Coexistence in ^{72}Kr

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I have applied a simple two-state mixing model to $E2$ strengths among low-lying states in ^{72}Kr. Solutions are found with basis states in ^{72}Kr similar to those in ^{74}Kr. As expected, the amount of the more collective basis state in the yrast states increases with J for $J = 0$, 2, and 4.

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I. INTRODUCTION

In a recent paper, Wimmer *et al.* [1] added to the information on $E2$ strengths in ^{72}Kr and improved our understanding of the structure of the low-lying states in that nucleus. They used inelastic scattering of 173.5 MeV/nucleon ^{72}Kr from targets of Be and Au to populate low-lying levels and then observed their gamma decays. They located the second 2^+ state, which was previously unknown, at 1148 keV, and obtained the $E2$ strength connecting it to the ground state (g.s.): $B(E2; 2_2 \rightarrow 0_1) = 133(19)\ e^2\,\mathrm{fm}^4$. They also reported an upper limit on its strength to the excited 0^+ state, $B(E2; 2_2 \rightarrow 0_2) < 367\, e^2\,\mathrm{fm}^4$. Their strength for $2_1 \rightarrow$ g.s. is $805(16)\, e^2\,\mathrm{fm}^4$; earlier values were 810(150) [2] and 999(130) [3]. Relevant $E2$ strengths are listed in Table I.

The g.s. of ^{72}Kr has long been considered to be predominantly oblate [3–5], with the yrast states becoming prolate as J increases [6–9]. The g.s. of ^{74}Kr consists of approximately equal mixtures of the two shapes [10,11]. Wimmer *et al.* analyzed their data in terms of two-state mixing. They used a variable moment of inertia model, assumed that higher-J states are not mixed and extrapolated the energies of high-spin states toward low spins in order to obtain estimates of energies of low-J basis states. From the energy differences of the perturbed and unperturbed states, they obtained the mixing amplitudes in a two-level mixing calculation for $J = 0$, 2, and 4. Chakraborty *et al.* [12] stressed the importance of *direc*t evidence in terms of $E2$ transition strengths, rather than *indirect* evidence involving energy patterns in treatments of coexistence and mixing.

More complete $E2$ data are available for ^{74,76,78}Kr [10,13]. Clément *et al.* [10] analyzed the ^{74,76}Kr data in terms of a similar two-state mixing model. I performed a similar analysis of all three isotopes [11]. My results agreed with those of Clément *et al.* for ^{74,76}Kr, and with the suggestion that a perceived insufficiency of the model for ^{76}Kr might be due to strong coupling between the 2_2^+ and 2_3^+ states. Such a complication was beyond the scope of both of the earlier analyses [10,11].

II. ANALYSIS AND RESULTS

I write

$$\Psi(0_1) = a\Phi(0_g) + b\Phi(0_e), \quad \Psi(0_2) = -b\Phi(0_g) + a\Phi(0_e),$$

$$\Psi(2_1) = A\Phi(2_g) + B\Phi(2_e), \quad \Psi(2_2) = -B\Phi(2_g) + A\Phi(2_e),$$

$$\Psi(4_1) = C\Phi(4_g) + D\Phi(4_e), \quad \Psi(4_2) = -D\Phi(4_g) + C\Phi(4_e).$$

$$\text{I define } M_g = \langle 0_g \| E2 \| 2_g \rangle,\ M_e = \langle 0_e \| E2 \| 2_e \rangle,$$
$$M_g' = \langle 2_g \| E2 \| 4_g \rangle,\ M_e' = \langle 2_e \| E2 \| 4_e \rangle.$$

Furthermore, I assume the g states are not connected to the e states by the $E2$ operator.

The experimental transition matrix elements are then given in terms of the mixing amplitudes and basis-state matrix elements. For example, we have $M_0 = aAM_g + bBM_e$, and similarly for the other transitions. Thus, four experimental values of M can be used to determine the four model parameters: two mixing amplitudes and two basis-state matrix elements.

Whenever all four of the $0 \leftrightarrow 2$ Ms are known, the solution is usually unique. If only $B(E2)$ are known, a sign ambiguity may arise from taking the square root. In some cases, consistency with the model can determine the sign. In my sign convention, M_0 and M_3 are positive, whereas M_1 and M_2 can have either sign, because they involve destructive interference.

TABLE I. $E2$ strengths of $0 \leftrightarrow 2$ transitions in ^{72}Kr.

Label	Initial	Final	$B(E2)(e^2\ \mathrm{fm}^4)$[a]	$M(E2)$ (eb)[b]
M_0	2_1	0_1	805(16)	0.634(6)
M_1	2_1	0_2	Unknown	
M_2	2_2	0_1	133(19)	±0.258(18)
M_3	2_2	0_2	<367	<0.43
M_0'	4_1	2_1	2720(550)	1.56(16)

[a]References [1,2].
[b]$M^2(E2) = (2J_i + 1)B(E2; i \rightarrow f)$.

TABLE II. Results of mixing for 0↔2 transitions in ^{72}Kr.

	Value	
Quantity	Present	[1]
a	0.582	0.345
A	0.801	0.863
M_g	1.14 eb[a]	$\beta_{\rm prol} = 0.45$
M_e	0.211 eb[a]	$\beta_{\rm obl} = 0.24$

[a]Taken from ^{74}Kr [11].

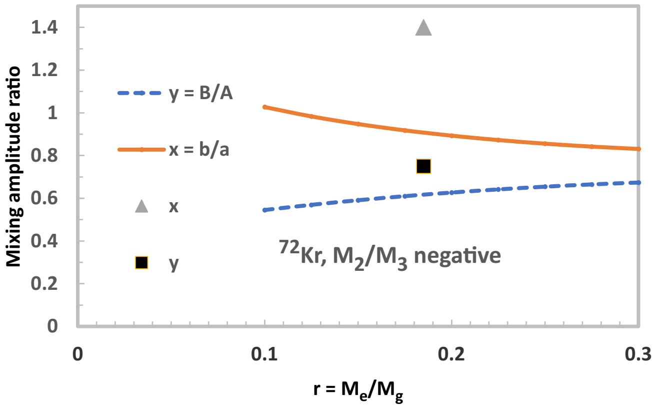

FIG. 1. Results of fitting M_0 and M_2, with M_2/M_3 negative, and with M_3 at its reported upper limit [1]. Curves of amplitude ratios $x = b/a$ (dashed) and $y = B/A$ (solid) are plotted vs $r = M_e/M_g$. Points labeled x and y are from the two-state mixing analysis that assumed that the basis states in ^{72}Kr and ^{74}Kr are the same.

The ^{72}Kr experimental situation is severely underdetermined. Only two Ms are known, with an upper limit for a third. To proceed will require some additional assumptions. One approach is to assume that the basis states g and e are the same in ^{72}Kr and ^{74}Kr. This is certainly not rigorously true, but it is a reasonable first approximation. This condition is relaxed later below. The analysis can be easily repeated with any other values of M_g and M_e.

Thus, I seek solutions with $M_g = 1.14$, $M_e = 0.211$ eb, as in ^{74}Kr. Any values of M_g and M_e near these requires M_2 to be negative. Results are listed in Table II. I return to this point later. I note that the present solution has most of the more collective basis state in the excited 0^+ state and in the first 2^+ state as in Ref. [1], but the mixing amplitudes are different from those of Ref. [1], especially for 0^+.

I have made no assumptions about the relative energies of g and e. If 0_g is higher in energy than 0_e, then b will turn out to be larger than a, as indeed is the case here. The fact that $A > B$ means that 2_g is lower than 2_e. These results are consistent with prior conclusions discussed in the introduction.

For a $K = 0$ rotational band, the ratio $M(E2;\ 4 \rightarrow 2)/M(E2;\ 2 \rightarrow 0)$ is $(18/7)^{1/2} = 1.60$ [14]. Thus, if the basis states g and e are taken to be $K = 0$ rotational bands, then $M'_g/M_g = M'_e/M_e = 1.60$, so that combining the 2^+ mixing derived above with the experimental $M'_0 = 1.56(16)$ eb allows a determination of the 4^+ mixing. It turns out that the central value of M'_0 is larger than obtained with any mixing, but any $C > 0.9$ reproduces M'_0 at the 1σ level. The result is then $C > 0.90$, $D < 0.44$, $M'_0 = 1.40$ to 1.46 eb. Thus, it is reasonable to conclude that the amount of basis state g in the first 4^+ state is larger than that for the first 2^+ state, which is larger than for the g.s. This is consistent with the long-standing understanding for ^{72}Kr.

Another approach is to examine the consequences of using the mixing amplitudes obtained from the energies in Ref. [1]. It is clear that the analysis in Ref. [1] took the positive square root for M_2. Of course, the absolute sign of M_2 has no meaning, but its sign relative to that of M_3 does have meaning. With the mixing amplitudes from Ref. [1], if I require agreement with M_0 and M_2, I can compute new values of M_g and M_e. These are $M_g = 1.21$, $M_e = 0.578$ eb. The ratio $M_e/M_g = 0.48$ is close to the $\beta_{\rm obl}/\beta_{\rm prol}$ ratio of 0.53 in Ref. [1], as it should be.

Wimmer *et al.* compared their $E2$ strengths with results of two different theoretical calculations [15–17]. One (HFB-5DCH) used the Hartree-Fock-Bogoliubov method in a five-dimensional collective Hamiltonian [15,16]; the other was a symmetry-conserving configuration-mixed calculation (SCCM) [17]. Table III lists the experimental and theoretical strengths, together with those obtained from the analysis above for all four transitions. My computed value of M_3 is more than twice the experimental upper limit, but all the others are even larger. The experimental upper limit can be

TABLE III. Experimental and theoretical $B(E2;\ 2_i \rightarrow 0_f)$ (e^2 fm^4) in ^{72}Kr.

					Two-state mixing	
Initial	Final	Experimental[a]	HFB-5DCH[b]	SCCM[c]	Standard	Hybrid[d]
2_1	0_1	805(16)	691	1603	805	805
2_1	0_2	Unknown	350	460	898	1548
2_2	0_1	133(19)	9	1	133	133
2_2	0_2	<367	1204	2123	853	1548
	Sum		2254	4187	2689	3599

[a]Reference [1].
[b]References [1,14,15].
[c]References [1,16].
[d]Using mixing amplitudes from Ref. [1] and requiring agreement for M_0 and M_2, with M_2 positive.

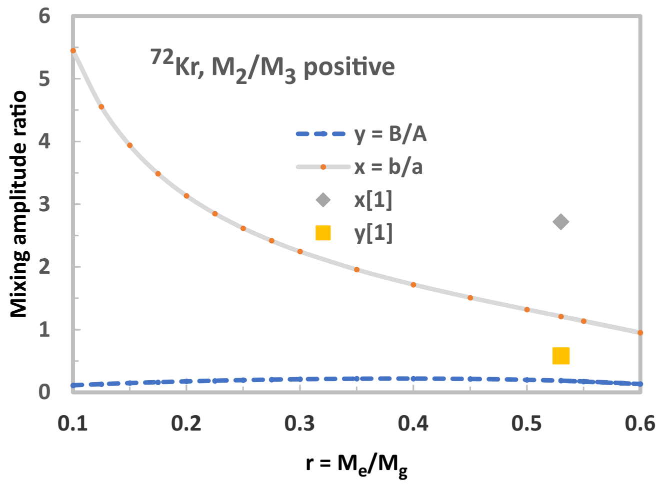

FIG. 2. Curves are as in Fig. 1, but for M_2/M_3 positive. Points labeled $x[1]$ and $y[1]$ represent the amplitudes obtained from the excitation energies in Ref. [1].

accommodated by small changes in the mixing amplitudes and by relaxing the assumption of equal M_g and M_e in ^{72}Kr and ^{74}Kr. For example, with $M_g = 0.911$, $M_e = 0.169$ eb, M_0 and M_2 are reproduced, and M_3 is at its upper limit. However, in this solution, the 0^+ states are almost maximally mixed.

The analysis can be extended by considering $r = M_e/M_g$ as an independent variable and expressing solutions in terms of r. If I set M_3 at its experimental upper limit, then I have three experimental quantities and, for a given r, three parameters to determine—two mixing amplitudes and M_g (with M_e given by rM_g). It is convenient to work in terms of amplitude ratios $x = b/a$ and $y = B/A$. I have performed this extended analysis for both signs of M_2/M_3. Results are displayed in the figures. The fits depicted here differ from the ones described above in two important aspects: (1) these include a value for M_3, at its experimental upper limit, and (2) no assumption is made about M_g and M_e.

Figure 1 displays the mixing amplitude ratios (curves) for M_2/M_3 negative. Individual points are from the analysis above that assumed the basis states in ^{72}Kr and ^{74}Kr are the same. Note that, for most of the displayed range, $x < 1$—which would mean that more than 50% of the basis state g is contained in the g.s. This is contrary to the long-held view of the g.s. I see three possibilities:

(1) the g.s. is not what it has long been thought to be;
(2) the basis states in ^{72}Kr and ^{74}Kr are quite different;
(3) the value of M_3 is larger than the reported upper limit.

In this regard, I note that, of the four results listed in Table III, they all have this $E2$ strength significantly larger

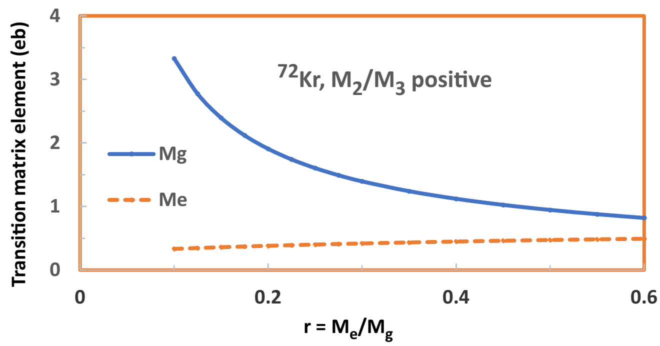

FIG. 3. Values of M_g and M_e that result from the fits in Fig. 2.

than the upper limit reported in Ref. [1]. Furthermore, in ^{74}Kr, the four $2 \to 0$ $B(E2)$ sum to $2680\, e^2\, \mathrm{fm}^4$, whereas the two known ones in ^{72}Kr sum to only $938\, e^2\, \mathrm{fm}^4$, leaving ample room for M_1 and/or M_3 to be large.

Figure 2 is as Fig. 1, but for M_2/M_3 positive. Here, the two points labeled $x[1]$ and $y[1]$ correspond to the amplitudes obtained in Ref. [1] from the excitation energies. This sign choice has $x > 1$ for virtually all the displayed range, in agreement with prior thinking. Figure 3 displays the values of M_g and M_e that result from this extended analysis. For a given value of r, the values of x and y from the curves vs r in Fig. 2 are used. For all values of r, either M_g or M_e is significantly different from the corresponding value in ^{74}Kr. Thus, here there are two possibilities:

(1) M_g and/or M_e are quite different in ^{72}Kr and ^{74}Kr;
(2) the value of M_3 is larger than the reported upper limit.

It would appear that an experiment designed to measure the $2_2 \to 0_2$ transition matrix element is severely needed. I would not be surprised if it turns out to be larger than the reported upper limit. It is possible that strong mixing with higher, presently unknown, 2^+ states could cause a problem with the current simple model. Future experiments on ^{72}Kr should illuminate the situation.

III. SUMMARY

I have applied a simple two-state mixing model to available $E2$ strengths in ^{72}Kr. The problem is underdetermined, but I was able to obtain a solution by assuming that the basis-state transition matrix elements are equal in ^{72}Kr and ^{74}Kr. A range of solutions exist near these values. Two questions remain: What is the ratio M_e/M_g? Is it near 0.2, as found for ^{74}Kr, or is it near 0.5, as obtained in Ref. [1]? What is the sign of M_2 relative to M_3? These two questions are connected. If M_3 can ever be measured, the ambiguities will be resolved. It would appear that a new experiment designed to measure the $2_2 \to 0_2$ transition matrix element is desirable.

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