

Secondary γ -ray decays from the partial-wave T matrix with an R -matrix application to $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}$

Carl R. Brune ^{*}*Edwards Accelerator Laboratory, Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA*R. James deBoer[†]*The Joint Institute for Nuclear Astrophysics, Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA*

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The secondary γ rays emitted following a nuclear reaction are often relatively straightforward to detect experimentally. Despite the large volume of such data, a practical formalism for describing these γ rays in terms of partial-wave T -matrix elements has never been given. The partial-wave formalism is applicable when R -matrix methods are used to describe the reaction in question. This paper supplies the needed framework, and it is demonstrated by the application to the $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}$ reaction.

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I. INTRODUCTION

We consider here a nuclear reaction sequence

$$A(a, b)B \text{ followed by } B \rightarrow C + \gamma,$$

where there are two nuclei in the initial (a and A) and final (b and B) states and the residual nucleus B is left in an excited state. Subsequent to the reaction, we have $B \rightarrow C$ by the emission of a single photon, which we refer to as a secondary γ ray. Note also that B and C correspond to different states in the *same* nucleus. The purpose of this paper is to describe how the emission of secondary γ rays is correlated with the incident beam direction and/or the direction of the particle b . We are particularly interested in situations where the transition matrix for the reaction process is available in partial wave form.

The R -matrix theory of nuclear reactions [1] can in principle provide a phenomenological description of any reaction process that only involves two-body channels. It provides the energy dependence of the partial-wave transition matrix in terms of resonance parameters. The use of R -matrix methods as a phenomenological tool for analyzing nuclear reaction data for nuclear astrophysics and other nuclear applications has become routine [2,3]. Despite the large volume of data involving secondary γ rays, a practical formalism for calculating secondary γ -ray emission has never been given and no currently available R -matrix code is capable of directly analyzing such data. This paper provides the necessary formalism to make this type of analysis possible.

From an experimental point of view, the detection of secondary γ rays offers many advantages compared to detection of the outgoing nuclei. For the detection of these γ rays, the

experimental energy resolution is only degraded by Doppler broadening and the intrinsic resolution of the detector. Outgoing nuclei are subject to many more types of kinematic broadening, such as energy loss effects in the case of charged particles, variation of the kinematics over the acceptance of the detector, and time-resolution effects in the case of neutron time-of-flight experiments. A secondary γ -ray measurement can use a thick target and detectors which subtend large solid angles without compromising resolution, leading to very efficient measurements. Examples where these experimental techniques have been applied include $^{12}\text{C}(n, n_1\gamma)^{12}\text{C}$ [4], $^{16}\text{O}(n, x\gamma)$ [5], and $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}$ [6].

However, we do not limit our consideration to cases where the outgoing nuclei are not detected, as the outgoing nuclei can provide considerable additional information. One experimental approach of this type is one in which information about the direction of the outgoing nuclei is inferred from the observed Doppler shift of the γ ray. These measurements are very feasible in light nuclei where the γ rays can be detected with ≈ 2 keV resolution and the range of the Doppler spread is ≈ 100 keV. The exact amount of Doppler spread depends on the kinematics of the particular reaction and the analysis requires that slowing down effects of the residual nucleus before it decays are negligible. When the γ rays are detected at 0° with respect to the incident beam direction, this analysis is particularly straightforward. Examples of experiments of this type are given in Refs. [6–11].

This paper is organized as follows. First, the general formalism for particle- γ correlations is presented. It is then specialized to the case when the transition matrix is available in partial wave form. We then further specialize the results to two common experimental scenarios: the differential cross section for the γ ray when the outgoing nuclei are not detected and the angular distribution of the reaction products when the γ ray is detected at 0° relative to the incident beam direction. Finally, we discuss the implementation of secondary γ -ray

^{*}brune@ohio.edu[†]rdeboer1@nd.edu

angular distributions in AZURE2 [3] and show an example application to the $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}$ reaction.

II. GENERAL FORMALISM

The angular distribution of the γ -ray radiation resulting from the decay $B \rightarrow C$ is dependent upon the magnetic substate population, or polarization, of the nucleus B before it decays. It is thus natural to analyze this information to learn about the mechanism(s) of such decays. Important early contributions to the analysis of angular correlations are provided by Satchler [12] and Devons and Goldfarb [13]. The formalism used for the calculation of polarization observables [14–18] is also closely related to this topic.

A. Angular correlation function

A significant paper documenting the understanding of particle- γ angular correlations is the review of angular correlations in inelastic neutron scattering by Sheldon [19]. This work is generalized by Rybicki *et al.* [20] to the case of arbitrary b - γ correlations, where the now standard formalism of Rose and Brink [21] for describing the γ ray is also utilized. The general approach to angular correlations is summarized by Satchler [22, Sec. 10.7] and provides the starting point for our work. The double differential cross section for detecting b and the γ is given by [22, Eq. (10.126)]

$$\frac{d^2\sigma}{d\Omega_b d\Omega_\gamma} = \frac{d\sigma}{d\Omega_b} \frac{W}{4\pi}. \quad (1)$$

Here, we assume the state B decays via γ -ray emission 100% of the time to state C . If this is not the case, the above expression must be multiplied by the appropriate branching-ratio factor.

This factorization assumes that the process is *sequential*, with the $A(a, b)B$ followed by photon emission. The situation would be much more complicated if photon emission from the alternative order of emission, i.e., $A(a, \gamma)[C + b]^*$, was significant. Such mechanisms could create continuum backgrounds and may also interfere quantum mechanically with the secondary γ ray from $A(a, b)B$. However, due to the relative weakness of photon emission, such scenarios are highly unlikely. More quantitatively, photon emission is typically four or more orders magnitude less likely than nuclear emissions and the photons would be spread over the energy range allowed phase space. Such effects could become important in a fine-tuned situation involving resonances where nuclear emissions are suppressed by isospin or Coulomb/angular-momentum barrier considerations and the initial-state photon decay energy matches that of the secondary photon decay. These order-of-emission effects are not an issue for any of the reactions mentioned in this paper.

The angular correlation function W is given by [22, Eqs. (10.127), (10.130), and (10.131)]

$$W = \sum_{kq} t_{kq}(I_B) R_k \left[\frac{4\pi}{2k+1} \right]^{1/2} Y_{kq}^*, \quad (2)$$

where $t_{kq}(I_B)$ is the polarization tensor of the nucleus B , R_k are the radiation parameters, and the Y_{kq} are spherical harmonics.

The notation I_X denotes the intrinsic spin of nuclear state X . The z axis is taken to be along the direction of the incident beam. The polarization tensor $t_{kq}(I_B)$ is a function of the spherical coordinates $\Omega_b = (\theta_b, \phi_b)$ defined in the center of mass of $b + B$. The spherical harmonic Y_{kq} is a function of the spherical coordinates $\Omega_\gamma = (\theta_\gamma, \phi_\gamma)$ defined in the center of mass of nucleus B .

B. Radiation parameters

The γ radiation parameters depend upon the intrinsic spins of nuclear states B and C and the multipolarities present, and are given in general by [22, Eqs. (10.153) and (10.154)]

$$R_k = \sum_{LL'} g_L g_{L'} R_k(LL' I_B I_C), \quad (3)$$

where

$$\begin{aligned} R_k(LL' I_B I_C) &= (2I_B + 1)^{1/2} (2L + 1)^{1/2} \\ &\times (2L' + 1)^{1/2} \times (-1)^{I_B - I_C + L - L' + k + 1} \\ &\times (L' L 1 - 1 | k 0) W(LL' I_B I_B; k I_C). \end{aligned} \quad (4)$$

Here, L and L' take on values of the multipolarities of the possible γ -ray transitions, $(LL' 1 - 1 | k 0)$ is a Clebsch-Gordan coefficient, and $W(LL' I_B I_B; k I_C)$ is a Racah coefficient. The relative multipole amplitudes g_L may be taken to be real and are normalized such that

$$\sum_L g_L^2 = 1, \quad (5)$$

which together with $R_0(LL' I_B I_C) = \delta_{LL'}$ implies $R_0 = 1$. Parity considerations require that k only take on even values, but note Eq. (4) is not necessarily zero for k odd [22, footnote 16, p. 389]. Additional properties of the $R_k(LL' I_B I_C)$ are discussed by Rose and Brink [21].

It is often the case that only one multipole is present, or that this assumption is a good approximation. In this situation, there is only a single L value and

$$\begin{aligned} R_k &= (2I_B + 1)^{1/2} (2L + 1) (-1)^{I_B - I_C + k + 1} \\ &\times (LL 1 - 1 | k 0) W(LL I_B I_B; k I_C). \end{aligned} \quad (6)$$

If two multipoles, L_1 and L_2 , are present, then one may define the mixing ratio $\delta = g_{L_2}/g_{L_1}$ and [20, Eq. (15)]

$$\begin{aligned} R_k &= [R_k(L_1 L_1 I_B I_C) + 2\delta R_k(L_1 L_2 I_B I_C) \\ &+ \delta^2 R_k(L_2 L_2 I_B I_C)] / (1 + \delta^2). \end{aligned} \quad (7)$$

C. Polarization tensors

The polarization tensors $t_{kq}(I_B)$ describe the polarization of the final nucleus B ; the precise definition is given in terms of expectation values of tensor operators $\tau_{kq}(I_B)$ in Ref. [22, Secs. 10.3.2 and 10.3.3]. Specifically, we have [22, Eq. 10.32]

$$t_{kq}(I_B) = \frac{\text{Tr}[\mathbf{T} \mathbf{T}^\dagger \tau_{kq}(I_B)]}{\text{Tr}[\mathbf{T} \mathbf{T}^\dagger]}, \quad (8)$$

where \mathbf{T} is the transition matrix for $A(a, b)B$ and the trace implies a sum over all spin projections of the incoming and

outgoing nuclei. To make this more explicit, we note [22, Eq. (10.25b)]

$$\langle I_B m_B | \tau_{kq}(I_B) | I_B m_B' \rangle = (2k+1)^{1/2} \langle I_B k m_B q | I_B m_B' \rangle \quad (9)$$

and take T in the individual particle spin basis to be $\mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b)$, which is also known as the scattering amplitude. We now have

$$t_{kq}(I_B) = (2k+1)^{1/2} \times \left[\sum_{m_b m_B m_B' m_a m_A} \langle I_B k m_B q | I_B m_B' \rangle \times \mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b) \mathcal{M}_{m_b m_B; m_a m_A}^*(\Omega_b) \right] / \sum_{m_b m_B m_a m_A} |\mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b)|^2, \quad (10)$$

where $m_B' = m_B + q$ is required for the Clebsch-Gordan coefficient to be nonzero. Also note that $t_{00} = 1$ and that the differential cross section for particle b is given by

$$\frac{d\sigma}{d\Omega_b} = \frac{1}{(2I_A+1)(2I_a+1)} \times \sum_{m_b m_B m_a m_A} |\mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b)|^2. \quad (11)$$

The ϕ_b dependence of the scattering amplitude \mathcal{M} is very simple and allows another scattering amplitude $f_{m_b m_B; m_a m_A}$ that only depends upon θ_b to be defined:

$$\mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b) = e^{i(m_b+m_B-m_a-m_A)\phi_b} f_{m_b m_B; m_a m_A}(\theta_b). \quad (12)$$

This is the form of the scattering amplitude calculated by the computer code FRESKO [23]. It is also easy to show that

$$t_{kq}(\Omega_b) = e^{iq\phi_b} t_{kq}(\theta_b, 0), \quad (13)$$

as expected for the tensor operator t_{kq} .

This form of the scattering amplitude may be sufficient for some calculations, e.g., if the scattering amplitudes from the aforementioned FRESKO code are available. For example, Eq. (1) may be integrated over Ω_b to yield the differential cross section for γ -ray emission:

$$\begin{aligned} \frac{d\sigma}{d\Omega_\gamma} &= \sum_k R_k P_k(\cos\theta_\gamma) \int_{-1}^1 \frac{d\cos\theta_b}{2} \frac{d\sigma}{d\Omega_b} t_{k0}(I_B) \quad (14) \\ &= \frac{1}{(2I_A+1)(2I_a+1)} \sum_k R_k P_k(\cos\theta_\gamma) \\ &\times \frac{(2k+1)^{1/2}}{2} \int_{-1}^1 d\cos\theta_b \\ &\times \sum_{m_b m_B m_a m_A} |f_{m_b m_B; m_a m_A}(\theta_b)|^2 \\ &\times \langle I_B k m_B 0 | I_B m_B \rangle, \quad (15) \end{aligned}$$

where P_k are the Legendre polynomials.

III. PARTIAL-WAVE T MATRIX

The T matrix can then in principle be used to calculate any experimental observable, with the calculation being independent of the model used to determine the T matrix. In this section, the T matrix is converted to partial wave form and specific γ -ray observables are calculated. The material in Secs. III B, III C, and III D represents the main results of this work. Note that in an R -matrix approach, the partial-wave T matrix is calculated from the R -matrix or level matrix.

The scattering amplitudes may be constructed from the partial-wave T matrix as follows. According to Lane and Thomas [1, Eq. VIII.2.3, p. 292], the scattering amplitudes connecting nonelastic channels are given in the channel spin basis by

$$\begin{aligned} A_{bB s' v'; aA s v} &= i \frac{\pi^{1/2}}{k_{aA}} \sum_{JM \ell' m'} (2\ell+1)^{1/2} \\ &\times (s\ell v 0 | JM) (s' \ell' v' m' | JM) \\ &\times Y_{\ell' m'}(\Omega_b) T_{bB s' \ell'; aA s \ell}^J, \quad (16) \end{aligned}$$

where k_{aA} is the center-of-mass wave number in the $a+A$ system and $T_{bB s' \ell'; aA s \ell}^J$ is the partial-wave T matrix. In the individual particle spin basis, the scattering amplitudes become

$$\begin{aligned} \mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b) &= i \frac{\pi^{1/2}}{k_{aA}} \\ &\times \sum_{JM \ell' m' s v s' v'} (2\ell+1)^{1/2} (s\ell v 0 | JM) \\ &\times (s' \ell' v' m' | JM) (I_a I_A m_a m_A | s v) \\ &\times \langle I_b I_B m_b m_B | s' v' \rangle Y_{\ell' m'}(\Omega_b) T_{bB s' \ell'; aA s \ell}^J, \quad (17) \end{aligned}$$

which is suitable for calculating the general particle- γ correlation function.

A. Differential cross section for particle b

The differential cross section for particle b can then in principle be calculated from $\mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b)$ using Eq. (11). This result can, in a certain sense, be simplified by introducing two sets of summing indices (one for each occurrence of \mathcal{M}), expressing the products of spherical harmonics as sums of single spherical harmonics, and contracting the sums over magnetic substates. The result is

$$\frac{d\sigma}{d\Omega_b} = \frac{1}{(2I_A+1)(2I_a+1)} \frac{\pi}{k_{aA}^2} \sum_{\mathcal{K}} G_{\mathcal{K}}(\theta_b), \quad (18)$$

where

$$\begin{aligned} G_{\mathcal{K}}(\theta_b) &= \sum_{J_1 J_2 \ell_1 \ell_2 \ell'_1 \ell'_2 s s'} (-1)^{s-s'} (2J_1+1)(2J_2+1) \\ &\times [(2\ell_1+1)(2\ell_2+1)(2\ell'_1+1)(2\ell'_2+1)]^{1/2} \\ &\times (\ell_1 \ell_2 00 | \mathcal{K} 0) (\ell'_1 \ell'_2 00 | \mathcal{K} 0) W(\ell_1 J_1 \ell_2 J_2; s \mathcal{K}) \\ &\times W(\ell'_1 J_1 \ell'_2 J_2; s' \mathcal{K}) \frac{P_{\mathcal{K}}(\cos\theta_b)}{4\pi} \\ &\times T_{bB s' \ell'_1; aA s \ell_1}^{J_1*} T_{bB s \ell_2; aA s \ell_2}^{J_2}, \quad (19) \end{aligned}$$

which agrees with Eq. VII.2.6 of Lane and Thomas [1, p. 292].

B. Differential cross section for γ -ray emission

The differential cross section for γ -ray emission may be found by integrating Eq. (1) over Ω_b :

$$\frac{d\sigma}{d\Omega_\gamma} = \frac{1}{(2I_A + 1)(2I_a + 1)} \frac{\pi}{k_{aA}^2} \sum_k (2k + 1)^{1/2} R_k \frac{P_k(\cos\theta_\gamma)}{4\pi} H_k, \quad (20)$$

where

$$H_k = \frac{k_{aA}^2}{\pi} \int_{4\pi} d\Omega_b \sum_{m_b m_B m_a m_A} |\mathcal{M}_{m_b m_B; m_a m_A}(\Omega_b)|^2 (I_B k m_B 0 | I_B m_B). \quad (21)$$

This equation may be “simplified” in a similar manner, except that the integration over Ω_b is carried out using the orthogonality of the spherical harmonics. This procedure results in

$$\begin{aligned} H_k = & \sum_{J_1 J_2 \ell_1 \ell_2 \ell' s_1' s_2'} (-1)^{k+s_2'-s_1'} (2J_1 + 1)(2J_2 + 1) [(2\ell_1 + 1)(2I_B + 1)(2s_1' + 1)(2s_2' + 1)]^{1/2} \\ & \times (k \ell_1 0 0 | \ell_2 0) W(k I_B s_2' I_b; I_B s_1') W(k s_1' J_2 \ell'; s_2' J_1) W(k J_1 \ell_2 s; J_2 \ell_1) T_{bB s_1' \ell': aA s \ell_1}^{J_1*} T_{bB s_2' \ell': aA s \ell_2}^{J_2}. \end{aligned} \quad (22)$$

Note that the factor of $(-1)^k$ in this expression does not actually come into play since k is required to be even. An equivalent expression with the arguments of the angular momentum coupling functions arranged in a more symmetrical and standard manner is

$$\begin{aligned} H_k = & \sum_{J_1 J_2 \ell_1 \ell_2 \ell' s_1' s_2'} (-1)^{\ell'+s-2s_1'-I_B+I_b-J_1-J_2} (2J_1 + 1)(2J_2 + 1)(2\ell_1 + 1)^{1/2} \\ & \times \left[\frac{(2\ell_2 + 1)(2I_B + 1)(2s_1' + 1)(2s_2' + 1)}{2k + 1} \right]^{1/2} (\ell_1 \ell_2 0 0 | k 0) W(I_B s_1' I_B s_2'; I_b k) \\ & \times W(s_1' J_1 s_2' J_2; \ell' k) W(\ell_1 J_1 \ell_2 J_2; s k) T_{bB s_1' \ell': aA s \ell_1}^{J_1*} T_{bB s_2' \ell': aA s \ell_2}^{J_2}. \end{aligned} \quad (23)$$

C. General correlation function

The general correlation function corresponding to Eq. (1) can be written

$$\frac{d^2\sigma}{d\Omega_b d\Omega_\gamma} = \frac{1}{(2I_A + 1)(2I_a + 1)} \frac{\pi}{k_{aA}^2} \sum_{kq} R_k \frac{Y_{kq}^*(\Omega_\gamma)}{(4\pi)^{1/2}} F_{kq}(\Omega_b), \quad (24)$$

where

$$F_{kq}(\Omega_b) = \frac{k_{aA}^2}{\pi} \sum_{m_b m_B m_a m_A} (I_B k m_B q | I_B m_B') \mathcal{M}_{m_b m_B; m_a m_A}^*(\Omega_b) \mathcal{M}_{m_b m_B'; m_a m_A}(\Omega_b). \quad (25)$$

The contraction of the magnetic substate quantum numbers is similar to that described above. This calculation is also closely related to the results derived in Refs. [15,16], but we do not rotate the t_{kq} to make the z axis along the direction of the scattered particle. We find

$$\begin{aligned} F_{kq}(\Omega_b) = & \sum_{J_1 J_2 \ell_1 \ell_2 \ell' s_1' s_2' \mathcal{K} \mathcal{L}} (-1)^{\ell_2'+s+s_2'-I_B+I_b+J_2} (2J_1 + 1)(2J_2 + 1) \left[\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2I_B + 1)(2\mathcal{K} + 1)}{2\mathcal{L} + 1} \right]^{1/2} \\ & \times [(2\ell_1' + 1)(2\ell_2' + 1)(2s_1' + 1)(2s_2' + 1)]^{1/2} (\ell_1 \ell_2 0 0 | \mathcal{K} 0) (\ell_1' \ell_2' 0 0 | \mathcal{L} 0) (\mathcal{K} k 0 q | \mathcal{L} q) W(I_B s_1' I_B s_2'; I_b k) W(\ell_1 J_1 \ell_2 J_2; s \mathcal{K}) \\ & \times \left\{ \begin{array}{ccc} \ell_1' & s_1' & J_1 \\ \mathcal{L} & k & \mathcal{K} \\ \ell_2' & s_2' & J_2 \end{array} \right\} \frac{Y_{\mathcal{L}q}^*(\Omega_b)}{(4\pi)^{1/2}} T_{bB s_1' \ell': aA s \ell_1}^{J_1*} T_{bB s_2' \ell_2': aA s \ell_2}^{J_2}, \end{aligned} \quad (26)$$

where $\{\}$ denotes the 9- J symbol. The previous results can be expressed as specializations of this formula:

$$\sum_{\mathcal{K}} G_{\mathcal{K}}(\theta_b) = F_{00}(\Omega_b) \quad (27)$$

and

$$H_k = \int_{4\pi} d\Omega_b F_{k0}(\Omega_b). \quad (28)$$

D. Correlation function for $\theta_\gamma = 0$

When the γ rays are detected at 0° with respect to the incident beam direction, their Doppler shift only depends on the angle θ_b of the ejectile. In this situation, it is straightforward to analyze the γ -ray energy spectrum to deduce the experimental correlation function [6–11]. When $\theta_\gamma = 0$, Eq. (24) becomes

$$\frac{d^2\sigma}{d\Omega_b d\Omega_\gamma}(\theta_\gamma = 0) = \frac{1}{(2I_A + 1)(2I_a + 1)} \frac{\pi}{k_{aA}^2} \times \sum_{kq} R_k \left(\frac{2k + 1}{4\pi} \right)^{1/2} F_{k0}(\theta_b). \quad (29)$$

The quantity $F_{k0}(\theta_b)$ is calculated using Eq. (26) with $q = 0$, where there is little simplification, except for the replacement

$$\frac{Y_{\mathcal{L}q}^*(\Omega_b)}{(4\pi)^{1/2}} \rightarrow \frac{(2\mathcal{L} + 1)^{1/2}}{4\pi} P_{\mathcal{L}}(\cos \theta_b). \quad (30)$$

IV. IMPLEMENTATION IN AZURE2

The capability of fitting the differential cross section for γ -ray emission has been implemented in the computer code AZURE2 [3]. This observable is calculated from the partial-wave T matrix using Eqs. (20) and (22). This calculation is very similar in structure to the calculation of the differential cross section for the emission of particle b given by Eq. (18), an observable that is already calculated in AZURE2. This modification thus proved to be a relatively simple addition to the computer code. An example is presented in the following section.

V. APPLICATION TO $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}^*$

Measurements of the $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}^*$ reaction in the vicinity of 1-MeV proton energy have been performed by Bray *et al.* [6]. Data for the total cross section and γ -ray angular distribution coefficients are given in Fig. 2 and Table 1 of their work. These coefficients are defined by

$$\frac{d\sigma}{d\Omega_\gamma} = \frac{\sigma_{\text{tot}}}{4\pi} \sum_k b_k P_k(\cos \theta_\gamma), \quad (31)$$

where σ_{tot} is the total cross section for populating the first excited state of ^{12}C and $b_0 \equiv 1$. The γ -ray transition from the first excited state to the ground state of ^{12}C is $2^+ \rightarrow 0^+$ and is a pure $E2$ transition. Equations (20) and (31) thus only receive contributions from $k = 0, 2$, and 4. The total cross section and coefficients b_2 and b_4 measured by Bray *et al.* [6] are shown by the points in Fig. 1.

Bray *et al.* [6] also performed an R -matrix fit to their $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}^*$ measurements and other $^{15}\text{N} + p$ reaction data. They included the b_k coefficients in their fit, but unfortunately no formulas or other descriptions of their procedures were given. They do, however, provide in Tables 2 and 3 their best fit R -matrix parameters, in the Lane and Thomas formalism [1]. Using these parameters in AZURE2, we have calculated σ_{tot} , b_2 , and b_4 , with the results being very close to the calculations given by Bray *et al.* [6] in Fig. 2 of their

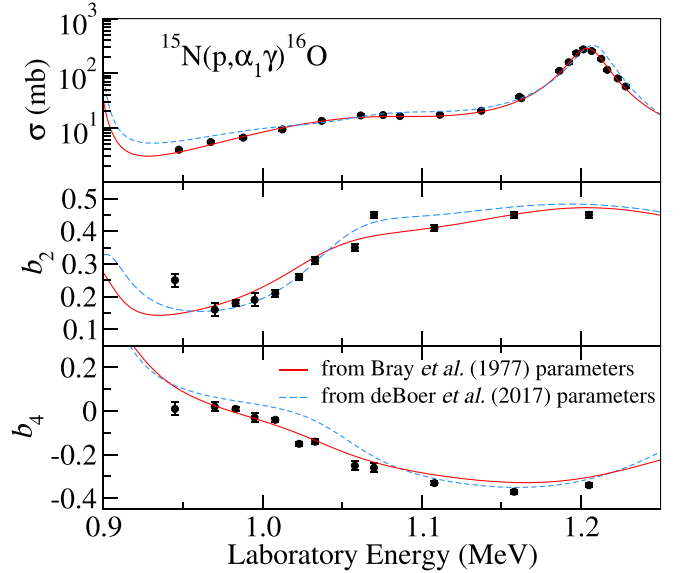


FIG. 1. The total cross section and γ -ray angular distribution coefficients for the $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}^*$ reaction. The experimental results of Bray *et al.* [6] are shown as points. Calculations using the Bray *et al.* [6] and deBoer *et al.* [24] R -matrix parameters are shown as the solid red and dashed blue curves, respectively.

paper. Our calculations are also shown as the solid curve in our Fig. 1, where they are seen to be very close to the experimental results of Bray *et al.* [6].

A comprehensive R -matrix analysis of the ^{16}O compound nucleus was recently published by deBoer *et al.* [24]. The angular distribution coefficients b_2 and b_4 defined above were not included in the fit or otherwise considered in the analysis. However, the R -matrix parameters given by Ref. [24] provide an excellent description of these coefficients. In this case, the R -matrix parameters are given in the alternative R -matrix formalism [25]. The σ_{tot} , b_2 , and b_4 resulting from using these parameters in an AZURE2 calculation are shown as the dashed curves in Fig. 1. These calculations are seen to be very close to both the Bray *et al.* [6] data and the calculations using their parameters. This example demonstrates that the R -matrix formalism can be used to predict the γ -ray angular distributions, provided the parameters can be suitably constrained using other reaction data.

Bray *et al.* [6] also presented γ -ray energy spectra measured at $\theta_\gamma = 0$ that were also well described by their R -matrix fit. Again, no formulas or other descriptions of their procedures were given. Their data are shown in their Fig. 3, and unfortunately are not presented in a manner that facilitates a simple comparison with other calculations of the correlation function for $\theta_\gamma = 0$.

VI. CONCLUSIONS

The general formalism for calculating the correlation of secondary γ rays with the beam direction and reaction products has been reviewed. The general results have been adapted to the partial-wave T -matrix formalism and specializations to common special cases have been presented. These

specializations are well suited for calculations when the reaction of interest is described by R -matrix theory. The case of the γ -ray angular distribution, with the other reaction products undetected, has been implemented in the R -matrix code AZURE2. This capability has been demonstrated with the example of the $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}^*$ reaction. We anticipate the application of this type of analysis to several other cases. Additional measurements have already been performed at the University of Notre Dame of the $^{15}\text{N}(p, \alpha_1\gamma)^{12}\text{C}^*$ reaction, extending the range of study to higher energies [26].

At last two extensions to this work can be envisioned. One additional possibility is the situation where the initial reaction

is a radiative capture reaction. Another is the case of γ -ray cascades following a reaction. This scenario can be addressed using the formalism given by Rose and Brink [21].

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