Signature of the N^* resonance in the mass spectrum of the $K\bar{K}N$ decay channels

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Three-body calculations of the $I = \frac{1}{2}$, $J^{\pi} = \frac{1}{2}^+$ state of $K\bar{K}N$ system were performed. Using separable potentials for two-body interactions in the Faddeev equation, different reaction processes for the $K\bar{K}N$ three-body system were studied. Within this method, the $\pi \Sigma K$ and $\pi \eta N$ mass spectra were extracted. Different types of $\bar{K}N$ - $\pi \Sigma$ potentials based on phenomenological and chiral SU(3) approach were used and the dependence of the mass spectra on the $\bar{K}N$ model of interaction was studied. It was shown that the $\pi \Sigma K$ and $\pi \eta N$ mass spectra could be a useful tool to study the properties of the N^* resonance.

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I. INTRODUCTION

The study of few-body systems, including antikaons, is an important issue in contemporary strangeness nuclear physics and has attracted continuous attention. The $\bar{K}N$ system is a building block of \overline{K} nuclear few-body systems. This is based on the fact that the $\bar{K}N$ interaction is strongly attractive and the binding energy of the $\bar{K}N$ quasibound state is about 10-30 MeV [1-11]. Of course, the binding energy of antikaonic systems is not so large in comparison with the typical hadron energy scale, and also the distance between constituent hadrons is larger than the typical size of them. Consequently, all hadronic constituents will keep their identity. Such quasibound states are usually called hadronic molecular states [12]. During the last two decades, various hadronic molecular states, including \bar{K} and K mesons, were studied by different groups [13–19]. Among these molecular states, the $\bar{K}NN$ bound state with quantum numbers $I = \frac{1}{2}$ and $J^{\pi} = 0^{-}$ has been the object of intense theoretical [20–33] and experimental [34–40] studies.

In addition to the $\bar{K}NN$, numerous theoretical works were also performed to study systems of two mesons and one baryon with strangeness S = 0, finding resonant states which could be identified with the existing baryonic resonances [13–16]. A variational calculation of the three-body $K\bar{K}N$ system was performed in Ref. [13] using effective potentials for $\bar{K}N$ and $K\bar{K}$ interactions. The $\Lambda(1405)$ resonance is generated as a quasibound state in the $\bar{K}N$ system and the scalar mesons $f_0(980)$ and $a_0(980)$ are reproduced as quasibound states of the $K\bar{K}$ system in I = 0 and I = 1 isospin channels, respectively. In this calculation, a quasibound state (*N*^{*} resonance) with quantum numbers $I = \frac{1}{2}$ and $J^{\pi} = \frac{1}{2}^{+}$ was found with a mass 1910 MeV and a width 90 MeV below all of the meson-baryon decay threshold energies of the $\Lambda(1405) + K$, $f_0(980) + N$, and $a_0(980) + N$ states. It was concluded that the $K\bar{K}N$ state can be understood by the structure of simultaneous coexistence of $\Lambda(1405) + K$ and $a_0(980) + N$ clusters and the \bar{K} meson is shared by both $\Lambda(1405)$ and $a_0(980)$ at the same time. This quasibound state was also studied in Refs. [14,15] by using the coupled-channel relativistic Faddeev equation and in Ref. [16] using fixed center approximation of the three-body Faddeev equation. The extracted pole energies were in agreement with the pole in Ref. [13]. A discussion on experimental observation of this N^* can be also found in Ref. [41].

The $K\bar{K}N$ quasibound state can be produced in γp and ppreactions, and the signal of the resonance may be observed in the mass spectrum of the final particles [41]. The investigation for the $K\bar{K}N$ quasibound state was explored through the γ incident reaction $\gamma p \rightarrow K^+ \Lambda$ by the CLAS experiment at Jefferson Lab (JLab) [42,43]. The $K\bar{K}N$ quasibound state could be also studied through the *pp* reaction (see Fig. 1). This reaction was performed as a HADES experiment at GSI [44]. This paper is devoted to investigating the pole structure of the $K\bar{K}N$ three-body system. It was studied how well the signal of the $K\bar{K}N$ quasibound state can be observed in the $\pi \Sigma K$ and $\pi \eta N$ mass spectrum resulting from the reaction under consideration. The few-body calculations for the $K\bar{K}N$ system were performed by using Faddeev AGS (Alt-Grassberger-Sandhas) equations [45]. The transition probabilities for the $(\bar{K}N)_{I=0} + K$ and $(K\bar{K})_{I=1} + N$ reactions were calculated. Within this method, the behavior of the transition probability was investigated. Different phenomenological and chiral based $\bar{K}N$ - $\pi\Sigma$ potentials were used [29,46] to investigate the sensitivity of the three-body observables to two-body inputs.

The paper is organized as follows: in Sec. II, I will explain the formalism used for the three-body $K\bar{K}N$ system and give a brief description of the transition probability formula for the reaction under consideration. Section III is devoted to the two-body inputs of the calculations and representation of the computed pole energies and mass spectra. In Sec. IV, I give conclusions.

II. FORMALISM AND BASIC INGREDIENTS

The present three-body calculations of the $K\bar{K}N$ system are based on the AGS form of the Faddeev equation [45].



FIG. 1. Diagram for the proton-proton reaction and formation of the $K\bar{K}N$ system. In the decay channel, when X is equal to K^+ the YZ pair is $\pi\Sigma$ and when X is equal to p the YZ pair should be $\pi\pi$ and $\pi\eta$ depending on the total isospin of the YZ pair.

To describe the two-body interactions, which are the basic ingredient of the calculations, separable potentials with the following form were used:

$$V^{I}_{\alpha\beta}(k_{\alpha},k_{\beta}) = g^{I}_{\alpha}(k_{\alpha})\,\lambda^{I}_{\alpha\beta}\,g^{I}_{\beta}(k_{\beta}),\tag{1}$$

where the quantities $g_{\alpha}^{I}(k_{\alpha})$ are the form factors of the interacting two-body subsystem with relative momentum k_{α} and isospin *I*. The strength parameters of the interaction are denoted by $\lambda_{\alpha\beta}^{I}$. To include the low-lying channels in $\bar{K}N$ and $K\bar{K}$ interactions, the potentials are labeled also by α and β indexes. The separable form of the two-body *T*-matrices is given by

$$T^{I}_{\alpha\beta}(k_{\alpha}, k_{\beta}; z) = g^{I}_{\alpha}(k_{\alpha}) \tau^{I}_{\alpha\beta}(z) g^{I}_{\beta}(k_{\beta}), \qquad (2)$$

where the operator $\tau_{\alpha\beta}^{I}(z)$ is the usual two-body propagator and z is the two-body energy. There are three different particles in the system under consideration. Therefore, defining the interacting pairs and their allowed spin and isospin quantum numbers, the $K\bar{K}N$ three-body system will have the following partitions:

(1):
$$(\bar{K}N)_{s=\frac{1}{2};I=0,1} + K$$
,
(2): $(KN)_{s=\frac{1}{2};I=0,1} + \bar{K}$, (3)
(3): $(K\bar{K})_{s=0;I=0,1} + N$.

For convenience, one can introduce the three rearrangement channels i = 1, 2, 3 of the $K\bar{K}N$ three-body system as shown in Fig. 2. The quantum numbers of the $K\bar{K}N$ are $I = \frac{1}{2}$ and $s = \frac{1}{2}$. Therefore, in actual calculations, when one includes isospin and spin indexes the number of configurations is equal to 6, corresponding to different possible two-quasiparticle partitions. Using separable potential for two-body interactions, the three-body Faddeev equation [24] in the AGS



FIG. 2. Diagrammatic representation of different partitions of the $K\bar{K}N$ system. Defining the interacting particles, there are three partitions, namely $K + (\bar{K}N)$, $\bar{K} + (KN)$, and $(K\bar{K}) + N$. The antikaon is defined by a red circle, the kaon by a blue circle and the nucleon by an orange circle.

form is given by

$$\mathcal{K}_{i,j}^{l_{i},l_{j}}(p_{i}, p_{j}; W) = \mathcal{M}_{i,j}^{l_{i},l_{j}}(p_{i}, p_{j}; W) + \sum_{k,l_{k}} \int d^{3}p_{k} \mathcal{M}_{i,k}^{l_{i},l_{k}}(p_{i}, p_{k}; W) \tau_{k}^{l_{k}} \times \left(W - \frac{p_{k}^{2}}{2\nu_{k}}\right) \mathcal{K}_{k,j}^{l_{k},l_{j}}(p_{k}, p_{j}; W).$$
(4)

Here, the operators $\mathcal{K}_{ij}^{l_i l_j}$ are the transition amplitudes which describe the elastic and rearrangement processes $i + (jk)_{l_i} \rightarrow j + (ki)_{l_j}$ [24] and the operators $\mathcal{M}_{ij}^{l_i l_j}$ are the corresponding Born terms. In this equation W is the three-body energy and $W - \frac{p_k^2}{2\nu_k}$ is the energy of the interacting pair (ij) where $\nu_i = m_i(m_j + m_k)/(m_i + m_j + m_k)$ is the reduced mass when particle *i* is a spectator. Faddeev partition indexes *i*, *j*, *k* = 1, 2, 3 denote simultaneously an interacting pair and a spectator particle. Depending on the spectator particle, the operator $\tau_k^{l_k}(W - \frac{p_k^2}{2\nu_k})$, *k* = 1, 2, 3 is given by

$$\tau_{1}^{I_{1}} = \tau_{K}^{I_{K}} = \tau_{\bar{K}N-\bar{K}N}^{I_{\bar{K}N}} \left(W - \frac{p_{K}^{2}}{2\nu_{K}} \right),$$

$$\tau_{2}^{I_{2}} = \tau_{\bar{K}}^{I_{R}} = \tau_{\bar{K}N-\bar{K}N}^{I_{\bar{K}N}} \left(W - \frac{p_{\bar{K}}^{2}}{2\nu_{\bar{K}}} \right),$$

$$\tau_{3}^{I_{3}} = \tau_{N}^{I_{N}} = \tau_{\bar{K}\bar{K}-\bar{K}\bar{K}}^{I_{\bar{K}\bar{K}}} \left(W - \frac{p_{N}^{2}}{2\nu_{N}} \right).$$
 (5)

The $\bar{K}N$ system is coupled with the $\pi\Sigma$ channel and the $K\bar{K}$ system is coupled with $\pi\pi$ and $\pi\eta$ systems in isospin I = 0 and I = 1 channels, respectively. In actual calculations, one should extend the Faddeev equation to include the low-lying channels. In the present calculations, the $\bar{K}N$ - $\pi\Sigma$ and $K\bar{K}$ - $\pi\pi$ - $\pi\eta$ couplings are not taken into account directly and the particle indexes are omitted for all three-body operators and the interactions between particles in low-lying channels are neglected. Thus, my coupled-channel three-body calculations with coupled-channel $\bar{K}N$ - $\pi\Sigma$ and $K\bar{K}$ - $\pi\pi$ - $\pi\eta$ interactions are equivalent to the one-channel three-body calculation using the so-called "exact optical" $\bar{K}N(-\pi\Sigma)$ and $K\bar{K}(-\pi\pi-\pi\eta)$ potentials [47]. The decay to the $K\pi\Sigma$, $\pi\eta N$, and $\pi\pi N$ channels is taken into account through the

imaginary part of the optical $\bar{K}N(-\pi\Sigma)$ and $K\bar{K}(-\pi\pi-\pi\eta)$ potentials.

Supposing the $(\bar{K}N)_{I=0} + K$ as the initial state of the $K\bar{K}N$ system, the three-body Faddeev AGS equations can be given by

$$\mathcal{K}_{K,K}^{I_{K},0} = \mathcal{M}_{K,N}^{I_{K},I_{N}} \tau_{N}^{I_{N}} \mathcal{K}_{N,K}^{I_{N},0} + \mathcal{M}_{K,\bar{K}}^{I_{K},I_{\bar{K}}} \tau_{\bar{K}}^{I_{\bar{K}}} \mathcal{K}_{\bar{K},K}^{I_{\bar{K}},0},
\mathcal{K}_{N,K}^{I_{N},0} = \mathcal{M}_{N,K}^{I_{N},0} + \mathcal{M}_{N,K}^{I_{N},I_{K}} \tau_{K}^{I_{K}} \mathcal{K}_{K,K}^{I_{K},0} + \mathcal{M}_{N,\bar{K}}^{I_{N},I_{\bar{K}}} \tau_{\bar{K}}^{I_{\bar{K}}} \mathcal{K}_{\bar{K},K}^{I_{K},0}, \qquad (6)
\mathcal{K}_{\bar{K},K}^{I_{\bar{K}},0} = \mathcal{M}_{\bar{K},K}^{I_{\bar{K}},0} + \mathcal{M}_{\bar{K},K}^{I_{\bar{K}},I_{K}} \tau_{K}^{I_{\bar{K}}} \mathcal{K}_{K,K}^{I_{\bar{K}},0} + \mathcal{M}_{\bar{K},N}^{I_{\bar{K}},I_{N}} \tau_{N}^{I_{N}} \mathcal{K}_{N,K}^{I_{N},0}.$$

To study the possible signature of the N^* resonance in the mass spectrum of the final particles in $K + (\bar{K}N)_{I=0}$ reaction, three different channels can be studied, given by

$$\begin{split} & K + (\bar{K}N)_{I=0} \to K\pi\Sigma, \\ & K + (\bar{K}N)_{I=0} \to \pi\eta N, \\ & K + (\bar{K}N)_{I=0} \to \pi\pi N, \end{split}$$

where the first two reactions are more probable [13]. To define the $K\pi\Sigma$ and $\pi\eta N$ mass spectrum, first one should define break-up amplitude. Since, the low-lying channels are not directly included in the calculations, the only Faddeev amplitudes which contribute in the scattering amplitude are $\mathcal{K}_{K,K}^{l,0}(p_K, \bar{P}_K; W)$ and $\mathcal{K}_{N,K}^{1,0}(p_N, \bar{P}_K; W)$ for extracting the $K\pi\Sigma$ and $\pi\eta N$ mass spectra, respectively. Therefore, the scattering amplitude can be expressed as

$$T_{(\pi\Sigma)+K\leftarrow(\bar{K}N)_{I=0}+K}(\vec{k}_{K},\vec{p}_{K},\bar{P}_{K};W)$$

$$=\sum_{I}g_{\pi\Sigma}^{I}(\vec{k}_{K})\tau_{K}^{I}\left(W-\frac{p_{K}^{2}}{2\nu_{K}}\right)\mathcal{K}_{K,K}^{I,0}(p_{K},\bar{P}_{K};W),$$

$$T_{(\pi\eta)+N\leftarrow(\bar{K}N)_{I=0}+K}(\vec{k}_{N},\vec{p}_{N},\bar{P}_{K};W)$$

$$=g_{\pi\eta}^{I=1}(\vec{k}_{N})\tau_{N}^{I=1}\left(W-\frac{p_{N}^{2}}{2\nu_{N}}\right)\mathcal{K}_{N,K}^{1,0}(p_{N},\bar{P}_{K};W),$$
(8)

where \vec{k}_i is the relative momentum between the interacting pair (jk) and \vec{P}_K is the initial momentum of spectator K in the $K\bar{K}N$ center of mass. The quantities $\mathcal{K}_{i,j}^{l_i,l_j}$ are the Faddeev amplitudes, which are derived from Faddeev Eq. (6).

Using Eq. (8), the transition probability of the $K + (\bar{K}N)_{I=0}$ reaction can be defined as follows:

$$w_{1}(\bar{P}_{K}, W) = \int d^{3}p_{K} \int d^{3}k_{K} \,\delta(W - Q_{1}(p_{K}, k_{K})) \\ \times \left| T_{(\pi\Sigma) + K \leftarrow (\bar{K}N)_{I=0} + K}(\vec{k}_{K}, \vec{p}_{K}, \bar{P}_{K}; W) \right|^{2}, \\ w_{2}(\bar{P}_{K}, W) = \int d^{3}p_{N} \int d^{3}k_{N} \,\delta(W - Q_{2}(p_{N}, k_{N})) \\ \times \left| T_{(\pi\eta) + N \leftarrow (\bar{K}N)_{I=0} + K}(\vec{k}_{N}, \vec{p}_{N}, \bar{P}_{K}; W) \right|^{2}, \quad (9)$$

where $Q_1(p_K, k_K)$ and $Q_2(p_N, k_N)$ are given by

$$Q_{1}(p_{K}, k_{K}) = \frac{p_{K}^{2}(m_{K} + m_{\pi} + m_{\Sigma})}{2m_{K}(m_{\pi} + m_{\Sigma})} - \frac{k_{K}^{2}(m_{\pi} + m_{\Sigma})}{2m_{\pi}m_{\Sigma}},$$

$$Q_{2}(p_{N}, k_{N}) = \frac{p_{N}^{2}(m_{N} + m_{\pi} + m_{\eta})}{2m_{N}(m_{\pi} + m_{\eta})} - \frac{k_{N}^{2}(m_{\pi} + m_{\eta})}{2m_{\pi}m_{\eta}}.$$
(10)

III. RESULTS AND DISCUSSIONS

Before I proceed to present the results, I should give a brief description of the two-body interactions which are used in the present calculations. The $K\bar{K}N$ three-body system have three different subsystems, which are $\bar{K}N$, $K\bar{K}$, and KN. The $\bar{K}N$ subsystem is coupled with $\pi \Sigma$ and $K\bar{K}$ subsystem is coupled with $\pi\pi$ and $\pi\eta$ in I = 0 and I = 1 channels, respectively. Therefore, in the full calculation of the $K\bar{K}N$ system, the $K\pi\Sigma$, $\pi\pi N$, and $\pi\eta N$ channels should be included too. In the present work, the low-lying channels are not included directly. Therefore, the πN , πK , ηN , and ΣK interactions in lowlying channels are neglected and the decays to $K\pi\Sigma$, $\pi\pi N$, and $\pi\eta N$ are included by using the so-called exact optical potentials. In the following, I will give a brief description of the $\bar{K}N$ - $\pi\Sigma$, $K\bar{K}$ - $\pi\pi$ - $\pi\eta$ and KN interactions.

The $\bar{K}N$ interaction, which is the fundamental ingredient to study few-body systems with an antikaon, is closely related to the structure of $\Lambda(1405)$ resonance in the isospin zero channel. The $\Lambda(1405)$ is located slightly below the $\bar{K}N$ threshold and decays into the $\pi\Sigma$ channel through the strong interaction. The $\Lambda(1405)$ resonance can be interpreted as a quasibound state of $\bar{K}N$ with a binding energy of 27 MeV [6]. On the other hand, the theoretical calculation based on chiral SU(3) dynamics claims that $\Lambda(1405)$ is dynamically generated by the meson-baryon interactions and consists of two poles coupled to the $\pi \Sigma$ and $\bar{K}N$ states [3,8]. According to the chiral models, the pole position in the Swave $\bar{K}N$ scattering amplitude is located at ≈ 1426 MeV. Thus, the $\bar{K}N$ interaction is expected to be weaker than that predicted by other phenomenological model calculations. In present calculation, different models of interaction were used to describe the coupled channel $\bar{K}N$ - $\pi\Sigma$ system. Two different phenomenological potentials plus one energy-dependent chiral based potential for \overline{KN} interaction [29,46] were used. The potentials yield the one- and two-pole structures of the $\Lambda(1405)$ resonance. The parameters of the phenomenological potentials are given in Ref. [46]. From now on I will refer to these potentials as SIDD¹ and SIDD², which have the one- and two-pole structures of the $\Lambda(1405)$ resonance, respectively. The SIDD potentials are adjusted to reproduce the results of the SIDDHARTA experiment. The parameters of the energydependent chiral potential are given in Ref. [29].

The $K\bar{K}$ system is coupled with $\pi\pi$ and $\pi\eta$ in I = 0 and I = 1 channels, respectively. A coupled-channel potential was constructed for the $K\bar{K}$ interaction in both isospin channels to take into account the decay of the $K\bar{K}$ system to $\pi\pi$ and $\pi\eta$ channels. A separable potential in the form given in Eq. (1) was used to describe the $K\bar{K}$ - $\pi\pi$ - $\pi\eta$ interaction. To define the strength parameter $\lambda_{\alpha\beta}^{I}$, I used the pole energy of $f_{0}(980)$ and $a_{0}(980)$ resonances and also the $K\bar{K}$ scattering length [48].

TABLE I. Range parameters $\Lambda^{I}_{K\bar{K}}$ (in fm⁻¹) and strength parameters $\lambda^{I}_{\alpha\beta}$ (in fm⁻²) of the $K\bar{K}$ - $\pi\pi$ - $\pi\eta$ potential. The range parameters in each isospin channel are independent of the two-body channel. For the α and $\beta = K\bar{K}$ one has $\alpha = \beta = 1$ and for $\alpha = \beta = \pi\pi$ and $\alpha = \beta = \pi\eta$ the value of the α and β is equal to 2.

	$\Lambda^0_{K\bar{K}}$	$\Lambda^1_{K\bar{K}}$	λ_{11}^0	λ_{12}^0	λ_{22}^0	λ_{11}^1	λ_{12}^1	λ^1_{22}
$V^{I}_{K\bar{K}}$	3.570	3.396	-1.757	2.975	1.187	-1.503	1.922	0.092

To determine the parameters of the $K\bar{K}$ interaction in both I = 0 and I = 1 channels, the mass 980 MeV and the width 60 MeV were taken for f_0 and a_0 resonances, which are close to the reported mass and width by the Particle Data Group [49]. The extracted parameters of the $K\bar{K}$ potential in the I = 0 and I = 1 channels, are given in Table I and the form factors are taken to be in Yamaguchi form [50].

To describe the repulsive KN interaction, I used a onechannel real potential in the form

$$V_{KN}^{I}(k,k') = g_{KN}^{I}(k) \lambda_{KN}^{I} g_{KN}^{J}(k'), \quad g_{KN}^{I}(k) = \frac{1}{\Lambda_{KN}^{2} + k^{2}}.$$
(11)

The range parameter of the *KN* potential, Λ_{KN} , was set to 3.9 fm⁻¹. The *KN* interaction with isospin I = 0 is very weak. Therefore, it would not change the results of the present work and can be neglected. The strength parameter in the I = 1 channel is adjusted to reproduce the *KN* scattering length. The experimental value of the scattering length for the I = 1 channel is $a_{KN}^{I=1} = -0.310 \pm 0.003$ fm [13,51,52]. Therefore, the value of the strength parameter is $\lambda_{KN}^{I=1} = 2.794$ fm⁻².

A. Pole position of the three-body $K\bar{K}N$ system

Before I proceed to represent the extracted mass spectra for the $K + (\bar{K}N)_{I=0}$ reaction, in this subsection I shall begin with a survey of the pole structure of the $K\bar{K}N$ system. The obtained results can be used as a guide to interpreting the behavior of the extracted mass spectra from the $K + (\bar{K}N)_{I=0}$ reaction. The solution of the Faddeev equation corresponding to bound states and resonance poles in the $(I, J^{\pi}) = (\frac{1}{2}, \frac{1}{2}^+)$ channel of the $K\bar{K}N$ three-body system was found by solving the homogeneous version of the Faddeev AGS equation, which is defined by

$$u_{n;i}^{I_i}(p_i; W) = \frac{1}{\lambda_n} \sum_{j, I_j} \int d^3 p_j (1 - \delta_{ij}) \mathcal{M}_{i,j}^{I_i, I_j}(p_i, p_j; W) \\ \times \tau_j^{I_j} \left(W - \frac{p_j^2}{2\nu_j} \right) u_{n;j}^{I_j}(p_j; W),$$
(12)

where λ_n and the form factors $u_{n;i}^{l_i}(p_i; W)$ are taken as the eigenvalues and eigenfunctions of the kernel of Eq. (4), respectively. In Fig. 3 (lower panel) the results of the present work for three-body $K\bar{K}N$ quasibound state are presented and sensitivity of the $K\bar{K}N$ pole position to the $\bar{K}N-\pi\Sigma$ interaction is investigated by using different potential models. In addition the $K\bar{K}N$ pole position, I also present the



FIG. 3. The sensitivity of the pole position(s) (in MeV) of the $\bar{K}N$ (up) and $K\bar{K}N$ (down) systems to the different phenomenological and chiral based models of the $\bar{K}N$ - $\pi\Sigma$ interaction is investigated. In the upper panel the quantities Z_1 and Z_2 stand for first and the second poles of $\Lambda(1405)$ resonance, respectively.

pole position of the quasibound states in the $\bar{K}N$ system for phenomenological and energy-dependent chiral potentials (upper panel).

The calculated energies of the quasibound state are -24.5, -38.3 and -43.6 MeV from the $K\bar{K}N$ threshold in the results with chiral, SIDD¹, and SIDD² potentials, respectively. The width of three-hadron decay is estimated to be 45-110 MeV. Jido *et al.* [13] made a variational calculation for the three-body $K\bar{K}N$ nuclear quasibound state using an effective interaction model for $\bar{K}N$, $K\bar{K}$, and KN interactions. In this calculation, a quasibound state with $I = \frac{1}{2}$ and $J^{\pi} = \frac{1}{2}^+$ was found with a binding energy about 19–41 MeV and a width 90–100 MeV below the threshold energy of the $K\bar{K}N$ state. The comparison of the present results for $K\bar{K}N$ obtained for coupled-channel $\bar{K}N$ - $\pi\Sigma$ and $K\bar{K}$ - $\pi\pi$ - $\pi\eta$ interactions with calculations in Ref. [13] within the variational method and effective $\bar{K}N$ and $K\bar{K}$ interactions shows that they are



FIG. 4. The $\pi \Sigma K$ mass spectra for the $(\bar{K}N)_{I=0} + K \rightarrow \pi \Sigma K$ reaction. Different types of $\bar{K}N$ - $\pi \Sigma$ potentials were used: the one- and two-pole versions of the SIDD potentials [46] and also one energy-dependent chiral potential [29]. The transition probabilities are calculated for different values of \bar{P}_K . In panels (a), (b), (c), and (d), the values of \bar{P}_K are 100, 200, 300, and 400 MeV/*c*, respectively. The blue dashed lines correspond to the mass spectra for the one-pole SIDD potential (SIDD¹) and the red dash-dotted lines show the mass spectra for the two-pole SIDD potential (SIDD²). The results corresponding to the chiral potential are depicted by black solid curves.

in the same range. However, in the case of the chiral lowenergy potential the extracted width is smaller than those by other models.

equation in the AGS form. To achieve this goal, one must solve the integral equations for the amplitudes defined in Eq. (4), and then construct the scattering amplitudes defined in Eq. (8).

B. Trace of N^* resonance in $(\bar{K}N)_{I=0} + K$ reaction

The calculated resonance energies presented in Fig. 3 give only pole positions of the three-body $K\bar{K}N$ system. However, we know that these results are not a quantity that can be directly measured in any experiment. The calculated results in Fig. 3 could be used as a guideline to study the $(\bar{K}N)_{I=0} + K$ reaction. To examine the existence of the quasibound state in the $K\bar{K}N$ system by experiments, one has to calculate the cross sections of $K\bar{K}N$ production reactions. As noted in Sec. I, the $K\bar{K}N$ quasibound state can be produced through the proton-proton reaction and the trace of the resonances would be seen in the mass spectrum of the final particles. In the present calculations, I have been studying how well the signature of the $K\bar{K}N$ system shows up in the observables of the three-body reactions by using the one-channel Faddeev The $\bar{K}N$ is coupled with the $\pi\Sigma$ channel and the $K\bar{K}$ system is coupled with $\pi\pi$ and $\pi\eta$ channels. Therefore, there are three decay channels for the $K\bar{K}N$ system, namely, $\pi\Sigma + K$, $\pi\pi + N$, and $\pi\eta + N$. In the present study, the first and the third processes are considered for the $(\bar{K}N)_{I=0} + K$ reaction. To remove the moving singularities in the kernel of AGS equations, the so called "point method" was used. The details of the point method are given in Refs. [53,54].

The transition probabilities for the $(\bar{K}N)_{I=0} + K \rightarrow \pi \Sigma K$ reaction are depicted in Fig. 4. To study the dependence of the mass spectra on the $\bar{K}N$ interaction, the three-body calculations are performed for chiral based and phenomenological potentials for the $\bar{K}N$ interaction. To investigate the energy dependence of the transition probability, I calculated $w_1(\bar{P}_K, W)$ for $\bar{P}_K = 100-400 \text{ MeV}/c$. The results suggest that the peak structure in the energy region around the $K\bar{K}N$ pole position



FIG. 5. The $\pi\eta N$ mass spectra for the $(\bar{K}N)_{l=0} + K \to \pi\eta N$ reaction. The explanations are same as in Fig. 4.

could be observed, regardless the momentum value and the class of the $\bar{K}N$ - $\pi\Sigma$ interaction. In the calculated mass spectra for the $\bar{K}N$ potentials having a two-pole structure of the $\Lambda(1405)$ resonance, the second pole of $\Lambda(1405)$ does not manifest itself the $\pi\Sigma K$ mass spectra. As one can see from Fig. 4, for phenomenological models a second peak structure can be seen below the $K\bar{K}N$ threshold. Its position depends on the model of interaction and it originates from a branch point in the complex plane [55–57], i.e., a threshold opening associated with the $\Lambda(1405)$ pole.

The results of the Faddeev calculations of the $(\bar{K}N)_{I=0} + K \rightarrow \pi \eta N$ reaction using different versions of $\bar{K}N$ - $\pi \Sigma$ potentials are shown in Fig. 5. Within this model, I have found two bump structures appearing in the $(\bar{K}N)_{I=0} + K \rightarrow \pi \eta N$ transition probabilities in the energy region around the $K\bar{K}N$ pole position and $W = M_K + M_{\Lambda(1405)}$. Comparing the results of the mass spectra with those presented in Fig. 4, one can see that the peak structures corresponding to the quasi-bound state in $K\bar{K}N$ system are more clear and the magnitudes of the transition probabilities are 2–4 times bigger than those in Fig. 4.

C. Trace of N^* resonance in the $(K\bar{K})_{I=1} + N$ reaction

As noted in Sec. I, the extracted results in Refs. [13–15] suggest that the $K\bar{K}N$ state can be understood by the structure of simultaneous coexistence of $\Lambda(1405) + K$ and $a_0(980) + N$ clusters, and the \bar{K} meson is shared by both $\Lambda(1405)$ and $a_0(980)$ at the same time. In the previous subsection, I supposed that the initial structure of the $K\bar{K}N$ system is $(\bar{K}N)_{I=0} + K$ and the $\pi \Sigma K$ and $\pi \eta N$ mass spectra were calculated for $(\bar{K}N)_{I=0} + K$ reaction. Now, I suppose that the initial structure of the $K\bar{K}N$ system is ($K\bar{K}$)_{I=1} + N and I will study the $\pi \eta N$ mass spectra in the $(K\bar{K})_{I=1} + N$ reaction. Supposing the $(K\bar{K})_{I=1} + N$ to be the initial channel of the $K\bar{K}N$ system, the three-body Faddeev AGS equations can be given by

$$\mathcal{K}_{K,N}^{I_{K},1} = \mathcal{M}_{K,N}^{I_{K},1} + \mathcal{M}_{K,N}^{I_{K},I_{N}} \tau_{N}^{I_{N}} \mathcal{K}_{N,N}^{I_{N},1} + \mathcal{M}_{K,\bar{K}}^{I_{K},I_{\bar{K}}} \tau_{\bar{K}}^{I_{\bar{K}}} \mathcal{K}_{\bar{K},N}^{I_{\bar{K}},1},
\mathcal{K}_{N,N}^{I_{N},1} = \mathcal{M}_{N,K}^{I_{N},I_{K}} \tau_{K}^{I_{K}} \mathcal{K}_{K,N}^{I_{K},1} + \mathcal{M}_{N,\bar{K}}^{I_{N},I_{\bar{K}}} \tau_{\bar{K}}^{I_{\bar{K}}} \mathcal{K}_{\bar{K},N}^{I_{\bar{K}},1},
\mathcal{K}_{\bar{K},N}^{I_{\bar{K}},1} = \mathcal{M}_{\bar{K},N}^{I_{\bar{K}},1} + \mathcal{M}_{\bar{K},K}^{I_{\bar{K}},I_{K}} \tau_{K}^{I_{K}} \mathcal{K}_{K,N}^{I_{K},1} + \mathcal{M}_{\bar{K},N}^{I_{\bar{K}},I_{N}} \tau_{N}^{I_{N}} \mathcal{K}_{N,N}^{I_{N},1},$$
(13)



FIG. 6. The $\pi\eta N$ mass spectra for the $(K\bar{K})_{l=1} + N \rightarrow \pi\eta N$ reaction. The explanations are same as in Fig. 4.

and, consequently, the scattering amplitude for the $N + (K\bar{K})_{l=1} \rightarrow \pi \eta + N$ reaction can be expressed as

$$T_{(\pi\eta)+N\leftarrow(K\bar{K})_{I=1}+N}(\vec{k}_{N},\vec{p}_{N},\bar{P}_{N};W)$$

= $g_{\pi\eta}^{I=1}(\vec{k}_{N}) \tau_{N}^{I=1}\left(W - \frac{p_{N}^{2}}{2\nu_{N}}\right) \mathcal{K}_{N,N}^{1,1}(p_{N},\bar{P}_{N};W).$ (14)

Using Eqs. (13) and (14), I define the transition probability of the $N + (K\bar{K})_{I=1} \rightarrow \pi \eta + N$ reaction as follows:

$$w_{3}(\bar{P}_{N}, W) = \int d^{3}p_{N} \int d^{3}k_{N} \,\delta(W - Q_{2}(p_{N}, k_{N})) \\ \times \left| T_{(\pi\eta) + N \leftarrow (K\bar{K})_{l=1} + N}(\vec{k}_{N}, \vec{p}_{N}, \bar{P}_{N}; W) \right|^{2}.$$
(15)

In Fig. 6, the $\pi\eta N$ mass spectra for the $(K\bar{K})_{I=1} + N \rightarrow \pi\eta N$ reaction were calculated. The $K\bar{K}N$, $a_0(980) + N$ thresholds and also the expected energy region for the quasibound state in the $K\bar{K}N$ system are shown using vertical lines. As one can see, the mass spectra are affected by two bump structures appearing in the $(K\bar{K})_{I=1} + N \rightarrow \pi\eta N$ transition probabilities in the energy region around the $K\bar{K}N$ pole position and $z = M_N + M_{a_0(980)}$, where the second

bump actually originates from a branch point in the complex plane. Here, the peak corresponding to the quasibound state in $K\bar{K}N$ is not pronounced in the $\pi\eta N$ mass spectra as in the $(\bar{K}N)_{I=0} + K \rightarrow \pi\eta N$ reaction.

In Ref. [41], the possible observation of the N^* was discussed. They provided a series of arguments which support the idea that the peak seen in the $\gamma p \rightarrow K^+ \Lambda$ reaction around 1920 MeV should correspond to the predicted bound state of $K\bar{K}N$ with a mixture of $f_0(980)$ and $a_0(980)$ components. It was said there that an ideal test of the nature of the N^* resonance is the study of the $\gamma p \rightarrow K^- K^+ p$ reaction close to threshold. It was concluded that the big asymmetry of the mass distribution with respect to phase space close to $M_{inv} = 2m_K$ is a consequence of the presence of the $f_0(980)$ or $a_0(980)$ below threshold. Furthermore, it was observed that the $\gamma p \rightarrow K^- K^+ p$ cross section is more pronounced at lower energies which is a consequence of the presence of the three-particle resonance below threshold. In the present paper, it was also shown that one can see the signal of the $K\bar{K}N$ quasibound state in the mass spectrum of the final particles and also the mass spectra are affected by branch points resulting from the resonances in two-body subsystems.

IV. CONCLUSION

In summary, the homogeneous Faddeev AGS equation for the $K\bar{K}N$ system was solved and the pole position of the $K\bar{K}N$ system for different types of $\bar{K}N$ - $\pi\Sigma$ potentials was calculated. The transition probabilities for the $(\bar{K}N)_{I=0} + K$ reaction were extracted and the possible observation of N^* in the mass spectrum of the decay products was studied. Based on the Faddeev approach, the $\pi\Sigma K$ and $\pi\eta N$ mass spectra for different types of $\bar{K}N$ and $K\bar{K}$ interactions were calculated. Within this model a bump was found, produced by the $K\bar{K}N$ system, appearing in the $(\bar{K}N)_{I=0} + K$ transition probabilities in the energy region around the $K\bar{K}N$ pole position for momentum $\bar{P}_K = 100$ –400 MeV/*c*. Furthermore, it was observed that the shapes and positions of the peaks in the transition

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probabilities are independent of the momentum \bar{P}_K of the initial $(\bar{K}N)_{I=0} + K$ channel. It was shown that not only can one see the signature of the $K\bar{K}N$ quasibound state, but also one can see the effect of the branch points which are resulting from $\Lambda(1405)$, $f_0(980)$, and $a_0(980)$ poles. The bump structures related to the branch points can affect the peak corresponding to the $K\bar{K}N$ quasibound state. Therefore, this reaction would also be helpful to reveal the dynamical origin of two-body resonances. The $K\bar{K}N$ system is mainly dominated by $\Lambda(1405) + K$ and $a_0(980) + N$ structures. Therefore, the $(K\bar{K})_{I=1} + N \rightarrow \pi \eta N$ reaction was also investigated and it was observed that the $\pi \eta N$ mass spectrum reveals the same behavior as in the case of the $(\bar{K}N)_{I=0} + K$ reaction. However, the magnitudes of the extracted mass spectra are considerably smaller than those resulting from the $(\bar{K}N)_{I=0} + K$ reaction.

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