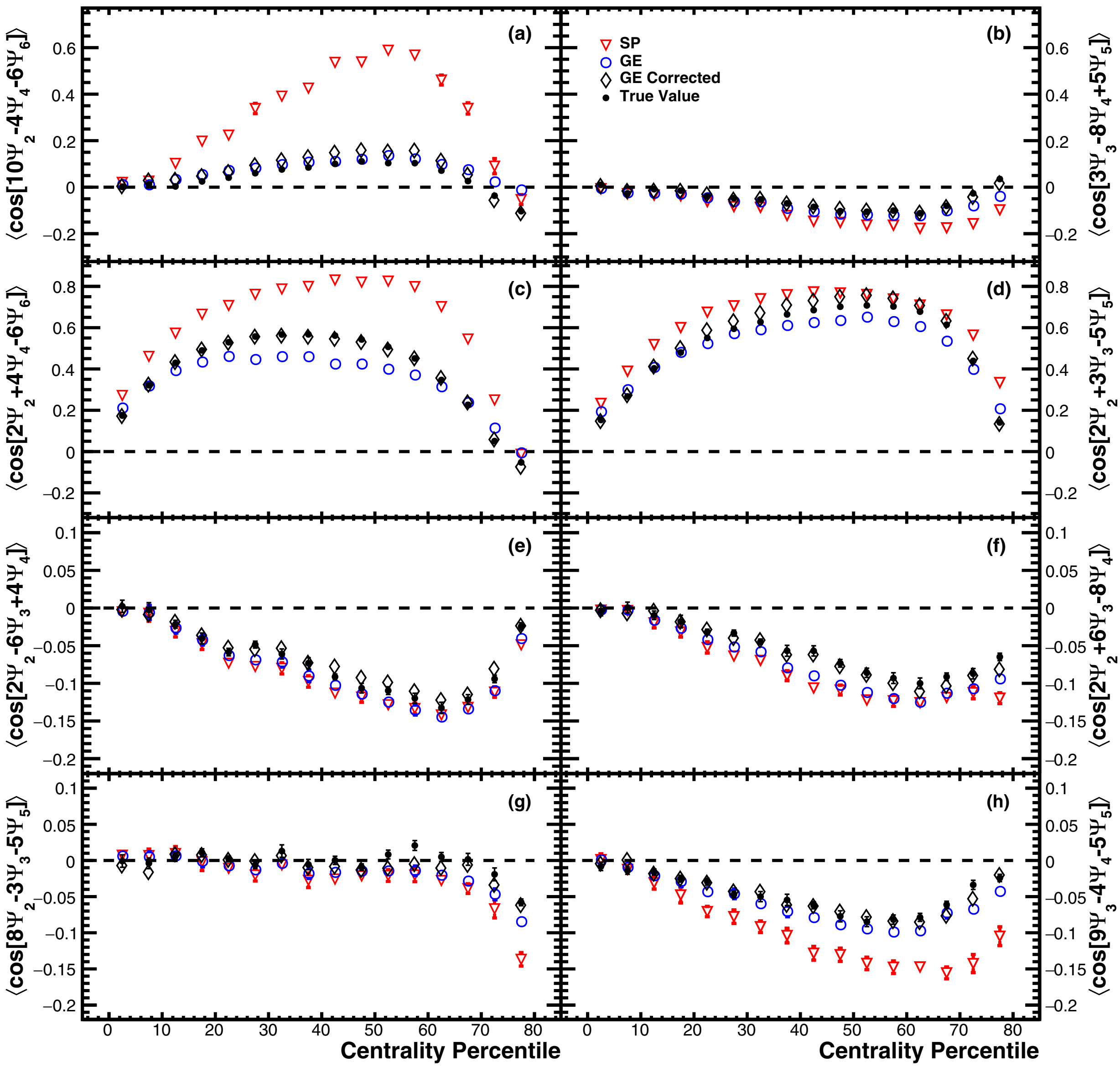

FIG. 2. Comparison of the GE and SP methods to the true value of the SPC between three symmetry planes iEBE-VISHNU.

there is a sharp peak at the center with a few events distributed around it. The tail is elongated in the x direction. Although there are far fewer events in the tail, it leads to inaccuracy in our Gaussian estimation. Specifically, the events are mostly concentrated symmetrically around the center, while the long tail in the x direction leads to a large difference between σ_x and σ_y. Also, it shifts the μ_x to the right. The GE would work better if we could fit the Gaussian distribution around the peak and remove the outliers.

Since we have no access to the 2D histograms in Fig. 4 experimentally, removing the outliers cannot be done easily. Here, we briefly point out a potential strategy for removing outliers by employing event shape engineering, which should be based on experimentally accessible quantities such as μ_x, σ_x, and σ_y. Dealing with more detailed experimental challenges and its efficiency will require a separate study in the future. Knowing the value of flow magnitudes and symmetry planes in each event in the simulation, one is easily able to remove the outliers. We can locate the peak in the histogram and fit a Gaussian distribution around it by ignoring events away from the peak with a certain criterion. Here, however, we try to introduce criteria that are model independent and applicable also in experiments where the histogram is not available. Before introducing our strategy, let us investigate our simulation further as follows. We first compute σ_x and σ_y from all events. After that we divide the events into two classes: the low-$\mathcal{R}$ class, with the condition $\mathcal{R} \leqslant \alpha\sigma_r$; and the rest, the high-$\mathcal{R}$ class. We have found that by ignoring the events at the tail of the distribution starting from twice the width σ_r ($\alpha = 2$), the GE is corrected very well, as we see shortly. After event classification, we compute μ_x and σ_r for the low-$\mathcal{R}$ class and estimate $\langle \cos\Theta \rangle$ by using Eq. (15). For the specific case shown in Fig. 4(a), the ratio σ_x/σ_y computed from all events in the given centrality class is around 3, while

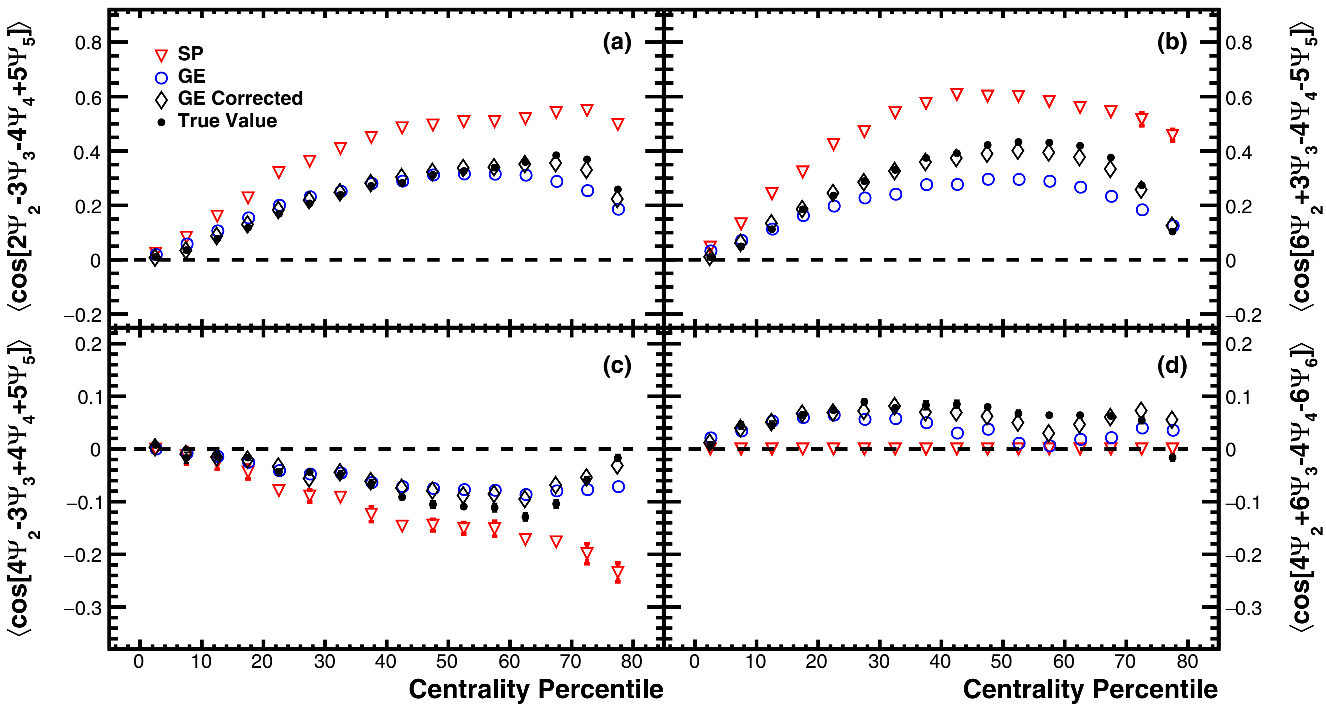

FIG. 3. Comparison of the GE and SP methods to the true value of the SPC between four symmetry planes iEBE-VISHNU.

if we compute the same ratio by using events in the low-$\mathcal{R}$ class this ratio decreases to 1.7. The corrected histogram with new μ_x, σ_x, σ_y, and σ_r values is depicted in Fig. 4(b). Employing this strategy without using direct information from histograms, the "corrected" Gaussian estimations $\langle\cos\Theta\rangle$ are obtained and shown in Figs. 1–3 by open diamonds, indicating an improvement in most cases. We should point out that $\alpha = 2$ is chosen for all observables, and we have not tuned it from one observable to the other.

Now we are in a position to introduce our experimental strategy. The low-$\mathcal{R}$/high-$\mathcal{R}$ classification is simple in the simulation because we are able to check the condition $\mathcal{R} \leqslant \alpha\sigma_r$ at each single event. In the experiment, however, we have no access to the single-event information and need to employ more sophisticated techniques such as event shape engineering [59]. It is noteworthy that the low-$\mathcal{R}$ class includes 94% of all events in our simulation. This means that by removing 6% of the high-$\mathcal{R}$ events the ratio σ_x/σ_y is reduced by a factor of approximately 2. According to the study mentioned above, the experimental strategy for finding the "corrected" GE is the following: one needs to classify events into low- and high-$\mathcal{R}$ classes, with ~6% of events in the high-$\mathcal{R}$ class. The classification percentile can be optimized by comparing the low-$\mathcal{R}$ class ratio σ_x/σ_y with that

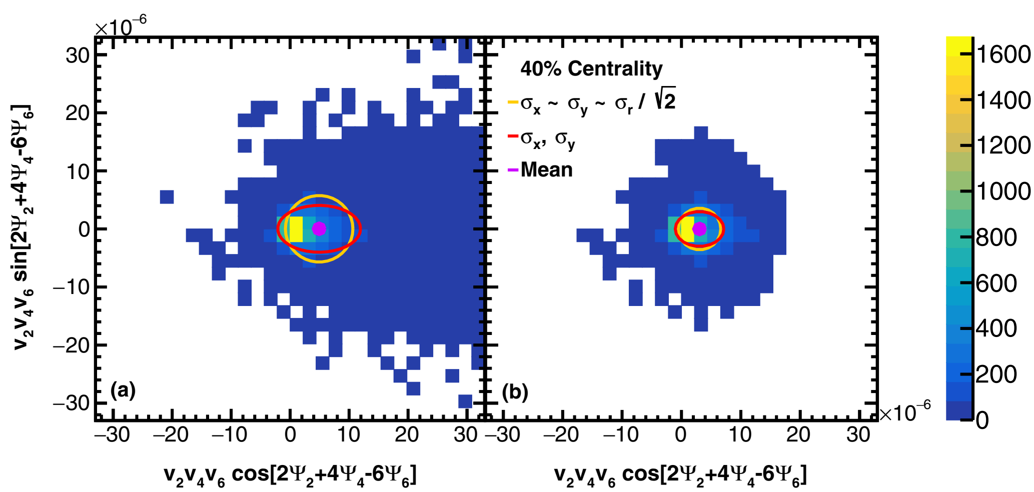

FIG. 4. Distributions of $v_2v_4v_6\cos[2\Psi_2+4\Psi_4-6\Psi_6]$ and $v_2v_4v_6\cos[2\Psi_2+4\Psi_4-6\Psi_6]$ before (left) and after (right) correction by rejection of events larger than $\mathcal{R} \leqslant \alpha\sigma_r$ ($\alpha = 2$).

obtained from all events in the given centrality class. The ratio σ_x/σ_y should be approximately robust against changes of the suitably chosen classification percentile criterion. The "corrected" GE is obtained from Eq. (15), where μ_x and σ_r are computed in the low-$\mathcal{R}$ class events.

V. CONCLUSIONS

After introducing the new procedure to correct for correlated flow fluctuations of different flow magnitudes, we have reduced significantly the systematic biases in the existing experimental techniques for symmetry plane correlations. This correction emerged from the modeling of experimentally accessible moments with a 2D Gaussian distribution. By using this new method, dubbed the Gaussian estimator, we have shown a significant improvement in estimating true values of SPCs over existing measurements in most cases of interest. We have demonstrated that, in combination with event shape engineering, this new estimator can be optimized even further.

The precise measurements of SPCs in the future must acknowledge the remaining small intercorrelation between flow amplitudes and symmetry planes, which can still cause a small bias in all available approximation methods for SPC measurements.

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APPENDIX A: BASIC PROPERTIES OF SYMMETRY PLANES

In this Appendix we outline in more detail the most important formal properties of symmetry planes. Besides the version of the Fourier series presented in Eq. (1) (Sec. I), the alternatively used form is

$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty}(c_n \cos n\varphi + s_n \sin n\varphi)\right], \tag{A1}$$

with

$$c_n = \int_0^{2\pi} f(\varphi)\cos(n\varphi)d\varphi\,, \tag{A2}$$

$$s_n = \int_0^{2\pi} f(\varphi)\sin(n\varphi)d\varphi. \tag{A3}$$

The Fourier series parametrizations in Eqs. (1) and (A1) are mathematically equivalent and can be interchanged by using the following relations:

$$v_n \equiv \sqrt{c_n^2 + s_n^2}, \tag{A4}$$

$$\Psi_n \equiv (1/n)\arctan\frac{s_n}{c_n}. \tag{A5}$$

Relation (A5) can be used as a definition of the symmetry plane Ψ_n. We discuss next some physical properties of symmetry planes and establish the connection between them and some commonly used observables in anisotropic flow analyses.

The symmetry plane Ψ_n has an obvious geometrical interpretation when the anisotropic distribution can be parametrized with only one harmonic n, since then one can show immediately that

$$f(\Psi_n + \varphi) = f(\Psi_n - \varphi)\,, \tag{A6}$$

i.e., a symmetry plane Ψ_n is a plane for which it is equally probable for a particle to be emitted above and below it. From Eq. (A5) one can see that symmetry planes are meaningful only when s_n and c_n are not both simultaneously 0. If the Fourier series permits only the sin term s_n (i.e., $c_n = 0$), the corresponding symmetry plane will be $\Psi_n = \pm\frac{\pi}{2n}$ (depending on whether s_n is positive or negative). In the case where the Fourier series permits only the cos term c_n (i.e., $s_n = 0$), all symmetry planes are equal to 0. However, if the flow amplitude v_n is 0 (because $c_n = s_n = 0$), the corresponding symmetry plane Ψ_n does not exist.

Another symmetry of interest is $f(\varphi) = f(-\varphi)$ due to which $s_n = 0$ for all n, and therefore from Eq. (A5) $\Psi_n = 0\ \forall n$, i.e., all symmetry planes are the same and equal to 0. Physically, this means that a heavy-ion collision was described in the laboratory frame with the coordinate system oriented such that the impact parameter vector is aligned with the x axis. The next symmetry which is satisfied to leading order in noncentral heavy-ion collisions is $f(\varphi) = f(\pi + \varphi)$, due to which $c_{2n+1},\ s_{2n+1} = 0$, and therefore only the even-symmetry planes Ψ_{2n} are well defined and nontrivial. In principle, one could also consider the symmetry $f(\varphi) = f(\pi - \varphi)$ in midcentral collisions, but we were not able to extract any new constraint on the symmetry planes which was not already covered by the other symmetries. Finally, since we assign to $f(\varphi)$ the probabilistic interpretation [which implies that $f(\varphi)$ must be a positive definite function], we do not consider symmetries like $f(\varphi) = -f(-\varphi)$, which otherwise could lead to additional constraints.

Another important physical interpretation of symmetry planes can be drawn from their relation with the Q vector [4,7,46], which is one of the most important objects in flow analyses. For a set of M azimuthal angles φ_i, the Q vector in harmonic n is defined as

$$Q_n \equiv \sum_{j=i}^{M} e^{\text{i}n\varphi_j} \equiv |Q_n|e^{\text{i}n\Psi_n}\,. \tag{A7}$$

With this definition, one can easily demonstrate that the angle of the Q vector is exactly the same as the symmetry planes Ψ_n from the Fourier series defined in Eq. (A5), since

$$\begin{aligned}(1/n)\arctan\frac{s_n}{c_n} &= (1/n)\arctan\frac{\langle\sin n\varphi\rangle}{\langle\cos n\varphi\rangle}\\ &= (1/n)\arctan\frac{M\text{Im}(Q_n)}{M\text{Re}(Q_n)}\\ &= (1/n)\arctan\frac{|Q_n|\sin n\Psi_n}{|Q_n|\cos n\Psi_n}\\ &= (1/n)\arctan\tan n\Psi_n\\ &= (1/n)n\Psi_n\\ &= \Psi_n\,.\end{aligned} \tag{A8}$$

This relation is utilized in the standard event plane method, where symmetry planes Ψ_n are estimated directly from Q vectors in each event [11].

APPENDIX B: CHOICE OF CORRELATORS

In this Appendix we start from the most general form of multiparticle correlators with nonunique harmonics, from which we find constraints such that these correlators are applicable for our GE method [Eq. (16)]. We see that from there on, constraints for the a_i will emerge naturally.

Consider two general multiparticle correlators $\langle k\rangle_{n_1,n_2,\ldots,n_k}$ (k-particle correlator with a set of nonunique harmonics $\{n_1, n_2,\ldots,n_k\}$) and $\langle l\rangle_{p_1,p_2,\ldots,p_l}$ (l-particle correlator with a set of nonunique harmonics $\{p_1, p_2,\ldots,p_l\}$). Focusing on the general form of the GE approximation [Eq. (16)], their ratio can in general be written as

$$\frac{\left\langle\langle k\rangle_{n_1,n_2,\ldots,n_k}\right\rangle}{\left\langle\langle l\rangle_{p_1,p_2,\ldots,p_l}\right\rangle} \propto \frac{\left\langle v_{n_1}\ldots v_{n_k} e^{i(n_1\Psi_{n_1}+\ldots+n_k\Psi_{n_k})}\right\rangle}{\sqrt{\left\langle v_{p_1}\ldots v_{p_l} e^{i(p_1\Psi_{p_1}+\ldots+p_l\Psi_{p_l})}\right\rangle}}. \tag{B1}$$

From this general ansatz the following constraints to achieve the desired SPC emerge:

$$\sum_{j=1}^{k} n_j = 0. \tag{B2}$$

$$\sum_{j=1}^{l} p_j = 0. \tag{B3}$$

$$\sum_{j=1}^{k} n_j\cdot\Psi_{n_j} \neq 0, \tag{B4}$$

$$\sum_{j=1}^{l} p_j\cdot\Psi_{p_j} = 0, \tag{B5}$$

$$\prod_{i=1}^{k} v_{n_i}^2 = \prod_{i=1}^{l} v_{p_i}\,. \tag{B6}$$

Constraints (B2) and (B3) satisfy the isotropy condition, which has to hold true for any nontrivial multiparticle correlator. Constraints (B4) and (B5) lead to a nonvanishing contribution of symmetry planes in the numerator, while the denominator does not depend on symmetry planes explicitly. Constraint (B6) ensures that the flow amplitudes in the numerator and denominator cancel each other exactly. Further, from constraint (B6) it follows that $l = 2k$. Therefore, when measuring a k-particle correlator in the numerator one has to measure a $2k$-particle correlator in the denominator, when using the GE approximation. To obtain the SPC one has to explicitly choose sets of correlators $\{n_1, n_2,\ldots,n_k\}$ and $\{p_1, p_2,\ldots,p_l\}$ which satisfy constraints (B2)–(B6). We elaborate on this now explicitly for the SPC between two symmetry planes, Ψ_m and Ψ_n, and demonstrate how the coefficients a_i used in the text [see, e.g., Eq. (2)] emerge naturally and which constraints a_i have to fulfill themselves. This formalism can be generalized for correlations between any number of symmetry planes.

1. Correlators between two symmetry planes

Focusing now on the SPC between two symmetry planes Ψ_m and Ψ_n and given constraints (B4) to (B6), the general sets of correlators in harmonics m and n (where $m \neq n$) are schematically

$$\Big\{\underbrace{m,\ldots,m}_{a_m\ \text{times}},\ \underbrace{-n,\ldots,-n}_{a_n\ \text{times}}\Big\}\quad \text{(numerator)}, \tag{B7}$$

$$\Big\{\underbrace{m,-m,\ldots,m,-m}_{2a_m\ \text{times}},\ \underbrace{n,-n,\ldots,n,-n}_{2a_n\ \text{times}}\Big\}\quad \text{(denominator)}, \tag{B8}$$

where $a_m, a_n \in \mathbb{N}$. Given constraints (B2) and (B3) the following constraints for a_m and a_n are valid:

$$\sum_{j=1}^{a_m} m + \sum_{k=1}^{a_n}(-n) = a_m m - a_n n = 0 \Rightarrow \frac{a_m}{n} = \frac{a_n}{m}\,, \tag{B9}$$

$$\sum_{j=1}^{2a_m}(-1)^j\cdot m + \sum_{k=1}^{2a_n}(-1)^k\cdot n = 0 \Rightarrow 2a_m \wedge 2a_n\ \text{even}, \tag{B10}$$

where $\wedge$ is the logical AND. This way, the constraints from Eqs. (B4) and (B5) are satisfied as well. We see that constraint (B10) will hold true for any $a_m,\ a_n$. Therefore, as a concrete example one can choose a_m and a_n as

$$a_m = \frac{l_{mn}}{m}, \tag{B11}$$

$$a_n = \frac{l_{mn}}{n}, \tag{B12}$$

where l_{mn} denotes the least common multiple between m and n. The order of the particle correlator in the numerator is given as

$$l_{mn}\left(\frac{1}{m}+\frac{1}{n}\right), \tag{B13}$$

and for the numerator twice the size, respectively. This method of using the least common multiple presents the lowest order of valid multiparticle correlators for the SPC between two symmetry planes. Any other method exhibits higher-order correlators. Given by this, the GE approach reads

$$\langle\cos[l_{mn}(\Psi_m-\Psi_n)]\rangle_{\rm GE} \propto \frac{\left\langle v_m^{a_m} v_n^{a_n}\cos[l_{mn}(\Psi_m-\Psi_n)]\right\rangle}{\sqrt{\left\langle v_m^{2a_m} v_n^{2a_n}\right\rangle}}. \tag{B14}$$

Although the method of the least common multiple exhibits the lowest possible order for an SPC with two planes, any multiple $k \in \mathbb{N}$ of this method represents a valid correlator as well. We can therefore always expand the set of correlators by changing $a_m \to ka_m$ and $a_n \to ka_n$ and then find in general

$$\langle\cos[kl_{mn}(\Psi_m - \Psi_n)]\rangle_{\rm GE} \propto \frac{\left\langle v_m^{ka_m} v_n^{ka_n} \cos[kl_{mn}(\Psi_m - \Psi_n)]\right\rangle}{\sqrt{\left\langle v_m^{2ka_m} v_n^{2ka_n}\right\rangle}}. \tag{B15}$$

2. Correlators between three symmetry planes

A general choice for the set of correlators for three unique harmonics m, n, and p are schematically

$$\left\{\underbrace{m}_{a_m \text{ times}}, \ldots, m, \underbrace{-n}_{a_n \text{ times}}, \ldots, -n, \underbrace{-p}_{a_p \text{ times}}, \ldots, -p\right\} \quad \text{(numerator)}, \tag{B16}$$

$$\left\{\underbrace{m, -m,}_{2a_m \text{ times}} \ldots, m, -m, \underbrace{n, -n,}_{2a_n \text{ times}} \ldots, n, -n, \underbrace{p, -p,}_{2a_p \text{ times}} \ldots, p, -p\right\} \quad \text{(denominator)}. \tag{B17}$$

Following the general constraints presented above we find the following constraints on a_m, a_n, and a_p:

$$\sum_{j=1}^{a_m} m + \sum_{k=1}^{a_n}(-n) + \sum_{l=1}^{a_p}(-p) = a_m m - a_n n - a_p p = 0, \tag{B18}$$

$$\sum_{j=1}^{2a_m}(-1)^j \cdot m + \sum_{k=1}^{2a_n}(-1)^k \cdot n + \sum_{l=1}^{2a_p}(-1)^l \cdot p = 0 \Rightarrow 2a_m \wedge 2a_n \wedge 2a_p \text{ even}. \tag{B19}$$

Again, the latter constraint is fulfilled trivially. In general these kind of correlators will be of a high order, therefore limiting experimental feasibility. We cannot reduce the problem of a 3-SPC into one single closed formula as has been the case for two planes, as now more combinatorial possibilities exist. As a trivial example, in cases where $m = n + p$ we can set trivially $a_m = a_n = a_p = 1$:

$$\langle\cos[m\Psi_m - n\Psi_n - p\Psi_p]\rangle_{\rm GE} \propto \frac{\langle v_m v_n v_p \cos[m\Psi_m - n\Psi_n - p\Psi_p]\rangle}{\sqrt{\left\langle v_m^2 v_n^2 v_p^2\right\rangle}}. \tag{B20}$$

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